Baryon resonances coupled to Pion-Nucleon states in lattice QCD

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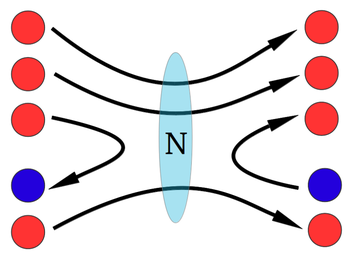
Motivation

Most of the QCD hadronic spectrum consists of resonances: states with a limited lifetime. They mainly decay under strong interaction processes into many-particle states. An example is the nucleon spectrum:

$N\pi$ S-wave		$N\pi$ P-wave	
$N(1535) \longrightarrow N\pi$	$35-55\%$	$N(1440) \longrightarrow N\pi$	$55-75\%$
$N(1535) \longrightarrow N\pi\pi$	$1-10\%$	$N(1440) \longrightarrow N\pi\pi$	$30-40\%$
$N(1650) \longrightarrow N\pi$	$50-90\%$	$N(1710) \longrightarrow N\pi$	$5-20\%$
$N(1650) \longrightarrow N\eta$	$5-15\%$	$N(1710) \longrightarrow N\pi\pi$	$40-90\%$
$N(1650) \longrightarrow N\pi\pi$	$10-20\%$	$N(1710) \longrightarrow N\eta$	$10-30\%$

This sector of the baryon spectrum mainly couples to $N\pi$ states, while the other channels $N\pi\pi$ and $N\eta$ are kinematically not accessible on our lattice. Since one of the tasks of lattice QCD is to reproduce the QCD spectrum, the resonant nature of the hadrons has to be taken into account.



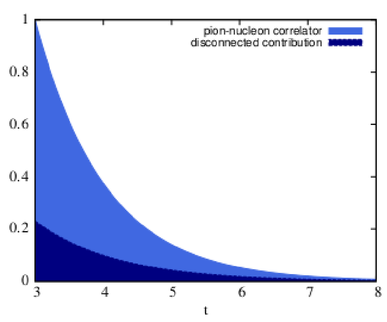
In our calculations we explicitly include pion-nucleon interpolators in order to give N^* the possibility of decaying into $N\pi$ states, which seem to couple too weakly with the qqq traditional interpolators.

Methods

In order to determine the energy levels we measure the Euclidean cross-correlation matrix $C(t)$ between several interpolators $O_i(t)$

$$C_{ij}(t) = \sum_n \langle O_i(t)|n\rangle e^{-E_n t}\langle n|\overline{O}_j(0)\rangle \qquad (1)$$

We extract the energy values E_n using the variational method and performing exponential fits. The correlation matrix entries are computed using the distillation method [2] which allows a reliable evaluation of partially disconnected diagrams.



$$C = \sum_{\dots}\sum_{i,j} \Gamma \dots v_i \, v_i^\dagger \, q\,\bar{q}\, v_j \, v_j^\dagger \dots \Gamma$$

Φ $\quad$ $\tau(i,j)$ $\quad$ $\Phi^\dagger$

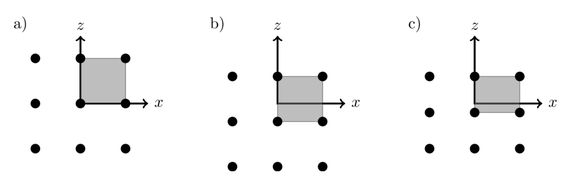
Instead of quark propagators between lattice sites one now computes propagators between eigenvectors of the 3D Laplacian: the perambulators τ.

Moving frames

In moving frames the ${}^2O_h$ symmetry of the lattice is broken and only a smaller group of symmetry survives [3, 4].

Group	d	Little group
${}^2O_h$	$(0,0,1)$	${}^2C_{4v}$
	$(1,1,0)$	${}^2C_{2v}$
	$(1,1,1)$	${}^2C_{3v}$

In the specific case of $\vec{p} \propto \vec{e_z}$ the little group is ${}^2C_{4v}$:



Γ	$dim\,\Gamma$	J	l
G_1	2	$\frac{1}{2}$	0
G_1	2	$\frac{1}{2}$	1

The irreducible representation G_1 mixes different angular momenta, therefore it is not possible to construct an interpolator with spin $1/2$ and definite l.

