The mass of the adjoint pion in $\mathcal{N}$=1 supersymmetric Yang-Mills theory

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The Model

$\mathcal{N}=1$ SUSY Yang-Mills Theory

Vector supermultiplet:

- Gauge field $A^a_\mu(x)$, $a=1,\dots,N_c^2-1$, "Gluon"
  Gauge group SU(N_c)
- Majorana-spinor field $\lambda^a(x)$, $\overline{\lambda}=\lambda^T C$, "Gluino"
  adjoint representation: $\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a+g\,f_{abc}A^b_\mu\lambda^c$
- (auxiliary field $D^a(x)$)

Lagrangean

$$\mathcal{L}=\int d^2\theta\,\mathrm{Tr}(W^A W_A)+\mathrm{h.c.}=\frac{1}{4}F_{\mu\nu}^{\ a}F_{\mu\nu}^{\ a}+\frac{1}{2}\overline{\lambda}^a\gamma_\mu(\mathcal{D}_\mu\lambda)^a+\frac{1}{2}D^aD^a$$

SUSY: (on-shell) $\delta A^a_\mu=-2\,\mathrm{i}\,\overline{\lambda}^a\gamma_\mu\varepsilon\,,\quad \delta\lambda^a=-\sigma_{\mu\nu}F_{\mu\nu}^{\ a}\varepsilon$

- Simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended standard model
- Similar to QCD

  Differences: λ: 1.) Majorana, "$N_f=\frac{1}{2}$"
  2.) adjoint representation of SU(N_c)
- Gluino mass term $m_{\tilde{g}}\,\overline{\lambda}^a\lambda^a$ breaks SUSY softly.

Motivation

Non-perturbative Problems

- Spontaneous breaking of chiral symmetry $Z_{2N_c}\to Z_2$
  $\longleftrightarrow$ Gluino condensate $<\lambda\lambda>\neq 0$
- Spectrum of bound states $\to$ Supermultiplets
- Confinement of static quarks
- Spontaneous breaking of SUSY?
- SUSY restoration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)

Spontaneous breaking of chiral symmetry

U(1)$_\lambda$: $\lambda'=\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\lambda\,,\ \overline{\lambda}'=\overline{\lambda}\,\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\leftrightarrow$ R-symmetry, $J_\mu=\overline{\lambda}\gamma_\mu\gamma_5\lambda$

Anomaly: $\partial^\mu J_\mu=\frac{N_c g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$ breaks U(1)$_\lambda\to Z_{2Nc}$

Spontaneous breaking $Z_{2N_c}\to Z_2$
by Gluino condensate $<\lambda\lambda>\neq 0$
$\leftrightarrow$ first order phase transition at $m_{\tilde{g}}=0$

$N_c=2:\quad <\lambda\lambda>=\pm C\Lambda^3$

SUSY on the Lattice

Lattice breaks SUSY. Restoration in the continuum limit?
Curci, Veneziano: use Wilson action, search for continuum limit with SUSY

$$S=-\frac{\beta}{N_c}\sum_p \mathrm{Re\,Tr}\,U_p$$

$$+\frac{1}{2}\sum_x\left\{\overline{\lambda}^a_x\lambda^a_x-\kappa\sum_{\mu=1}^4\left[\overline{\lambda}^a_{x+\hat{\mu}}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x+\overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}}\right]\right\}$$

$\beta=\frac{2N_c}{g^2}$, $\kappa=\frac{1}{2m_0+8}$ hopping parameter, m_0 : bare gluino mass

$V_{ab,x\mu}=2\,\mathrm{Tr}\,(U^\dagger_{x\mu}T_aU_{x\mu}T_b)$, adjoint link variables

$$S_f=\frac{1}{2}\overline{\lambda}Q\lambda=\frac{1}{2}\lambda M\lambda\,,\qquad M\equiv CQ$$

$$\int[d\lambda]\,\mathrm{e}^{-S_f}=\mathrm{Pf}(M)=\pm\sqrt{\det Q}$$

Effective gauge field action

$$S_{\mathrm{eff}}=-\frac{\beta}{N_c}\sum_p\mathrm{Re\,Tr}\,U_p-\frac{1}{2}\log\det Q[U]$$

Include sign Pf(M) in the observables. Gauge group SU(2).

