Systematics analyses on nucleon isovector observables in 2+1-flavor dynamical domain-wall lattice QCD near physical mass

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RBC and UKQCD collaborations have been generating dynamical Domain-Wall Fermions (DWF) ensembles:

- good chiral and flavor symmetries,

that allowed us do a lot of good pion and kaon physics as well as nucleon.

We are now running at physical pion mass: Sergey Syritsyn's talk.

This talk: puzzling and persistent deficit seen in the isovector axial charge, g_A, using

- light, $m_\pi \sim 171$ and 248 MeV, quarks ($m_{ud}a = 0.001$ and 0.0042, and $m_{\rm res}a \sim 0.002$),
- a large, $(4.6\text{fm})^3$, volume ($a^{-1} \sim 1.371(10)$ GeV),

made possible by Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action, and by Meifeng Lin, Yasumichi Aoki, Tom Blum, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Eigo Shintani, Takeshi Yamazaki, ...

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Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_5\gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

Related to mean-squared charge radii, anomalous magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

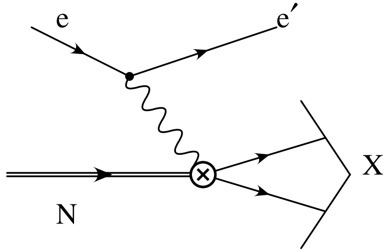
On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings



$$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + \ldots,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + \ldots$$

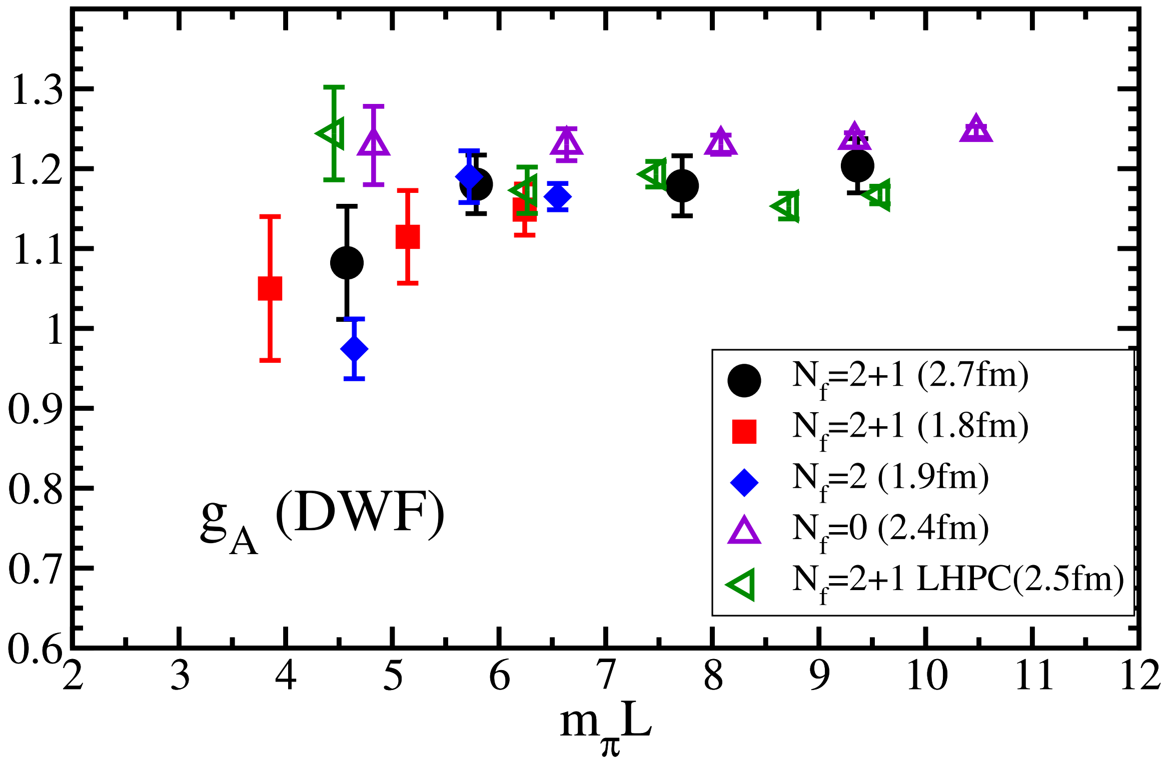
Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

We can optimize the source so that A_1 is small, and we take sufficiently large $t_{\rm sep}$.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

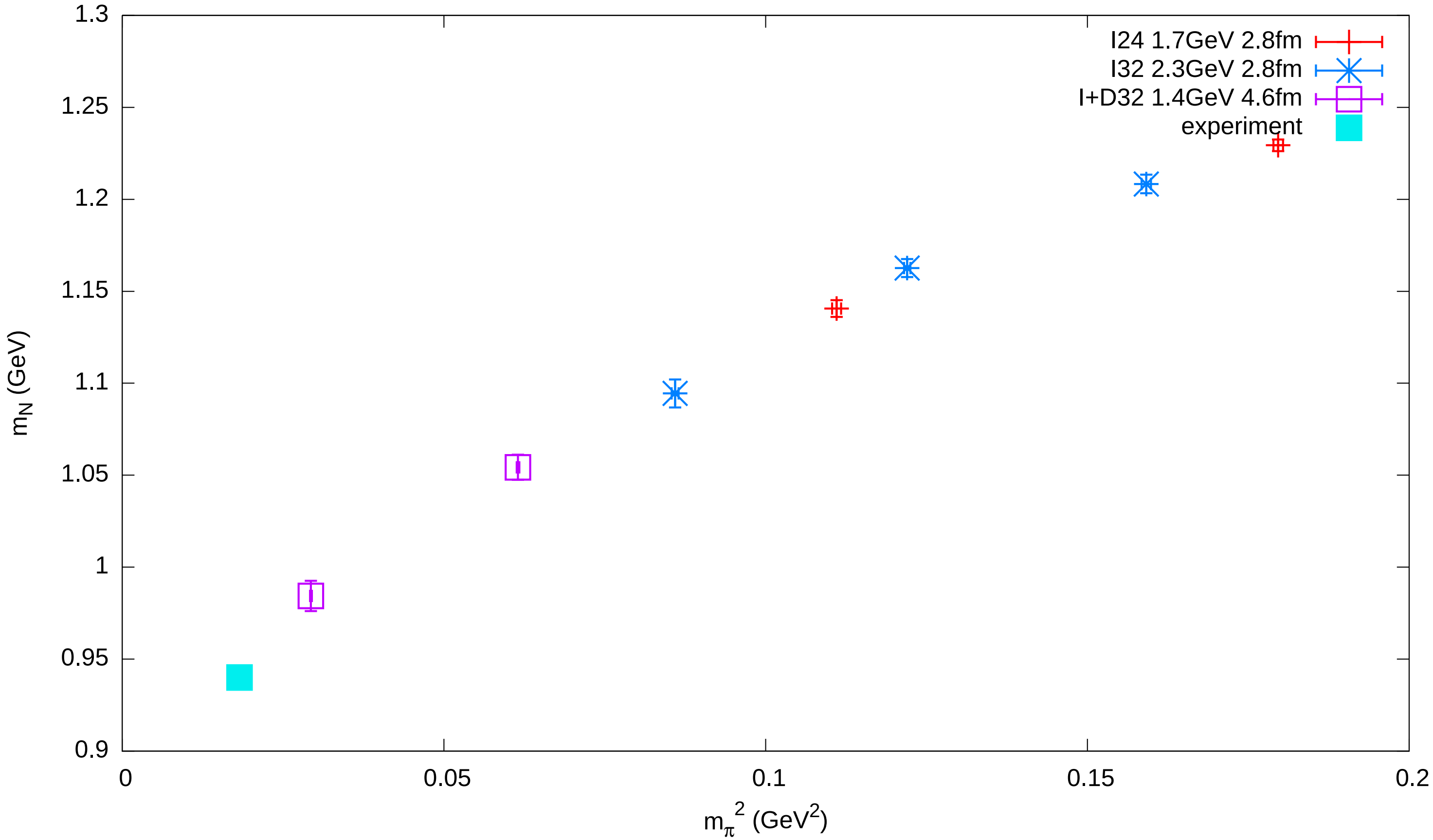
- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron β decay, decides neutron life.



- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

Many in the past pointed out this is a fragile quantity as pion mass is set light: Adkins+Nappi+Witten, Jaffe, Kojo+McLerran+Pisarski, ...

Nucleon mass: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles have been reanalyzed for nucleon physics.



with $a^{-1} = 1.371(10)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.

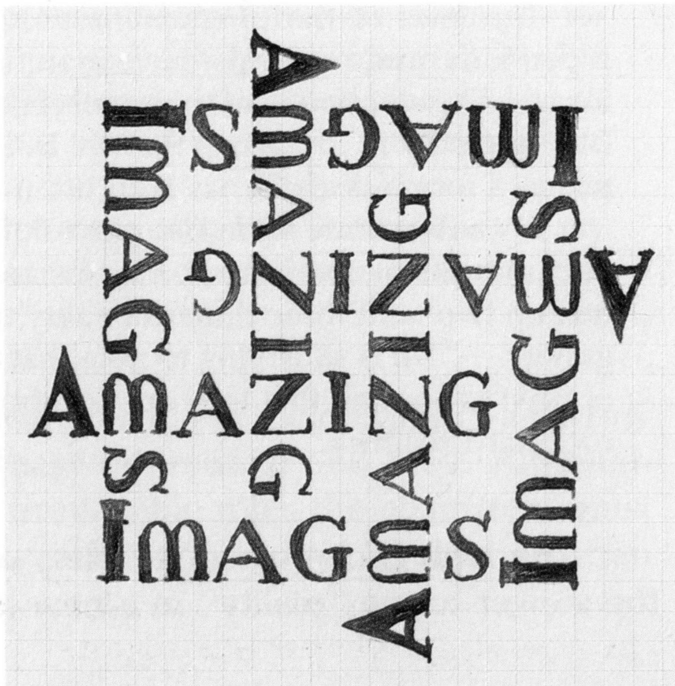
Closer to physical mass, $m_\pi = 170$ and 250 MeV, $m_N < 1.0$ GeV,

.

