π-π Scattering with $N_f = 2 + 1 + 1$ Twisted Mass Fermions

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Motivation

- Scattering lengths are fundamental quantities in QCD and ingredients for EFTs and interesting for nuclear physics
- Most particles in the hadron spectrum are resonances described by their mass and decay width
- Some states, e.g. the roper resonance, are not even qualitatively described by naive quark model
- Need non-perturbative method from first principles $\rightarrow$ lattice QCD

Scattering at low Energies

- Some interesting scattering channels:
  - π-π scattering for $I = 0, 1, 2 \rightarrow$ e.g. the ρ meson
  - K-π scattering for $I = {}^1/_2, {}^3/_2 \rightarrow$ e.g. the $K^*(892)$ and κ meson
  - D-meson scattering $\rightarrow X$, Y, Z states (talk by Liuming Liu on Friday)
- At low energies details of potentials are not important for scattering
- In the partial wave expansion of the scattering process, only the lowest partial waves contribute, here only s-wave

π-π scattering

- The easiest possible scattering to calculate is π-π scattering with $I = 2$ and pions at rest $\rightarrow$ no disconnected contributions
- The scattering phase-shift δ_s can be related to the scattering length a_s

$$\lim_{k \to 0} k \cot(\delta_s(k)) = -\frac{1}{a_s}$$

- Lüscher[1]: two particles in a box cause energy shift due to interaction
- Energy shift δE is related to the scattering length of the particles

$$\delta E_{\pi\pi}^{I=2} = -\frac{4\pi a_{\pi\pi}^{I=2}}{m_\pi L^3} \left\{ 1 + c_1 \frac{a_{\pi\pi}^{I=2}}{L} + c_2 \frac{\left(a_{\pi\pi}^{I=2}\right)^2}{L^2} \right\} + \mathcal{O}(L^{-6})$$

[1] M. Lüscher, Commun. Math. Phys. **105**, 153 (1986)

Laplacian Heaviside Smearing[2]

- Fermion smearing: $\widetilde{\psi}(n) = \mathcal{S}(n,m)\psi(m)$ with $\mathcal{S} = \Theta\left(\sigma_s^2 + \Delta\right)$

  Heaviside function: $\Theta(x)$

  Laplace operator: Δ

  cutoff for spectrum of Δ: σ_s^2

- Decomposition into eigenvalues $\Lambda_\Delta = \mathrm{diag}(\lambda_1, \ldots, \lambda_\Delta)$:

$$\Delta = V_\Delta^\dagger \Lambda_\Delta V_\Delta \quad \rightarrow \quad \mathcal{S} = V_\Delta^\dagger \Theta\left(\sigma_s^2 + \Lambda_\Delta\right) V_\Delta = V_s^\dagger V_s$$

- V_s contains N_v lowest eigenvectors which are used as sources

- Inversions are stored in perambulator: $V_s^\dagger \Omega^{-1} V_s$

[2] M. Peardon *et al.*, Phys. Rev. D **80**, 054506 (2009)

Stochastic LapH[3]

- Introduce N_R random vectors, ρ, in T, D and V_s
  $E(\rho) = 0$ and $E(\rho\rho^\dagger) = \mathbb{1}$
- ρ must be different for each quark line to avoid bias
- Dilution of random vectors, $P^{(b)}\rho$, to zero many off-diagonal elements
  $P^{(b)}$ dilution matrix, N_D number of dilution vectors
- Statistical errors of correlation functions
  - Random vectors $\propto \frac{1}{\sqrt{N_R}}$
  - Dilution vectors $\propto \frac{1}{N_D}$

  $\Rightarrow$ Find balance between N_R and N_D for best signal in dependence of number of inversions, N_I

[3] C. Morningstar *et al.*, Phys. Rev. D 83, 114505 (2011)

Simulation Details

- We want all-to-all propagators for everything because:
  - Fierz rearrangement
  - Twisted mass: $D_u^{-1} = \left[\gamma_5 D_d^{-1} \gamma_5\right]^\dagger$
  - Same perambulators for connected and disconnected diagrams
  - Temporal extent not too large: $T = 48, 64, 96$

- block dilution in time with size 2 or 3

$\Rightarrow$ - interlace dilution in LapH-space with size 4 or 6

- full dilution in Dirac space

- Number of random vectors: 5
- Number of eigenvectors: $L = 24 : 120$; $L = 32 : 220$; $L = 48 : 660$

