# Fine lattice simulations with the Ginsparg-Wilson fermions

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JLQCD Collaboration

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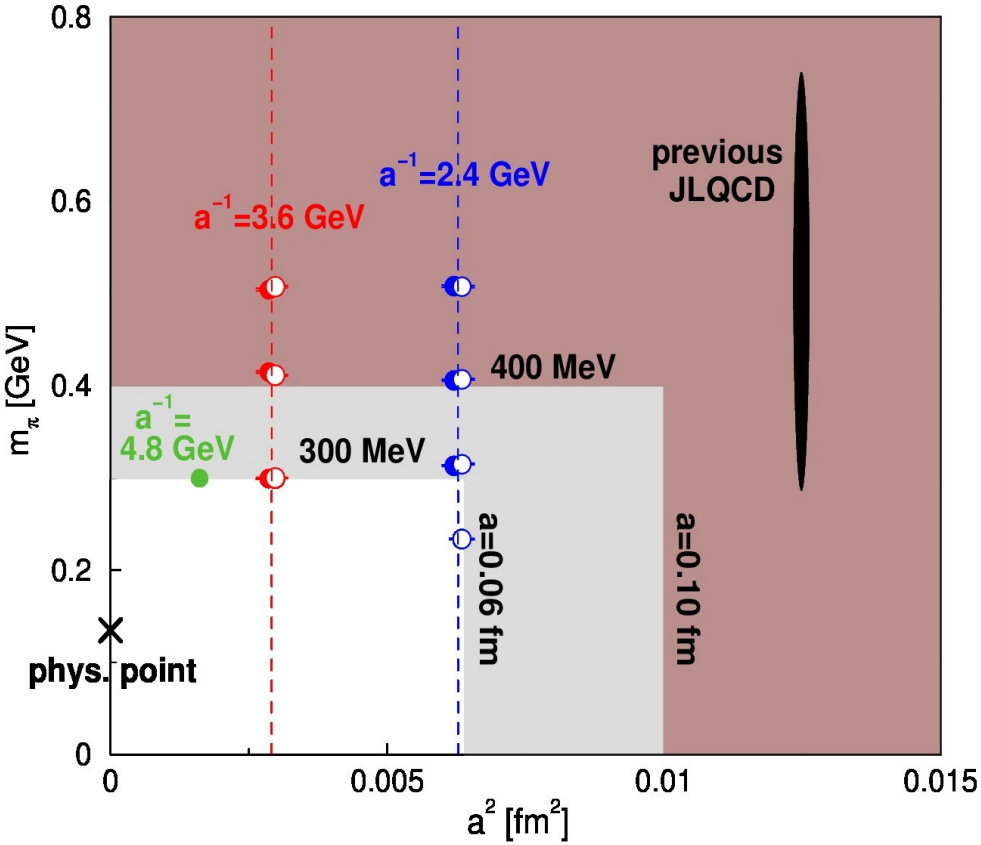
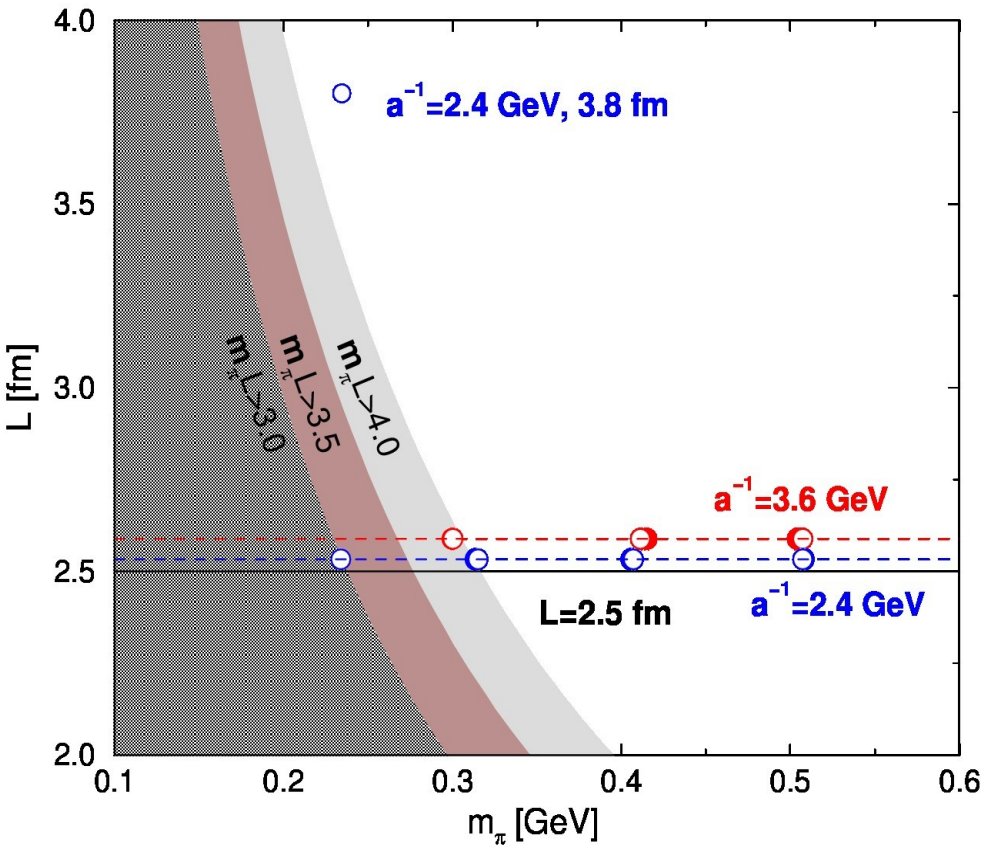
# Flavor physics with GW fermions

- Heavy quark physics for NP search
  - need experimental / theoretical studies at high-precision
  - LQCD: matrix elements with comparable precision
- $N_f$ = 2+1 simulation with better control of systematic errors
  - chiral symmetry to avoid unwanted effect of its violation *i.e.* operator mixing etc. RBC Collab., 1998-
    - Domain-Wall fermions (Möbius kernel), $m_{res} \leq m_{ud} \times 0.1$
  - continuum extrapolation ($a^{-1}$ = 2.4, 3.6, 4.8 GeV)
  - light quarks ($m_\pi$= 500, 400, 300 MeV and lighter)
  - lattice volume satisfying $m_\pi L > 4$
  - generated configs may be useful for other quantities, especially for those requiring good chiral symmetry.

