Effective Polyakov Loop Theories on the Lattice

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Lattice 2014, New York

Motivation

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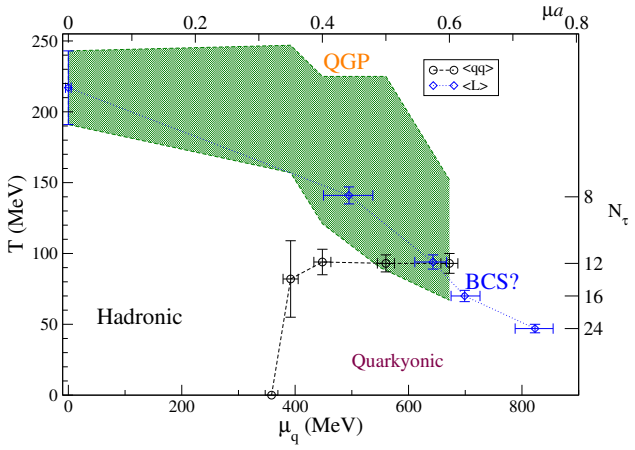
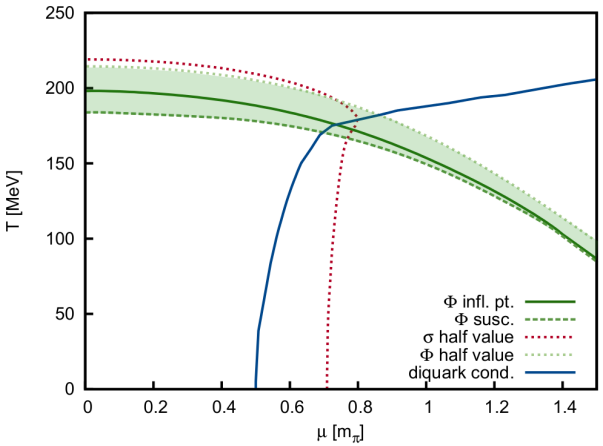
- Exploration of the deconfinement transition of QCD
- Use of effective Polyakov loop potentials in functional methods
- Effects of unquenching
- Exploration of the BEC Phase of two-color QCD



Boz, Cotter, Fister, Mehta, Skullerud [1303.3223]



Strodthoff, von Smekal [1306.2897]

Setup

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- Per-site probability distribution $P(L)$ via histogram-method from QC_2D simulations with 2 flavors of staggered quarks
- Per-site constrained potential

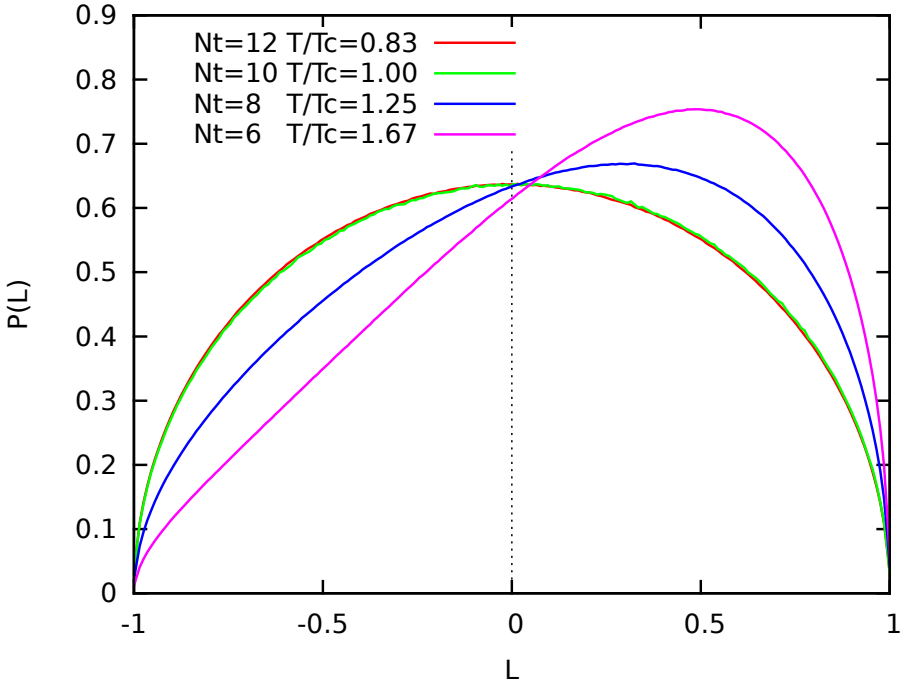
$$V_0(L) = -\log P(L)$$

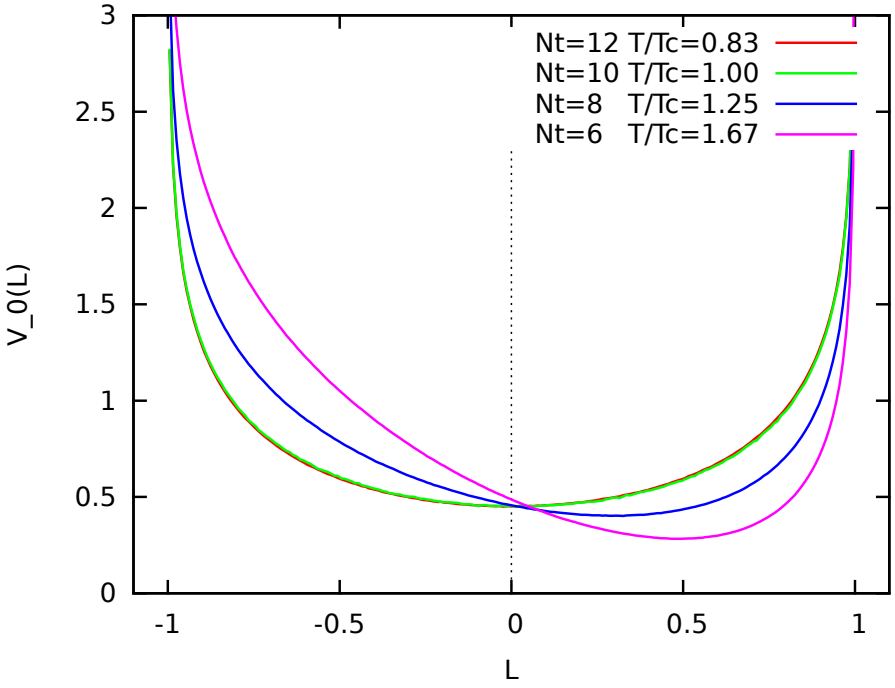
- Obtain actual per-site effective potential by Legendre transform

