Conserved currents and results from 2+1f dynamical Mobius DWF simulations at the physical point

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for the RBC and UKQCD collaborations

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Outline

Conserved currents

- Exactly conserved vector current of approximate overlap operators [PAB]
- Axial ward identity defect [PAB]

Status of RBC-UKQCD 2+1f chiral fermion simulation [RBC-UKQCD]

- Global fits to physical point data [C. Kelly]
- Quark masses [C. Kelly, N. Garron, J. Frison]
- B_K [C. Kelly, J. Frison]
- $K \to \pi\pi$ [T. Janowski]
- Summary

Scaled Shamir/Mobius fermions

- RBC-UKQCD has generated two physical point ensembles using the scaled Shamir kernel, tanh approximation to obtain good residual chiral symmetry breaking
- Compared to earlier DWF simulations the approximation to the overlap sign function is made more accurate
- Difference in exponentially small terms in action
  Construction of conserved currents now differs from Furman and Shamir setup.

$$S^5 = \bar{\psi} D^5_{GDW} \psi \tag{1}$$

We define

$$D_+ = (bD_W + 1) \quad ; \quad D_- = (1 - cD_W)$$

$$\tilde{D} = (D_-)^{-1} D_+$$

and take

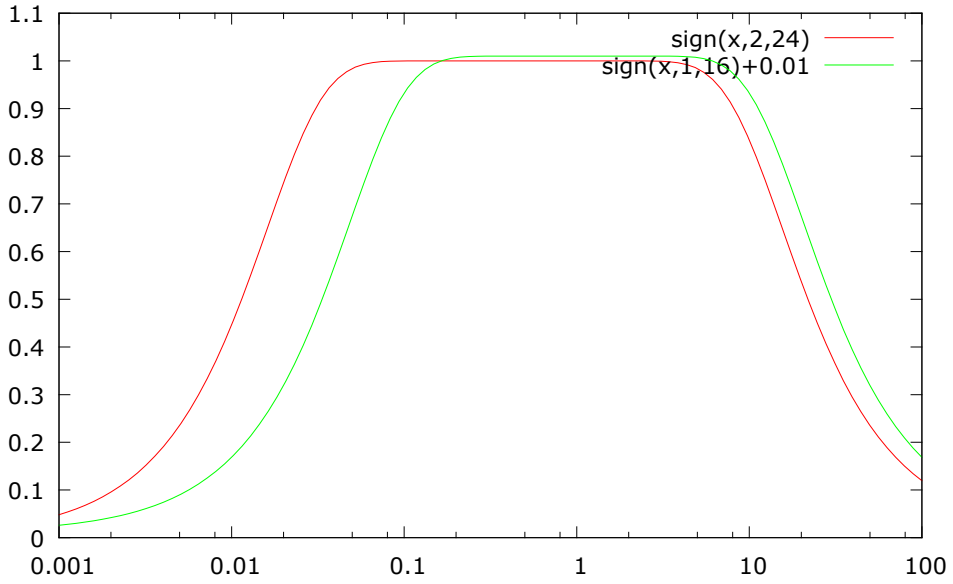
$$D^5_{GDW} = D_-^{-1} D^5_{Brower} = \begin{pmatrix} \tilde{D} & -P_- & 0 & \dots & 0 & mP_+ \\ -P_+ & \ddots & \ddots & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \ddots & \ddots & -P_- \\ mP_- & 0 & \dots & 0 & -P_+ & \tilde{D} \end{pmatrix}$$

Physical fields remain surface fields

$$\begin{array}{lllllll} q_R & = & P_+ \psi_{L_s} & ; & q_L & = & P_- \psi_1 \\ \bar{q}_R & = & \bar{\psi}_{L_s} P_- & ; & \bar{q}_L & = & \bar{\psi}_1 P_+ \end{array}$$

and mass term and chiral rotations are obvious in this field basis.

Change to approximate sign function



Surface to bulk propagator

Following results are well known in literature and arise from LDU decomposition in fifth dimension.

We define the Scaled Shamir (Mobius) kernel in terms a scale factor s $H^M = \gamma_5 \frac{sD_W}{2+D_W}$

Now, if we choose $b + c = s$ and $b - c = 1$, then the transfer matrix is $T^{-1} = -[H_M - 1]^{-1}[H_M + 1]$
Effective 4d overlap operator is

$$D_{ov} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{T^{-L_s} - 1}{T^{-L_s} + 1} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \epsilon(H_M)$$

Propagator into the bulk from a surface field q is:

$$\begin{aligned}
\langle Q_s \bar{q} \rangle &= \left[\mathcal{P}^{-1} D^5_{GDW}(m)^{-1} R_5 \mathcal{P}\right]_{s1} \\
&= \frac{1}{1-m}\left\{\mathcal{P}^{-1} D^5_{GDW}(m)^{-1} D_{GDW}(1)\mathcal{P} - \mathbb{1}\right\}_{s1} \\
&= \left(\begin{array}{c|c} \frac{1}{1-m}\left(D_{ov}^{-1}(m) - \mathbb{1}\right) & 0 \\ \hline T^{-(L_s-1)}[1 + T^{-L_s}]^{-1} D_{ov}^{-1}(m) & \\ \vdots & 0 \\ T^{-1}[1 + T^{-L_s}]^{-1} D_{ov}^{-1}(m) & \end{array}\right)_{s1}
\end{aligned}$$

Normal 5d DWF propagator is

$$\begin{aligned}
G_q &= \mathcal{P}\langle Q_s \bar{q} \rangle \\
&= [P_+ + P_- T^{-1}] \begin{pmatrix} T^{-(L_s-1)} \\ T^{-(L_s-2)} \\ \vdots \\ T^{-1} \\ 1 \end{pmatrix} [1 + T^{-L_s}]^{-1} D_{ov}^{-1}
\end{aligned}$$

Surface to bulk propagator

NB contact term already subtracted in surface-to-surface propagator for DWF/Cayley Tanh representation of overlap

$$S(x) = \frac{1}{1-m}\left(D_{ov}^{-1}(m) - \mathbb{1}\right)$$

The source vector η may be used to eliminate this by forming

$$(1-m)S(x)\eta + \eta = D_{ov}^{-1}(m)\eta = [1+T^{-L_s}][1+T^{-L_s}]^{-1}D_{ov}^{-1}\eta$$