With the new statistical technique, "AMA" offer ×10–20 acceleration: by allowing

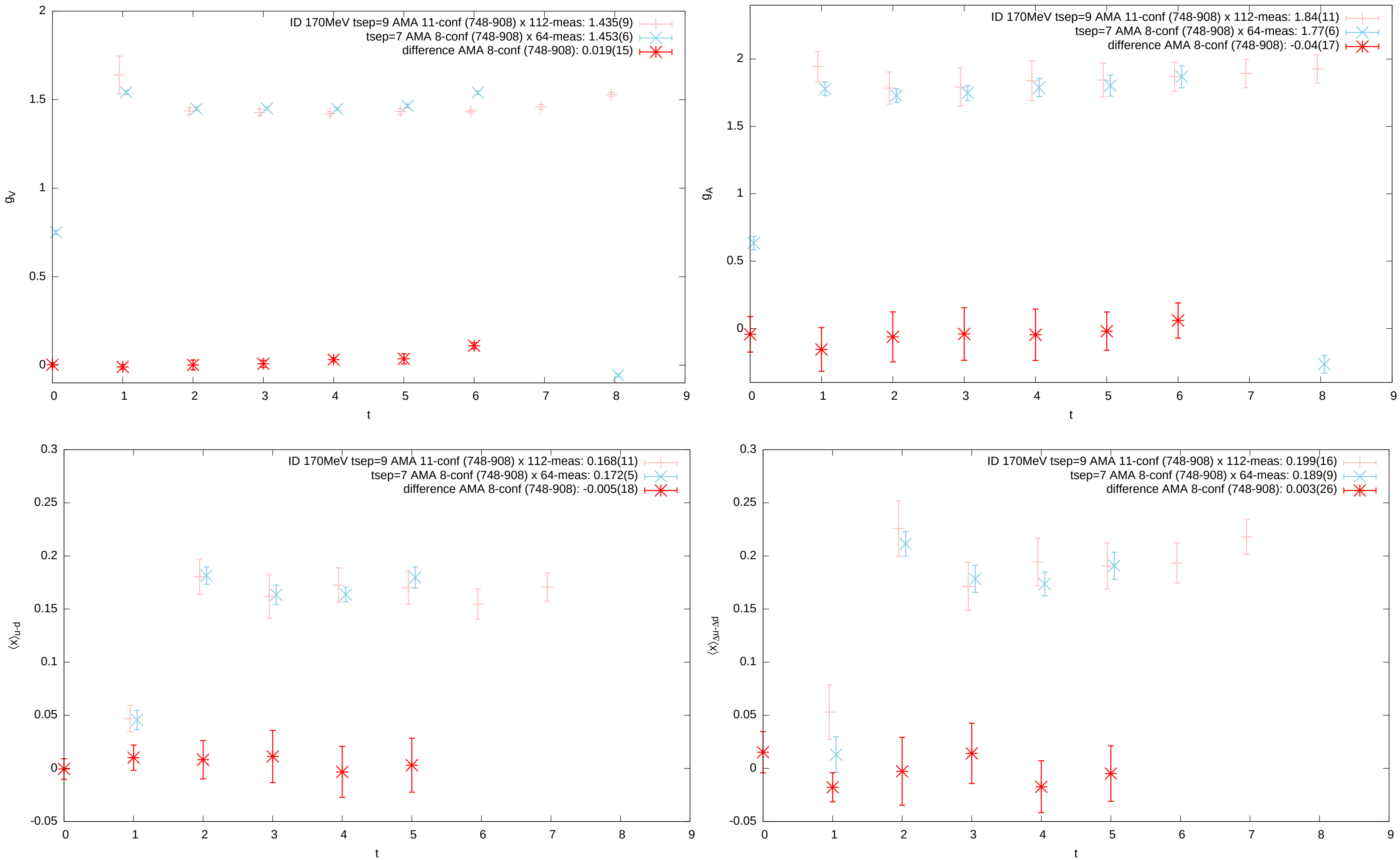
- cruder,
- but cheaper,

independent statistical sampling at much higher frequency, by taking advantage of point-group symmetries of the lattice to organize many such cruder but independent and equivalent measurements:



$$\langle O \rangle_{\mathrm{AMA}} = \frac{1}{N_{\mathrm{sloppy}}} \sum_{s}^{N_{\mathrm{sloppy}}} \langle O \rangle^{s}_{\mathrm{sloppy}} + \frac{1}{N_{\mathrm{accurate}}} \sum_{a}^{N_{\mathrm{accurate}}} \left(\langle O \rangle^{a}_{\mathrm{accurate}} - \langle O \rangle^{a}_{\mathrm{sloppy}} \right)$$

With the AMA we established no excited-state contamination is present in any of our 170-MeV calculations:



When compared with the same configurations, the difference is always consistent with 0.

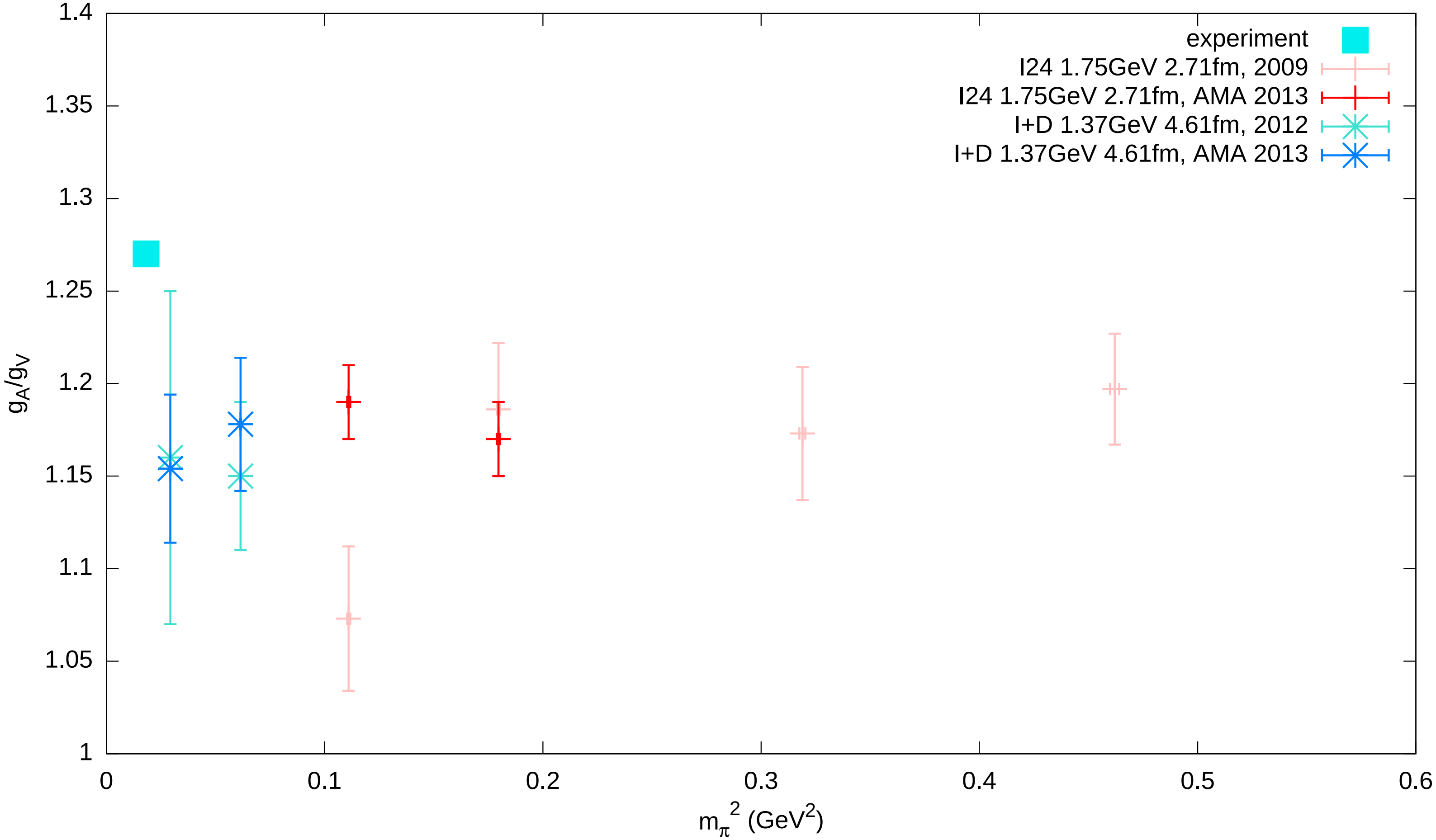
$A_1\langle 1|O|0\rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0\rangle$.

In agreement with many other groups' experiences in controlling this systematics.

This talk would not be possible without AMA:

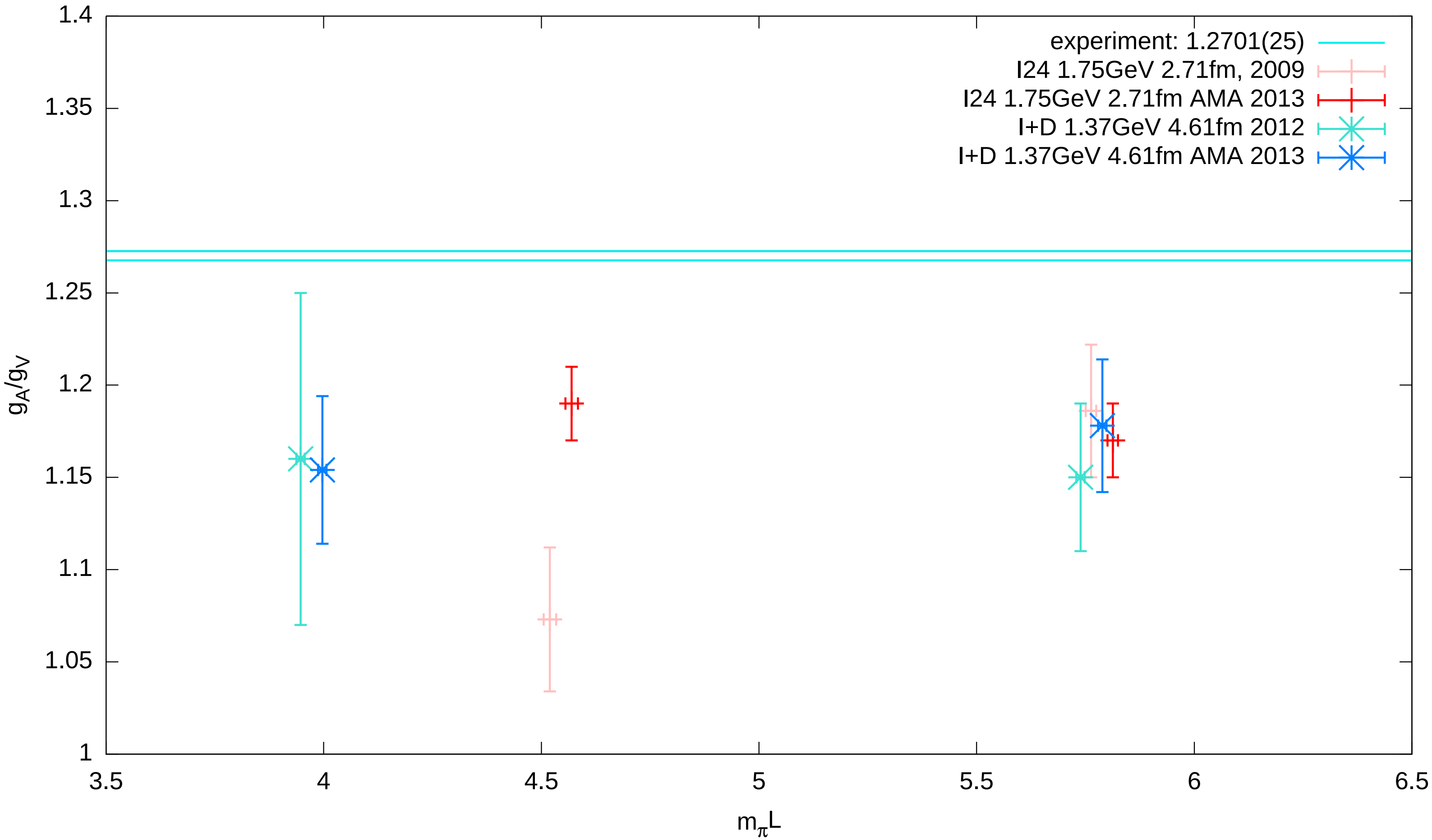
observable	fit range	non AMA	AMA
g_V	2-7	1.445(14)	1.449(8)
	3-6	1.439(14)	1.447(8)
g_A	2-7	1.8(2)	1.67(5)
	3-6	1.8(2)	1.66(6)
g_A/g_V	2-7	1.26(13)	1.15(4)
	3-6	1.28(15)	1.15(4)
$\langle x \rangle_{u-d}$	3-6	0.13(2)	0.146(7)
	4-5	0.11(3)	0.145(8)
$\langle x \rangle_{\Delta u-\Delta d}$	3-6	0.19(4)	0.165(9)
	4-5	0.20(5)	0.167(10)
$\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$	3-6	0.64(13)	0.86(5)
	4-5	0.5(2)	0.83(6)
$\langle 1 \rangle_{\delta u-\delta d}$	3-6	1.7(2)	1.42(4)
	4-5	1.7(2)	1.41(5)

