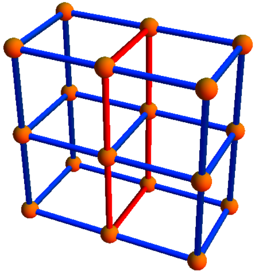
Phase structure of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory at finite temperature



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Supersymmetry - The project

DESY-Münster collaboration:
$\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice *(Investigation of non-perturbative aspects of a supersymmetric Yang-Mills theory)*
Members of the collaboration:

- G. Münster (WWU), I. Montvay (DESY)
- G. Bergner (Frankfurt), P. Giudice (WWU)
- Phd students: U. D. Özurgurel, S. Piemonte, D. Sandbrink (WWU)

Recent publication:

- *G. Bergner, P. Giudice, G. Münster, S. Piemonte, D. Sandbrink*: Phase structure of the N=1 supersymmetric Yang-Mills theory at finite temperature, *arXiv:1405.3180 [hep-lat]*

Why SUSY at finite temperature on the lattice?

Supersymmetry (SUSY) relates boson particles to fermion particles:

$$
\begin{aligned}
Q|\text{boson}\rangle &= |\text{fermion}\rangle \\
Q|\text{fermion}\rangle &= |\text{boson}\rangle
\end{aligned}
$$

SUSY appears as a challenge/solution to problems in:

1. Particle physics: hierarchy problem, AdS/CFT, ...
2. Astrophysics/thermodynamics: cosmological constant problem, quasi-conformal regime of QCD, dark matter ...
3. Theoretical aspects of field theory: divergences in QFT, formulation of SUSY on the lattice, ...

On the lattice, we try to study the thermodynamics in the limit of exact SUSY, but exact supersymmetry *is not* realized in nature ...

Why SUSY at finite temperature on the lattice?

"It is plausible that finite temperature QCD and finite temperature supersymmetric gauge theory are not all that different."

Barton Zwiebach in *An introduction to String Theory*

SYM → FINITE TEMPERATURE → QCD

"Ideal" world → FINITE TEMPERATURE → "Our" world

SUSY is broken at finite temperature, fermions and bosons "feel differently" the temperature: maybe Supersymmetric Yang-Mills theories (SYM) and QCD share the same properties at finite temperature.

The continuum action

We study on the lattice the $\mathcal{N} = 1$ SYM based on gauge group SU(2). In the continuum the on-shell action can be written as:

$$S = \int d^4x \left\{ \frac{1}{4}(F^a_{\mu\nu} F^a_{\mu\nu}) + \frac{1}{2}\bar{\lambda}_a \gamma^\mu D^{ab}_\mu \lambda_b \right\}$$

where λ is a Majorana fermion in the adjoint representation:

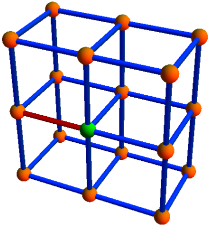
$$\begin{aligned} \bar{\lambda}_a &= \lambda_a^T C \\ D^{ab}_\mu \lambda_b &= \partial_\mu \lambda_a + igA^c_\mu (T^A_c)^{ab} \lambda_b \end{aligned}$$

The global supersymmetry relates bosonic gauge fields to fermion fields:

$$\begin{aligned} A_\mu &\rightarrow A_\mu - 2i\bar{\lambda}\gamma_\mu \epsilon \\ \lambda^a &\rightarrow \lambda^a - \sigma_{\mu\nu} F^a_{\mu\nu} \epsilon \end{aligned}$$

SUSY on the lattice

To perform Monte Carlo simulations, we introduce a finite lattice spacing a, but it breaks SUSY explicitly:



$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu C)_{\alpha\beta} P_\mu$$

The propagation of a gluino in the space-time is related to the inverse of the Dirac-Wilson operator D_W:

$$S_f = \frac{1}{2}\bar{\lambda}(D_W[V_\mu] + m)\lambda$$

The mass $m \neq 0$ is a renormalization parameter of the theory. The links V_μ in D_W are in the adjoint representation:

$$V_\mu(x)_{ab} = 2\mathrm{Tr}(U_\mu^\dagger(x) T_a^F U_\mu(x) T_b^F)$$

SUSY on the lattice

After the integration of the Majorana field λ, the full partition function of theory reads:

$$Z(g, m) = \int DU_\mu \mathrm{Pf}(D_W + m) \exp\left(-S_g\right)$$

Monte-Carlo simulations are able to handle only positive quantity, so:

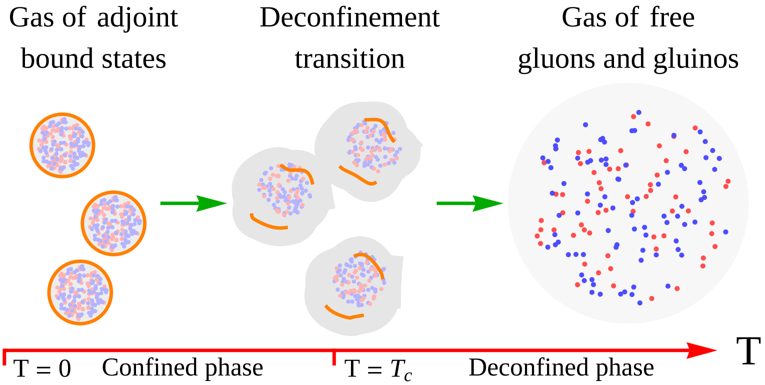
$$\mathrm{Pf}(D_W + m) \rightarrow \mathrm{Sgn}(\mathrm{Pf}(D_W + m)) \times |\mathrm{Pf}(D_W + m)|$$

Sign problem

The sign of the Pfaffian is inserted in the observables, separate computation needed!

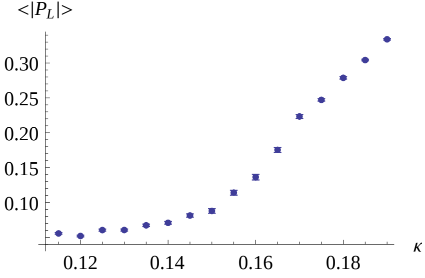
Deconfinement phase transition

At zero temperature the theory has a bound spectrum of mesons, glueballs and gluino-glue, at high temperature deconfinement is expected to occur.

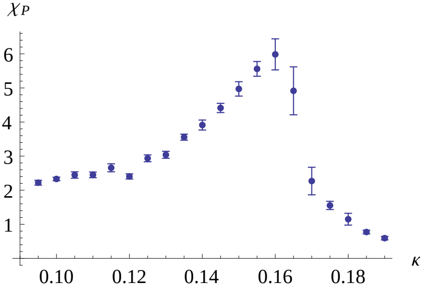


The deconfinement phase transition

The Polyakov loop is an exact order parameter in SYM for the deconfinement phase transition even for $m_g \neq \infty$.



(a) Polyakov Loop $\langle P_L \rangle$, lattice $12^3 \times 4$



(b) Polyakov Loop Susc. χ_L, lattice $12^3 \times 4$

Figure: Graphics of the expectation value of the Polyakov loop and of its susceptibility for the lattices $12^3 \times 4$ at $\beta = 1.65$.

The order of the deconfinement phase transition

The peak of the Polyakov loop susceptibility scales as $\frac{\chi_P(V_1)}{\chi_P(V_2)} = \left(\frac{V_1}{V_2}\right)^x$ with $x = 0,\ 1,\ 0.657$ respectively for a crossover, first order or second order phase transition in the universality class of the Z_2 Ising model.

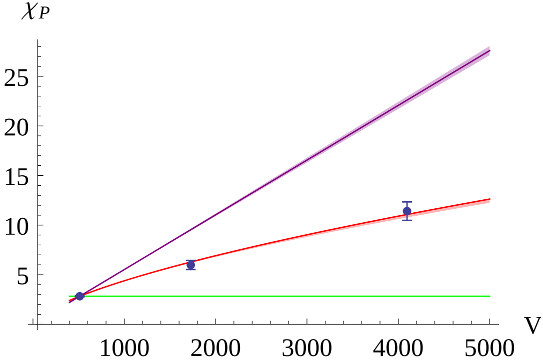
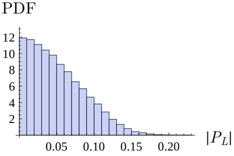


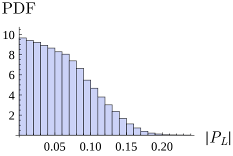
Figure: Finite size scaling for lattices $4 \times V$ at $\beta = 1.65$ and $\kappa = 0.16$. Red line: second order. Green line: cross-over. Purple line: first order.