# Overview

- Design of our numerical simulation




- HMC: $a^{-1}$ = 2.4 GeV has been finished. 90% done for 3.6 GeV.
- $a^{-1}$ = 4.8 GeV is running

# Plan of this talk

- Numerical simulation
  - HMC
  - basic measurements
- Scale setting with gradient flow
- Baseline studies of generated configs
  - thermalization / autocorrelation
  - correlation with topology
- Summary & outlook

# Related Presentations

- Y. Cho (in collab. with Southanpton), heavy quark physics
- G. Cossu, finite temperature
- H. Fukaya, topology issues
- M. Tomii, renormalization of local operators
- A. Tomiya, Dirac spectrum at finite temperature

# Numerical Simulation

# Domain-Wall (Möbuis) fermions

Kaplan 1992; Shamir 1994; Borici 1997; Chiu 1998; Brower et al. 2001

- 5D representation

$$D_{DW}^{(5)}(m) = 1 + b(4+M)D_W - (1 - c(4+M)D_W)\cdot \begin{bmatrix} 0 & P_- & & & -mP_+ \\ P_+ & 0 & P_- & & \\ & \ddots & \ddots & \ddots & \\ & & P_+ & 0 & P_- \\ -mP_- & & & P_+ & 0 \end{bmatrix}$$

$D_W$: Wilson Dirac op with mass $-M$

- 4D effective operator

5D projection: $\mathcal{P} \equiv \begin{bmatrix} P_- & P_+ & & & \\ & P_- & \ddots & & \\ & & \ddots & \ddots & \\ & & & P_- & P_+ \\ P_+ & & & & P_- \end{bmatrix}$

$$D_{DW}^{(4)}(m) = \left[\mathcal{P}^{-1} D_{DW}^{(5)}(m=1)^{-1} D_{DW}^{(5)}(m)\mathcal{P}\right]_{11}$$

$$= \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \underline{\tanh(L_s \tanh^{-1} H_M)}$$

$L_s \to \infty$

sign function approx.

- scaled Shamir kernel: $H_M = \gamma_5 \dfrac{bD_W}{2 + cD_W}$ we set $b$ = 2, $c$ = 1

- stout link smearing
  - smaller residual mass, faster inversion JLQCD 2013

# HMC

- Symanzik gauge + Möbius Domain-Wall ($N_f$ = 2+1)
  - tree-level Symanzik action
  - 3-level stout smearing
- standard RHMC with Omelyan integrator
  - performance @BG/Q : 16 → 30 GFlops/node (HMC),

    45 GFlops/node (meas)

thanks to P. Boyle !

Hitachi SR16k M1, 57TFlops peak

IBM BG/Q, 1.2PFlops peak

IROIRO++

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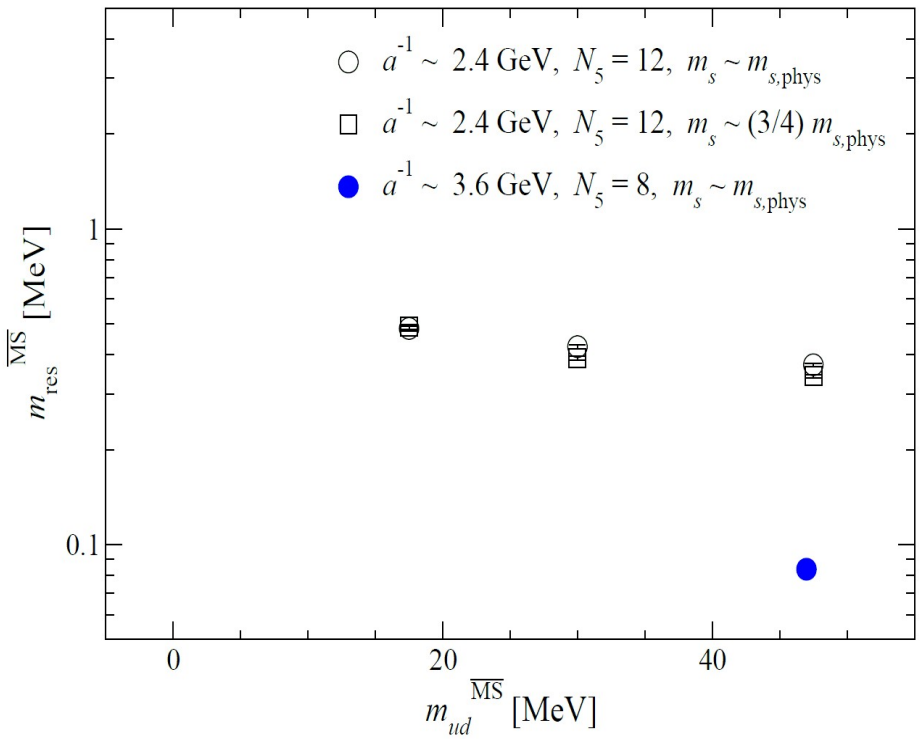
# Chiral symmetry

- residual mass Kaneko et al (JLQCD), Lattice 2013




- chiral symm. satisfied with good accuracy
  - $a^{-1}$ = 2.4 GeV:

    $L_s = 12 \rightarrow m_{\mathrm{res}} \sim m_{\mathrm{ud}} \times 0.1$
  - $a^{-1}$ = 3.6 GeV:

    $L_s = 8 \rightarrow m_{\mathrm{res}} \sim m_{\mathrm{ud}} \times 0.02$

# Gauge ensembles: status

- β = 4.17, $a^{-1}$~2.4 GeV

$32^3$x64x12

| $m_{ud}$ | $m_\pi$[MeV] | MD time |
|---|---|---|
| $m_s$=0.030 | | |
| 0.007 | 310 | 10,000 |
| 0.012 | 410 | 10,000 |
| 0.019 | 510 | 10,000 |
| $m_s$=0.040 | | |
| 0.0035 | 230 | 10,000 |
| 0.007 | 320 | 10,000 |
| 0.012 | 410 | 10,000 |
| 0.019 | 510 | 10,000 |
| $m_s$=0.040, $48^3$x96 | | |
| 0.0035 | 240 | 2,890 |