$$W(h) = \log \int dL \; \exp(-V_0(L) + hL)$$

$$V_{\text{eff}}(\hat{L}) = \sup_h (\hat{L}h - W(h))$$

Polyakov Loop Distributions and Effective Potentials at β = 2.577856

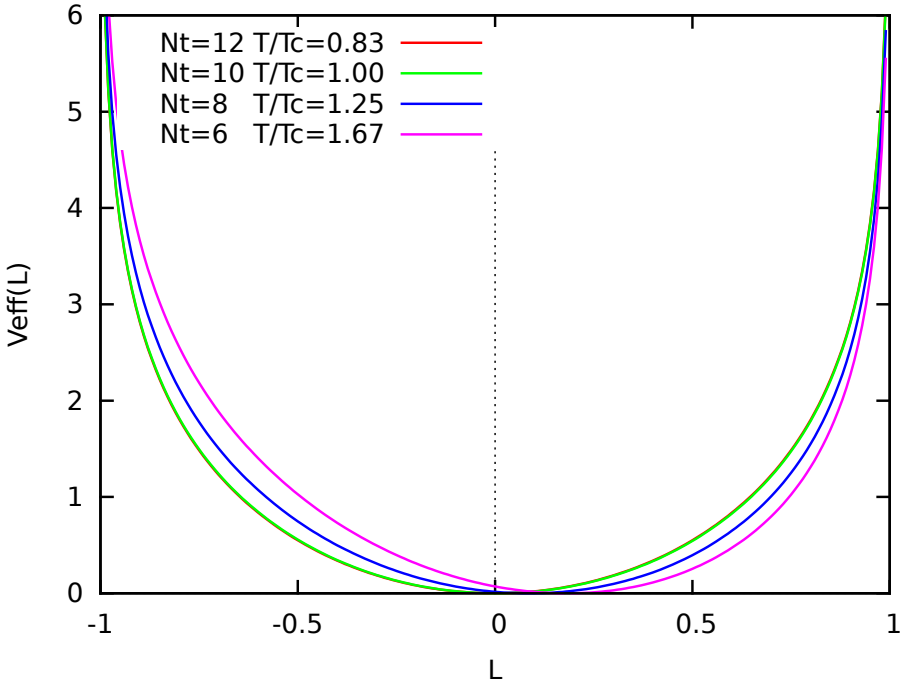


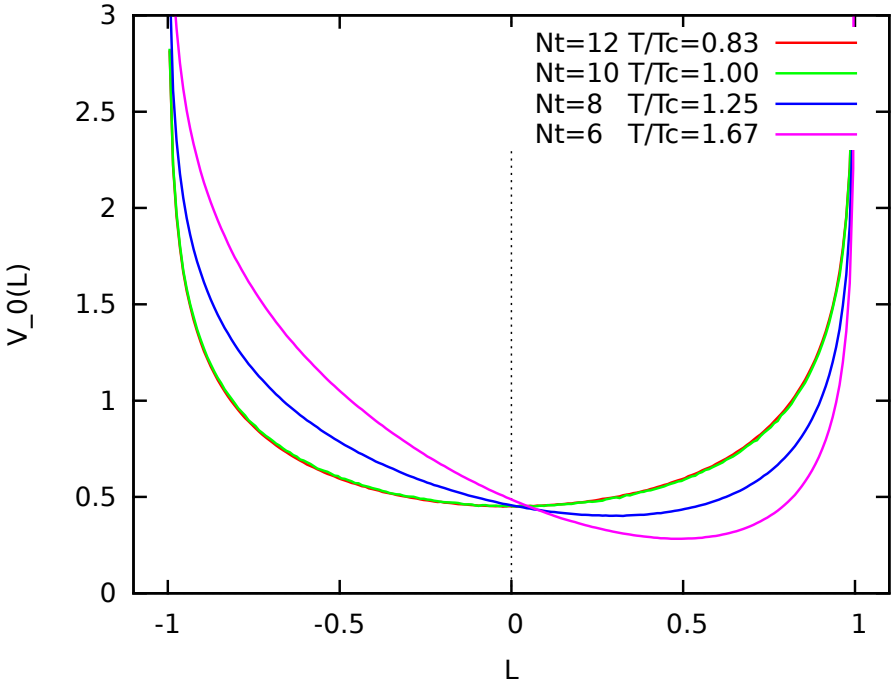


pure gauge results by Smith, Dumitru, Pisarski, von Smekal [1307.6339]

- Fixed scale approach

Polyakov Loop Distributions and Effective Potentials at β = 2.577856

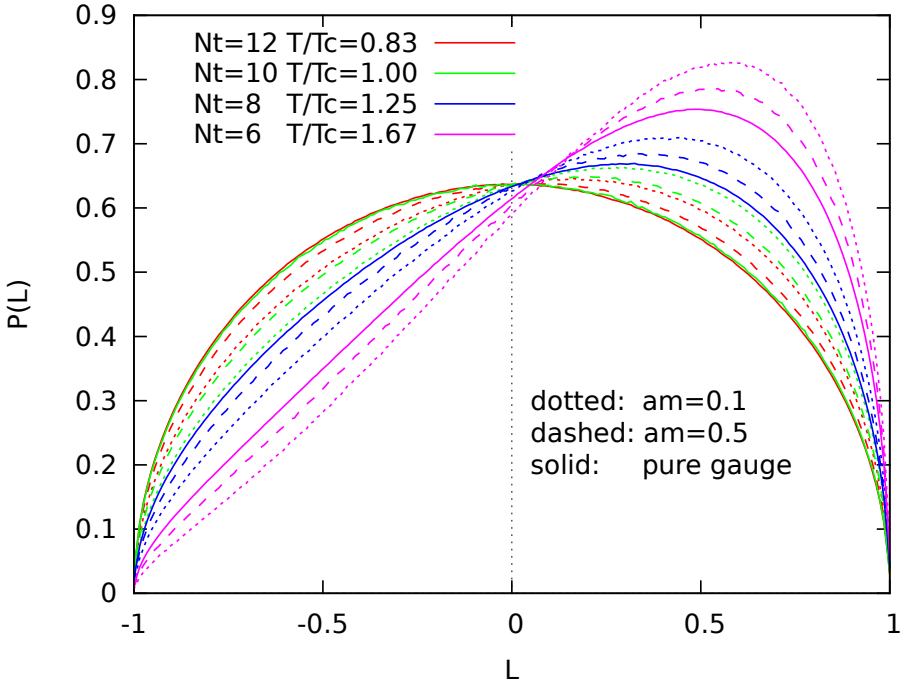


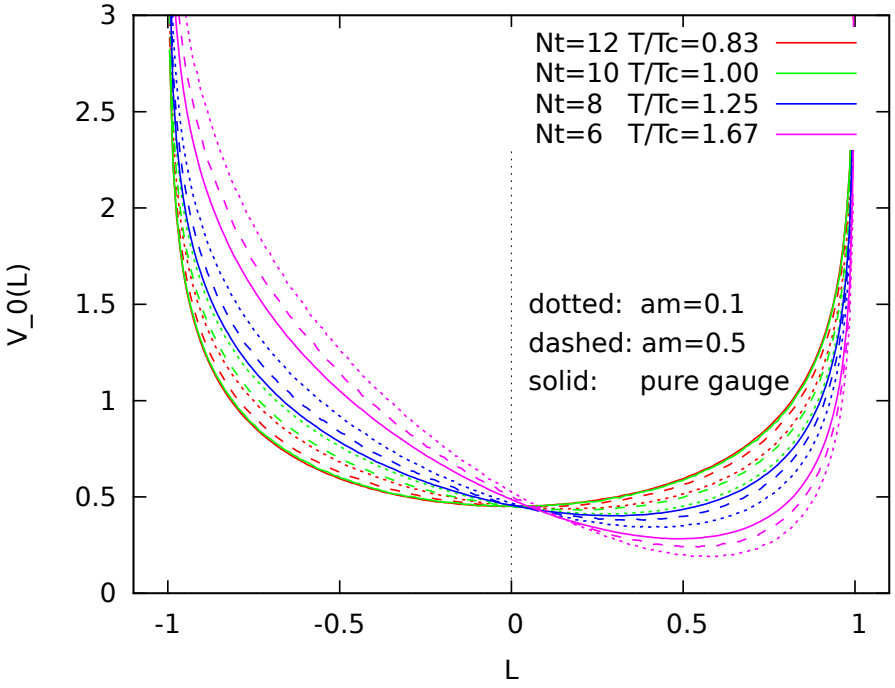


pure gauge results by Smith, Dumitru, Pisarski, von Smekal [1307.6339]