Software Details

- *Petsc* and *Slepc* for eigenvector computation
  - Lanczos with thick restart
  - Chebyshev acceleration
  - 3 steps of 3-dim HYP smearing in Laplace operator
- *Eigen* for all matrix related computations
- tmLQCD library with CG and EigCG for inversions
  - CG on GPUs with MPI and OpenMP for $L = 24/32$
  - EigCG on Juqueen with MPI and OpenMP for $L = 32/48$

Overview over Ensembles

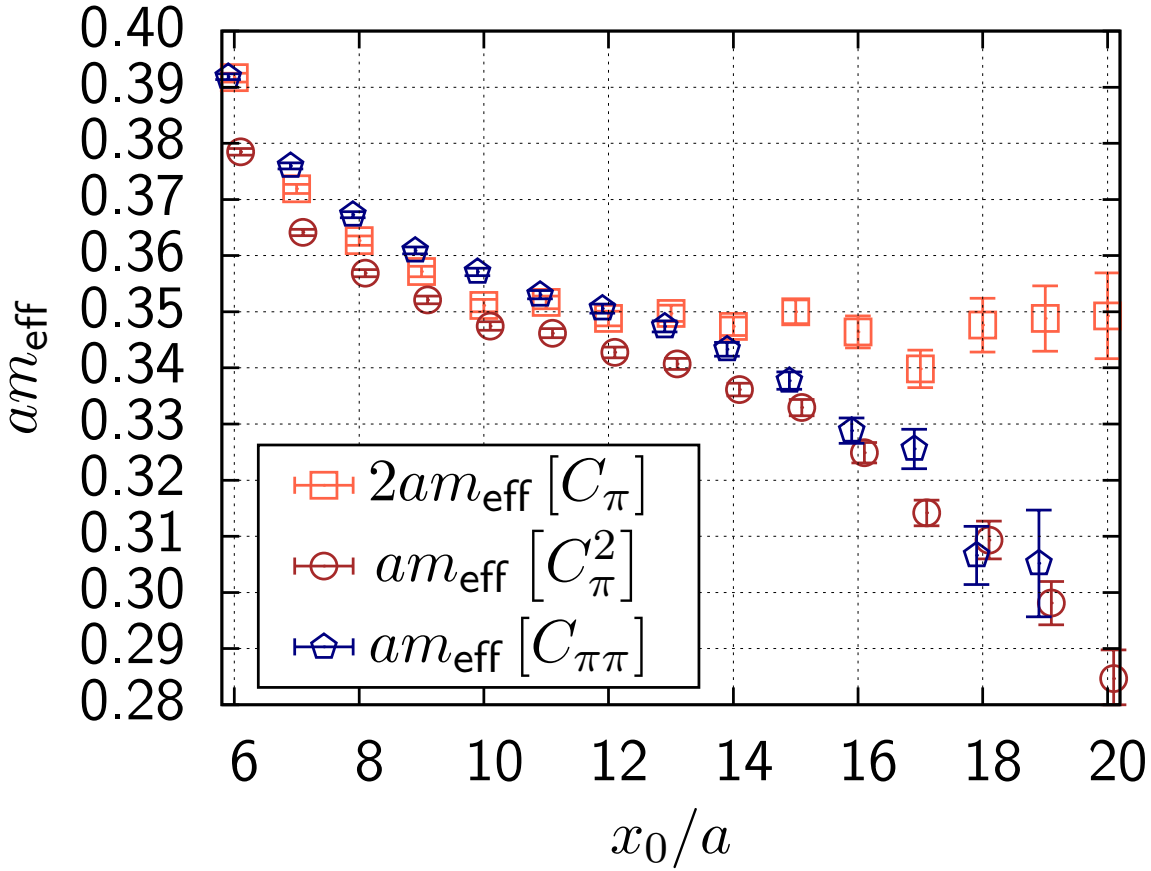
- Ensembles are generated by the European Twisted Mass Collaboration[4]

name	L_s	L_t	am_π	af_π	# conf	m_π[MeV]
A30.32	32	64	0.12395	0.06451	100	284
A40.32	32	64	0.14142	0.06791	150	324
A40.24	24	48	0.14492	0.06568	200	332
A40.20	20	48	0.14927	0.06198	150	342
D45.32	32	64	0.12087	0.04799	50	384
B55.32	32	64	0.15518	0.06557	50	372
A60.24	24	48	0.17275	0.07169	200	396
A80.24	24	48	0.19875	0.07623	300	455
A100.24	24	48	0.22293	0.07926	300	510