- β = 4.35, $a^{-1}$~3.6 GeV

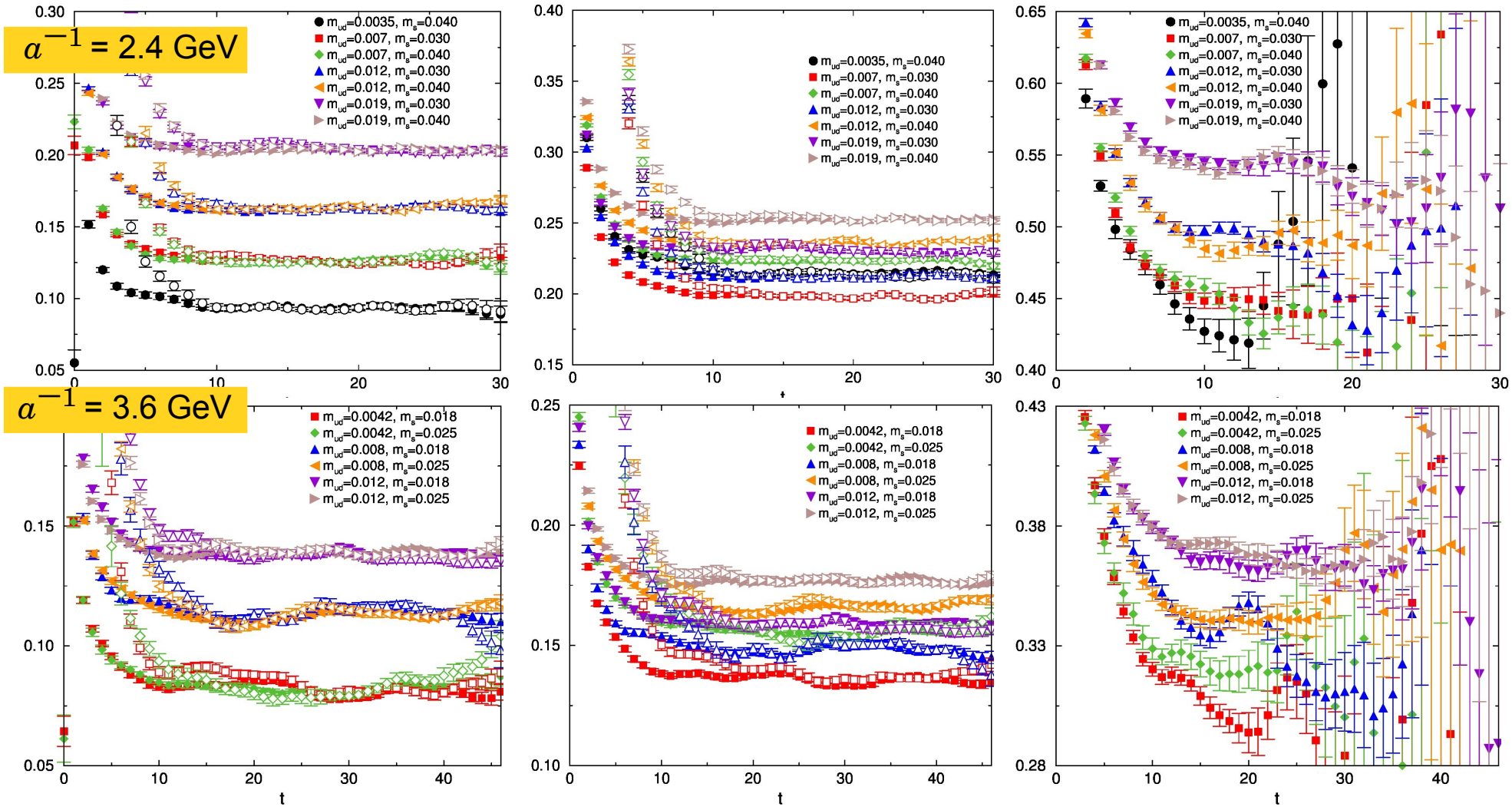
$48^3$x96x8

| $m_{ud}$ | $m_\pi$[MeV] | MD time* |
|---|---|---|
| $m_s$=0.018 | | |
| 0.0042 | 300 | 10,000 |
| 0.0080 | 410 | 4,260 |
| 0.0120 | 500 | 10,000 |
| $m_s$=0.025 | | |
| 0.0042 | 300 | 10,000 |
| 0.0080 | 410 | 10,000 |
| 0.0120 | 510 | 10,000 |

* 1 traj. = 2 MD time

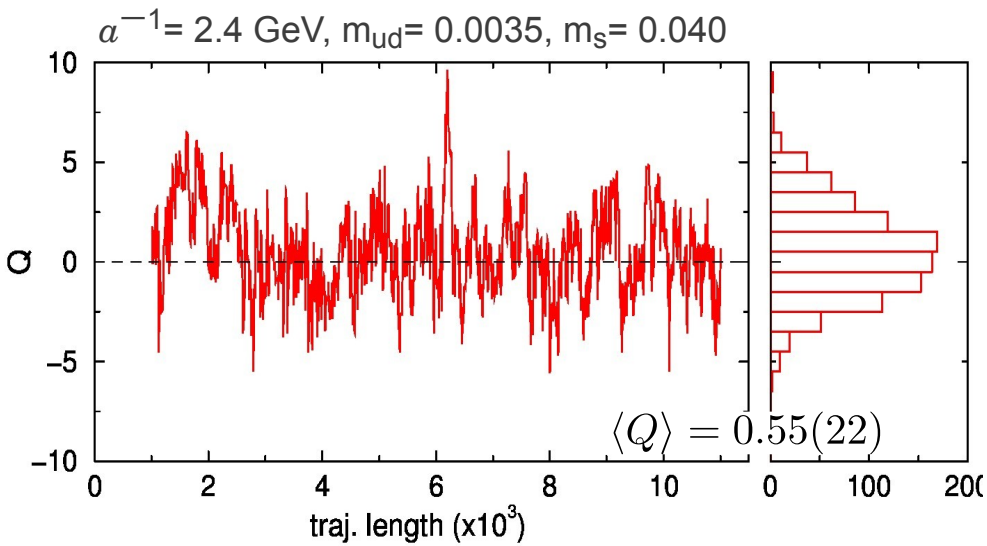
# Basic measurements

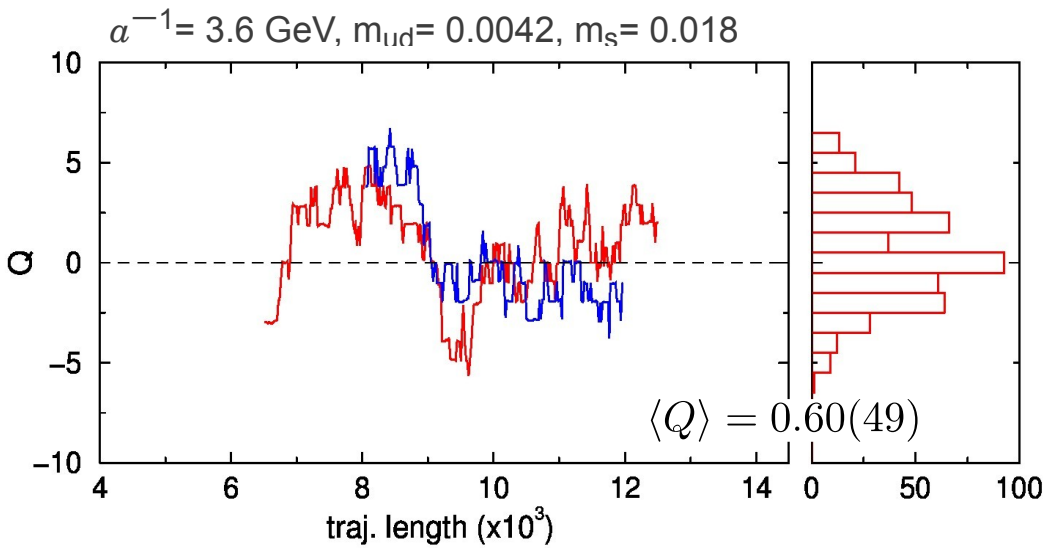
- light hadron correlators (local-local, smeared-local, 1or 2 source locations)
  - effec. mass of pion kaon nucleon