By applying $P_+ and P_-$ we find we have the following set of vectors

$$\begin{pmatrix} P_+ \\ P_- T^{-L_s} \\ P_+[1+T^{-L_s}] \\ P_-[1+T^{-L_s}] \end{pmatrix} [1+T^{-L_s}]^{-1}D_{ov}^{-1}$$

Eliminate to form L_s+1 vectors from a 4d source η containing all powers of transfer matrix

$$T(s) = \begin{pmatrix} 1 \\ T^{-1} \\ \vdots \\ T^{-L_s} \end{pmatrix} \left[1 + T_1^{-1} \cdots T_{L_s}^{-1}\right]^{-1} D_{ov}^{-1}(m)\eta$$

Conserved currents from Fermion field rotation

Standard textbook work (e.g. Montvay and Munster!):

$$\psi'_y = \psi_y - i\alpha\psi_y$$

$$\bar{\psi}'_y = \bar{\psi}_y + i\bar{\psi}_y\alpha$$

leaves partition function invariant

$$\begin{aligned} Z' &= \int d\bar{\psi}' d\psi' e^{-S[\bar{\psi}',\psi']} \\ &= \int d\bar{\psi} d\psi e^{-S[\bar{\psi},\psi]} \left\{ 1 - i\alpha \left[\frac{\delta S}{\delta \psi_y}\psi_y - \bar{\psi}_y \frac{\delta S}{\delta \bar{\psi}_y} \right] \right\} \\ &= Z \end{aligned}$$

Therefore:

$$\left[\frac{\delta S}{\delta \psi_y}\psi_y - \bar{\psi}_y \frac{\delta S}{\delta \bar{\psi}_y} \right] = 0$$

For one-hop stencial δS involves terms from all sites $y \pm \mu$ connected by the lattice Dirac operator to y.
Generates the familiar nearest neighbour backwards difference term in the Wilson conserved vector current
Eight terms with a ψ at site y and eight terms with a $\bar{\psi}$ at site y.

Conserved current from gauge rotation

Gauge invariance leaves action under simultaneous transform

$$U_\mu(y) \to (1 + i\alpha)U_\mu(y)$$

$$U_\mu(y - \hat{\mu}) \to U_\mu(y - \hat{\mu})(1 - i\alpha)$$

$$\psi_y \to (1 + i\alpha)\psi_y$$

$$\bar{\psi}_y \to \bar{\psi}_y(1 - i\alpha).$$

Change variables of all fermion fields at site y simultaneous with this gauge transform:

$$\psi'_y = (1 + i\alpha)\psi_y$$

$$\bar{\psi}'_y = \bar{\psi}_y(1 - i\alpha)$$

leaves action invariant. Phase associated with the Fermion is absorbed, and we can view the change in action as being associated with the *unabsorbed* phases on the eight gauge links connected to site y.

$$\sum_\mu \left[\frac{\delta S}{\delta U_\mu(y)^{ij}} U_\mu(y)^{ij} - \frac{\delta S}{\delta [U^\dagger_\mu(y)]^{ij}} [U^\dagger_\mu(y)]^{ij} - \frac{\delta S}{\delta U_\mu(y-\mu)^{ij}} U_\mu(y-\mu)^{ij} + \frac{\delta S}{\delta [U^\dagger_\mu(y-\mu)]^{ij}} [U^\dagger_\mu(y-\mu)]^{ij} \right]$$

For the nearest neighbour Wilson action this generates the same sixteen terms entering $\Delta^\mu_- J^\mu = 0$.

Where

$$J_\mu(x) = \frac{\delta S}{\delta U_\mu(x)^{ij}} U_\mu(x)^{ij} - \frac{\delta S}{\delta [U^\dagger_\mu(x)]^{ij}} [U^\dagger_\mu(x)]^{ij}$$

If the action is translational invariant $J_\mu(x)$ is a conserved current, as we expect.

Wilson conservation law

δ indicates the variation of terms under the simultaneous infinitesimal rotation of all gauge links connected to site y.

$$\delta(\textstyle\sum_x(\bar\psi D_W\psi)(x)) = \Delta^-_\mu J^W_\mu\Big|_y \quad = \quad \textstyle\sum_\mu \begin{bmatrix} -\bar\psi_y \frac{1-\gamma_\mu}{2} U_\mu(y)\psi_{y+\hat\mu} \\ +\bar\psi_{y-\hat\mu} \frac{1-\gamma_\mu}{2} U_\mu(y-\hat\mu)\psi_y \\ -\bar\psi_y \frac{1+\gamma_\mu}{2} U^\dagger_\mu(y-\hat\mu)\psi_{y-\hat\mu} \\ +\bar\psi_{y+\hat\mu} \frac{1+\gamma_\mu}{2} U_\mu(y)^\dagger\psi_y \end{bmatrix}$$

$$= \quad \Delta^-_\mu \left[\bar\psi_y \frac{1-\gamma_\mu}{2} U_\mu(y)\psi_{y+\hat\mu} - \bar\psi_{y+\hat\mu} U^\dagger_\mu(y)\frac{1+\gamma_\mu}{2}\psi_y\right]$$

- If we consider action formed as product of Wilson matrices

$$S = \sum_{xyzw} \bar\psi_x D_W(x,y) D_W(y,z) D_W(z,w)\psi(w)$$

- The link variation approach gives three terms, each of which are conserved under a nearest neigbour difference divergence:

$$\delta(\bar\psi D_W D_W D_W\psi)\psi = \psi\left[(\delta D_W)D_W D_W + \mathbf{D_W}(\delta\mathbf{D_W})\mathbf{D_W} + D_W D_W(\delta D_W)\right]\psi$$

- Interior insertions are the integral of a divergence; required to recognise the backwards divergence current conservation law

Exact vector current for approximate overlap operator

$$D_{ov} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{T^{-L_s} - 1}{T^{-L_s} + 1}$$