With AMA and other statistical improvements, g_A/g_V vs m_π^2 then looked like the following:



Moves away from the experiment as m_π approaches the experimental value.

About 10-% deficit in g_A/g_V seems solid except perhaps for $O(a^2)$ error:



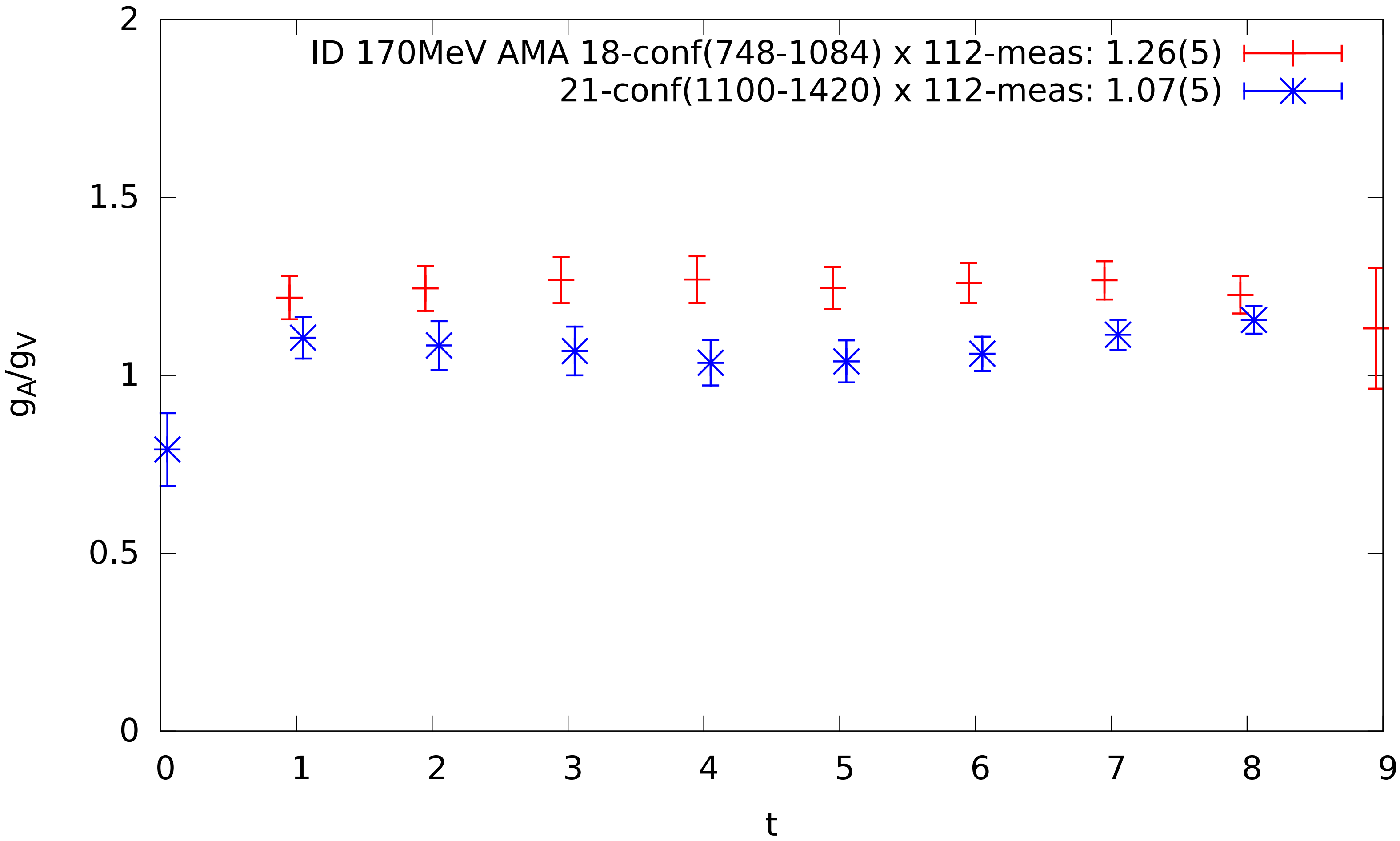
Excited-state contamination now is unlikely the cause.

Appears like monotonically decreasing with $m_\pi L$.

In agreement with the great majority of other groups.

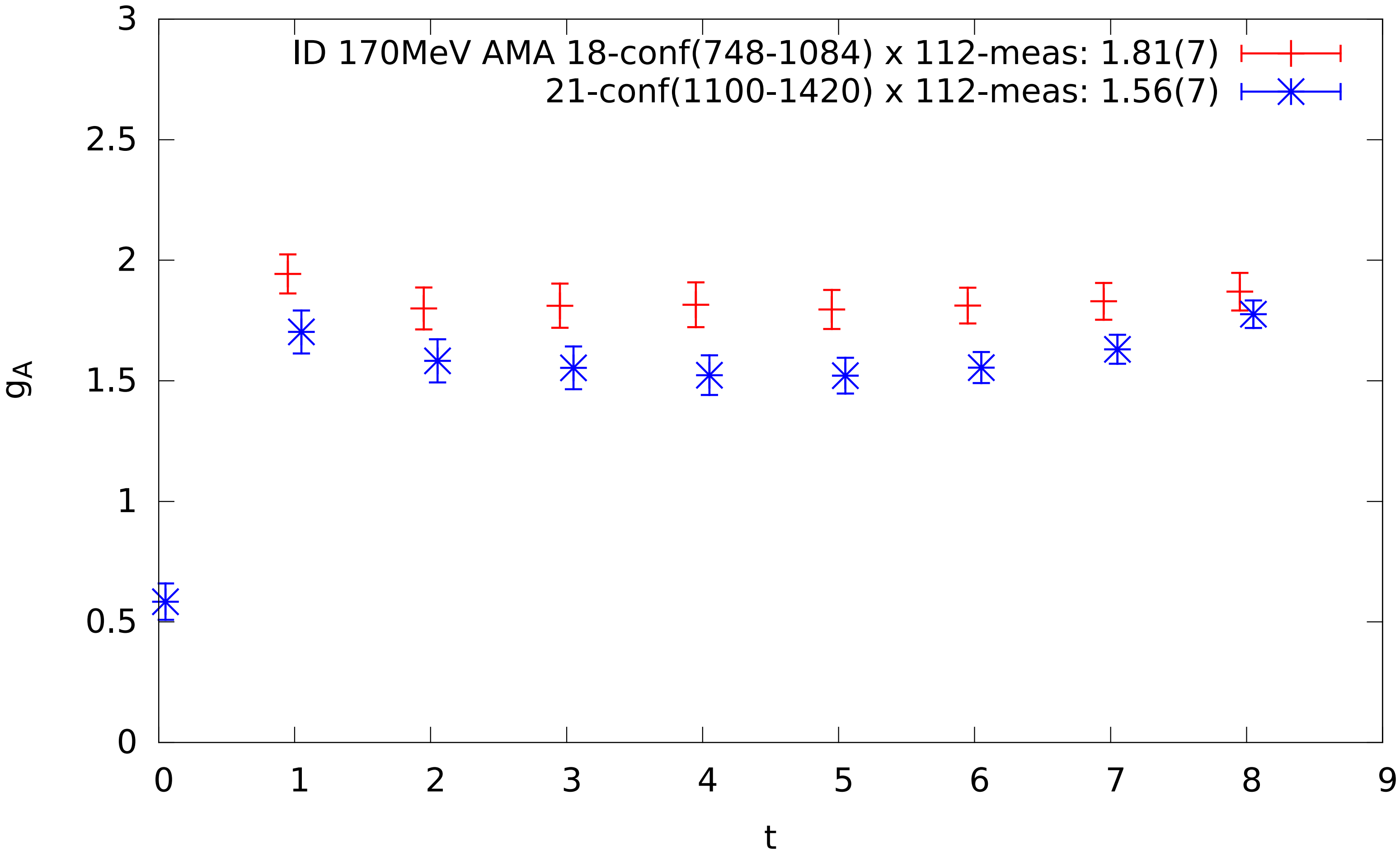
Why?

Long-range auto-correlation seen in g_A/g_V at $m_\pi = 170$ MeV:



Indicative of inefficient sampling.