- Fixed scale approach

Unquenched Polyakov Loop Distributions and Effective Potentials at β = 2.577856

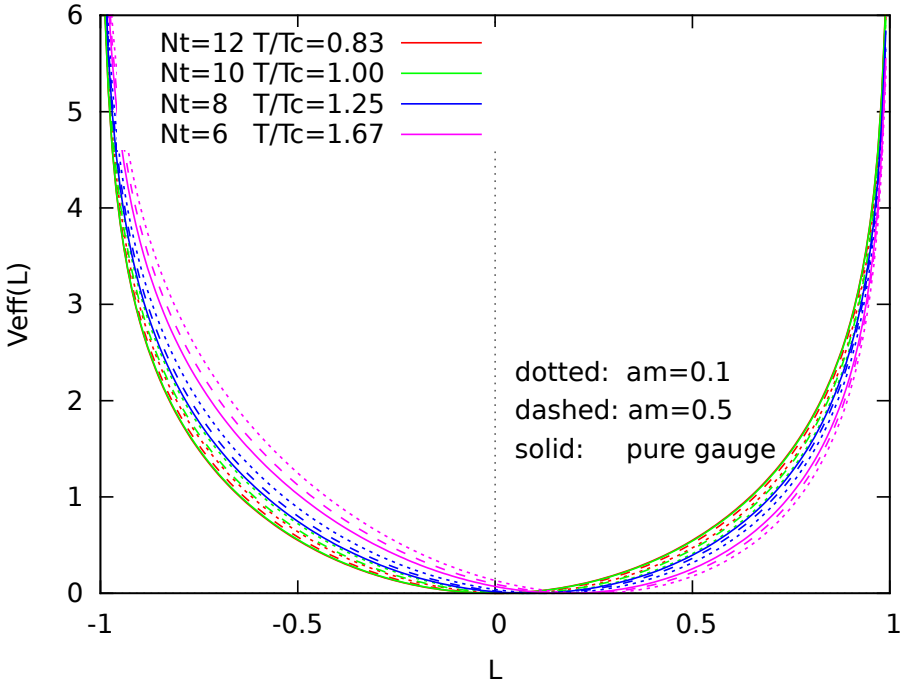


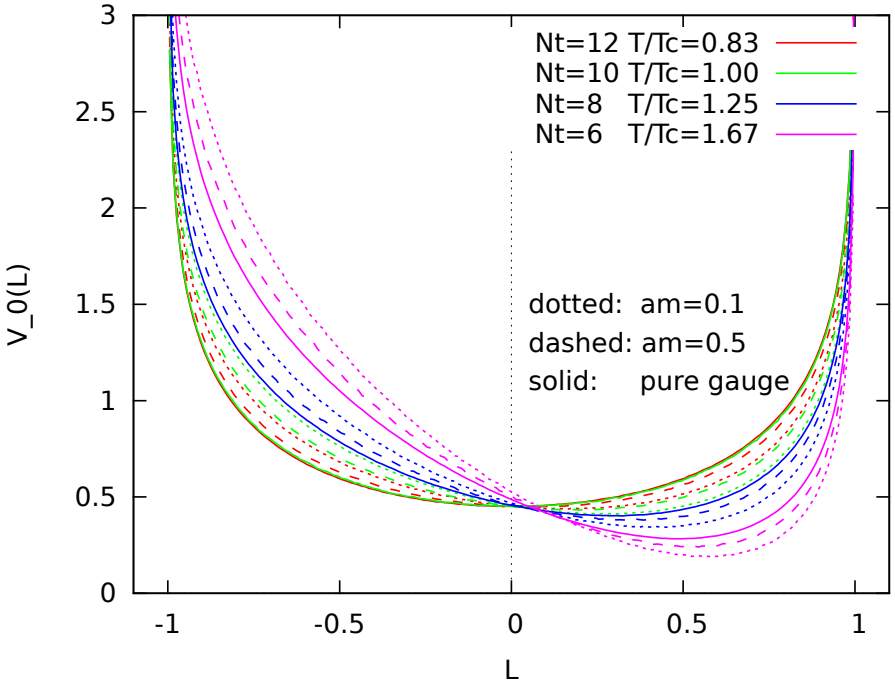


Scheffler, PS, Smith, von Smekal in preparation

- With N_f = 2 staggered quarks, neglect scale change through quark masses

Unquenched Polyakov Loop Distributions and Effective Potentials at β = 2.577856





Scheffler, PS, Smith, von Smekal in preparation

- With N_f = 2 staggered quarks, neglect scale change through quark masses

Effective Polyakov Loop Theories

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- 3d SU(N) spin models sharing universal behavior at deconfinement transition with underlying gauge theory
- Can be derived from combined strong coupling and hopping expansion
- Less computational cost, especially with dynamical fermions
- Finite density $\longrightarrow$ Worm Algorithm

Effective Action

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most general form:

$$S = \sum_{ij} L_i \, K^{(2)}(i,j) \, L_j + \sum_{ijkl} L_i L_j \, K^{(4)}(i,j,k,l) \, L_k L_l + \ldots + \sum_i h \, L_i + \ldots$$

can also contain loops in adjoint or higher representations.

B. Svetitsky, Phys. Rept. 132 (1986), Heinzl et al., Phys. Rev. D. 72 (2005)

Truncate Series: e.g. nearest neighbor interactions, resum generalized Polyakov loops:

$$\boxed{S = -\sum_{\langle ij \rangle} \log(1 + \lambda L_i L_j) - 4N_f \sum_i \log(1 + hL_i + h^2)}$$

Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012)

Comparing the Models

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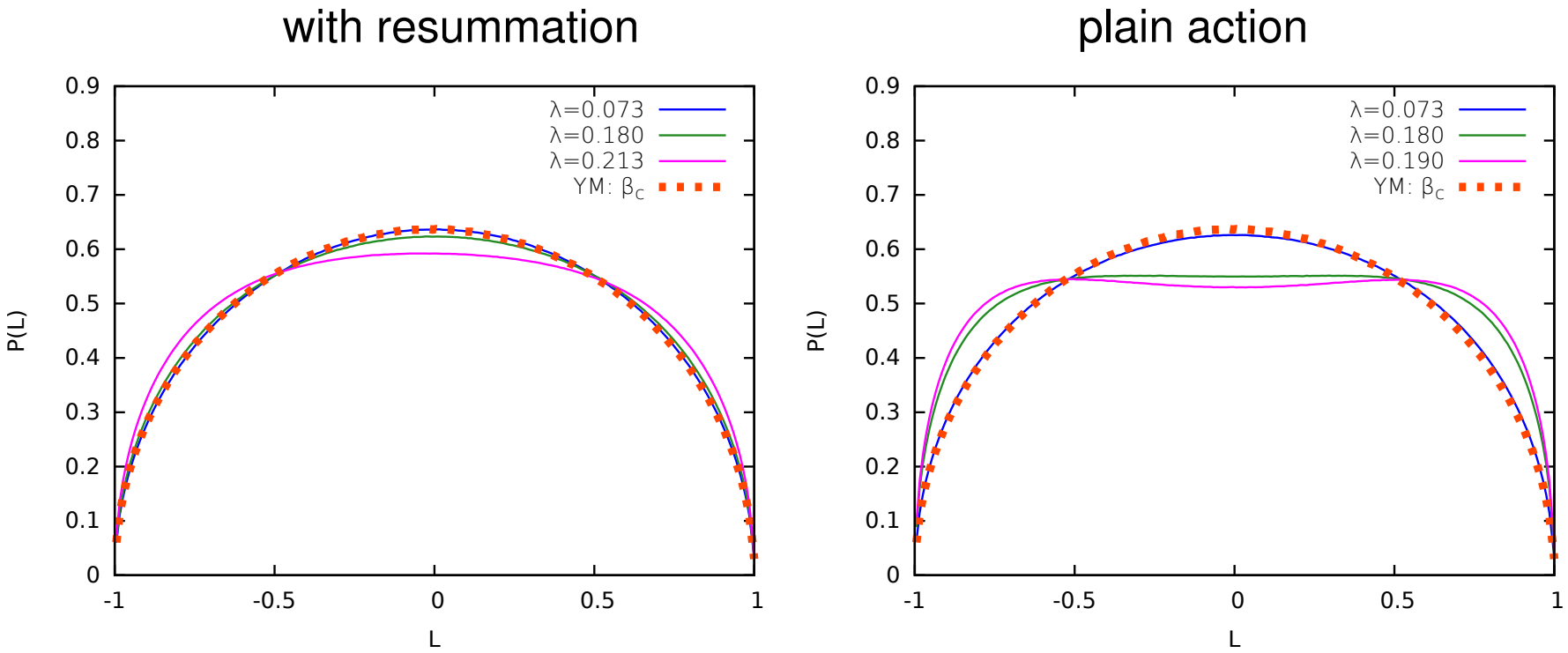

- Strong coupling limit: distributions match pure gauge theory
- $\lambda \to \lambda_c$: distributions get deformed, resummed theory reproduces pure gauge theory better