Derivative of D_{ov} with respect to links:

$$\begin{aligned}
\delta D_{ov} &= \frac{1-m}{2}\gamma_5 \left\{ \delta\left(\frac{1}{1+T^{-L_s}}\right)[1 - T^{-L_s}] + \frac{1}{1+T^{-L_s}}\delta(1 - T^{-L_s}) \right\} && (2)\\
&= \frac{1-m}{2}\gamma_5 \left\{ \delta\left(\frac{1}{1+T^{-L_s}}\right) - \frac{1}{1+T^{-L_s}}\delta(T^{-L_s}) \left(1 - \frac{T^{-L_s}}{1+T^{-L_s}}\right) \right\} && (3)\\
&= (1-m)\gamma_5 \delta\left(\frac{1}{1+T^{-L_s}}\right) && (4)
\end{aligned}$$

The two point function of this current is then

$$\gamma_5 \tilde{D}_{ov}^{-\dagger} \gamma_5 \gamma_5 [1 + T^{-L_s}]^{-1} \left\{ \sum_{s=0}^{L_s-1} T^{-s} \delta(T^{-1}) T^{-(L_s-1-s)} \right\} [1 + T^{-L_s}]^{-1} D_{ov}^{-1} \qquad (5)$$

Exact vector current for approximate overlap operator

Can show:

$$(b+c)\delta(T^{-1}) \quad = \quad \left[b[P_+ - T^{-1}P_-] + c[T^{-1}P_+ - P_-]\right]\delta(D_W)\left[b[P_+ + P_- T^{-1}] + c[P_+ T^{-1} + P_-]\right]$$

Earlier result for $L_s + 1$ powers of T^{-1} may be used to construct for $s \in \{0 \ldots L_s - 1\}$:

$$\left[b[P_+ + P_- T^{-1}] + c[P_+ T^{-1} + P_-]\right] T^s$$

Ccontracting these vectors through the Wilson conserved current the Mobius matrix element can be formed a *very* similar manner to the standard DWF conserved vector current.

$$\gamma_5 \tilde{D}_{ov}^{-\dagger} \gamma_5 \gamma_5 [1 + T^{-L_s}]^{-1} \left\{ \sum_{s=0}^{L_s-1} T^{-s} \delta(T^{-1}) T^{-(L_s-1-s)} \right\} [1 + T^{-L_s}]^{-1} D_{ov}^{-1} \tag{6}$$

$c = 0$: Matrix element reduces to being *identical* to that for the Furman and Shamir vector current.
The exactly conserved vector current of the approximate overlap operator is the Furman and Shamir 5d vector current.
$c \neq 0$: simple Furman & Shamir like approach to produce the conserved vector currents for Mobius Fermions with no additional cost.
Following Furman and Shamir introduce chiral Fermion field rotation

$$\psi(x,s) \to \begin{cases} e^{i\alpha\Gamma(s)}\psi(x,s) & ; \quad x = x_0 \\ \psi(x,s) & ; \quad x \neq x_0 \end{cases} \quad \text{where } \Gamma(s) \to \begin{cases} -1 & ; \quad 0 \leq s < L_s/2 \\ 1 & ; \quad L_s/2 \leq s \end{cases}$$

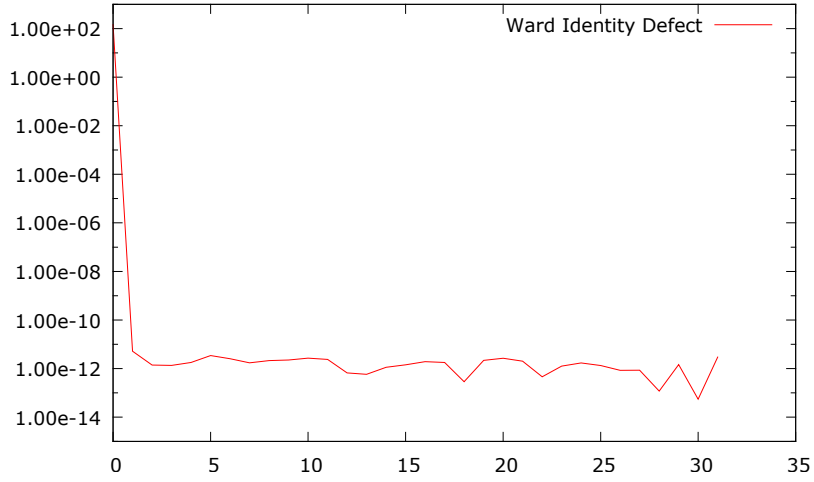
Construct (almost) conserved axial current whose pseudoscalar matrix element is

$$\mathrm{Tr}\left(\gamma_5 [\gamma_5 \eta^\dagger \tilde{D}_{ov}^{-\dagger} \gamma_5][1 + T^{-L_s}]^{-1} \left\{ \sum_{s=0}^{L_s-1} T^{-s} \Gamma(s) \delta(T^{-1}) T^{-(L_s-1-s)} \right\} [1 + T^{-L_s}]^{-1} D_{ov}^{-1} \eta\right) \tag{7}$$

The exact vector current conservation induces the same J_{5q} midpoint density defect.

Compute Axial Ward Identity defect (limited by convergence error on propagator).

$$\langle P | \Delta^{-}_{\mu} A^{c}_{\mu} - 2mP - 2J_{5q} \rangle$$



Compared to Brower et al current, treatment of $D\_$ field rotation is:

- Manifestly hermitian
- Single inversion required
- I have not checked this is identical to Brower's current, but my axial WTI is satisfied

Conserved axial and vector currents enable

- Furman and Shamir determination of Z_{A^L}
- Interpretation mres as the pionic matrix element of J_{5q} gives point of vanishing pion mass
- g-2 calculations to proceed

Last lattice conference RBC-UKQCD used

$$\frac{\langle P | 2mP + 2J_{5q} \rangle}{\langle P | \Delta^{-}_{0} A^{L}_{0} \rangle}$$

to determine Z_A without an explicit construction of the axial current. This is now fixed.