Long-range auto-correlation seen in g_A at $m_\pi = 170$ MeV:



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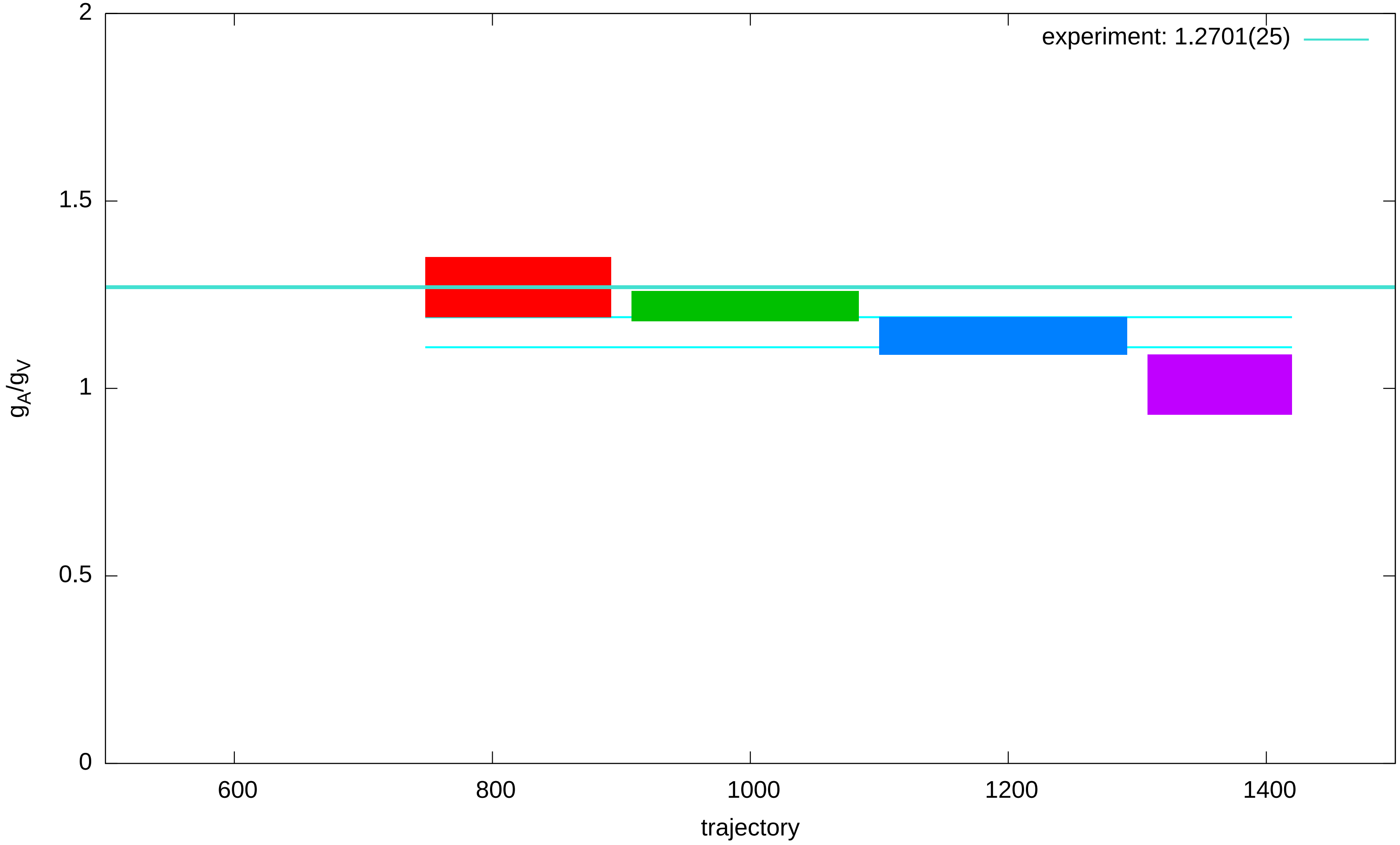
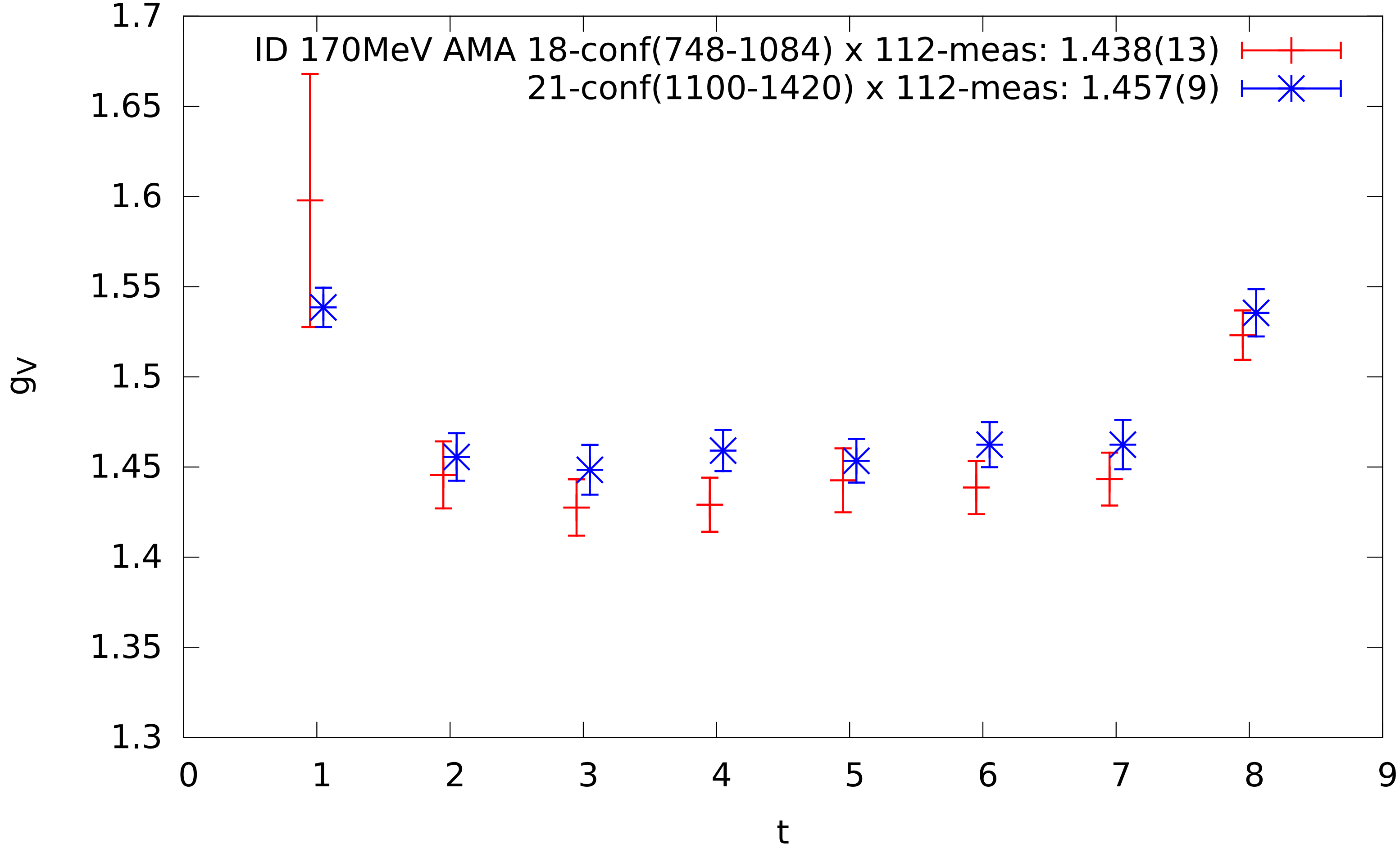


Indicative of inefficient sampling, but only in g_A and g_A/g_V.

But no such auto-correlation is seen in other observables, g_V, $\langle x \rangle_{u-d}$ or $\langle x \rangle_{\Delta u - \Delta d}$:



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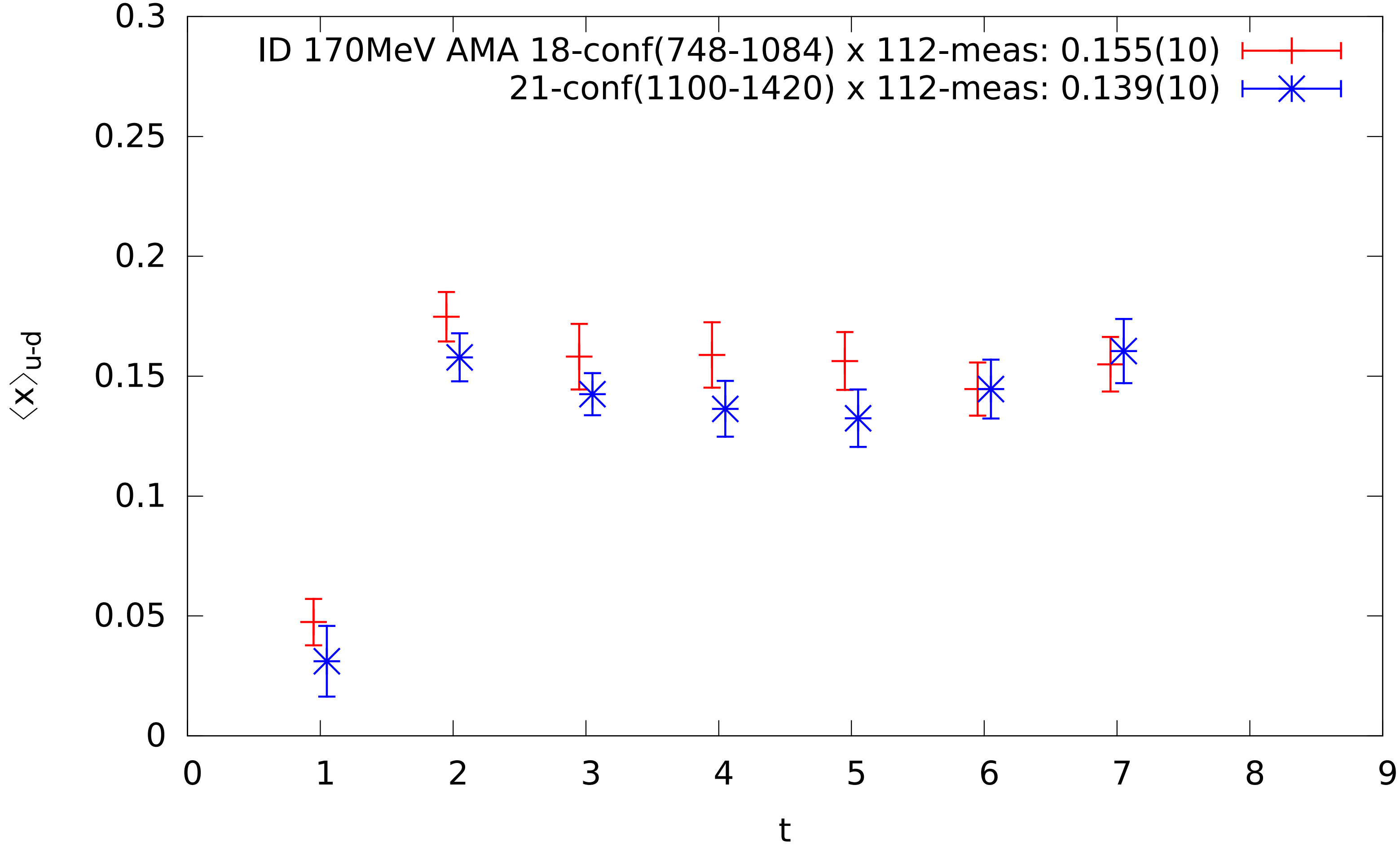