Polyakov Loop Distributions, Effective Theory

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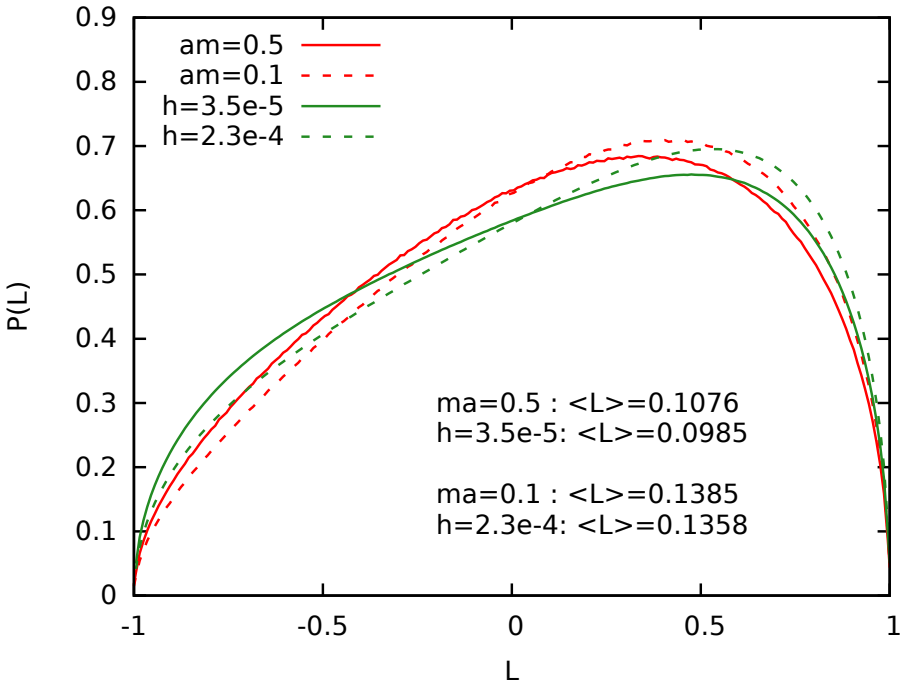
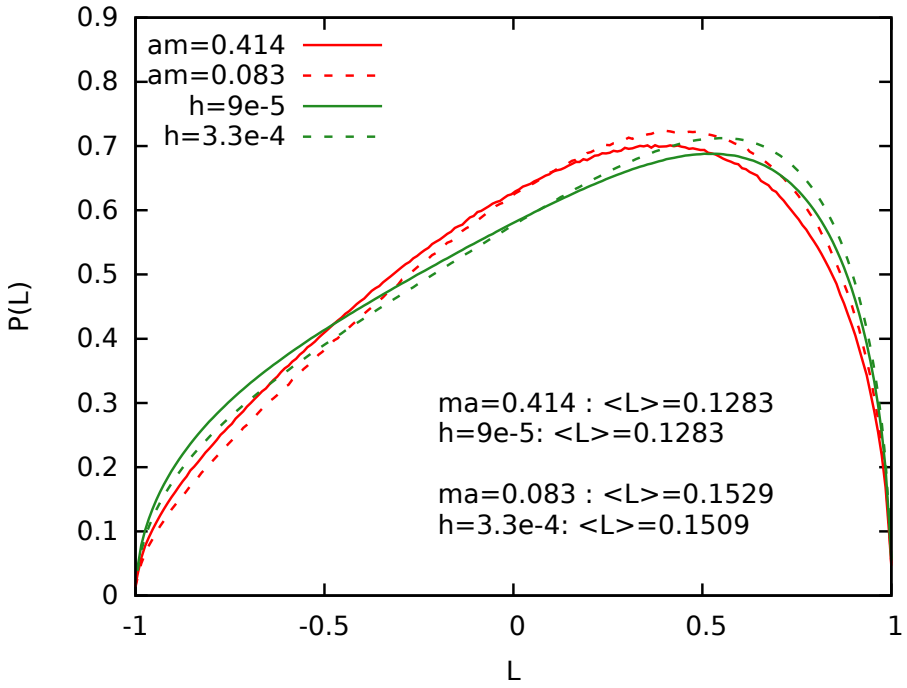
$T/T_c = 1.25$



$T/T_c = 1.5$



Scheffler, PS, Smith, von Smekal in preparation

- Couplings λ and h are matched to reproduce the correct $\langle L \rangle$

Polyakov Loop Distributions, Effective Theory

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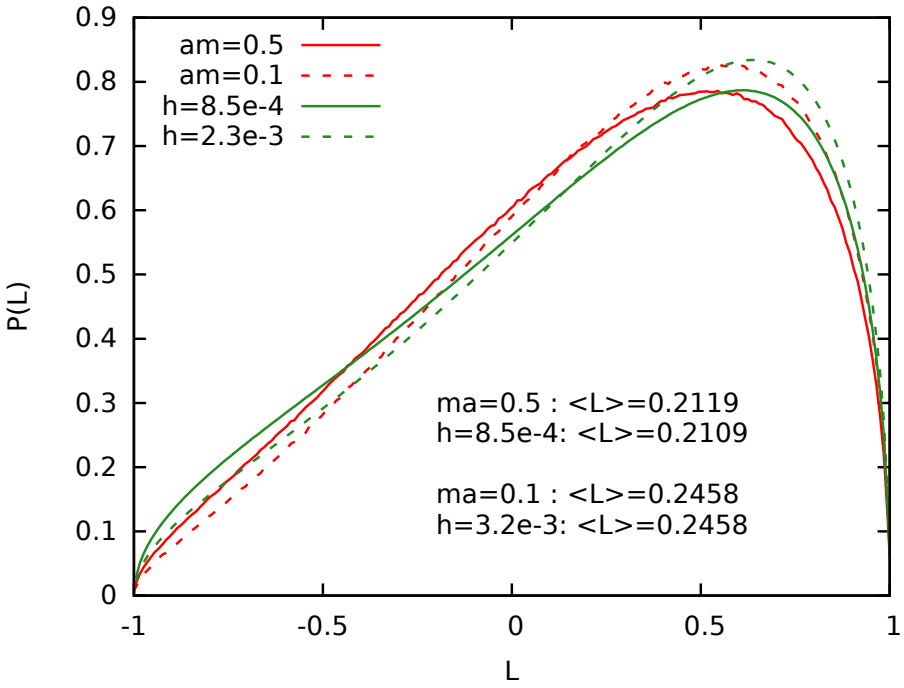
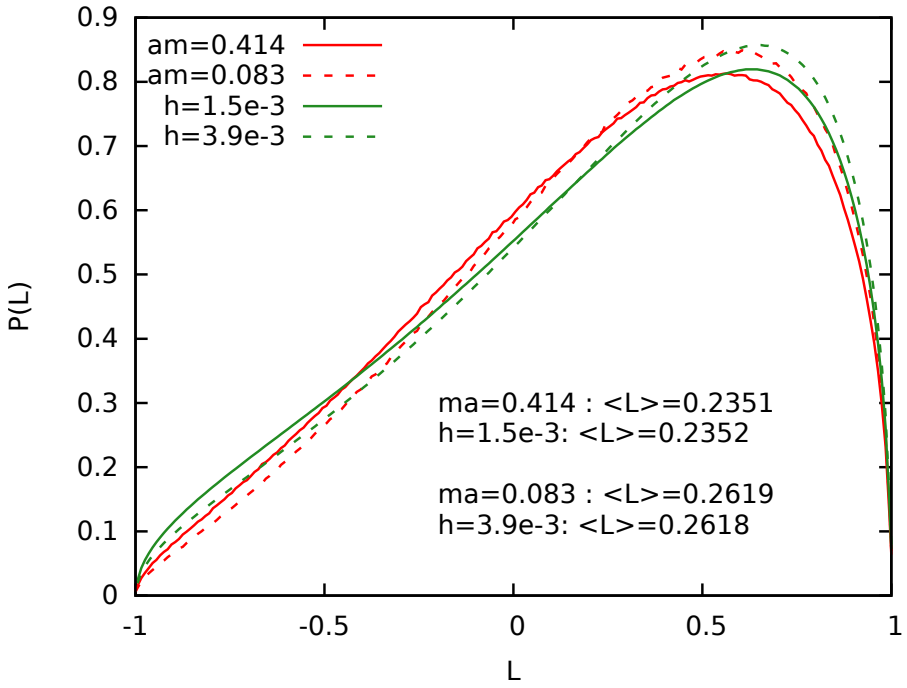
$T/T_c = 1.67$



$T/T_c = 2.0$



Scheffler, PS, Smith, von Smekal in preparation

- Couplings λ and h are matched to reproduce the correct $\langle L \rangle$

Effective Polyakov Loop Potentials, Effective Theory

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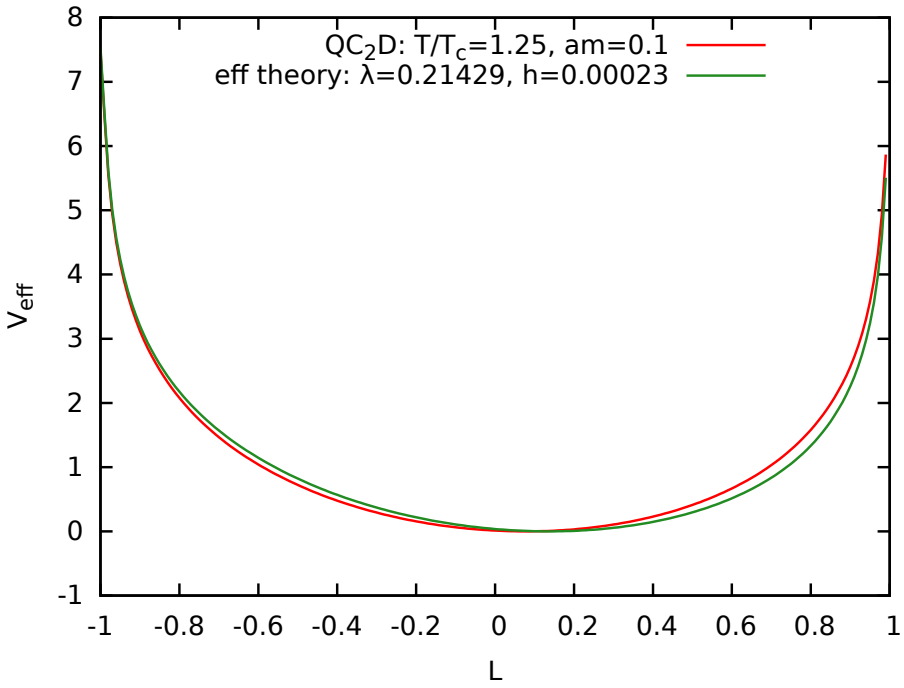
$T/T_c = 1.25$