RBC-UKQCD DWF project

UKQCD
Rudy Arthur (Odense)
Peter Boyle (Edinburgh)
Luigi Del Debbio (Edinburgh)
Shane Drury (Southampton)
Jonathan Flynn (Southampton)
Julien Frison (Edinburgh)
Nicolas Garron (Dublin)
Jamie Hudspith (Toronto)
Tadeusz Janowski (Southampton)
Andreas Juettner (Southampton)
Ava Kamseh (Edinburgh)
Richard Kenway (Edinburgh)
Andrew Lytle (TIFR) Marina Marinkovic (Southampton)
Brian Pendleton (Edinburgh)
Antonin Portelli (Southampton)
Thomas Rae (Mainz)
Chris Sachrajda (Southampton)
Francesco Sanfilippo (Southampton)
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Taku Izubuchi (RBRC/BNL)
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Chulwoo Jung (BNL)
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Jasper Lin (Columbia)
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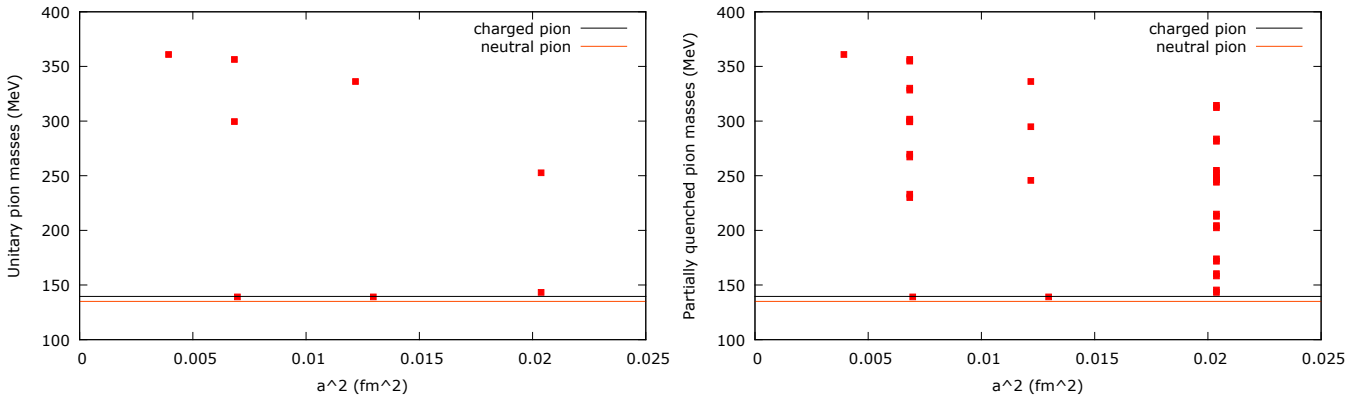
Greg McGlynn (Columbia)
David Murphy (Columbia)
Shigemi Ohta (KEK)
Eigo Shintani (Mainz)
Amarjit Soni (BNL)
Sergey Syritsyn (RBRC)
Oliver Witzel (BU)
Hantao Yin (Columbia)
Jianglei Yu (Columbia)
Daiqian Zhang (Columbia)

RBC-UKQCD 2+1f ensembles

Unitary pion masses included in fits:

Gauge action	β	Mobius scale	Volume	a^{-1} (GeV)	m_π (MeV)
Iwasaki	2.13	2.0	$48^3 \cdot 96 \cdot 24$	1.730(4)	139
	2.13	1.0	$24^3 \cdot 64 \cdot 16$	1.785(5)	336
	2.25	2.0	$64^3 \cdot 128 \cdot 12$	2.358(7)	139
	2.25	1.0	$32^3 \cdot 64 \cdot 12$	2.383(9)	299
	2.25	1.0	$32^3 \cdot 64 \cdot 12$	2.383(9)	356
	2.37	1.0	$32^3 \cdot 64 \cdot 12$	3.144(18)	360
Iwasaki+DSDR	1.75	1.0	$32^3 \cdot 64 \cdot 32$	1.380(7)	143
Iwasaki+DSDR	1.75	1.0	$32^3 \cdot 64 \cdot 32$	1.380(7)	250

Unitary (left) and partially quenched (right) pion masses in RBC-UKQCD data set



Valence analysis

Preliminary

- Use EigCG and *all mode averaging* to evaluate gauge fixed wall source propagators from all time planes. $\times$ 2 twists.
- 8 timeplanes evaluated *exactly*
- 96,128 timeplanes evaluated *inexactly*.
- Combine high statistics estimation of inexact CF and low statistics estimate of the small correction
- Target f_K, f_π, m_K, m_π, K_{l3}, B_K, $K \to (\pi\pi)_{I=2}$.
- Also, new run under way using HDCG targetting: BSM $K^0 - \bar{K}^0$ mixing, HVP, $K - \pi$ scattering, D & D_s two point and bag params with DWF charm.

Bare results in the 0.1% to 0.4% accuracy range

	$48^3 \times 96$	$64^3 \times 128$
m_π	0.08058(15)	0.05907(12)
m_K	0.28851(17)	0.21523(16)
m_Ω	0.9706(16)	0.71788(90)
m'_Ω	1.246(39)	0.939(13)
f_π	0.07592(11)	0.055535(74)
f_K	0.090464(99)	0.066477(86)
Z_A	0.71180(13)	0.743360(70)
m_{res}	0.0006105(33)	0.0003117(19)