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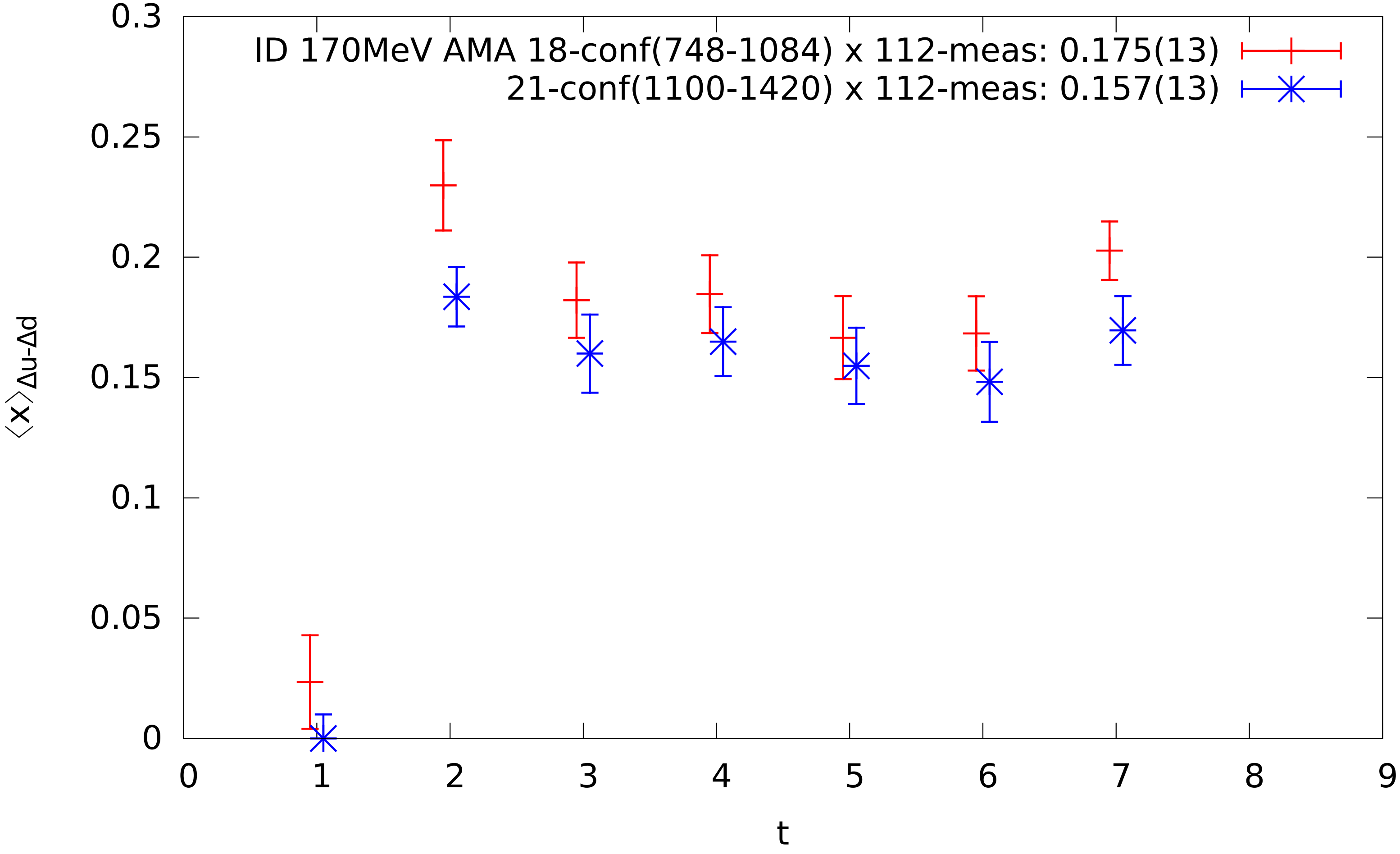

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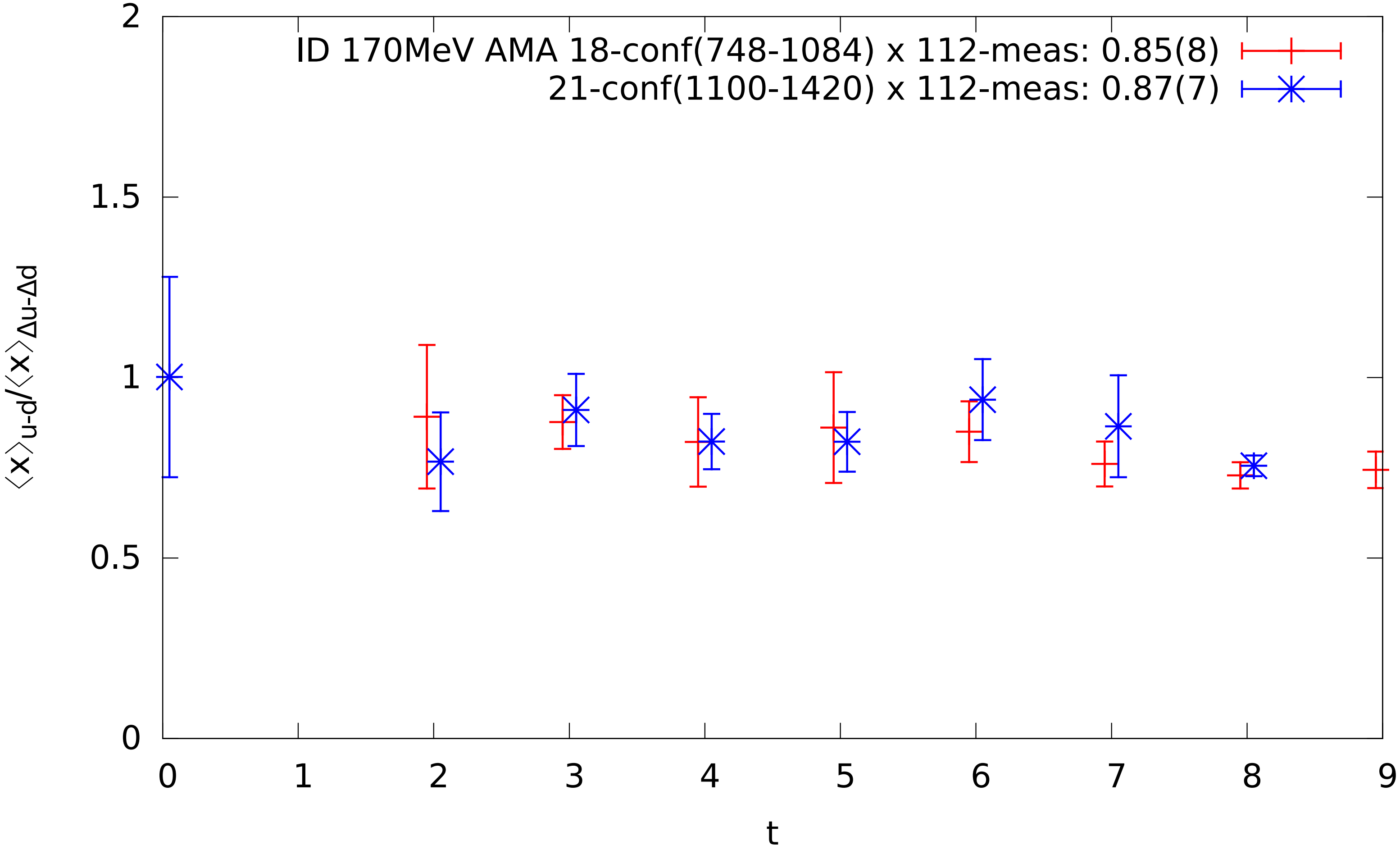
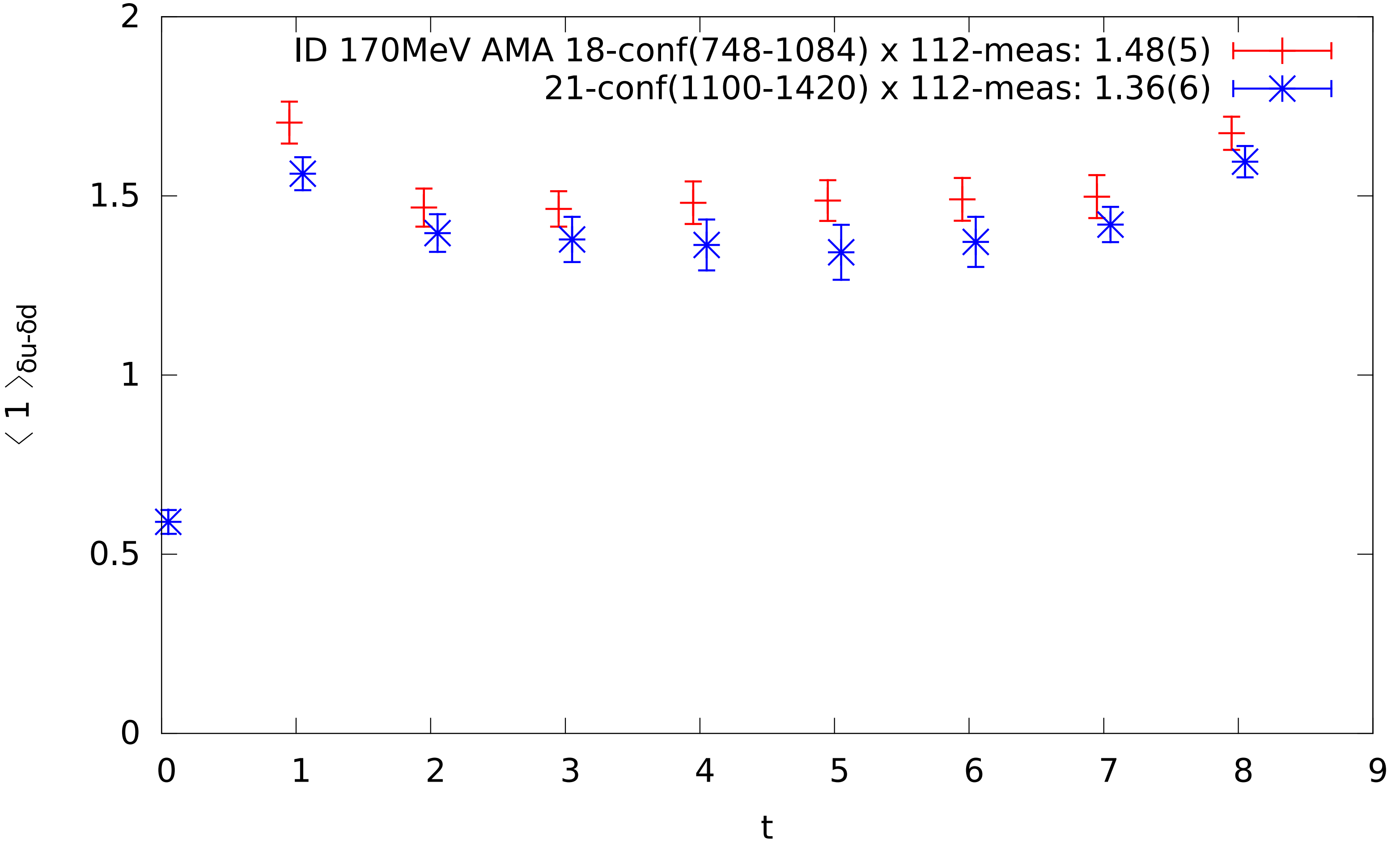