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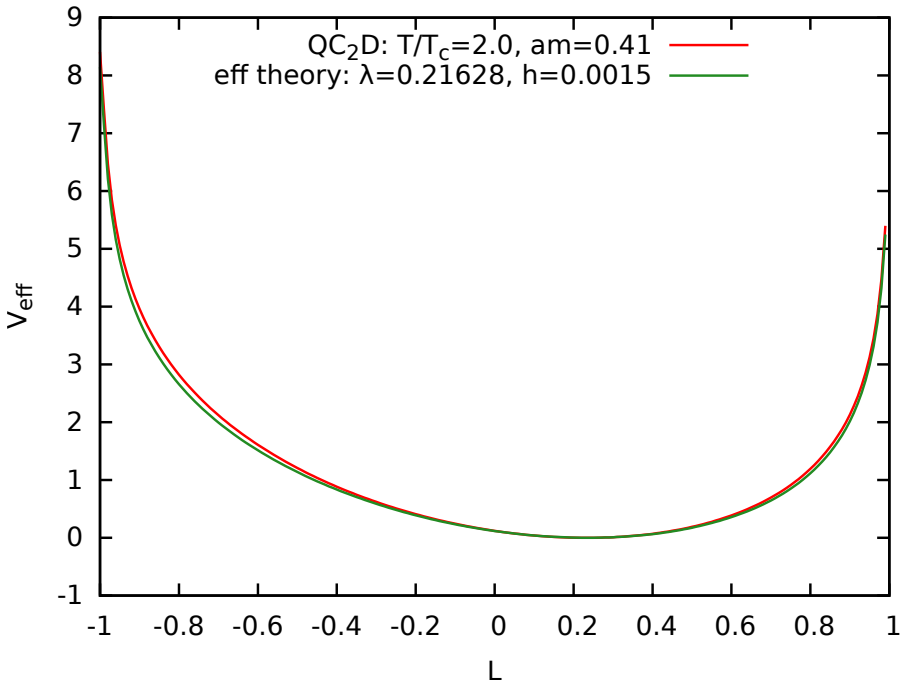

Scheffler, PS, Smith, von Smekal in preparation

- very good agreement between effective theory and full two-color QCD

Further Results

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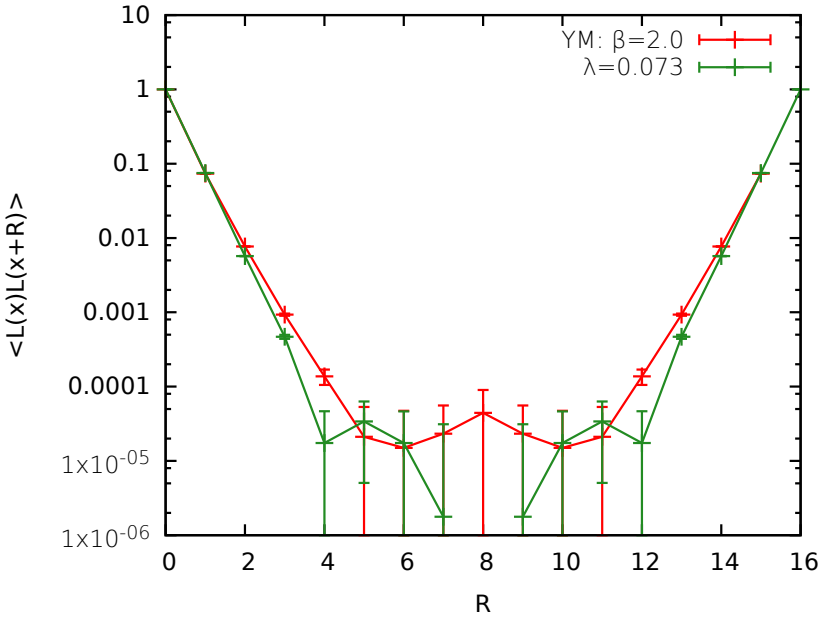
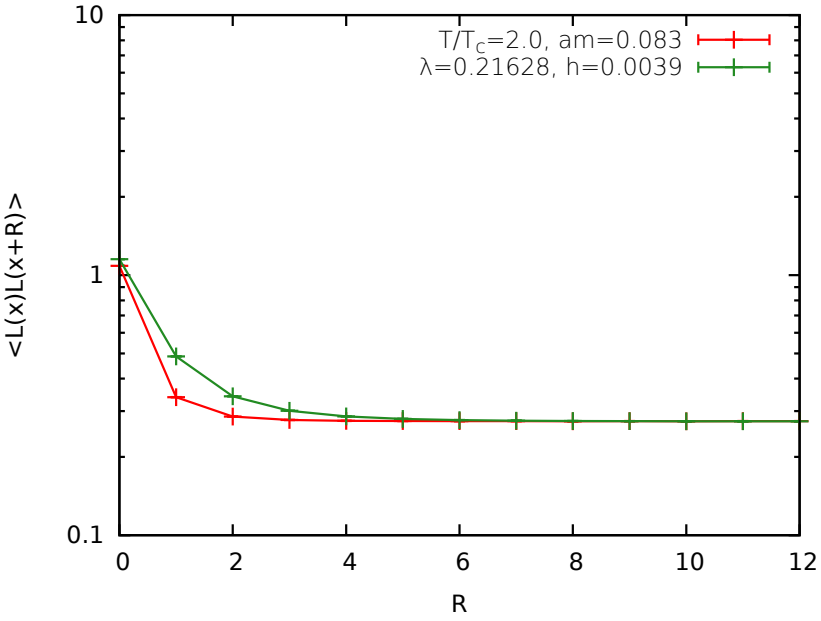
- Adjoint loops: strong fluctuations around λ_c, no benefit at $\lambda \neq \lambda_c$
- Correlation functions:





Cold and Dense QC_2D with Heavy Quarks

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- Use effective theory at low temperature and finite chemical potential with heavy quarks $\longrightarrow$ strong coupling regime
- Combined strong coupling and hopping expansion to order $\mathcal{O}(\kappa^n, u^m)$, $n + m = 4$ compare: *Langelage, Neuman, Philipsen [1403.4162]*
- Low temperature, strong coupling $\longrightarrow \lambda \leq 10^{-16}$
- Fermionic partition function
- Calculate density:

$$n = \frac{T}{V} \frac{\partial \log Z}{\partial \mu}$$

Effective Action

$$-S_{\text{eff}} = \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2$$

Effective Action

$$-S_{\text{eff}} = \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

Effective Action

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$$
\begin{aligned}
-S_{\text{eff}} = &\sum_{\vec{x}} \log(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
&+ 2\frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
&+ \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2}\mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}}\mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
&+ 2\frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2}\mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}}\mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
&+ \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}\mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}}
\end{aligned}
$$

Effective Action

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$$
\begin{aligned}
-S_{\text{eff}} = & \sum_{\vec{x}} \log(1 + h\mathrm{Tr}W_{\vec{x}} + h^2)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
& + 2\frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
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& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + \kappa^4 N_\tau^2 \sum_{x,i} \frac{h^4}{(1 + hL_x + h^2)(1 + hL_{x+i} + h^2)} \,.
\end{aligned}
$$