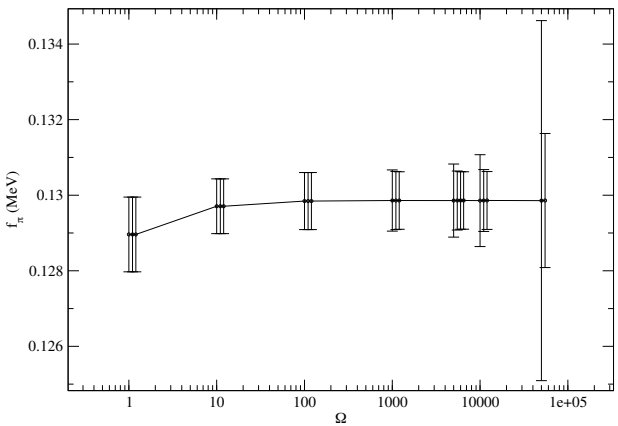
Global fits

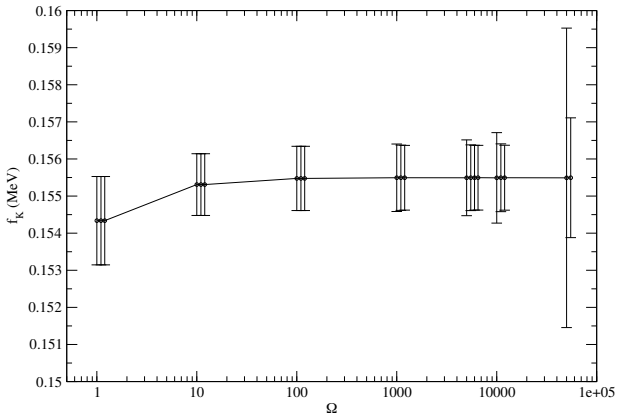
Preliminary

- Must make small correction from simulated $m_\pi = 139$ MeV to the neutral pion $m_{\pi^0} = 135$ MeV.
  - Scaling trajectory: defined by remove lattice cut-off with m_π, m_K, m_Ω held at continuum values
  - Determines m_{ud}, m_s and a.
- Three fits ansatze used to assess systematic error in making this small correction.
  - NLO ChPT
  - NLO ChPT with finite volume log
  - Analytic Taylor expansion around physical point
- Overweight physical point data
  - Wish to only determine correction to our model independent all order results near physical point
    Ansatze have a *limited/unknown* range of applicability.
    There are many more data points in this at large mass regime
  - Insert constraint that each ansatz must go through the new physical point data
  - Implement this constraint *constraint* by over weighting; Minimise

$$\chi^2 = \sum_{j \in \mathrm{unphysical}} \frac{(f_j - y_j)^2}{\sigma_j^2} + \Omega \sum_{i \in \mathrm{physical}} \frac{(f_i - y_i)^2}{\sigma_i^2}$$

  Where $\omega = 1$ for non-physical point simulations.
- Expect $\sum_{i \in \mathrm{physical}} \frac{(f_i - y_i)^2}{\sigma_i^2} \to 0$
- Expect fit results saturate Ω independent limit as constrain is applied





Global fits

Preliminary

- Express expansion of quantity Q in physical renormalised masses and lattice spacing

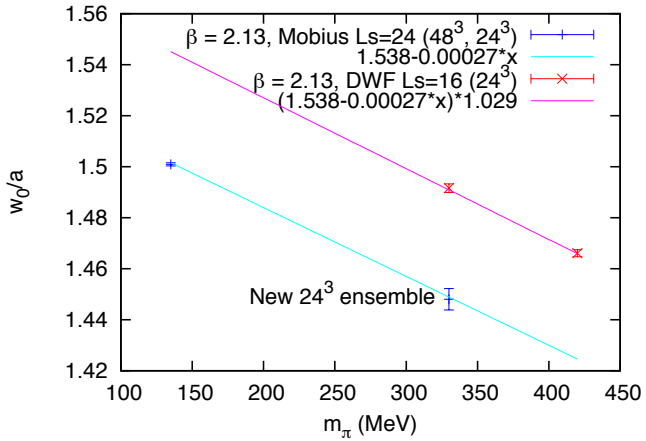
$$Q = Q(a^2, m_l^r, m_h^r)$$

- Perform double expansion in masses and a^2; ignore terms of $O(ma^2)$.
- Relate lattice spacing and dimensionalful masses of ensemble e those of a reference ensemble r

$$a^r = R_a^e a^e \quad ; \quad m_l^r = Z_l^e m_l^e$$

- Z_l^e and R_a^e are global fit parameters associated with each ensemble.

Expected DWF $\leftrightarrow$ Mobius to be a negligible effect.
Relevant and irrelevant couplings in Symanzik expansion changed at $O(m_{\rm res})$
Larger than expected effect for at least one relevant coupling!



Scheme	Mobius	DWF
Z_m (SMOM-γ)	1.5179(7)	1.511(2)
Z_m (SMOM-q)	1.4334(6)	1.428(2)

2% shift in lattice spacing, 0.3% shift in quark mass renormalisation.

Perhaps take CL holding NPR renormalised mass fixed to prevent coupling hadronic noise to scaling trajectory?