Isovector transversity, $\langle 1 \rangle_{\delta u - \delta d}$, may show similar trend, but much milder at most:

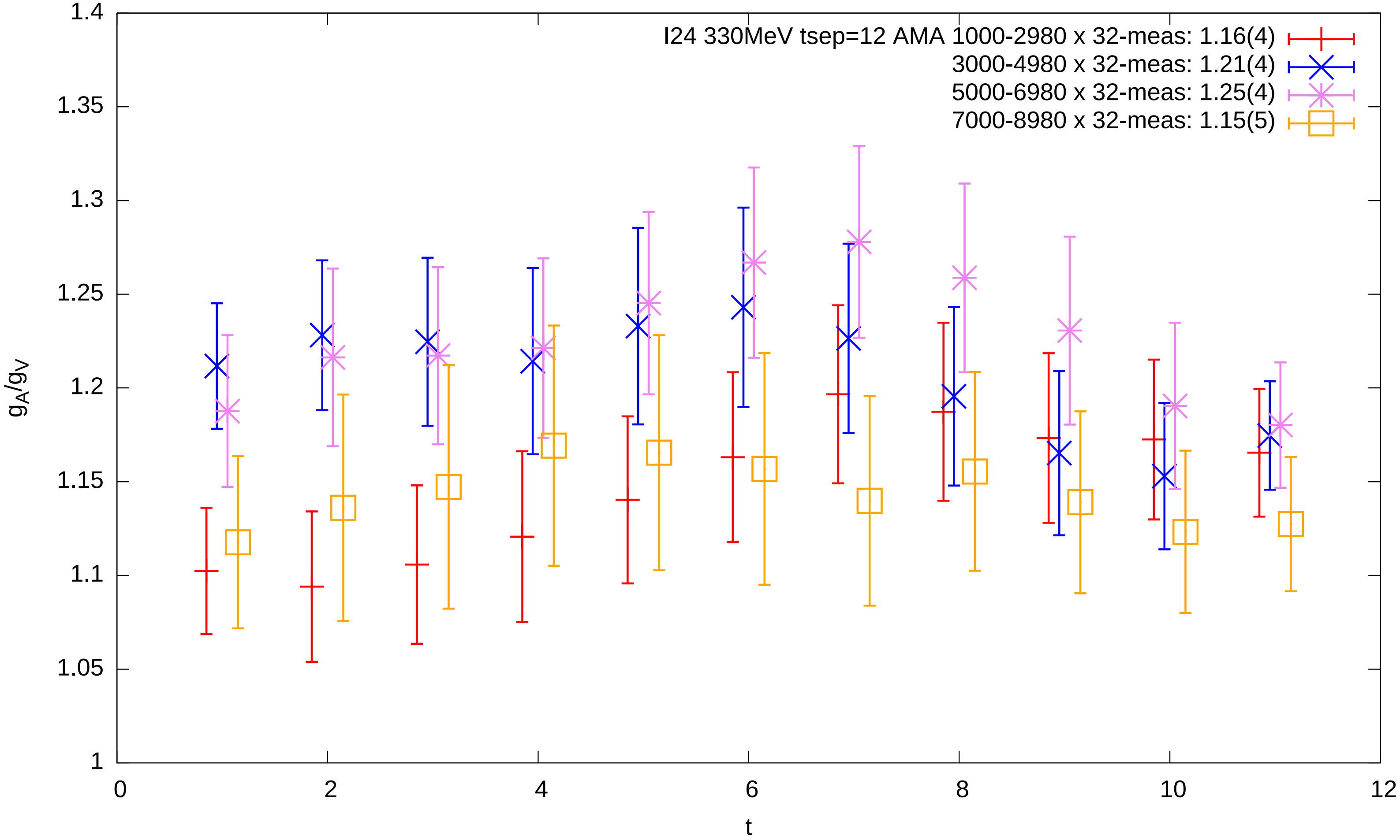


There appear long-range autocorrelations in axial charge but not in others:

Blocked jackknife analysis				
	bin size			
	1	2	3	4
g_V	1.447(8)	1.447(6)	-	-
g_A	1.66(6)	1.66(7)	1.71(8)	1.65(4)
g_A/g_V	1.15(4)	1.15(5)	1.15(6)	1.14(3)
$\langle x\rangle_{u-d}$	0.146(7)	0.146(8)	0.146(8)	-
$\langle x\rangle_{\Delta u-\Delta d}$	0.165(9)	0.165(11)	0.165(10)	-
$\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$	0.86(5)	0.86(4)	-	-
$\langle 1\rangle_{\delta u-\delta d}$	1.42(4)	1.42(6)	1.42(6)	1.41(3)

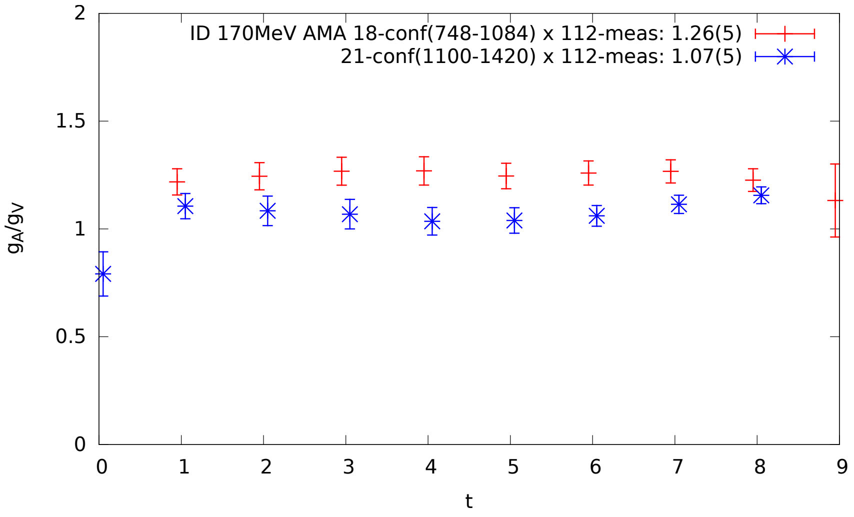
except in perhaps transversity.
But the difference may be hard to notice by standard blocked jackknife analysis.

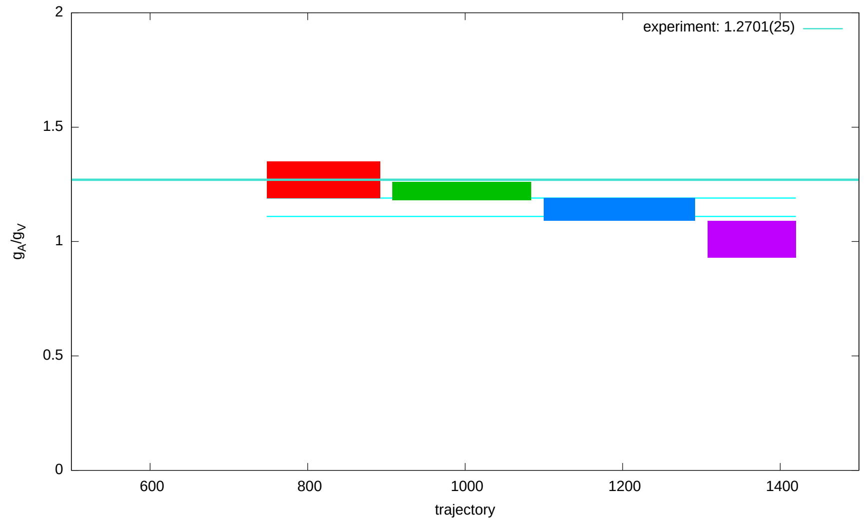
Long-range auto-correlation also seen in g_A/g_V also at $m_\pi = 330$ MeV:



but not at any larger $m_\pi L$.
Indicative of insufficient spatial volume.

Long-range auto-correlation seen in g_A/g_V:





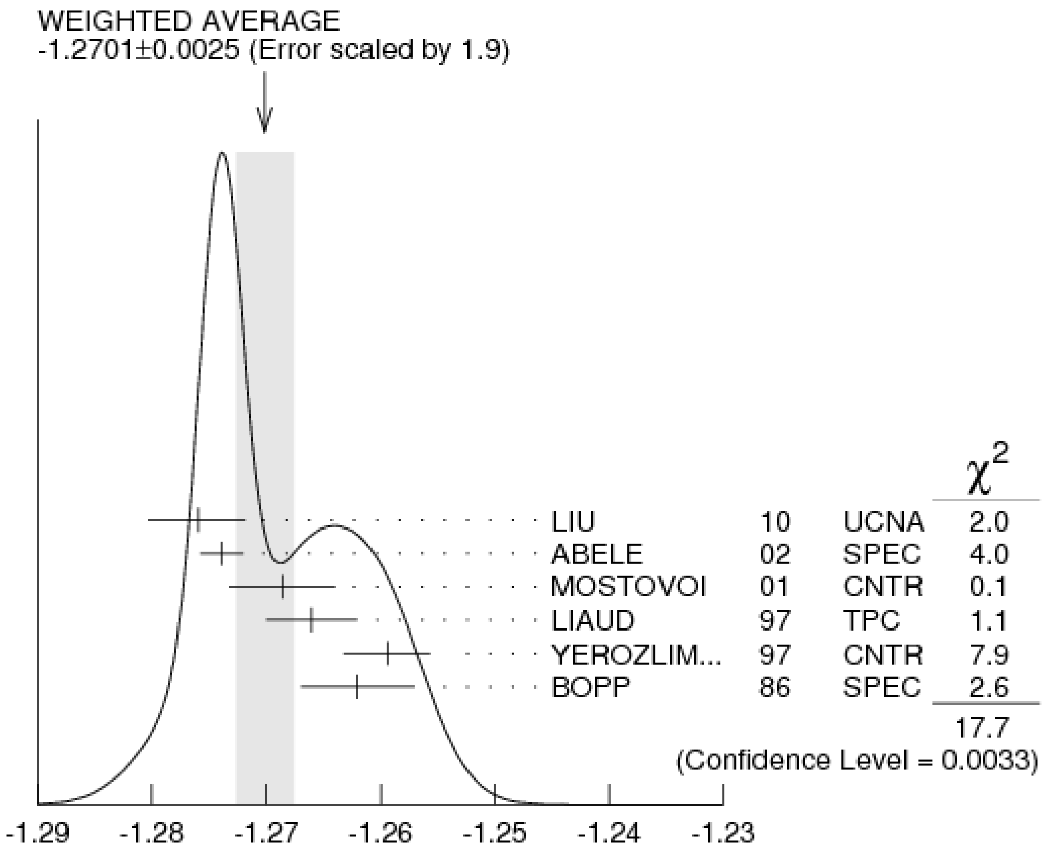
Non-AMA analyses are much noisier but not inconsistent with these:

Indicative of inefficient sampling, but only in g_A and g_A/g_V.