Cold and Dense QC_2D with Heavy Quarks

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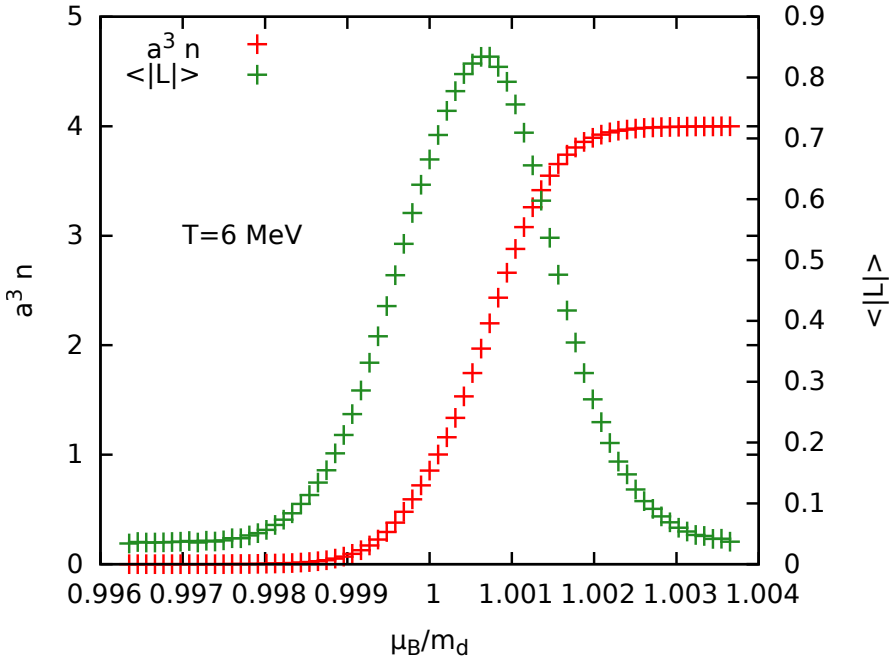
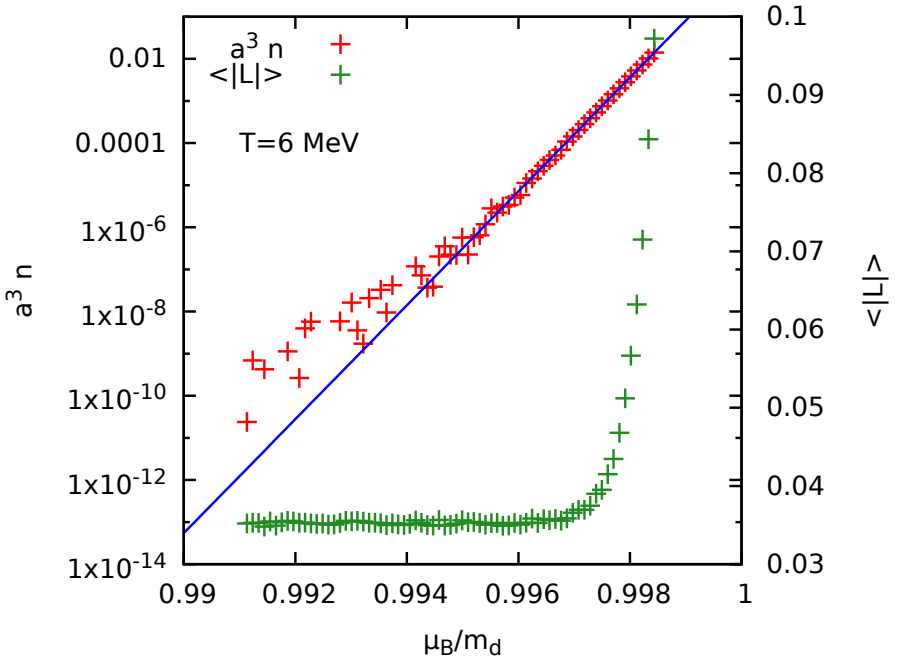
- Very heavy quarks: m_d = 19.17 GeV
- Deconfinement transition with unphysical lattice saturation





Cold and Dense QC_2D with Heavy Quarks

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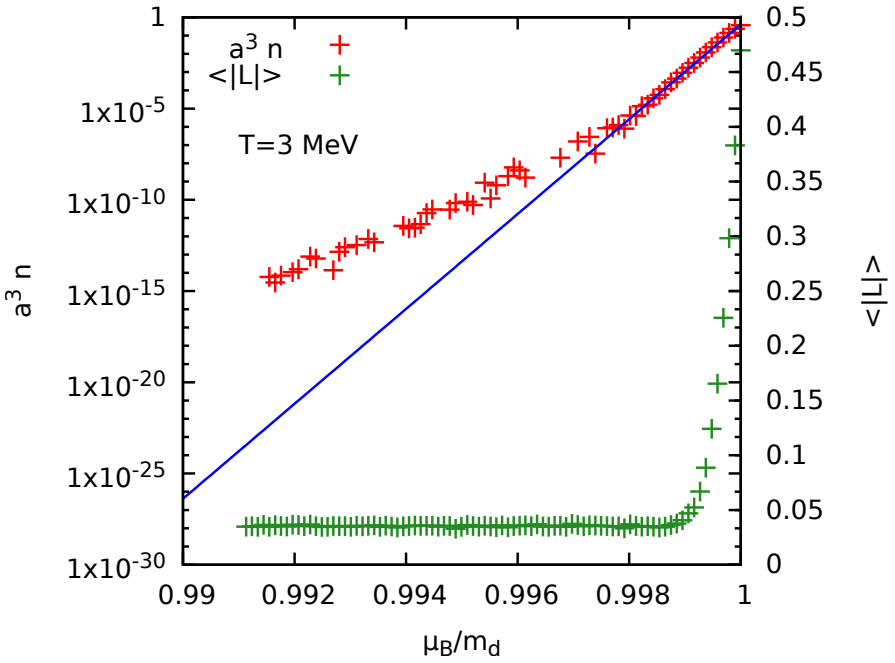
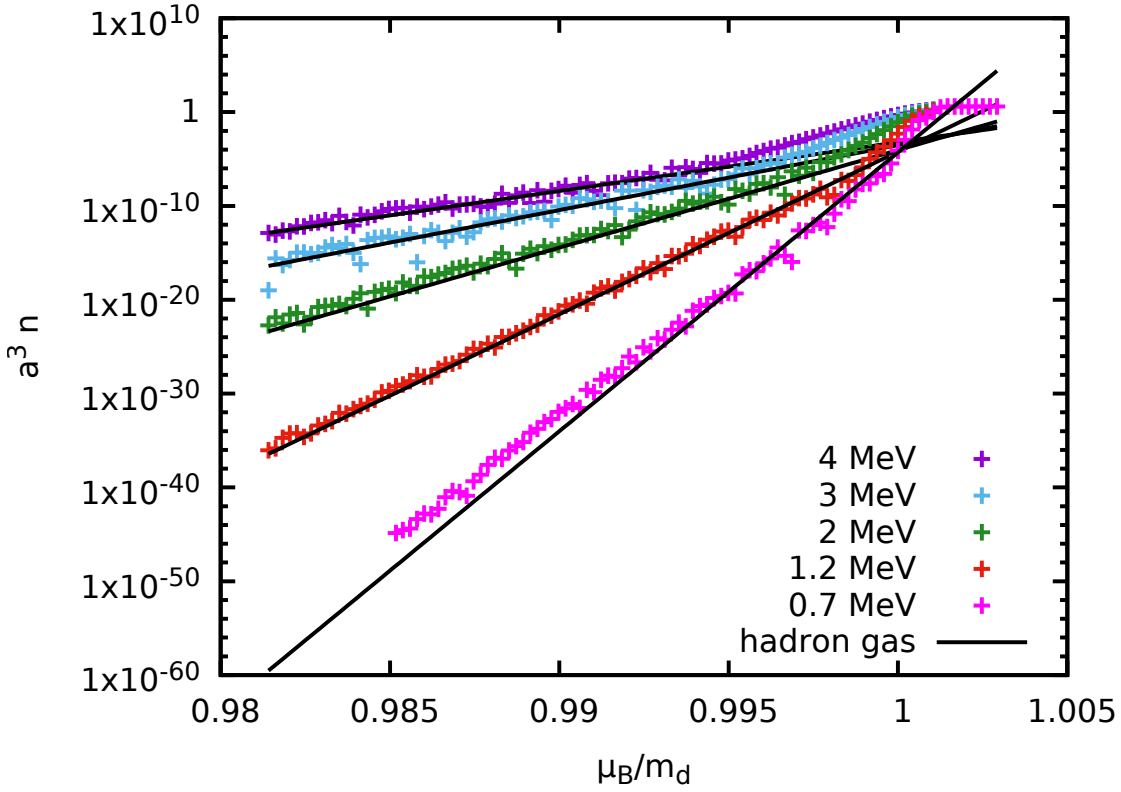


- Diquark BEC??
- What happens for smaller quark masses?

Cold and Dense QC_2D with Heavy Quarks



- below μ_c: hadron gas $\propto \exp\left(\frac{\mu_B - m_d}{T}\right)$

Summary and Outlook

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Summary

- Unquenched Polyakov loop potentials from full QC_2D and effective theory
- Small T, finite μ: deconfiment transition with unphysical lattice saturation
- Possible diquark BEC?? $\rightarrow$ difference between two and three colors?