- Lattice spacings: A $= 0.086$ fm, B $= 0.082$ fm, D $= 0.062$ fm

[4] R. Baron *et al.*, PoS LATTICE **2010**, 123 (2010) and R. Baron *et al.*, JHEP **1006**, 111 (2010)

Thermal States



Removal of Thermal states[5]

- Taking the ratio: $$\frac{C_{\pi\pi}(t)}{C_\pi^2(t)} \propto \exp(-\delta E_{\pi\pi}^{I=2} t)$$

  $\rightarrow$ Thermal states do not cancel in the ratio
- Use derivative method

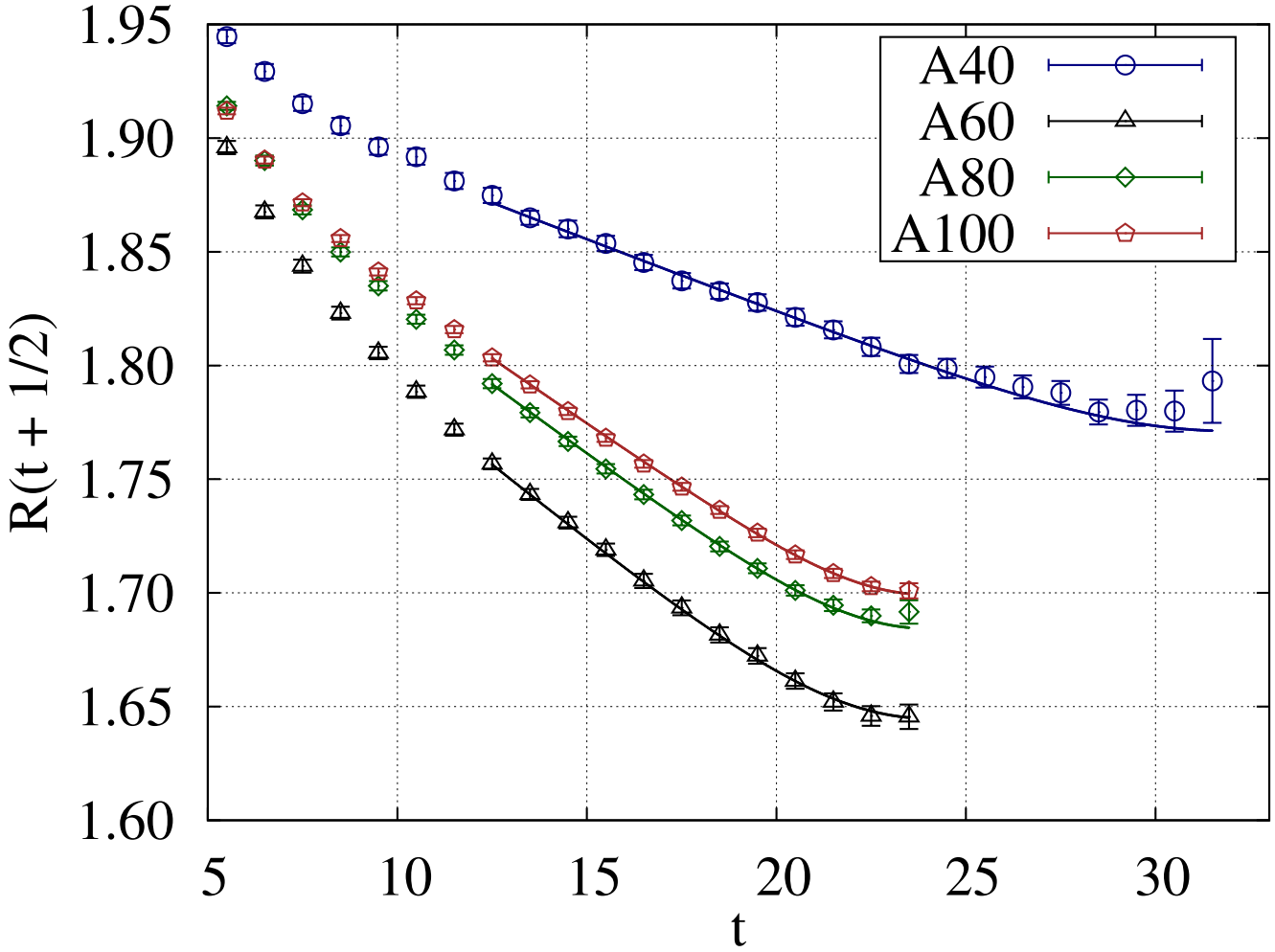
$$\begin{aligned} R(t + {}^1\!/_2) &= \frac{C_{\pi\pi}(t) - C_{\pi\pi}(t+1)}{C_\pi^2(t) - C_\pi^2(t+1)} \\ &= A\left(\cosh(\delta E_{\pi\pi}^{I=2} t') + \sinh(\delta E_{\pi\pi}^{I=2} t') \coth(2 m_\pi t')\right) \end{aligned}$$