# Basic measurements (contd)

- Yang-Mills gradient flow Lüscher, 2010
  - on each stored config. $V_{x\mu}(0) = U_{x\mu}, \quad \left.\frac{dV_{x\mu}}{dt}\right|_t = -g_{0\mu}^2 S_g[V]\, V_{x\mu}$
  - monitoring topological charge $Q = \frac{1}{16\pi^2}\sum \mathrm{tr} F_{\mu\nu}\tilde{F}_{\mu\nu}$




- long autocorrelation, finer lattice tends to freeze.→ no problem, talk by Fukaya

  - lattice scale by the energy density $E(t) \equiv \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}|_t$

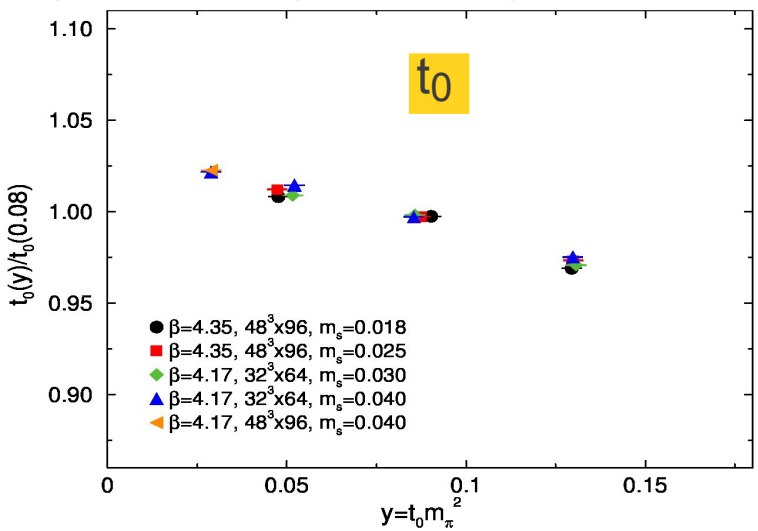
# Scale setting

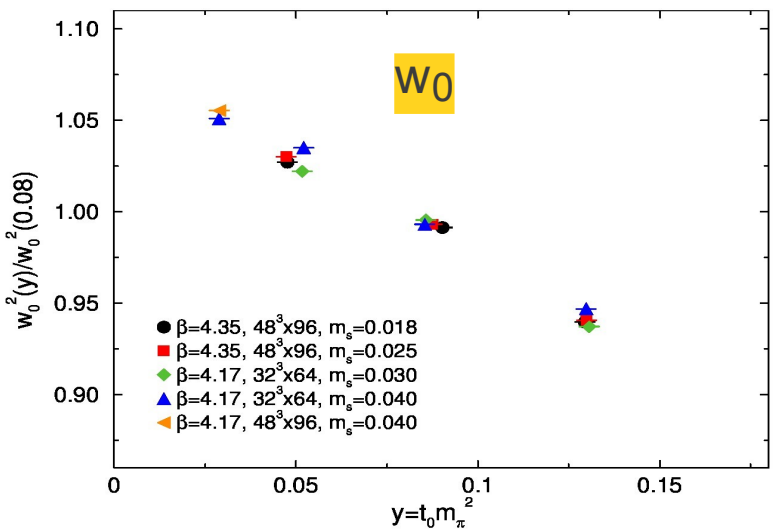
# Reference quantities

- $t_0$ , $w_0$ : $t^2\langle E\rangle\Big|_{t=t_0} = 0.3$ , $t\frac{d}{dt}[t^2\langle E\rangle]\Big|_{t=w_0^2} = 0.3$

BMW 2012: $t_0^{1/2}$= 0.1465 fm, $w_0$= 0.1755 fm

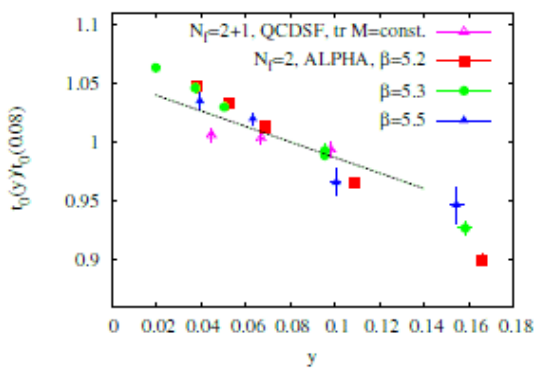
  - mass dependence (vs $t_0\, m_\pi^2$)

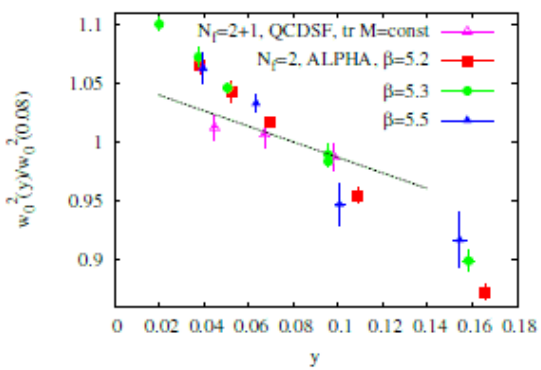



cf. Sommer, 2013

consistent with known behavior

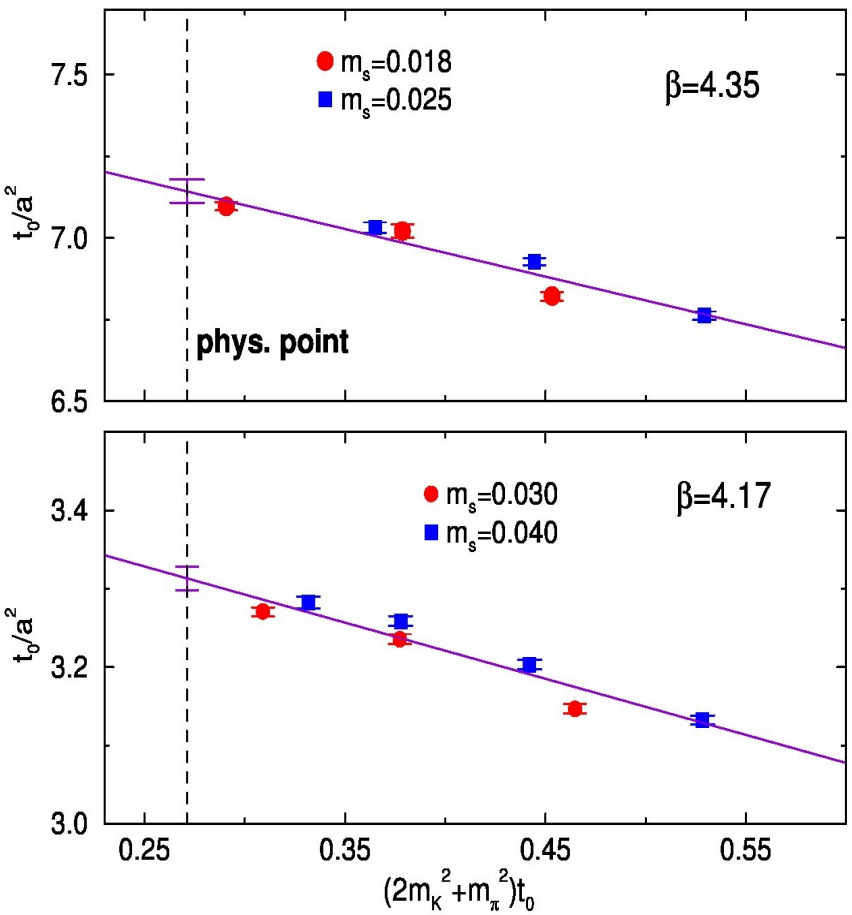



  - $t_0$: milder mass dependence, small statistical error → scale determination by $t_0$