Outlook

- Simulations of effective theory and full Q_2CD at finite density
- Main goal: effective Polyakov loop potentials at finite density
- Cold and dense regime: go to smaller quark masses $\rightarrow$ higher order in hopping expansion

Backup Slides

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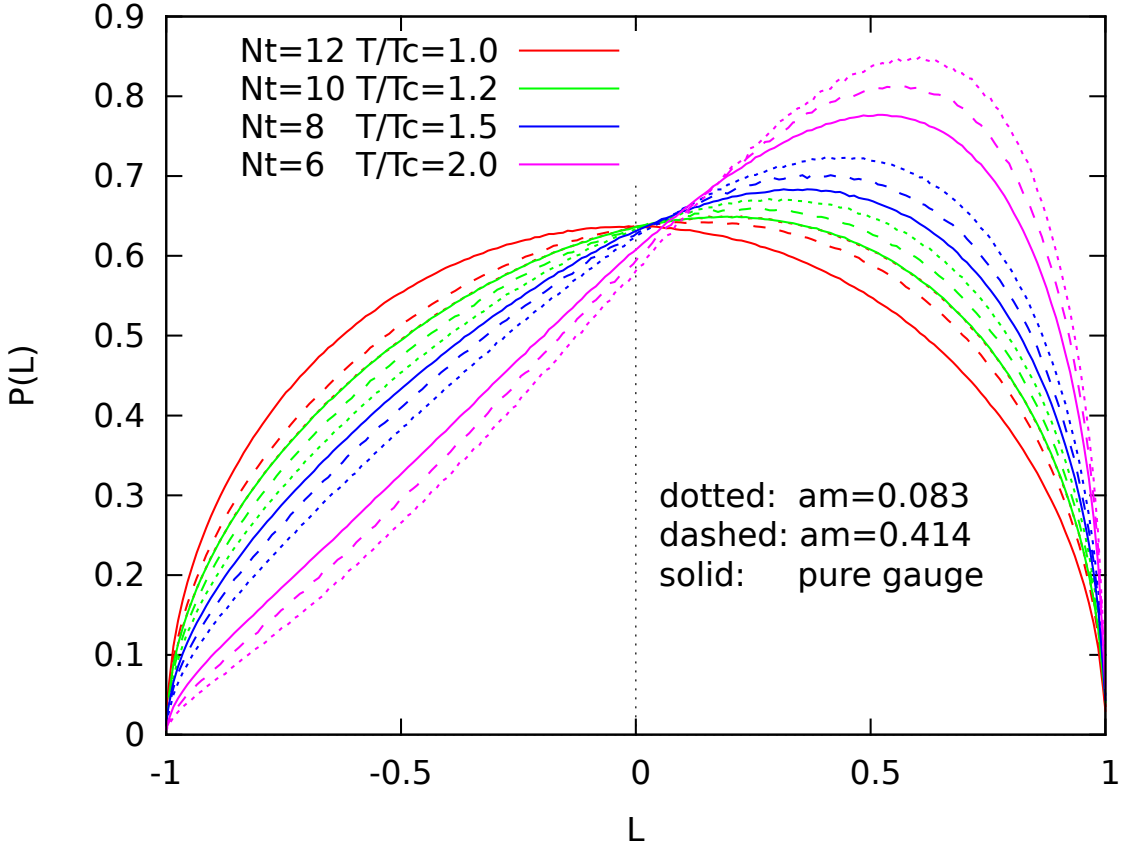
Polyakov Loop Distributions