with $t' = t + \frac{1}{2} - \frac{T}{2}$

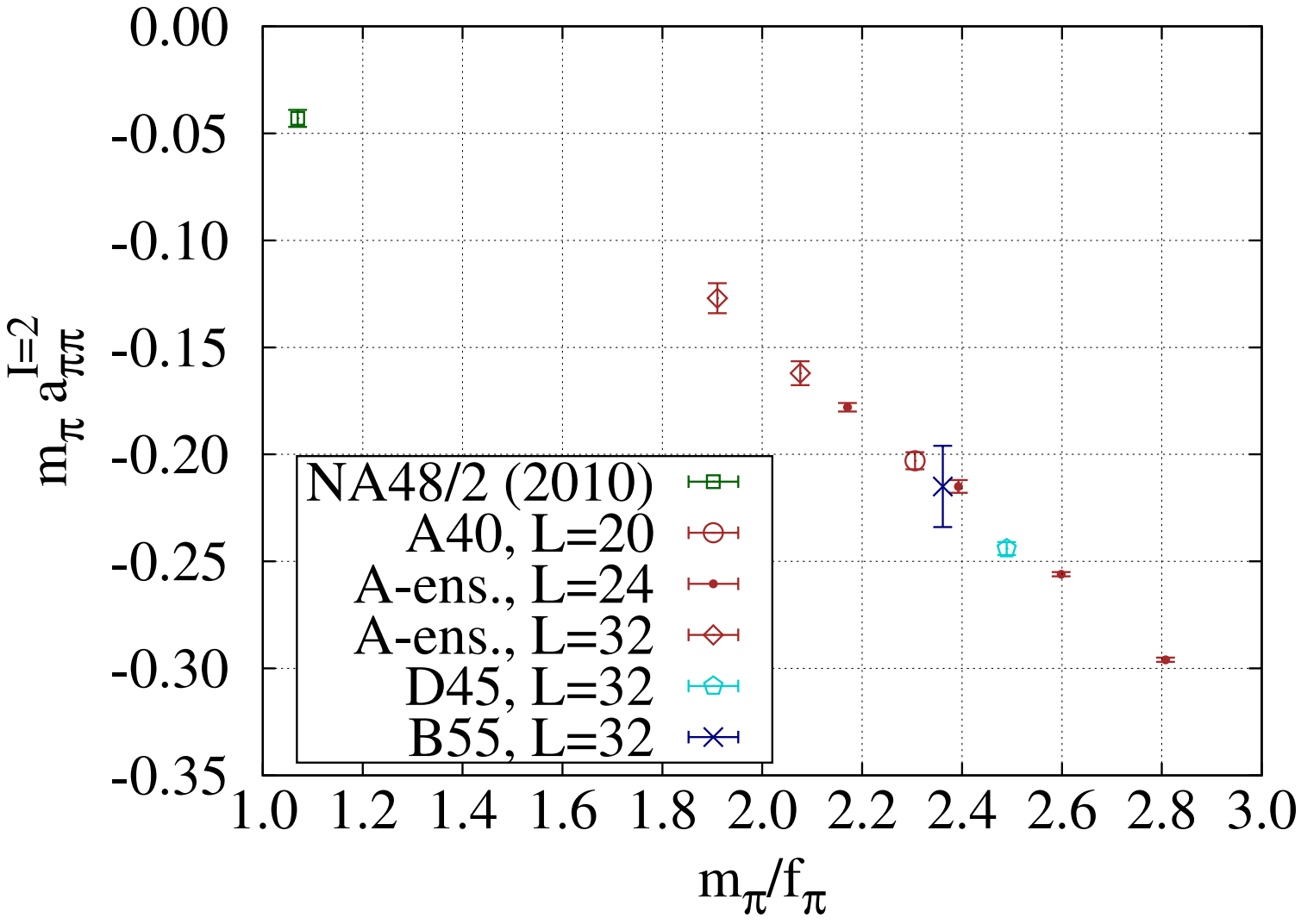
- Extract $\delta E_{\pi\pi}^{I=2}$ by fitting $R(t + {}^1\!/_2)$

