Effective Polyakov Loop Theories on the Lattice

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Motivation

- Exploration of the deconfinement transition of QCD
- Use of effective Polyakov loop potentials in functional methods
- Effects of unquenching
- Exploration of the BEC Phase of two-color QCD
Setup

- Per-site probability distribution $P(L)$ via histogram-method from QC$_2$D simulations with 2 flavors of staggered quarks

- Per-site constrained potential

\[ V_0(L) = - \log P(L) \]

- Obtain actual per-site effective potential by Legendre transform

\[ W(h) = \log \int dL \exp(-V_0(L) + hL) \]

\[ V_{\text{eff}}(\hat{L}) = \sup_h (\hat{L}h - W(h)) \]
Polyakov Loop Distributions and Effective Potentials at $\beta = 2.577856$

Pure gauge results by Smith, Dumitru, Pisarski, von Smekal [1307.6339]

- Fixed scale approach
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Scheffler, PS, Smith, von Smekal in preparation

- With $N_f = 2$ staggered quarks, neglect scale change through quark masses
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Effective Polyakov Loop Theories

- 3d SU(N) spin models sharing universal behavior at deconfinement transition with underlying gauge theory
- Can be derived from combined strong coupling and hopping expansion
- Less computational cost, especially with dynamical fermions
- Finite density $\rightarrow$ Worm Algorithm
Effective Action

most general form:

\[ S = \sum_{ij} L_i K^{(2)}(i, j) L_j + \sum_{ijkl} L_i L_j K^{(4)}(i, j, k, l) L_k L_l + \ldots + \sum_i h L_i + \ldots \]

can also contain loops in adjoint or higher representations.


Truncate Series: e.g. nearest neighbor interactions, resum generalized Polyakov loops:

\[ S = - \sum_{\langle ij \rangle} \log(1 + \lambda L_i L_j) - 4N_f \sum_i \log(1 + h L_i + h^2) \]

*Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012)*
Comparing the Models

- Strong coupling limit: distributions match pure gauge theory
- $\lambda \to \lambda_c$: distributions get deformed, resummed theory reproduces pure gauge theory better
Polyakov Loop Distributions, Effective Theory

$T / T_c = 1.25$

$T / T_c = 1.5$

Couplings $\lambda$ and $h$ are matched to reproduce the correct $\langle L \rangle$. 

Scheffler, PS, Smith, von Smekal in preparation
Polyakov Loop Distributions, Effective Theory

\[ T / T_c = 1.67 \]

\[ T / T_c = 2.0 \]

\[ \text{Scheffler, PS, Smith, von Smekal } \text{ in preparation} \]

- Couplings \( \lambda \) and \( h \) are matched to reproduce the correct \( \langle L \rangle \)
Effective Polyakov Loop Potentials,

**Effective Theory**

\[ T/T_c = 1.25 \]

\[ T/T_c = 2.0 \]

![Graph showing effective potential for different temperatures](image)

Scheffler, PS, Smith, von Smekal  in preparation

- very good agreement between effective theory and full two-color QCD
Further Results

- Adjoint loops: strong fluctuations around $\lambda_c$, no benefit at $\lambda \neq \lambda_c$

- Correlation functions:
Cold and Dense QC$_2$D with Heavy Quarks

- Use effective theory at low temperature and finite chemical potential with heavy quarks $\longrightarrow$ strong coupling regime
- Combined strong coupling and hopping expansion to order $O(\kappa^n, u^m), n + m = 4$
  
  compare: Langelage, Neuman, Philipsen [1403.4162]
- Low temperature, strong coupling $\longrightarrow \lambda \leq 10^{-16}$
- Fermionic partition function
- Calculate density:
  
  $$ n = \frac{T}{V} \frac{\partial \log Z}{\partial \mu} $$
Effective Action

$$-S_{\text{eff}} = \sum_{\tilde{x}} \log(1 + \hbar \text{Tr} \mathcal{W}_{\tilde{x}} + h^2)^2$$
Effective Action

\[-S_{\text{eff}} = \sum_{x} \log(1 + h \text{Tr} W_{x} + h^2)^2 - 2h_2 \sum_{x,i} \text{Tr} \frac{hW_{x}}{1 + hW_{x}} \text{Tr} \frac{hW_{x+i}}{1 + hW_{x+i}}\]
Effective Action

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\[+ 2 \frac{\kappa^4 N_T^2}{N_C^2} \sum_{\bar{x},i} \text{Tr} \frac{hW_{\bar{x}}}{(1 + hW_{\bar{x}})^2} \text{Tr} \frac{hW_{\bar{x}+i}}{(1 + hW_{\bar{x}+i})^2} \]

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\[ + \kappa^4 N^2_T \sum_{x,i} \frac{h^4}{(1 + hL_x + h^2)(1 + hL_{x+i} + h^2)}. \]
Cold and Dense QC$_2$D with Heavy Quarks

- Very heavy quarks: $m_d = 19.17$ GeV

- Deconfinement transition with unphysical lattice saturation
Cold and Dense QC$_2$D with Heavy Quarks

▶ Diquark BEC??

▶ What happens for smaller quark masses?
below $\mu_c$: hadron gas $\propto \exp\left(\frac{\mu_B - m_d}{T}\right)$
Summary

- Unquenched Polyakov loop potentials from full QC\(_2\)D and effective theory
- Small \(T\), finite \(\mu\): deconfinement transition with unphysical lattice saturation
- Possible diquark BEC?? \(\rightarrow\) difference between two and three colors?

Outlook

- Simulations of effective theory and full Q\(_2\)CD at finite density
- Main goal: effective Polyakov loop potentials at finite density
- Cold and dense regime: go to smaller quark masses \(\rightarrow\) higher order in hopping expansion
Polyakov Loop Distributions

at $\beta = 2.635365$
Polyakov Loop Effective Potential

at $\beta = 2.635365$
Fit Coefficients

at $\beta = 2.635365$

\[ V_0(l) = V^{(T_c)}(l) + a(T) - b(T)l + c(T)l^2 \]
Finite Volume Test

$N_t = 8, \ am = 0.5$

finite size comparison for $N_t=8, \ ma=0.5$

finite size comparison for $N_t=8, \ am=0.5$

beta=2.577856

distribution Polyakov loop value

finite size comparison for $N_t=8, \ am=0.5$
Adjoint Coupling

\[ \lambda = 0.2141 \]
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Analytic relations for Cold and Dense regime

\[ h = \exp \left[ N_\tau \left( a\mu + \ln 2\kappa + 6\kappa^2 \frac{u - u^{N_\tau}}{1 - u} \right) \right], \]

\[ a m_\pi = -2 \ln(2\kappa) - 6\kappa^2 - 24\kappa^2 \frac{u}{1 - u} + 6\kappa^4 + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5). \]