Fine lattice simulations with the Ginsparg-Wilson fermions

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Flavor physics with GW fermions

- Heavy quark physics for NP search
  - need experimental / theoretical studies at high-precision
  - LQCD: matrix elements with comparable precision

- $N_f = 2+1$ simulation with better control of systematic errors
  - chiral symmetry to avoid unwanted effect of its violation i.e. operator mixing etc.  
    - Domain-Wall fermions (Möbius kernel), $m_{res} \leq m_{ud} \times 0.1$
  - continuum extrapolation ($a^{-1} = 2.4, 3.6, 4.8$ GeV)
  - light quarks ($m_{\pi} = 500, 400, 300$ MeV and lighter)
  - lattice volume satisfying $m_{\pi}L > 4$
  - generated configs may be useful for other quantities, especially for those requiring good chiral symmetry.
Overview

- Design of our numerical simulation

- HMC: $a^{-1} = 2.4$ GeV has been finished. 90% done for 3.6 GeV.

- $a^{-1} = 4.8$ GeV is running
Plan of this talk

- Numerical simulation
  - HMC
  - basic measurements
- Scale setting with gradient flow
- Baseline studies of generated configs
  - thermalization / autocorrelation
  - correlation with topology
- Summary & outlook
Related Presentations

- Y. Cho (in collab. with Southampton), heavy quark physics
- G. Cossu, finite temperature
- H. Fukaya, topology issues
- M. Tomii, renormalization of local operators
- A. Tomiya, Dirac spectrum at finite temperature
Numerical Simulation
Domain-Wall (Möbius) fermions

5D representation

\[ D_{DW}^{(5)}(m) = 1 + b(4 + M)D_W - (1 - c(4 + M)D_W) \]

\[ D_W: \text{Wilson Dirac op with mass } -M \]

4D effective operator

\[ D_{DW}^{(4)}(m) = [P^{-1}D_{DW}^{(5)}(m = 1)^{-1}D_{DW}^{(5)}(m)P]_{11} \]

\[ = \frac{1 + m}{2} + \frac{1 - m}{2} \gamma_5 \tanh(L_s \tanh^{-1} H_M) \]

5D projection:

\[ P = \begin{bmatrix} P_- & P_+ \\ P_- & P_+ \\ \vdots & \vdots \\ P_+ & P_- \end{bmatrix} \]

\[ L_s \to \infty \]

sign function approx.

scaled Shamir kernel:

\[ H_M = \gamma_5 \frac{bD_W}{2 + cD_W} \]

we set \( b = 2, c = 1 \)

stout link smearing

- smaller residual mass, faster inversion

JLQCD 2013
HMC

- Symanzik gauge + Möbius Domain-Wall ($N_f = 2+1$)
  - tree-level Symanzik action
  - 3-level stout smearing
- standard RHMC with Omelyan integrator
  - performance @BG/Q: $16 \rightarrow 30$ GFlops/node (HMC), $45$ GFlops/node (meas)
  - thanks to P. Boyle!

Hitachi SR16k M1, 57TFlops peak
IBM BG/Q, 1.2PFlops peak
Chiral symmetry

- residual mass Kaneko et al (JLQCD), Lattice 2013

- chiral symm. satisfied with good accuracy
  - $a^{-1} = 2.4$ GeV:
    \[ L_S = 12 \rightarrow m_{\text{res}} \sim m_{\text{ud}} \times 0.1 \]
  - $a^{-1} = 3.6$ GeV:
    \[ L_S = 8 \rightarrow m_{\text{res}} \sim m_{\text{ud}} \times 0.02 \]
Gauge ensembles: status

- $\beta = 4.17, \ a^{-1} \sim 2.4 \text{ GeV}$

32$^3 \times 64 \times 12$

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<th>$m_{ud}$</th>
<th>$m_\pi$ [MeV]</th>
<th>MD time</th>
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<tr>
<td>0.019</td>
<td>510</td>
<td>10,000</td>
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$m_s=0.040, \ 48^3 \times 96$

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- $\beta = 4.35, \ a^{-1} \sim 3.6 \text{ GeV}$

48$^3 \times 96 \times 8$

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<tr>
<td>0.0120</td>
<td>510</td>
<td>10,000</td>
</tr>
</tbody>
</table>

* 1 traj. = 2 MD time
Basic measurements

- light hadron correlators (local-local, smeared-local, 1 or 2 source locations)

  - effective mass of pion
  - kaon
  - nucleon

\[ a^{-1} = 2.4 \text{ GeV} \]

\[ a^{-1} = 3.6 \text{ GeV} \]
Basic measurements (contd)

- **Yang-Mills gradient flow**  
  Lüscher, 2010
  
  \[ V_{x\mu}(0) = U_{x\mu}, \quad \left. \frac{dV_{x\mu}}{dt} \right|_t = -g^2_{\mu} S_g[V] V_{x\mu} \]