The deconfinement transition

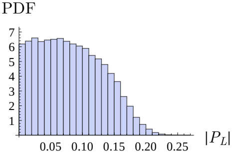
The distributions of the Polyakov loop at different values of κ show a slow emergence of a new peak (lattice $8^3 \times 4$ and $\beta = 1.65$):



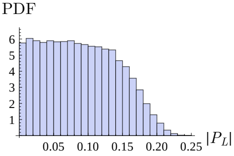
(a) $\kappa = 0.100$



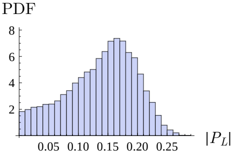
(b) $\kappa = 0.125$



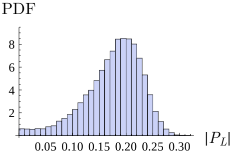
(c) $\kappa = 0.145$



(d) $\kappa = 0.150$



(e) $\kappa = 0.160$



(f) $\kappa = 0.165$

Deconfinement towards the supersymmetric limit

SUSY and chiral symmetry are broken on the lattice, extrapolation to $a = 0$ and $m_g^R = 0$ is the only solution available:

$$\frac{T_c(\mathrm{SYM})}{T_c(\mathrm{Pure\ Gauge})} = 0.822(17) \quad \rightarrow \quad T_c = 197(4)\ \mathrm{MeV}$$

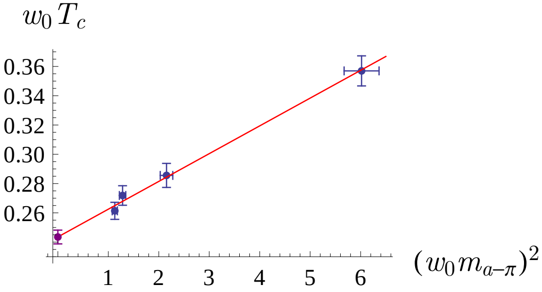
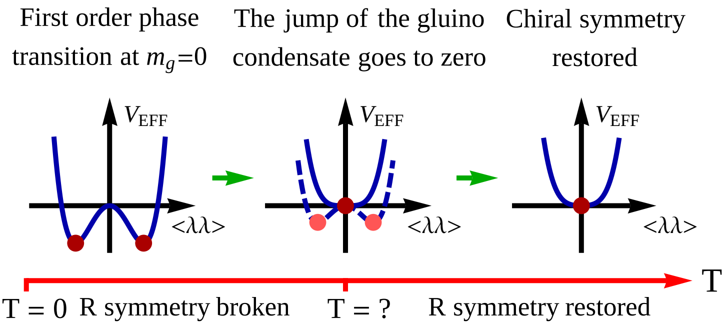


Figure: Extrapolation to the chiral limit $m_g = 0$ of the deconfinement temperature with lattices 4×12, 5×15 at $\beta = 1.65, 1.62, 1.60$.

Chiral phase transition

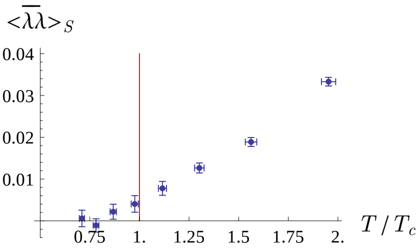
Classical $U_A(1)$ chiral (or R) symmetry $\lambda \to \exp\{i\theta\gamma_5\}\lambda$ is broken by anomaly. A remaining Z_{2N_c} symmetry is spontaneously broken to Z_2 by a non vanishing value of the gluino condensate $\langle\lambda\lambda\rangle$:

$$\mathrm{U}(1) \to Z_{2N_c} \to Z_2$$



Chiral phase transition

Simulations on a lattice $12^3 \times N_t$ at $\beta = 1.7$, $\kappa = 0.192$ and $aM_\pi = 0.388(9)$ seem to indicate that chiral symmetry is restored when deconfinement occurs, but need extrapolation to the chiral limit!



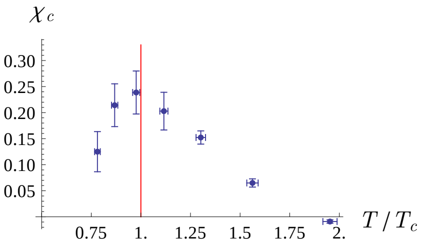


Figure: Subtracted gluino condensate and its susceptibility. The red line marks the expected deconfinement phase transition occuring at T_c.

Finite temperature supersymmetry on the lattice

Lattice simulations of the $\mathcal{N} = 1$ SU(2) SYM have shown that:

- The deconfinement phase transition occurs with a clear signal at a temperature lower than in pure gauge theory
- The deconfinement phase transition is of second order in the regime explored
- Chiral symmetry seems to get restored near the deconfinement phase transition

Future works:

- Critical scaling for the chiral symmetry phase transition
- Smaller pion mass and lattice spacing thermodynamics
- Periodic boundary conditions "thermodynamics"