Why?

Why?

Difficult history:



Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path.

Why?

Difficult history:

Non-relativistic quark model: 5/3. Very bad, but some "large-N_c" conform?
And with absurd "relativistic" correction: 5/4, really?

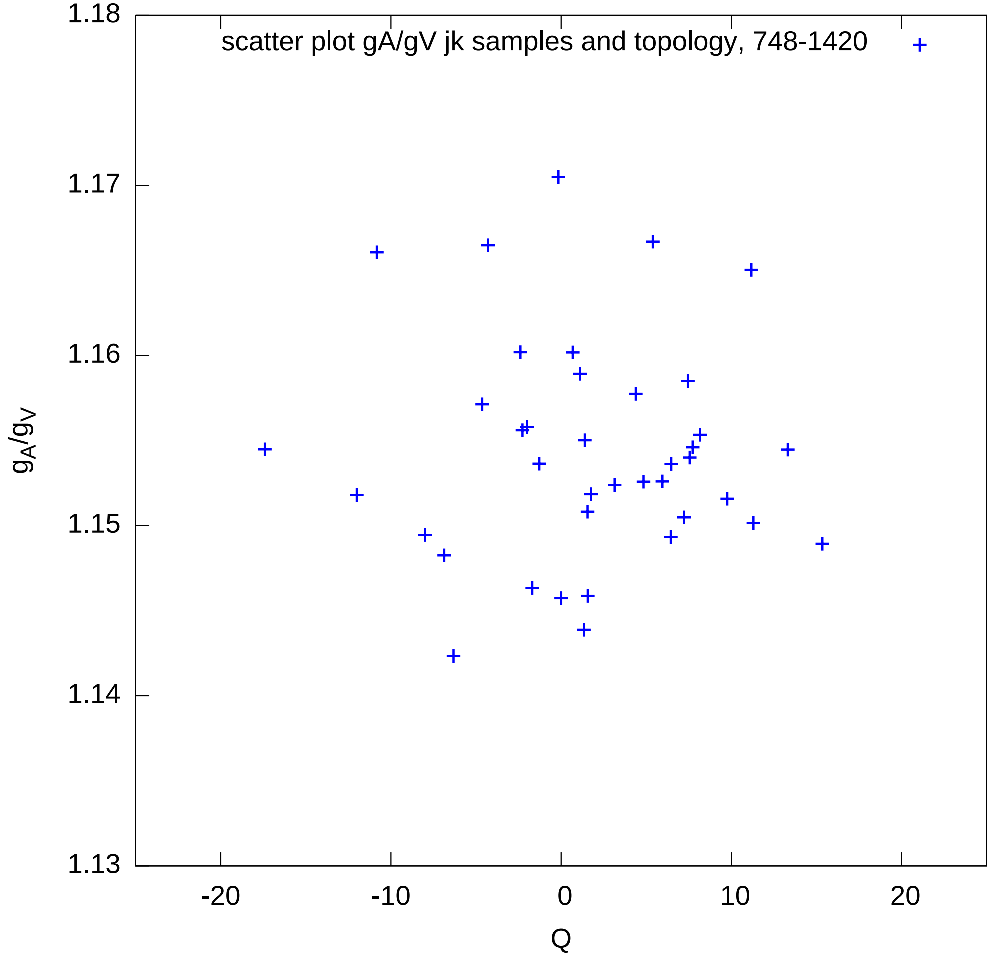
Without pion,
MIT bag model: 1.09, as good(!) as lattice but when experiment was 1.22.[1]

With only pion,
Skyrmion: 0.61(!) with a peculiar geometry but when experiment was 1.23.

Accurate reproduction of the 'pion cloud' geometry seems essential.

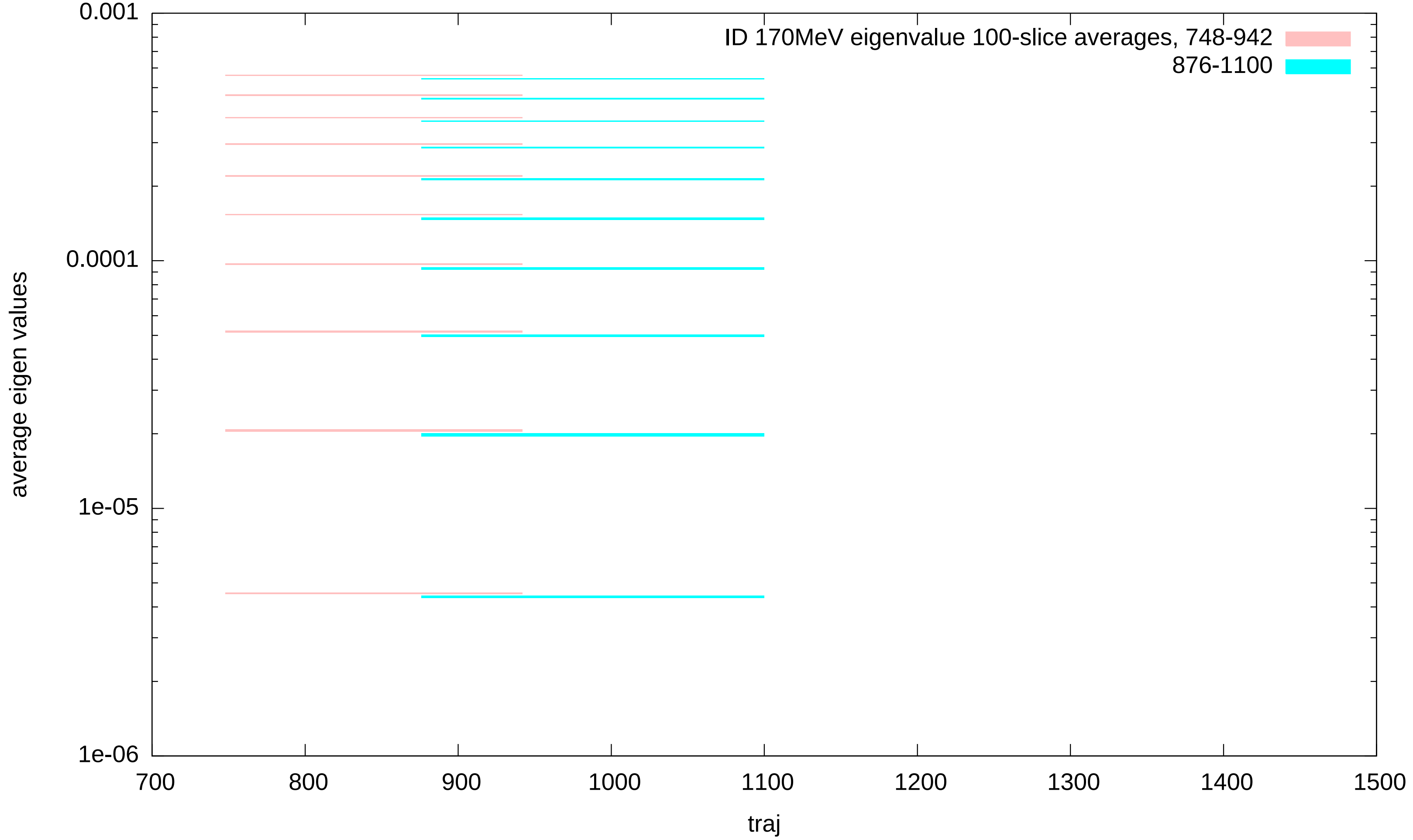
[1] Assuming a growth rate of 0.001 per year.

If the error stops growing after bin size of 3–4,
then the corresponding integrated autocorrelation time would be 20–30 MD time units,
similar to the known integrated autocorrelation time of topological charge.