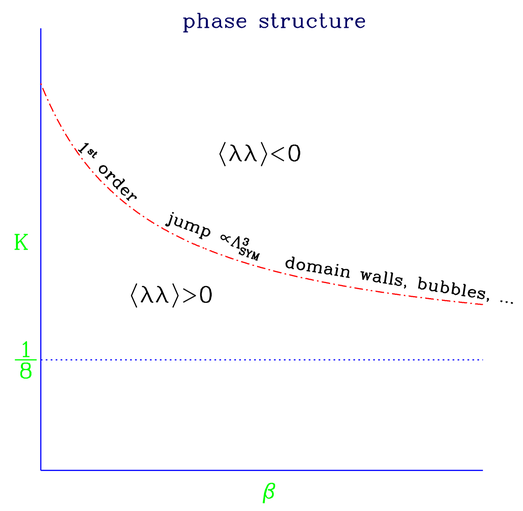
Numerical simulations by the Münster-DESY-Frankfurt group: see talks by P. Giudice and S. Piemonte.

Main Objectives:

- Spectrum of bound states $\to$ Supermultiplets
- SUSY restoration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)
- Behaviour at finite temperatures
- Topology

The Goals

Phase transition for SU(2)



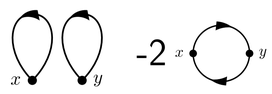
Line $\kappa=\kappa_c(\beta)$: first order phase transition at zero gluino mass.

The adjoint pion

Tuning to κ_c: $m_{\text{a}-\pi}\to 0$

What is the adjoint pion a–π?

Meson a–η' : $0^-\leftrightarrow\overline{\lambda}\gamma_5\lambda$
Correlator has disconnected pieces



Correlator of a–π $\doteq$ connected part of the a–η' correlator

The a–π is not a physical particle in the Hilbert space of the theory.

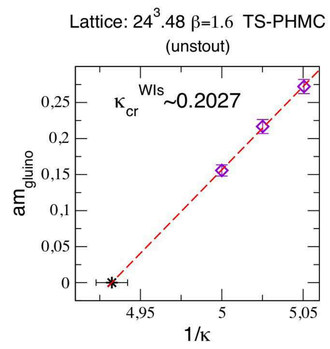
Assumption: $\boxed{m^2_{\text{a}-\pi}\propto m_{\tilde{g}}}$

analogously to the Gell-Mann-Oakes-Renner (GOR) relation of QCD:

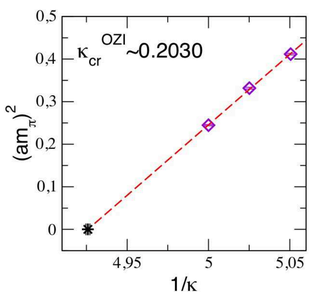
$$m^2_\pi\propto m_q$$

Arguments:

- OZI-approximation of SYM (Veneziano,Yankielowicz)
- Numerical investigations of both $m_{\tilde{g}}$ from supersymmetric Ward identities and $m_{\text{a}-\pi}$



SUSY Ward identities



OZI-arguments

renormalized gluino mass

$$am_{\tilde{g}}Z_S=\frac{1}{2}\left(\frac{1}{\kappa}-\frac{1}{\kappa_c}\right)\qquad (am_{\text{a}-\pi})^2\simeq A\left(\frac{1}{\kappa}-\frac{1}{\kappa_c}\right)$$

Goals

- Define the adjoint pion a–π properly
- Establish $m^2_{\text{a}-\pi}\propto m_{\tilde{g}}$

Approach

Idea: chiral perturbation theory
But: SUSY Yang-Mills theory does not have a continuous chiral symmetry.