Global fit results

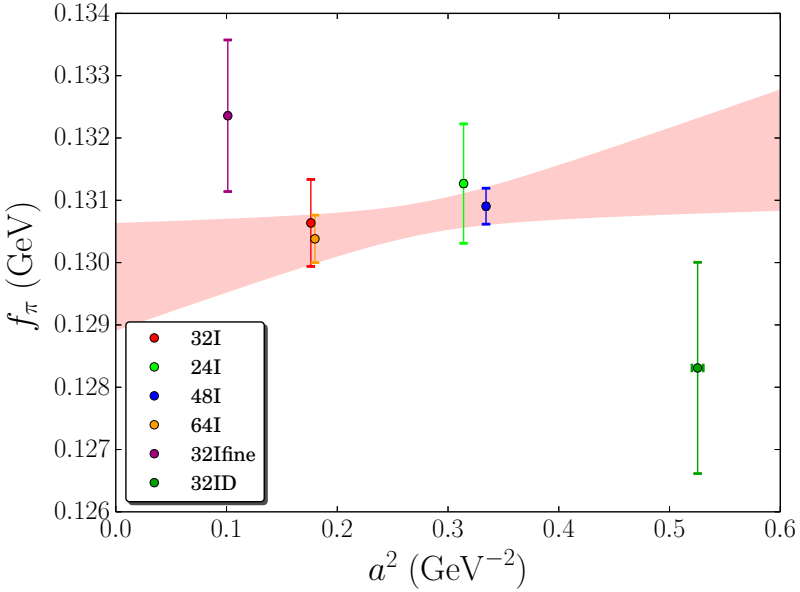
Preliminary

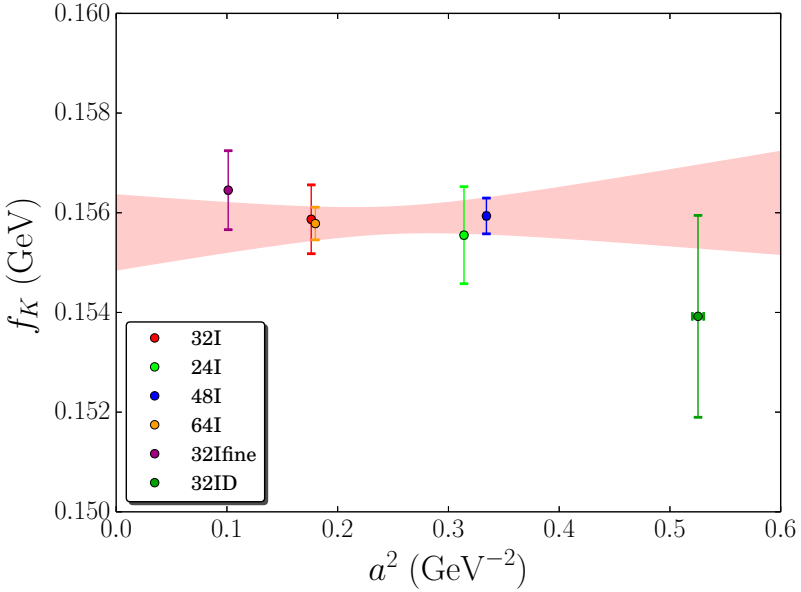
Fit	1	2	3	4
$Z_l^{24\mathrm{I}}$	0.9691(54)	0.9701(51)	0.9746(131)	0.9745(134)
$Z_l^{48\mathrm{I}}$	0.9691(54)	0.9701(51)	0.9699(52)	0.9658(197)
$Z_l^{32\mathrm{I}}$	1.0000	1.0000	1.0000	1.0000
$Z_l^{64\mathrm{I}}$	1.0000	1.0000	1.0000	0.9960(191)
$Z_h^{24\mathrm{I}}$	0.9608(47)	0.9613(43)	0.9643(98)	0.9649(99)
$Z_h^{48\mathrm{I}}$	0.9608(47)	0.9613(43)	0.9612(44)	0.9620(172)
$Z_h^{32\mathrm{I}}$	1.0000	1.0000	1.0000	1.0000
$Z_h^{64\mathrm{I}}$	1.0000	1.0000	1.0000	1.0005(166)
$R_a^{24\mathrm{I}}$	0.7334(26)	0.7493(40)	0.7511(65)	0.7510(68)
$R_a^{48\mathrm{I}}$	0.7334(26)	0.7333(25)	0.7332(25)	0.7289(118)
$R_a^{32\mathrm{I}}$	1.0000	1.0000	1.0000	1.0000
$R_a^{64\mathrm{I}}$	1.0000	1.0000	1.0000	0.9945(148)
f_π (GeV)	0.1299(8)	0.1299(8)	0.1299(8)	0.1299(8)
f_K (GeV)	0.1555(9)	0.1555(9)	0.1555(9)	0.1555(9)

$$
\begin{aligned}
a_{32\mathrm{I}}^{-1} &= 2.383(9)(0)(1)\ \mathrm{GeV} \\
a_{64\mathrm{I}}^{-1} &= 2.358(7)(0)(1)\ \mathrm{GeV} \\
a_{24\mathrm{I}}^{-1} &= 1.785(5)(4)(1)\ \mathrm{GeV} \\
a_{48\mathrm{I}}^{-1} &= 1.730(4)(0)(1)\ \mathrm{GeV} \\
a_{32\mathrm{Ifine}}^{-1} &= 3.144(18)(17)(2)\ \mathrm{GeV} \\
a_{32\mathrm{ID}}^{-1} &= 1.380(7)(0)(1)\ \mathrm{GeV}
\end{aligned}
\tag{8}
$$

Decay constants

Preliminary





$$\begin{aligned} f_\pi &= 0.1298(9)(0)(2)\ \mathrm{GeV} \\ f_K &= 0.1556(8)(0)(2)\ \mathrm{GeV} \\ f_K/f_\pi &= 1.199(5)(6)(1) \end{aligned} \tag{9}$$

Errors are statistical/chiral/finite-volume
Chiral errors on f_π and f_K set to zero because they could not be statistically resolved.
Were able to resolve the difference between the ChPTFV and analytic fits in the ratio to 2σ.

Wilson flow scale

Preliminary

	Analytic	ChPT	ChPTFV
w_0^{32IW}	0.8716(29)	0.8718(20)	0.8715(21)
w_0^{24IW}	0.8691(16)	0.8691(15)	0.8690(15)
w_0^{32ID}	0.8687(17)	0.8687(16)	0.8687(17)
w_0^{48IW}	0.8716(28)	0.8717(20)	0.8714(20)
$w_0^{32IWfine}$	0.8730(41)	0.8732(30)	0.8728(31)
w_0^{64IW}	0.8839(45)	0.8835(37)	0.8839(37)
$w_0^{\mathrm{continuum}}$	0.8749(59)	0.8751(44)	0.8746(45)

$$\begin{aligned} t_0^{1/2} &= 0.729(4)(0)(1)\ \mathrm{GeV}^{-1} \\ w_0 &= 0.875(5)(0)(1)\ \mathrm{GeV}^{-1} \end{aligned} \tag{10}$$

Step scaling over flavour threshold

Preliminary
See talk by J. Frison:
Take $0.8\mathrm{GeV} \sim \mu_0 < \mu_1 \ldots < m_c^{SMOM} < \ldots \mu_n \sim 5\mathrm{GeV}$
Define threshold step scaling functions:

$$\sigma(\mu_n, \mu_{n+1}, m_c) = \lim_{a \to 0} \left[\Lambda^{2+1+1}(a, \mu_{n+1}, m_c)\right]^{-1} \Lambda^{2+1+1}(a, \mu_n, m_c)$$