# Lattice scale

- mass dependence of $t_0/a^2$




$\propto 2m_{ud} + m_s$

- chiral expansion to NLO

$$(t_0/a^2) = (t_{0,\mathrm{ch}}/a^2)\left[1 + \frac{k_1}{(4\pi f)^2}(2m_K^2 + m_\pi^2) + \mathcal{O}(M^4)\right]$$

$$= (t_{0,\mathrm{ch}}/a^2)\left[1 + \frac{k_1}{(4\pi f)^2 t_{0,\mathrm{ch}}}(2m_K^2 + m_\pi^2)t_0 + \mathcal{O}(M^4)\right]$$

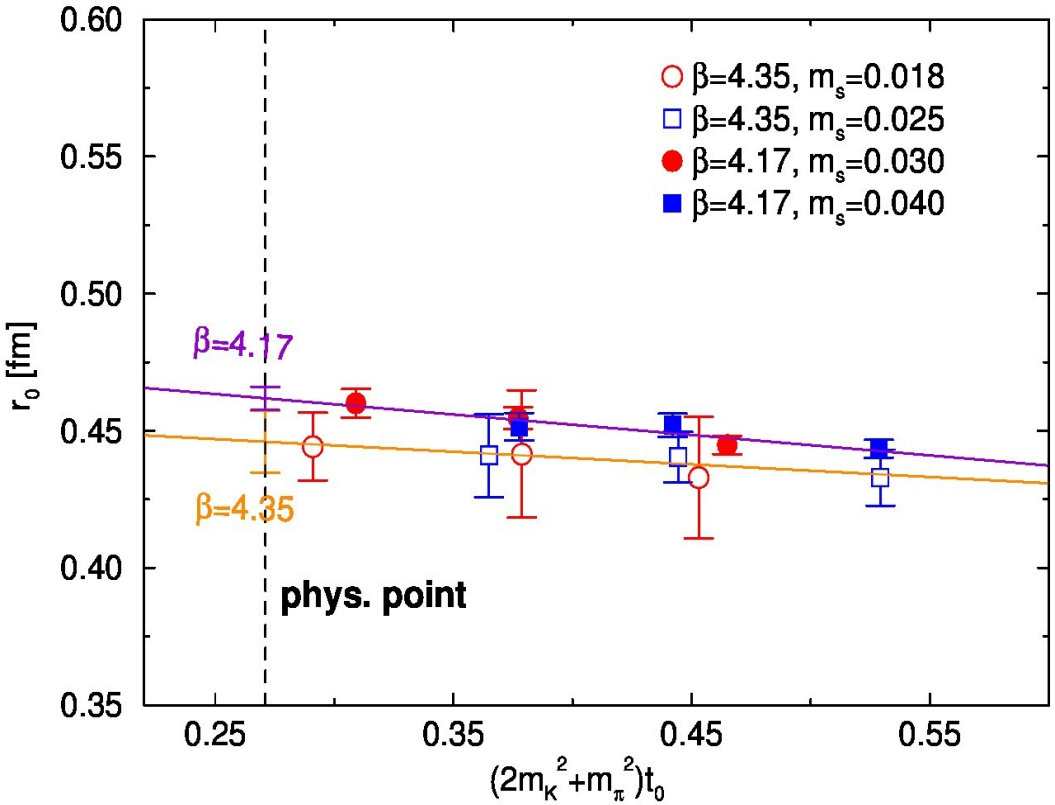
small higher-order effect Bär-Golterman, 2013

- β=4.17 lightest: L= 2.5 fm vs 3.8 fm
  - consistent $t_0 \to$ FVE not significant
- preliminary results

  (input: $m_\pi$= 135 MeV, $m_K$= 495 MeV)
  - β = 4.17: $a^{-1}$ = 2.492(6) GeV
  - β = 4.35: $a^{-1}$ = 3.660(9) GeV

# Consistency with HQ potential

- $r_0$ at the physical point




β = 4.17: $r_0$ = 0.462(4) fm

β = 4.35 (very preliminary):