[5] X. Feng, K. Jansen and D. B. Renner, Phys. Lett. B **684**, 268 (2010)

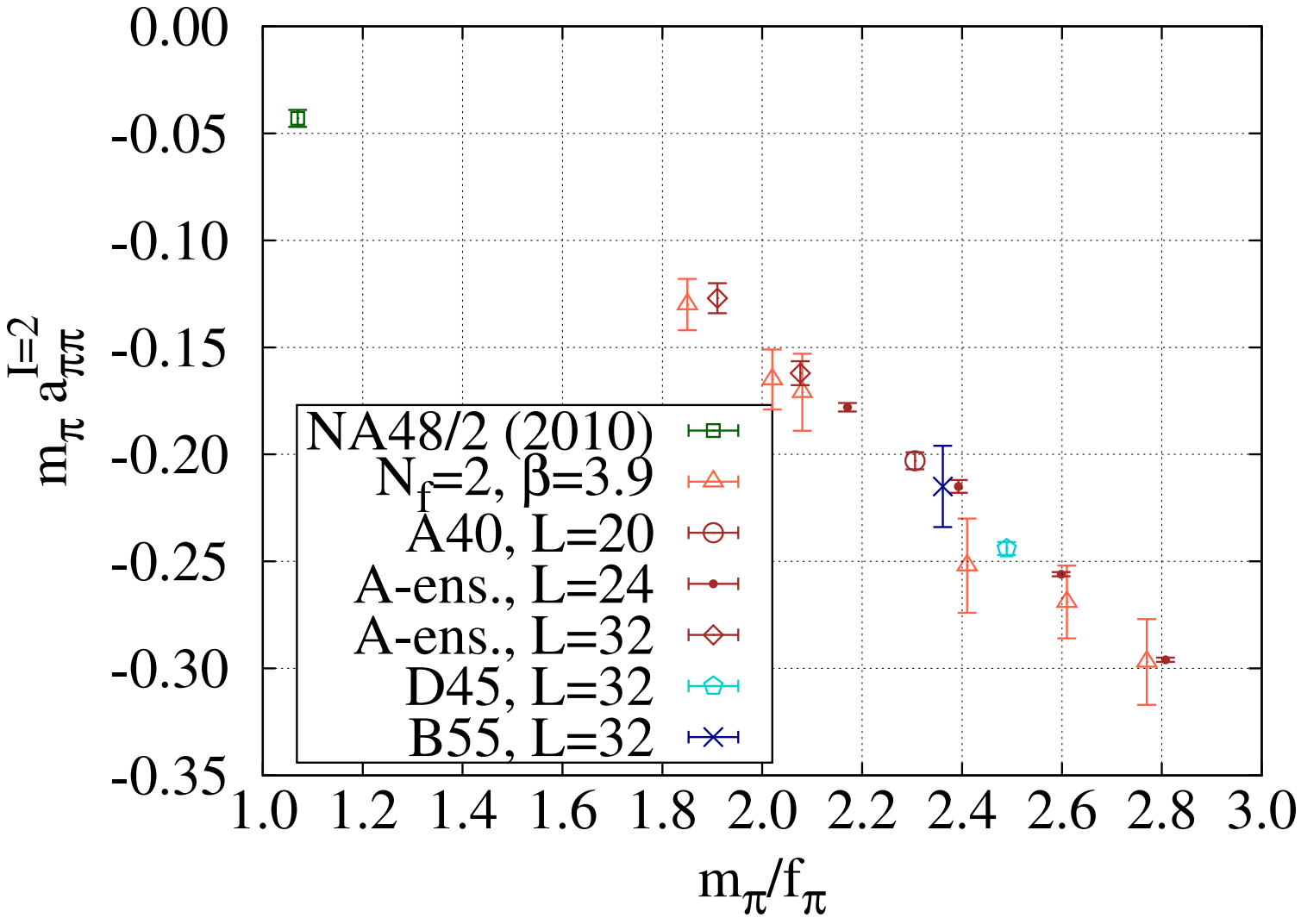
The ratio $R(t + {}^1\!/_2)$



Overall data

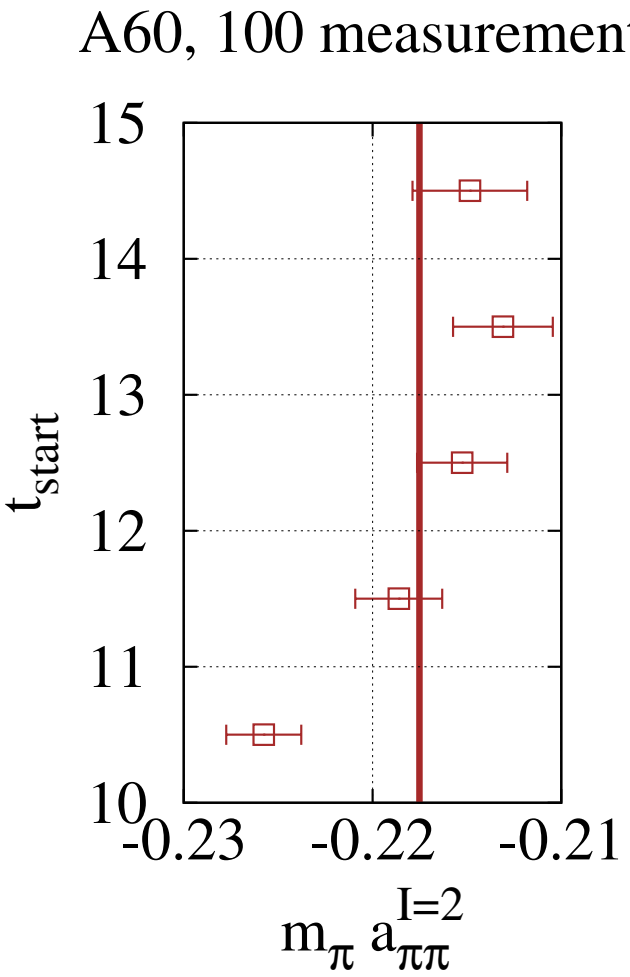


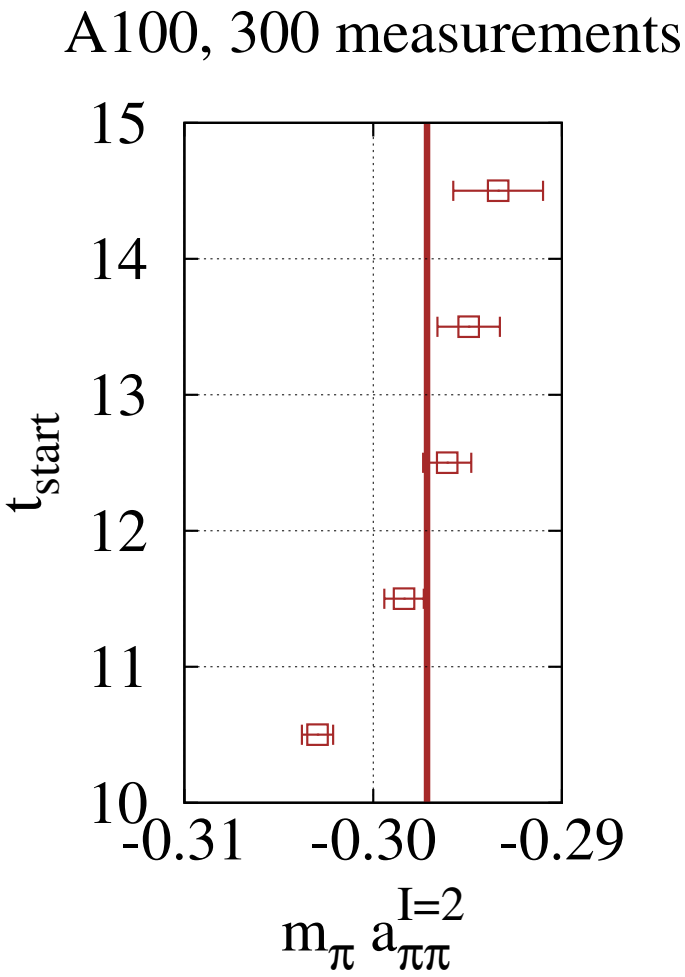
Overall data - comparison to $N_f = 2$