Then

$$\langle \mathcal{O}(\mu_1, m_c)\rangle_{ren}^{2+1+1} = \Pi_n \sigma(n, n+1) \langle \mathcal{O}(\mu_0)\rangle_{ren}^{2+1}$$

- Choose scale from W_0 at suff. IR Wilson flow time that we match the IR limit of 2+1+1 flav theory to the 2+1f theory.
- For $mu0 >> ms, mu, md$ this is equivalent to matching massless mu,d,s.
- Fix m_c to its physical value, defined by NPR in a small volume by taking hierarchy of scales:

$$\mu_{d/s} < \mu_0 < m_c < \mu_n$$

- Run from off-shell amplitudes in approx massless 3f theory to off shell amplitudes in approx massless 4f theory.
- Treat charm threshold effects treated non-perturbatively, and the charm at its physical mass at all stages.
- Mass independence of Z_m in RI schemes is satisfied if

$$p, a^{-1} \gg \Lambda, m_q$$

- Do *not* need $m_q \to 0$

Quark masses

Preliminary

$$m_{ud}(\overline{\mathrm{MS}}, 3.0\mathrm{GeV}) = 3.014(39)(0)(5)(35)\mathrm{MeV} \tag{11}$$

$$m_s(\overline{\mathrm{MS}}, 3.0\mathrm{GeV}) = 82.27(92)(0)(6)(95)\mathrm{MeV} \tag{12}$$

Errors are statistical, chiral, finite-volume and from the perturbative truncation respectively.
In the RGI scheme these correpond to

$$\hat{m}_{ud} = 8.67(11)(0)(1)(10)\ \mathrm{MeV} \tag{13}$$

and

$$\hat{m}_s = 236.7(26)(0)(2)(27)\ \mathrm{MeV}. \tag{14}$$

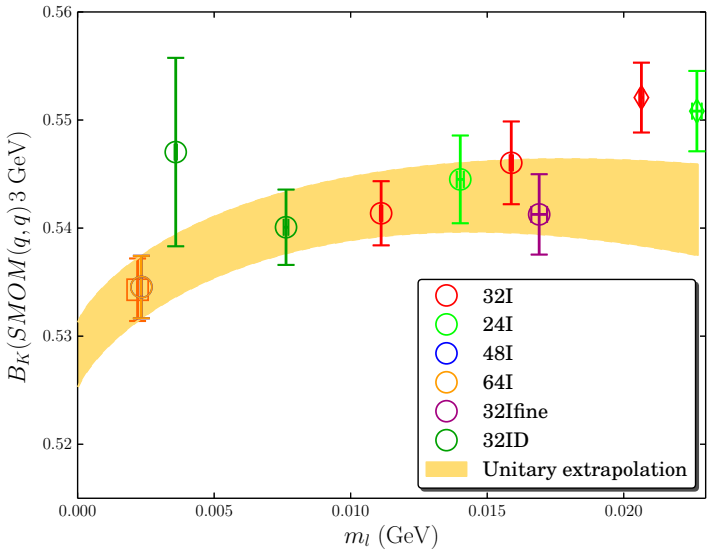
The quark mass ratio is

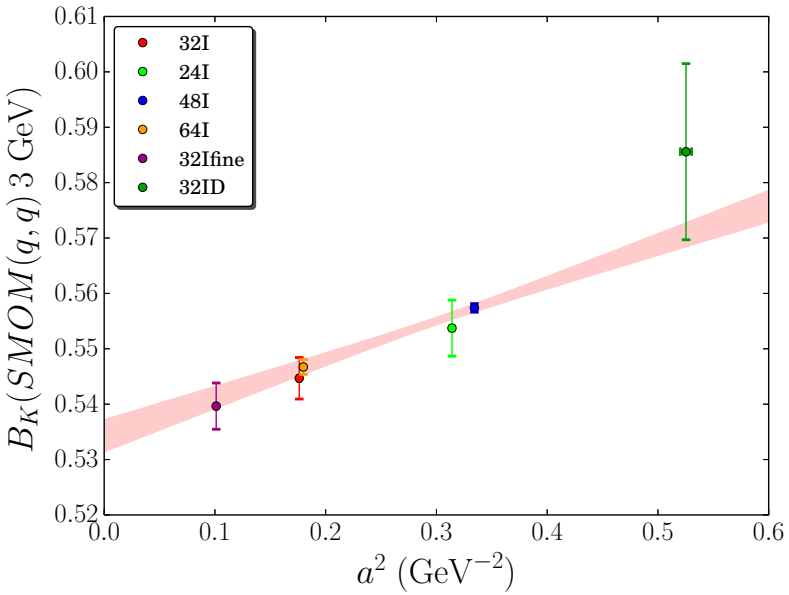
$$m_s / m_{ud} = 27.29(34)(3)(0). \tag{15}$$

Plan: step scale these across charm threshold to 5+ GeV.

Neutral Kaon mixing

Preliminary





Introduce step scaling *across charm threshold*. See talk by Julien Frison WHEN.

Theory	Scale (GeV)	Ansatz	$(\not{q}, \not{q})$	(γ^μ, γ^μ)
2+1f	3.0	Analytic	0.5364(30)	0.5188(29)
		ChPT	0.5345(29)	0.5170(28)
		ChPTFV	0.5343(29)	0.5168(28)
2+1+1f	5.0	Analytic	0.5184(29)	0.5074(28)
		ChPT	0.5166(28)	0.5057(28)
		ChPTFV	0.5164(28)	0.5055(28)

Neutral Kaon mixing

Preliminary
Errors are statistical, chiral and finite-volume:

$$B_K^{2+1f}(\mathrm{SMOM}(\not{q}, \not{q}), 3\ \mathrm{GeV}) = 0.5343(29)(21)(2)\ , \tag{16}$$

$$B_K^{2+1+1f}(\mathrm{SMOM}(\not{q}, \not{q}), 5\ \mathrm{GeV}) = 0.5164(28)(20)(2)\ . \tag{17}$$

Take the full difference between the results obtained using the two RI-SMOM intermediate schemes, and use the SMOM($\not{q}$, $\not{q}$) result for our central value.

$$B_K^{2+1f}(\overline{\mathrm{MS}}, 3\ \mathrm{GeV}) = 0.5296(29)(20)(2)(107)\ , \tag{18}$$

$$B_K^{2+1+1f}(\overline{\mathrm{MS}}, 5\ \mathrm{GeV}) = 0.5125(28)(20)(2)(52)\ . \tag{19}$$

Factor of two improvement in the truncation systematic when renormalizing at 5 GeV.
O(1%) total error.