Add additional flavours of gluinos,

$$\lambda_i(x),\quad i=2,\dots,N,$$

which are quenched, in order to keep SUSY.

$\longrightarrow$ Partially Quenched Chiral Perturbation Theory (PQChPT)

Adjoint pions: $\bar{\lambda}_i\gamma_5(\tau_\alpha)_{ij}\lambda_j,\qquad i,j=1,2$

The Calculation

Adding gluinos

Add additional flavours of gluinos: $\lambda_i(x),\quad i=2,\dots,N$

Chiral symmetry group: $G=\mathrm{SU}(N)$
(transformations of N Weyl fermions)

Spontaneous breakdown to $H=\mathrm{SO}(N)$

Consider $N=2$.

Goldstone manifold $G/H=\mathrm{SU}(2)/\mathrm{SO}(2)\sim S^2$
can be parameterised by $u=\exp(\mathrm{i}\alpha_1T_1+\mathrm{i}\alpha_3T_3)$.

Nonlinear Goldstone boson field defined by
$U(x)=u(x)^2=u(x)u(x)^T\doteq\exp\left(\mathrm{i}\frac{\phi(x)}{F}\right)$
transforms as

$$U(x)\to U'(x)=VU(x)V^T\,,\qquad V\in\mathrm{SU}(2).$$

Leading order effective Lagrangean

$$\mathcal{L}_2=\frac{F^2}{4}\mathrm{tr}(\partial_\mu U\partial^\mu U^\dagger)+\frac{F^2}{4}\mathrm{tr}(\chi U^\dagger+U\chi^\dagger)$$

with

$$\chi=2B_0m_{\tilde{g}}\mathbf{1}$$

PQChPT

Introduce ghost gluino $\rho(x)$, compensating the contribution of the additional gluino.

Chiral symmetry group: SU(2|1)

Goldstone boson field: graded matrix field

$$\phi=\begin{pmatrix}\phi_{ss}&\phi_{sv}&\phi_{sg}\\ \phi_{vs}&\phi_{vv}&\phi_{vg}\\ \phi_{gs}&\phi_{gv}&\phi_{gg}\end{pmatrix}$$

s, v and g stand for sea, valence and ghost.

The adjoint pion is represented by ϕ_{sv}.

Leading order effective Lagrangean

$$\mathcal{L}_2^{PQ}=\frac{F^2}{4}\mathrm{str}(\partial_\mu U\partial^\mu U^\dagger)+\frac{F^2}{4}\mathrm{str}(\chi U^\dagger+U\chi^\dagger).$$

PQChPT to one-loop yields

$$m^2_{\text{a}-\pi}=M^2_{sv}=2B_0\,m_{\tilde{g}}+\frac{(2B_0\,m_{\tilde{g}})^2}{F^2}(30L_8-2L_4-7L_5+8L_6).$$

Results

- Adjoint pion a–π is defined in PQChPT
- $m^2_{\text{a}-\pi}=2B_0\,m_{\tilde{g}}$
  in leading order PQChPT

References

References

[1] F. Farchioni, A. Feo, T. Galla, C. Gebert, R. Kirchner, I. Montvay, G. Münster and A. Vladikas:
"The Supersymmetric Ward identities on the lattice,"
Eur. Phys. J. **C 23** (2002) 719
[arXiv:hep-lat/0111008].

[2] G. Bergner, I. Montvay, G. Münster, U. D. Özugurel and D. Sandbrink:
"Towards the spectrum of low-lying particles in supersymmetric Yang-Mills theory,"
JHEP **1311** (2013) 061
[arXiv:1304.2168 [hep-lat]].

[3] G. Münster and H. Stüwe:
"The mass of the adjoint pion in $\mathcal{N}=1$ supersymmetric Yang-Mills theory,"
JHEP **1405** (2014) 034
[arXiv:1402.6616 [hep-th]].