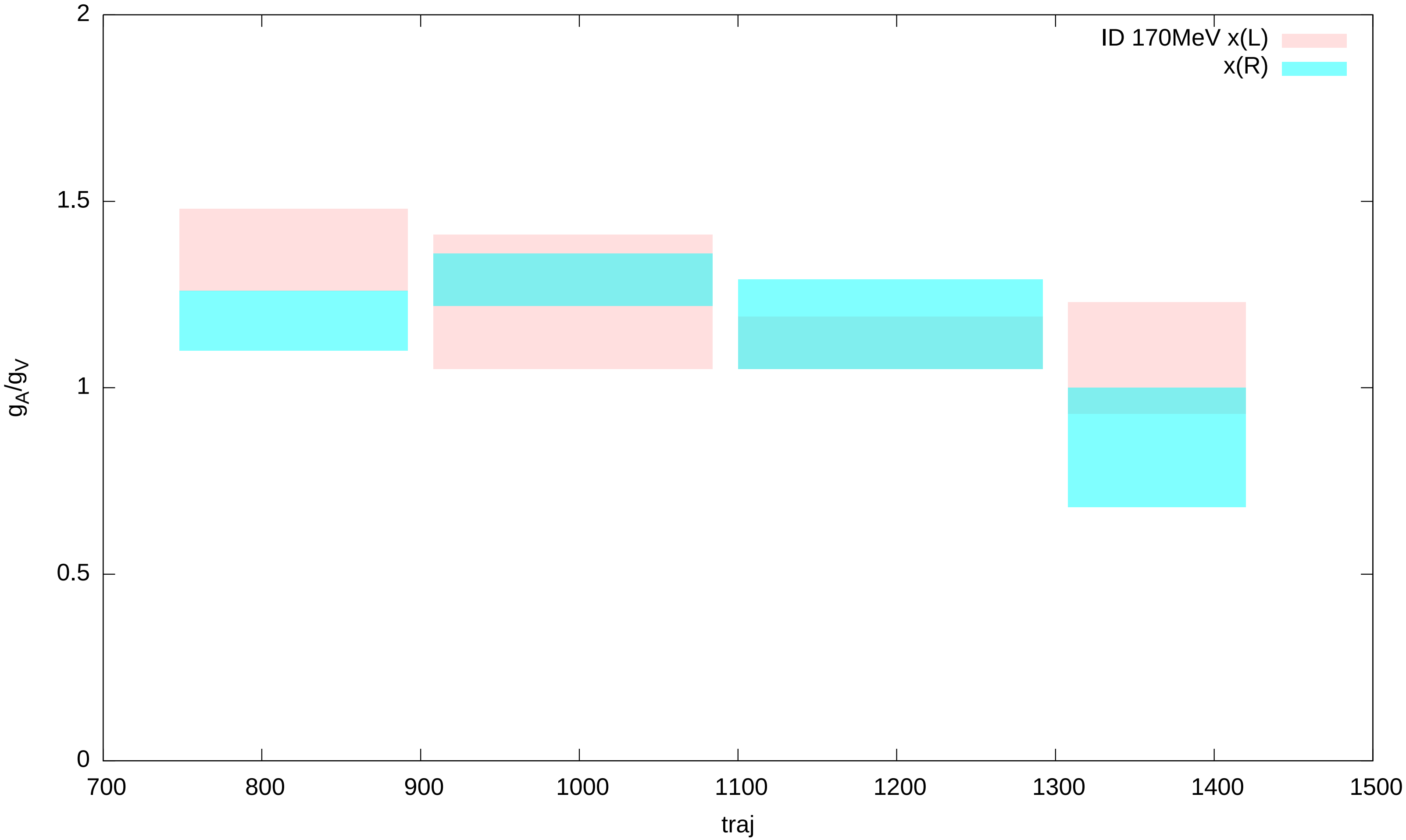
But no correlation is seen with topological charge.

Or low-mode deflation? Statstics is limited to 748–1100:

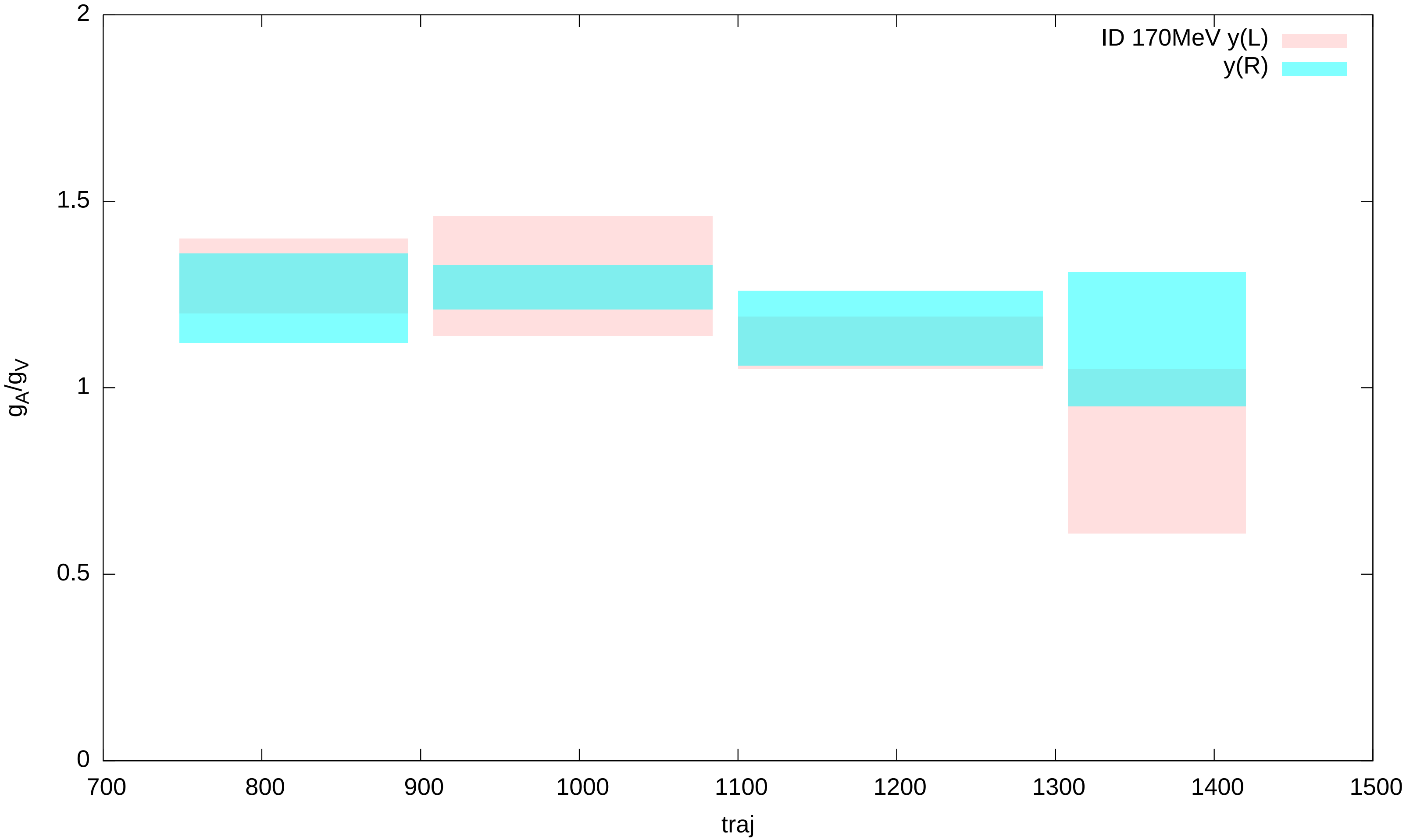


The lowest 100 modes are not different.
The higher the more different,
but not much different?

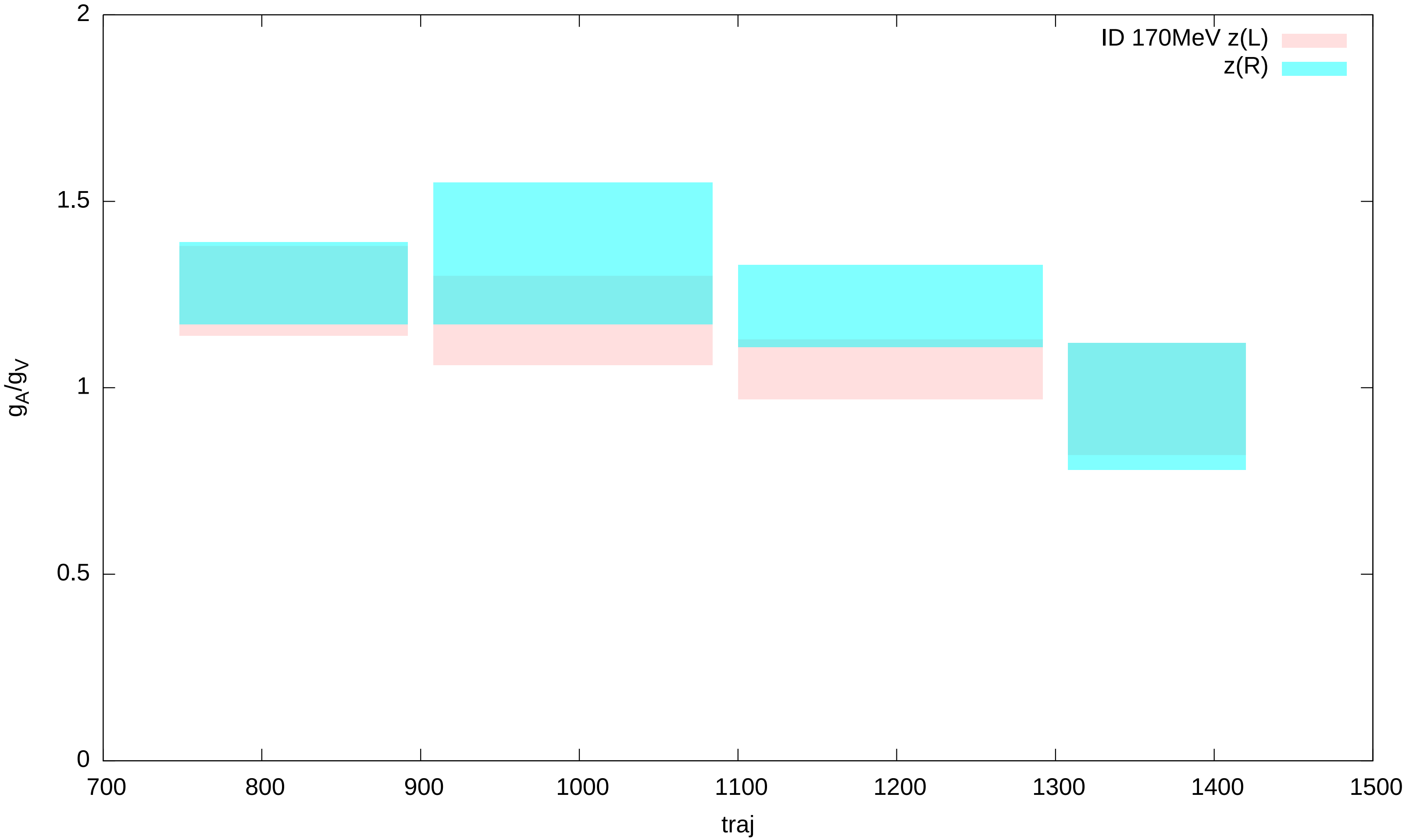
Uneven spatial distribution? Perhaps in x direction, ...



Uneven spatial distribution? Perhaps in y direction, ...



Uneven spatial distribution? Perhaps not in z, the spin, direction, ...



Summary

Systematics are explored in nucleon isovector observables using 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries up to $O(a^2)$, $m_{\text{res}}a \sim 0.002$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.98$ and 1.05 GeV,

jointly generated by RBC and UKQCD Collaborations.

Serious systematics in the axial charge, about 10-% deficit in g_A/g_V, with long-range autocorrelation,

- but does not appear correlated with topological charge;
- appears unevenly distributed in space some of the MD time;
- does not appear affected by low-mode deflation issues.

No such serious systematics is seen in other observables,

- except perhaps in transversity, where it is at most shorter-range and milder.

If indeed accurate reproduction of the 'pion cloud' geometry is essential,
larger-volume study with high statistics is desired.