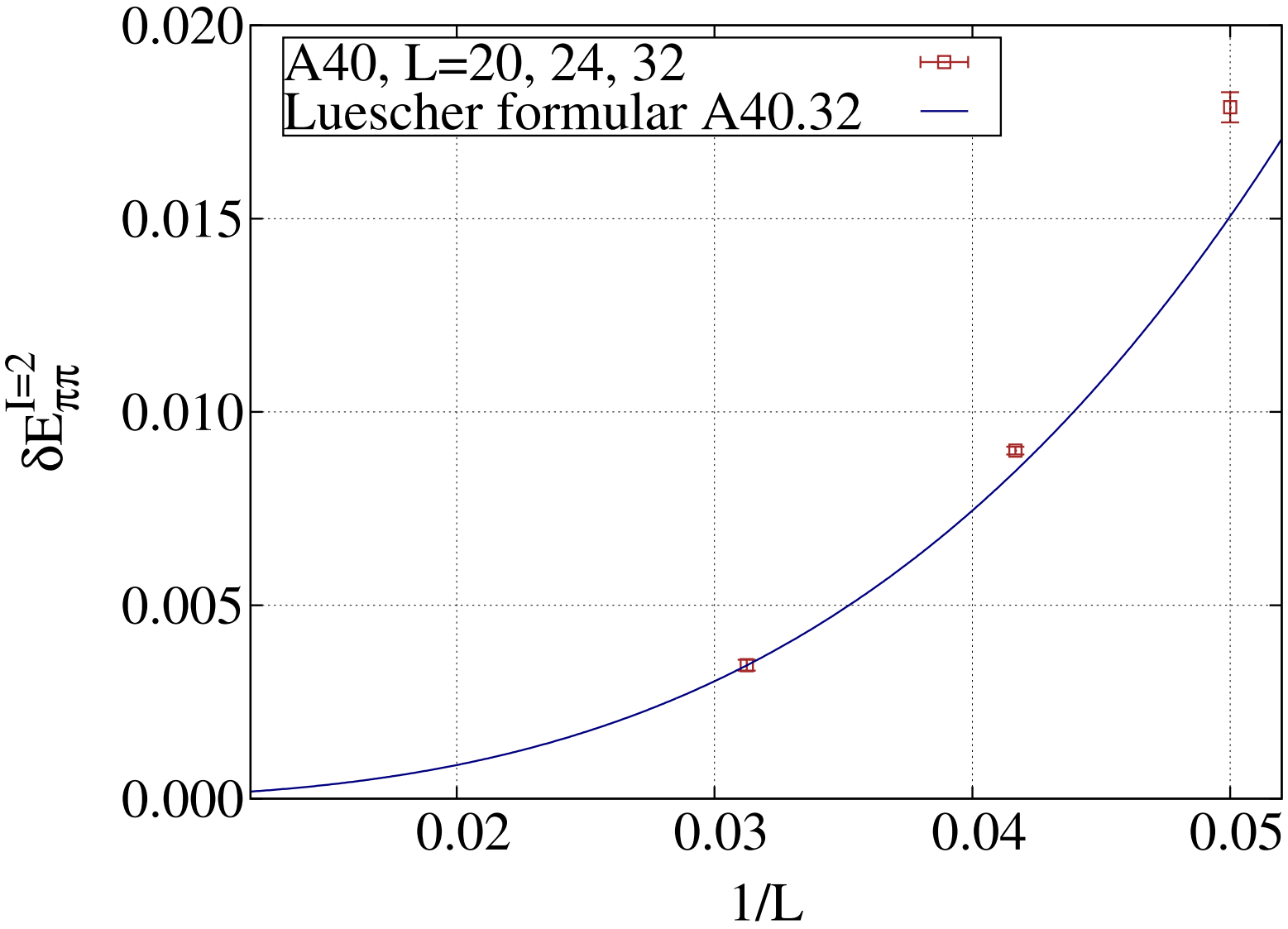
Dependence on fit range

- Fitrange: t_{start} - 22.5





Volume Effects on A40



Conclusions and outlook

- First attempt to extract scattering parameters with $N_f = 2 + 1 + 1$ twisted mass fermions
- Test of viability of stochastic LapH method on large lattices
- Go on to larger lattices: $L = 48$ and smaller pion masses $\rightarrow$ approaching the physical point
- Include momenta and displacements
- Closer investigation of systematic effects - might become quite demanding
- Investigation of other scattering processes ...

Thank you!