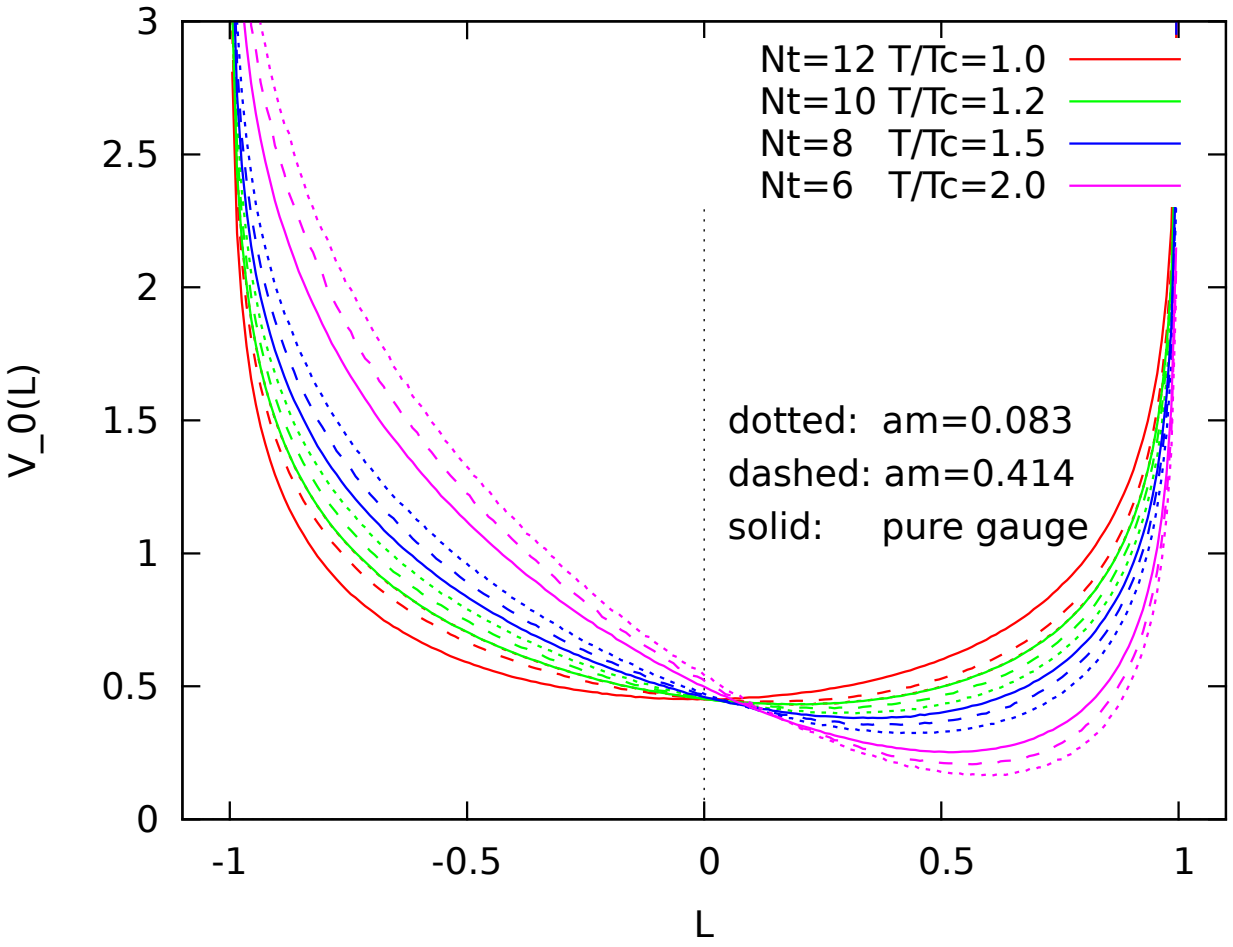
at β = 2.635365



◀ β = 2.577856

Polyakov Loop Effective Potential

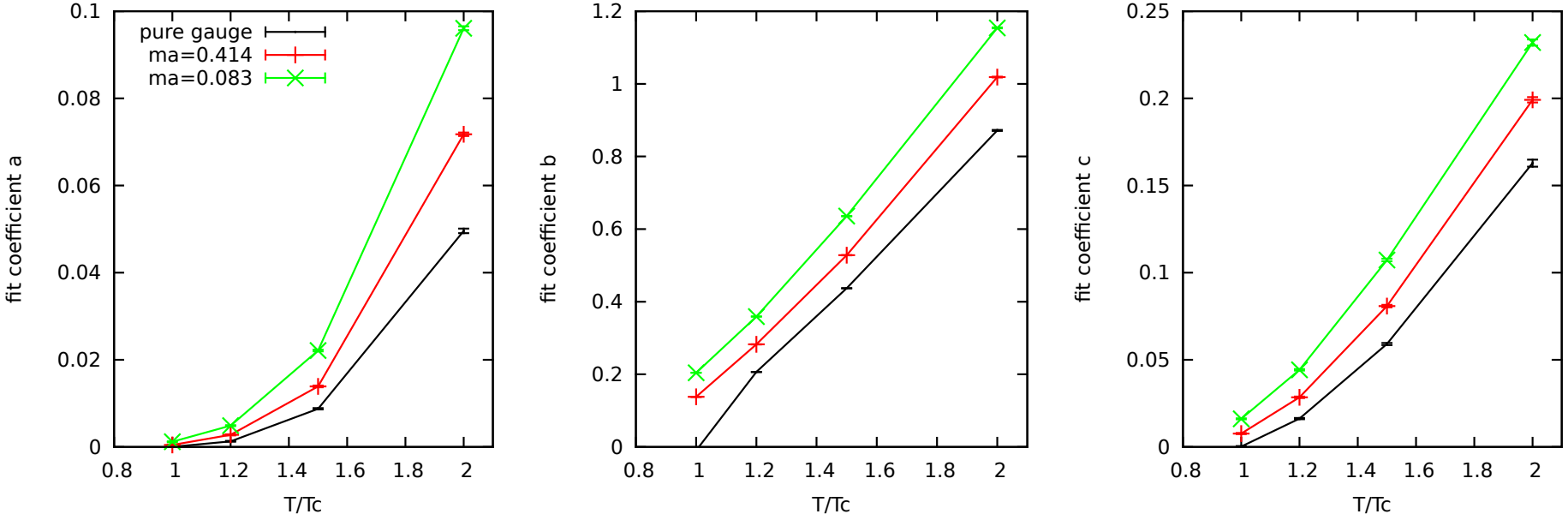
at β = 2.635365



◂ β = 2.577856

Fit Coefficients

at β = 2.635365

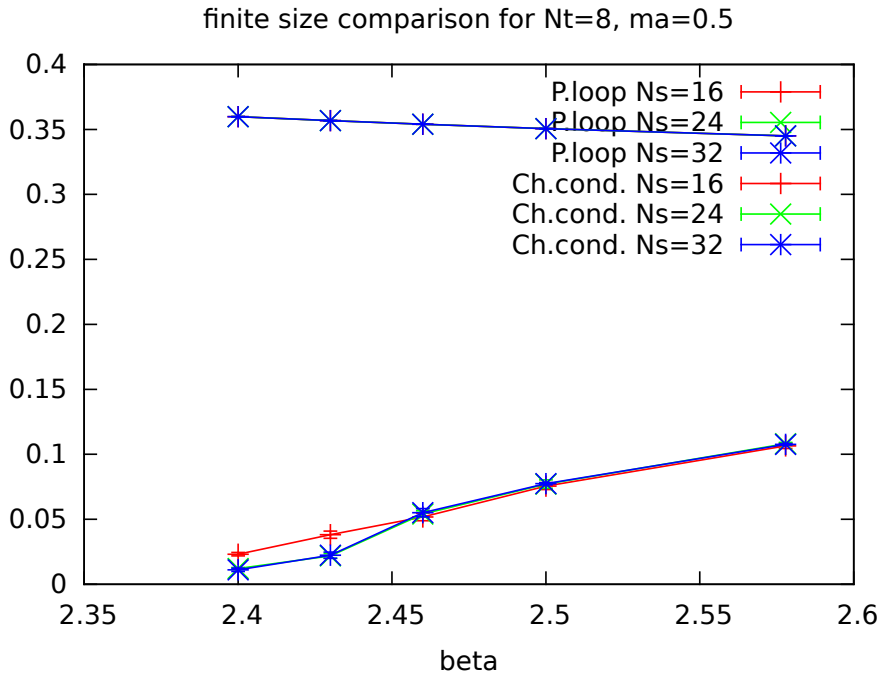


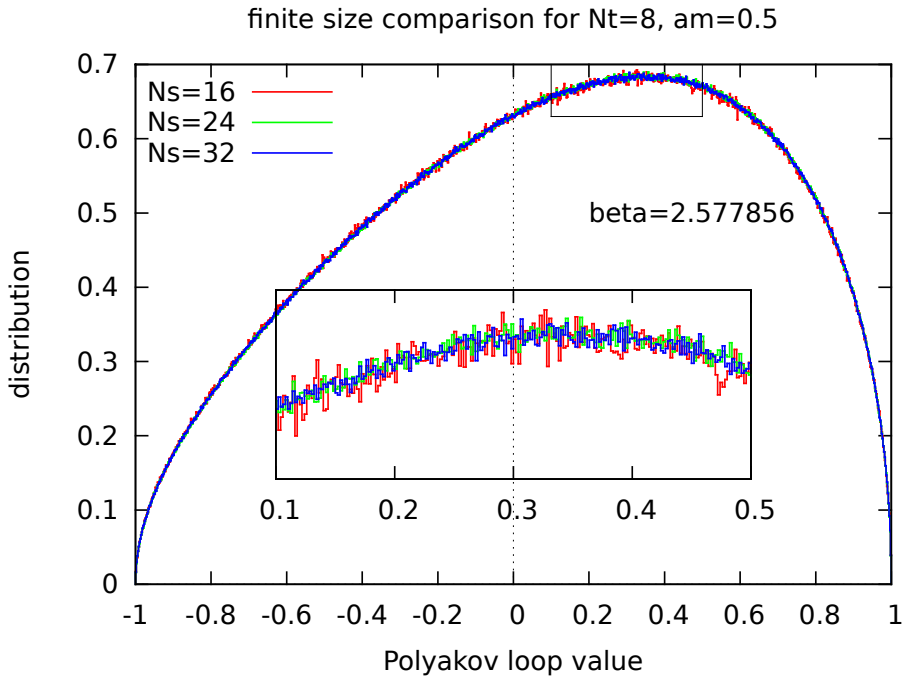
$$V_0(l) = V^{(T_c)}(l) + a(T) - b(T)l + c(T)l^2$$

◀ β = 2.577856

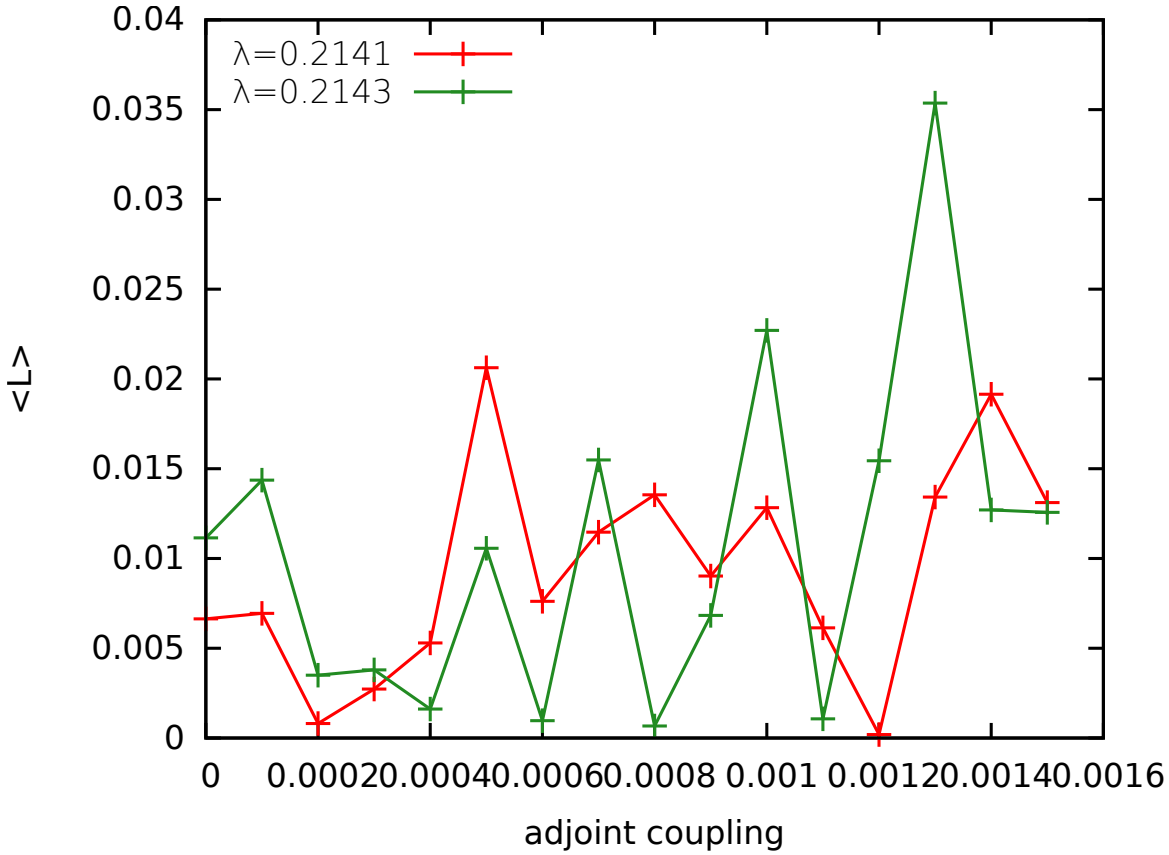
Finite Volume Test

$N_t = 8, am = 0.5$





Adjoint Coupling



Analytic relations for Cold and Dense regime

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$$h = \exp\left[N_\tau\left(a\mu + \ln 2\kappa + 6\kappa^2\frac{u - u^{N_\tau}}{1-u}\right)\right] ,$$

$$am_\pi = -2\ln(2\kappa) - 6\kappa^2 - 24\kappa^2\frac{u}{1-u} + 6\kappa^4 + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5) .$$