$r_0$ = 0.45(1) fm

cf. $r_0$ = 0.466(4) fm HPQCD

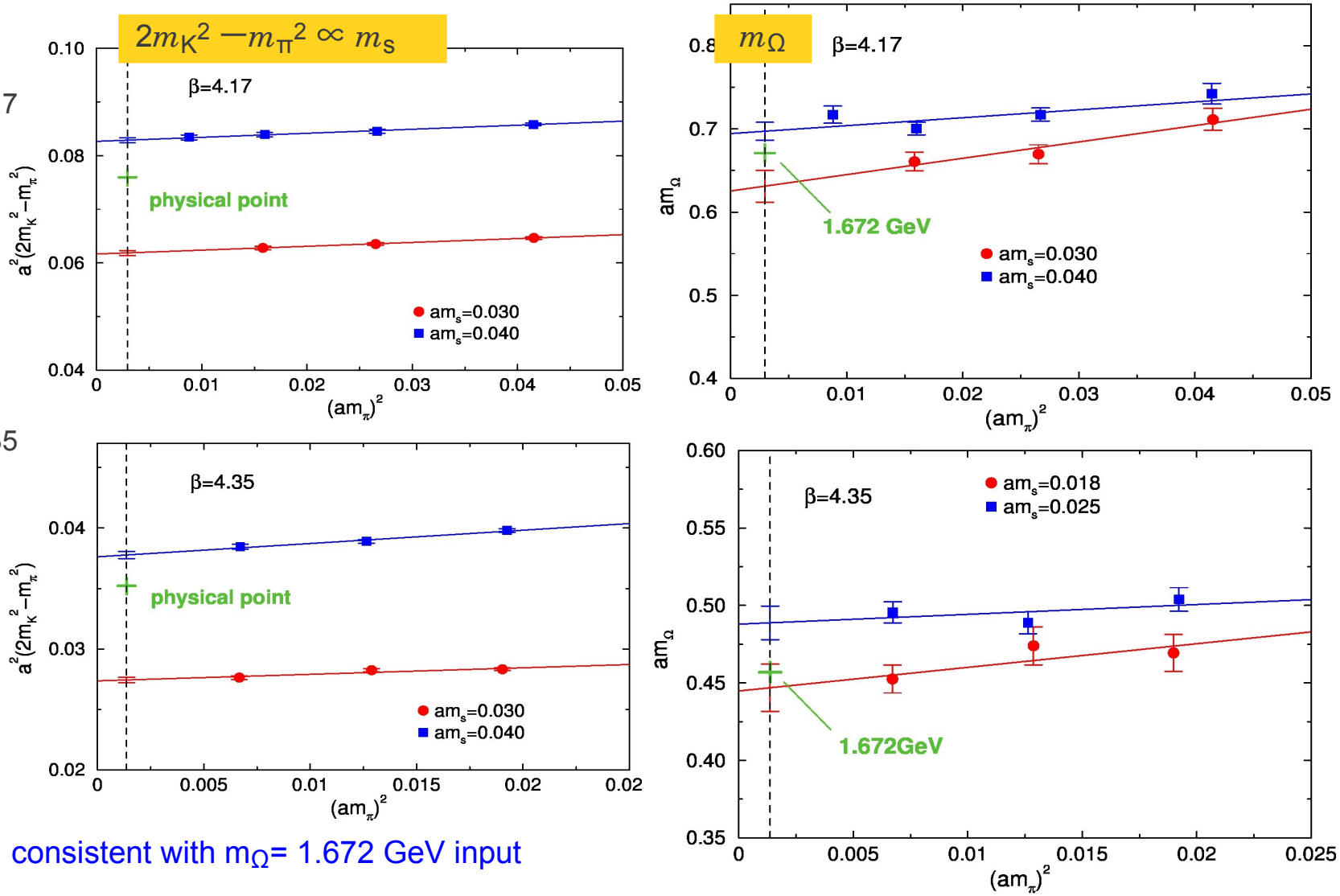
$r_0$ = 0.492(10) fm PACS-CS

# Consistency with Omega mass

- $m_\Omega$ at the physical point

▶ β=4.17

▶ β=4.35



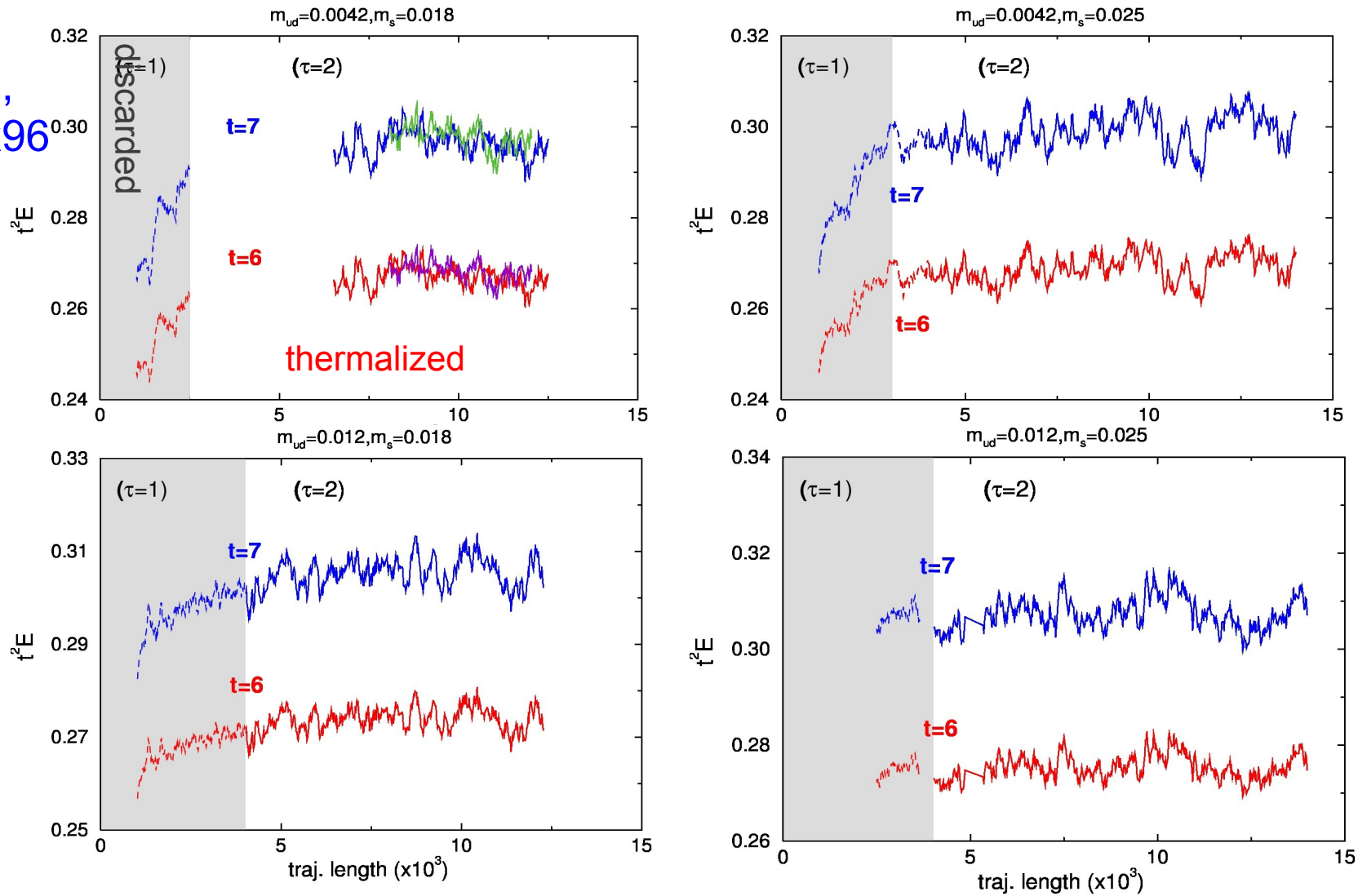

consistent with $m_\Omega$= 1.672 GeV input