- monitoring topological charge
  \[ Q = \frac{1}{16\pi^2} \sum \text{tr} F_{\mu\nu} \tilde{F}_{\mu\nu} \]

\[ a^{-1} = 2.4 \text{ GeV}, m_{ud} = 0.0035, m_s = 0.040 \]
\[ a^{-1} = 3.6 \text{ GeV}, m_{ud} = 0.0042, m_s = 0.018 \]

- long autocorrelation, finer lattice tends to freeze. \( \rightarrow \) no problem, talk by Fukaya

- lattice scale by the energy density
  \[ E(t) \equiv \frac{1}{4} F^{\alpha}_{\mu\nu} F_{\mu\nu}^\alpha |_t \]
Scale setting
Reference quantities

- $t_0$, $w_0$: $t^2\langle E \rangle \bigg|_{t=t_0} = 0.3$, $t \frac{d}{dt}[t^2\langle E \rangle] \bigg|_{t=w_0^2} = 0.3$

**mass dependence (vs $t_0 m_{\pi}^2$)**

BMW 2012: $t_0^{1/2} = 0.1465$ fm, $w_0 = 0.1755$ fm

consistent with known behavior

$t_0$: milder mass dependence, small statistical error $\rightarrow$ scale determination by $t_0$
Lattice scale

- mass dependence of $t_0/a^2$

- chiral expansion to NLO

$$(t_0/a^2) = (t_{0,\text{ch}}/a^2) \left[ 1 + \frac{k_1}{(4\pi f)^2} (2m_K^2 + m_{\pi}^2) + O(M^4) \right]$$

$$= (t_{0,\text{ch}}/a^2) \left[ 1 + \frac{k_1}{(4\pi f)^2 t_{0,\text{ch}}} (2m_K^2 + m_{\pi}^2) t_0 + O(M^4) \right]$$

small higher-order effect Bär-Golterman, 2013

- $\beta = 4.17$ lightest: $L = 2.5$ fm vs $3.8$ fm
  - consistent $t_0 \rightarrow$ FVE not significant

- preliminary results
  - (input: $m_\pi = 135$ MeV, $m_K = 495$ MeV)
    - $\beta = 4.17$: $a^{-1} = 2.492(6)$ GeV
    - $\beta = 4.35$: $a^{-1} = 3.660(9)$ GeV

$\propto 2m_{ud} + m_s$
Consistency with HQ potential

- \( r_0 \) at the physical point

\[ \beta = 4.17: \quad r_0 = 0.462(4) \text{ fm} \]

\[ \beta = 4.35 \text{ (very preliminary)}: \quad r_0 = 0.45(1) \text{ fm} \]

\[ \text{cf. } r_0 = 0.466(4) \text{ fm \ HPQCD} \]

\[ r_0 = 0.492(10) \text{ fm \ PACS-CS} \]
Consistency with Omega mass

$m_\Omega$ at the physical point

$2m_K^2 - m_\pi^2 \propto m_S$

$\beta = 4.17$

$\beta = 4.35$

consistent with $m_\Omega = 1.672$ GeV input
Studies of generated configurations
Thermalization through Wilson flow

- history of $t^2 <E>$ (around $t_0$)  JLQCD, 2013

$\beta = 4.35$, $48^3 \times 96$

Autocorrelation of mesons

- Pion effective mass (at specific time slice)

\[ a^{-1} = 2.4 \text{ GeV} \]

\[ a^{-1} = 3.6 \text{ GeV} \]

- Autocorrelation time is shorter than that of \( t^2 \langle E \rangle \)
Effect of topology

- Observables are correlated with topology?
  - pion effective mass vs $Q^2$ distribution (at the heaviest sea quarks)

\[ \alpha^{-1} = 2.4 \text{ GeV} \]
\[ \alpha^{-1} = 3.6 \text{ GeV} \]

no significant correlation observed
Conclusions

- $N_f = 2+1$ simulation with Möbius Domain-Wall fermions
  - precise control of systematics with chiral symm. / discretization / finite volume
  - 10,000 MD-times generated at $a^{-1} = 2.4, 3.6$ GeV
  - scale setting by YM gradient flow
    - study of $r_0$ at the physical point
    - consistent with $m_\Omega = 1.672$ GeV input
  - baseline study of generated configurations
    - total check of autocorrelations by several observables
    - correlation of physical results and topology
- We are ready to physics calculations
  - application to chiral dynamics,
    - charm & bottom quark physics, etc.
Backup
First look at the mass spectrum

$\beta = 4.17$

- PCAC relation in pseudo-scalar
- Other hadrons

Further improve the signal: increase the trajectories and/or low-eigenmodes-averaging