Simulation setting

180 configurations, 2 mass-degenerate light quarks, tree level improved Wilson-Clover action. $M_\pi = 266$ MeV, $a = 0.12$ fm, $V = 16^3 \times 32$. [1]

Interpolators

The single particle interpolators are smeared combining $N_v = 32, 64$.

$$N^{(i)}(\vec{p}) = \sum_{\vec{x}} \epsilon_{abc}\, P_\pm\, \Gamma_1^{(i)}\, u_a(\vec{x})\, u_b^T(\vec{x})\, \Gamma_2^{(i)}\, d_c(\vec{x})\;\; e^{-i\vec{x}\vec{p}} \qquad (2)$$

$$\pi^+(\vec{p}) = \sum_{\vec{x}} \bar{d}_a(\vec{x})\gamma_5 u_a(\vec{x})\;\; e^{-i\vec{x}\vec{p}}.$$

The two particle interpolators read

$$N\pi(\vec{p}) = \gamma_5 N(\vec{p})\pi(\vec{0}) \qquad (3)$$

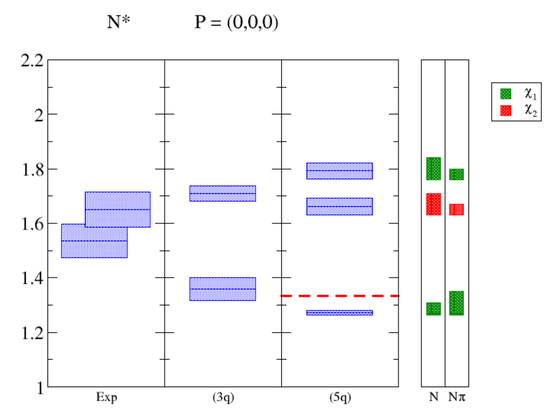
and we project to isospin $\frac{1}{2}$ by choosing the combination

$$O_{N\pi} = p\pi^0 + \sqrt{2}\, n\pi^+. \qquad (4)$$

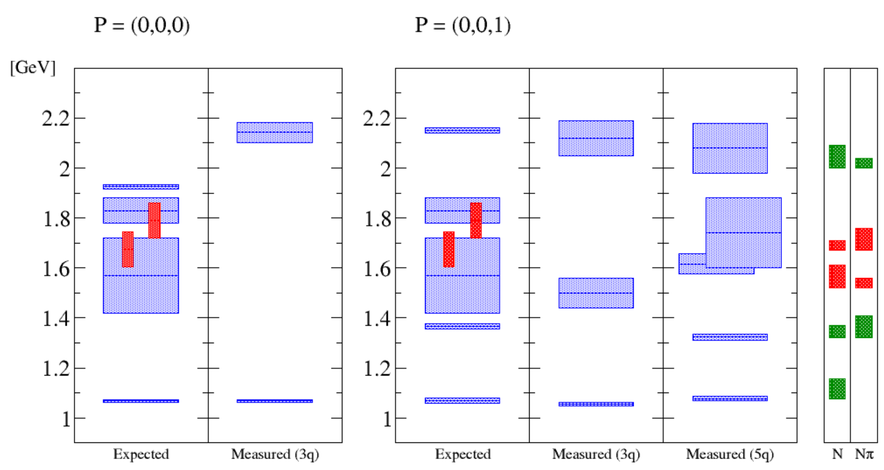
Since parity is not a good quantum number in moving frames, no parity projection is performed on our interpolators.

Results

Due to the fact that the interpolators are parity unprojected, we need to use our information on the negative parity sector obtained from the $\vec{p} = (0,0,0)$ study of $N\pi$ states in S-wave. [5]



When the $N\pi$ are taken into account, the nucleon spectrum in $\vec{p} = (0,0,1)$ changes: two new states appear and the signals become more stable.
In order to disentangle the positive and the negative sector we analyze the eigenvector components of the different state (rightmost columns).



The 1st and the 2nd state is the ground state nucleon and the pion-nucleon state respectively. The states 3 and 5 are most likely N^* states since they couple with our interpolators accordingly. The 4th state seems not to belong to the negative parity sector. It predominantly couples with $\chi_2\pi$ and χ_2 and it is our best candidate for a N^+ resonance. Once more it seems that the inclusion of the decay channels in the calculation is necessary for a reliable extraction of the excited states.

References

[1] A. Hasenfratz *et al.*, Phys. Rev. D **78**, 054511 (2008), [arXiv:0806.4586],
[2] M. Peardon *et al.*, Phys. Rev. D **80**, 054506 (2009), [arXiv:0905.2160],
[3] M. Göckeler *et al.*, Phys.Rev. D **86** 094513 (2012), [arXiv:1206.4141],
[4] L. Leskovec, S. Prelovsek, Phys.Rev. D **85** 114507 (2012), [arXiv:1202.2145],
[5] C. B. Lang, V. Verduci, Phys.Rev. D **87** 054502 (2013),[arXiv:1212.5055].