# Studies of generated configurations

# Thermalization through Wilson flow

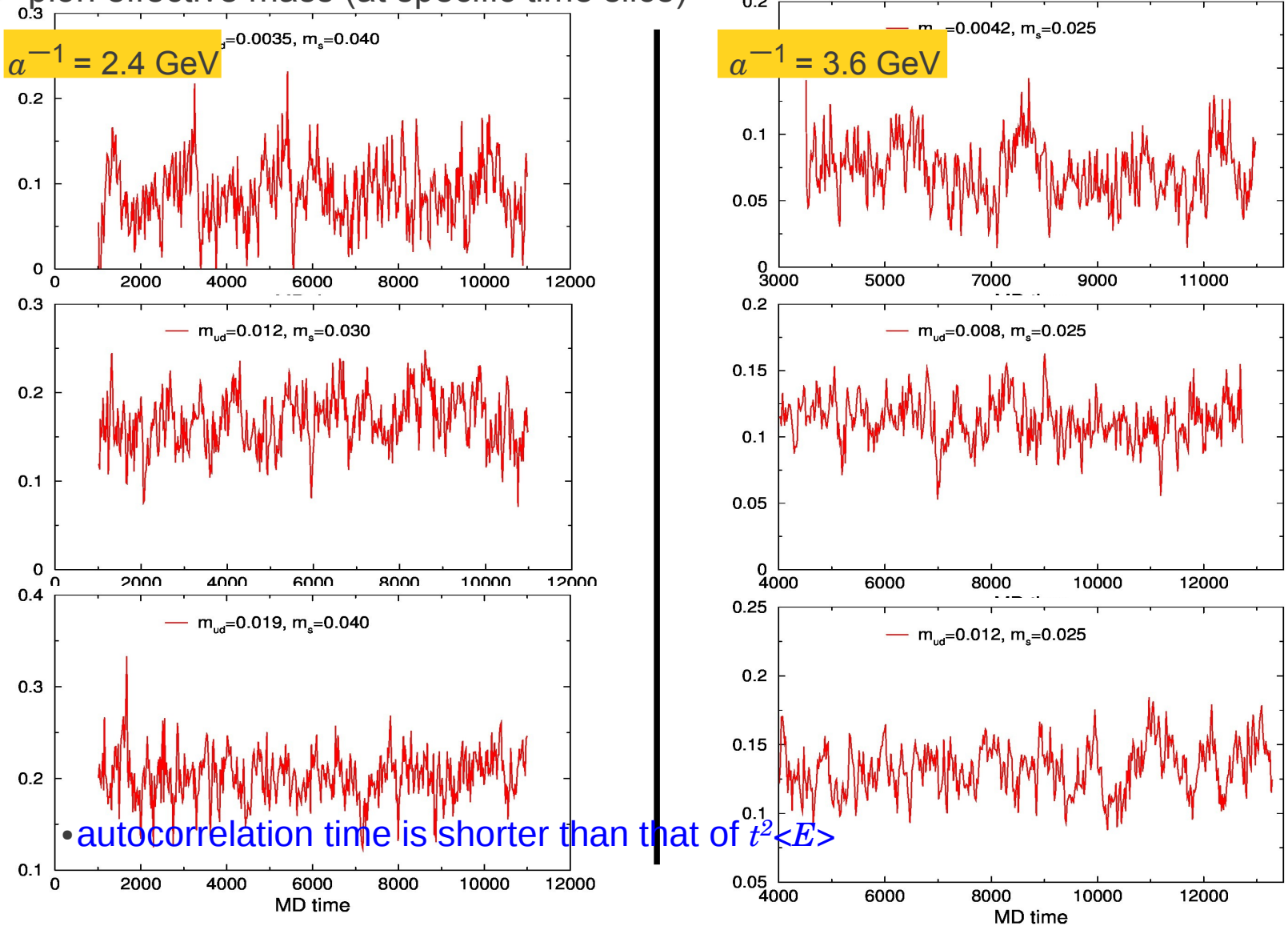
- history of $t^2\langle E\rangle$ ( around $t_0$) JLQCD, 2013

β= 4.35,
$48^3$x96



# Autocorrelation of mesons

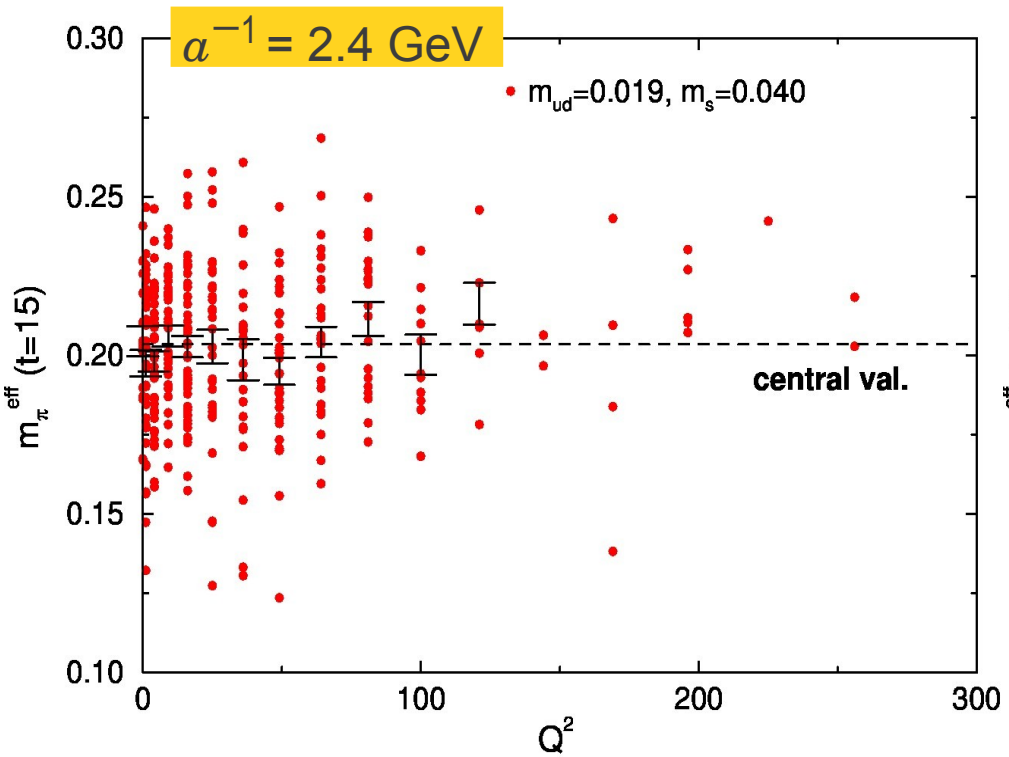
- pion effective mass (at specific time slice)