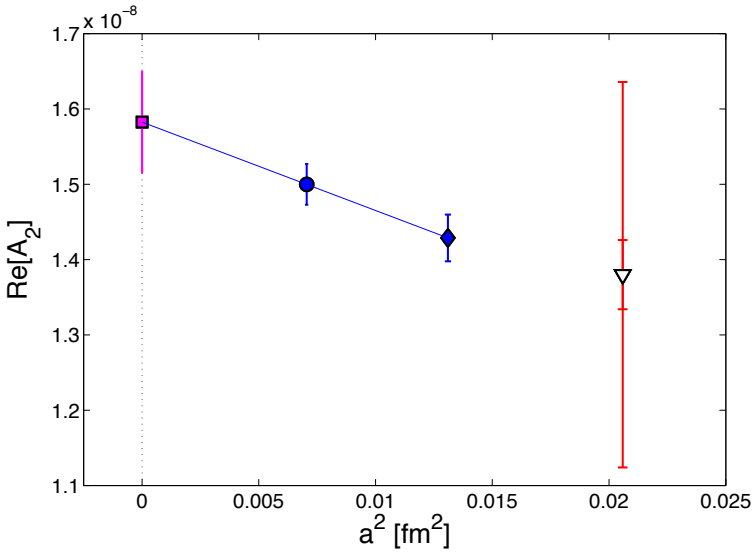
$K \to (\pi\pi)_{I=2}$ Decays

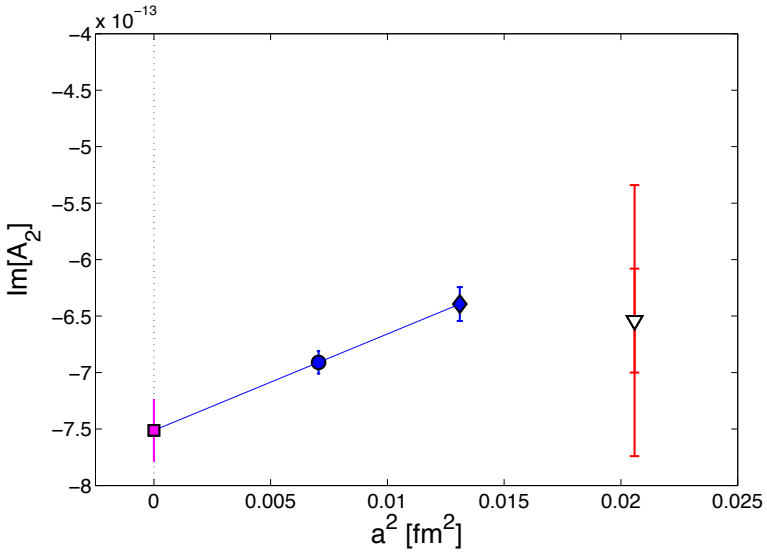
Preliminary

2012 [Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*]

$\mathrm{Re}\, A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}}\, 10^{-8}\ \mathrm{GeV}$ $\mathrm{Im}\, A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}$

2014 [RBC-UKQCD Work in progress]





Preliminary

see also talk by T.Janowski @ lat'13 [Janowski, Sachrajda, Boyle, Christ, Mawhinney, Yin, Zhang, N.G., Lytle]

- Journal paper in *very* late draft
- Chiral extrapolation and discretisation systematics have been almost eliminated
- Renormalisation systematic will be dominant around 8%

Summary

- Developed conserved vector and axial current implementation for Mobius Tanh approx
- 1% accurate physical point simulations for f_π agree with experiment
- Step scaling across charm threshold enables 1% accurate renormalised B_K
  - Required flavour threshold step scaling to 5GeV and multiple schemes to do this believably.
  - See talk by J. Frison
- K_{l3} form factor determined in talk by D. Murphy - preliminary but O(0.3%) error predicted
- $K \to (\pi\pi)_{I=2}$: $\mathrm{Re}(A_2)$, $\mathrm{Im}(A_2)$ determined to perhaps 8%
- Quark masses determined, step scaling to 2+1+1 under way
- HVP, BSM kaon bag params, $K - \pi$ scattering, D physics underway

Non-local actions

- A nonlocal Dirac action is a sum of bilinear non-local chains

$$\sum_{\text{chain}} \bar{\psi}(x) U^{\text{chain}}(x, y) \psi(y)$$

- From Fermion rotation at site y sum over all chains starting and ending on y are induced:

$$\frac{\delta S}{\delta \psi_y} \psi_y - \bar{\psi}_y \frac{\delta S}{\delta \bar{\psi}_y} = 0$$

- This generates Kikukawa and Yamada's *non-local kernel* $K_x(y, z)$, in original overlap conserved currents work.

$$\begin{aligned} \delta_S &= i\alpha_x \textstyle\sum_y \left[\bar{\psi}_y D^{ov}_{(y,x)} \psi_x - \bar{\psi}_x D^{ov}_{(x,y)} \psi_y \right] \\ &= i\alpha_x \textstyle\sum_{yz\mu} \partial_\mu \bar{\psi}_y K_{x,\mu}(yz) \psi_z \end{aligned}$$

- Partitioning of this sum of terms, into *discrete divergence operator* and *current* not simple.
  Ambiguity in moving terms between divergence operator and current defs (Mandula?)
  Ambiguity fixed if we specify a nearest neighbour backwards difference; must add and subtract terms
- Differentiating with respect to the links yields additional terms, looking like a backwards divergence that are summed (integrated) over volume.

For a non-local action the link differentiation generates the terms than the Fermion field differentiation misses