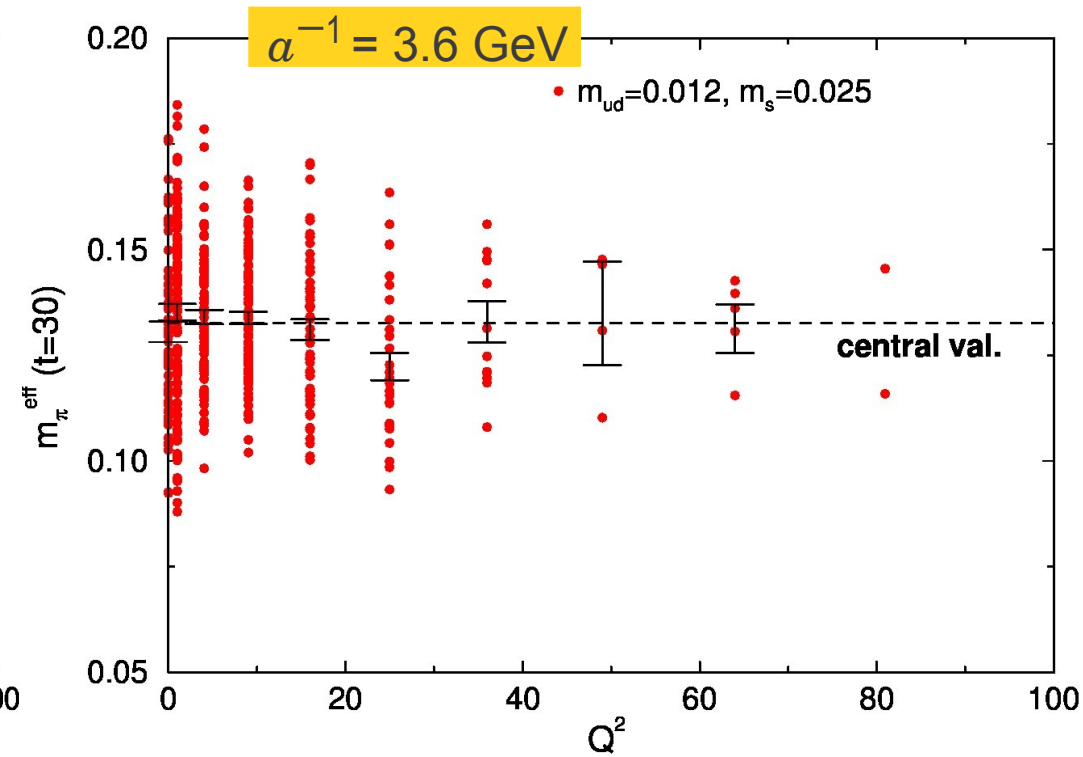



- autocorrelation time is shorter than that of $t^2\langle E\rangle$

# Effect of topology

- Observables are correlated with topology?
  - pion effective mass vs $Q^2$ distribution (at the heaviest sea quarks)




no significant correlation observed

# Conclusions

- $N_f$ = 2+1 simulation with Möbius Domain-Wall fermions
  - precise control of systematics with chiral symm. / discretization / finite volume
  - 10,000 MD-times generated at $a^{-1}$ = 2.4, 3.6 GeV
  - scale setting by YM gradient flow
    - study of $r_0$ at the physical point
    - consistent with $m_\Omega$ = 1.672 GeV input
  - baseline study of generated configurations
    - total check of autocorrelations by several observables
    - correlation of physical results and topology
- We are ready to physics calculations
  - application to chiral dynamics,

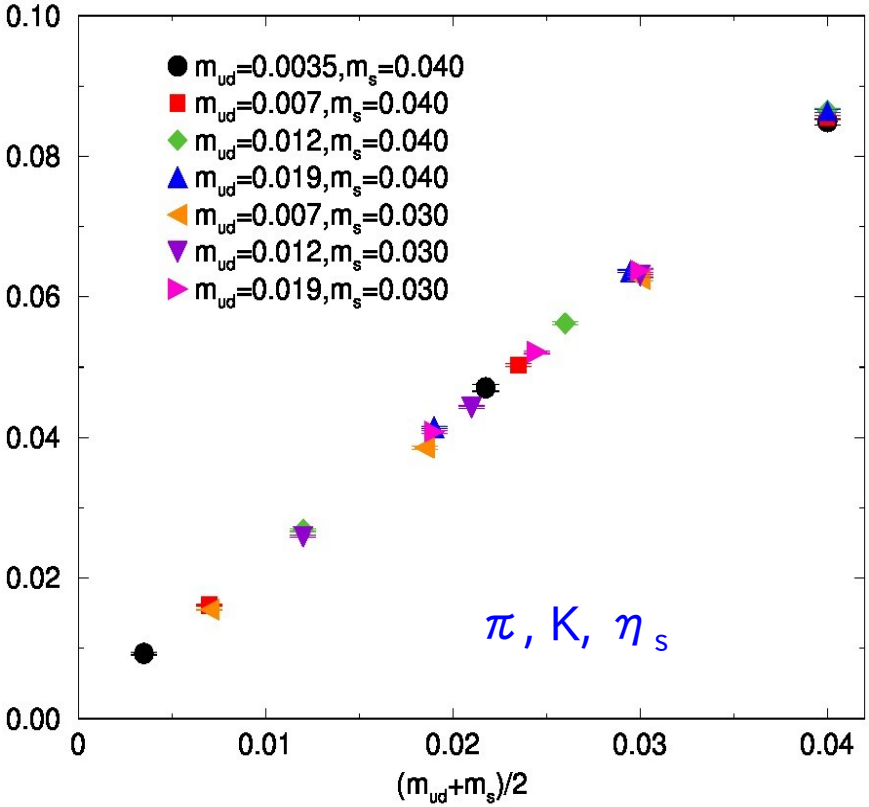
    charm & bottom quark physics, etc.

# Backup

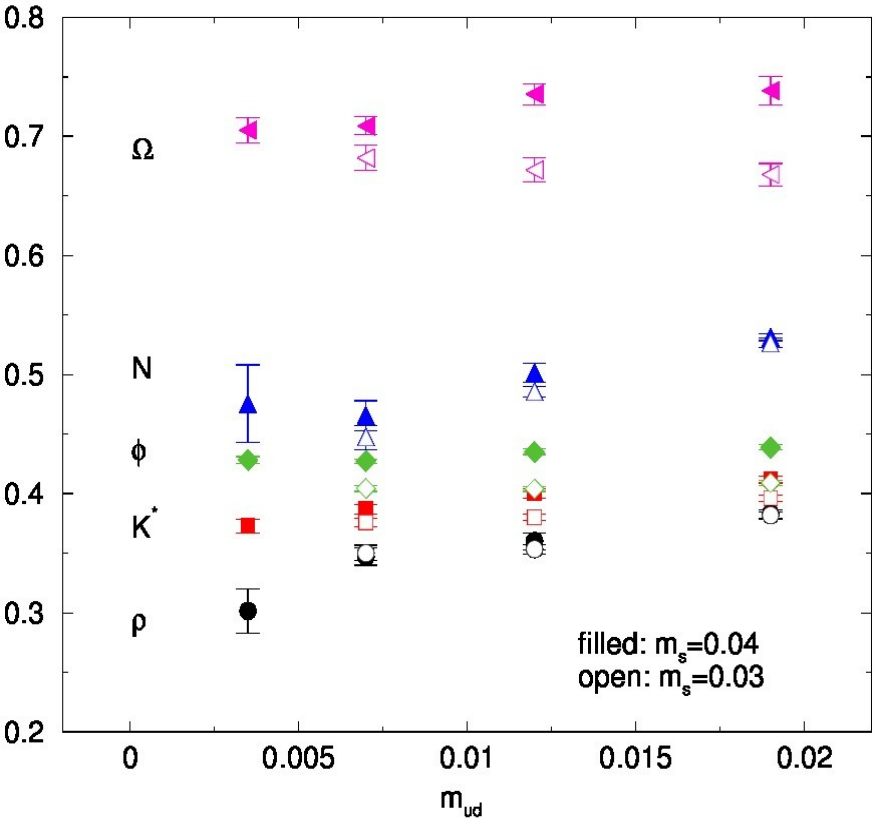
# First look at the mass spectrum

$\beta = 4.17$

- PCAC relation in pseudo-scalar




- other hadrons




further improve the signal: inclease the trajectories
and/or low-eigenmodes-averaging