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Screening without dynamical fermions

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- **Screening with dynamical quarks**
- **Nontrivial spectra of two dimensional gauge theories**
- **Lattice: partition function and its continuum limit**
- **Adding external charges:**

  Wilson loops and Polyakov lines

  continuum limit

  interpretation

  theta states

  screening and effective fractional charge
- **Fractional charges on a lattice and the new/classical continuum limit**
- **Nonabelian case**

I. Screening *with* dynamical fermions

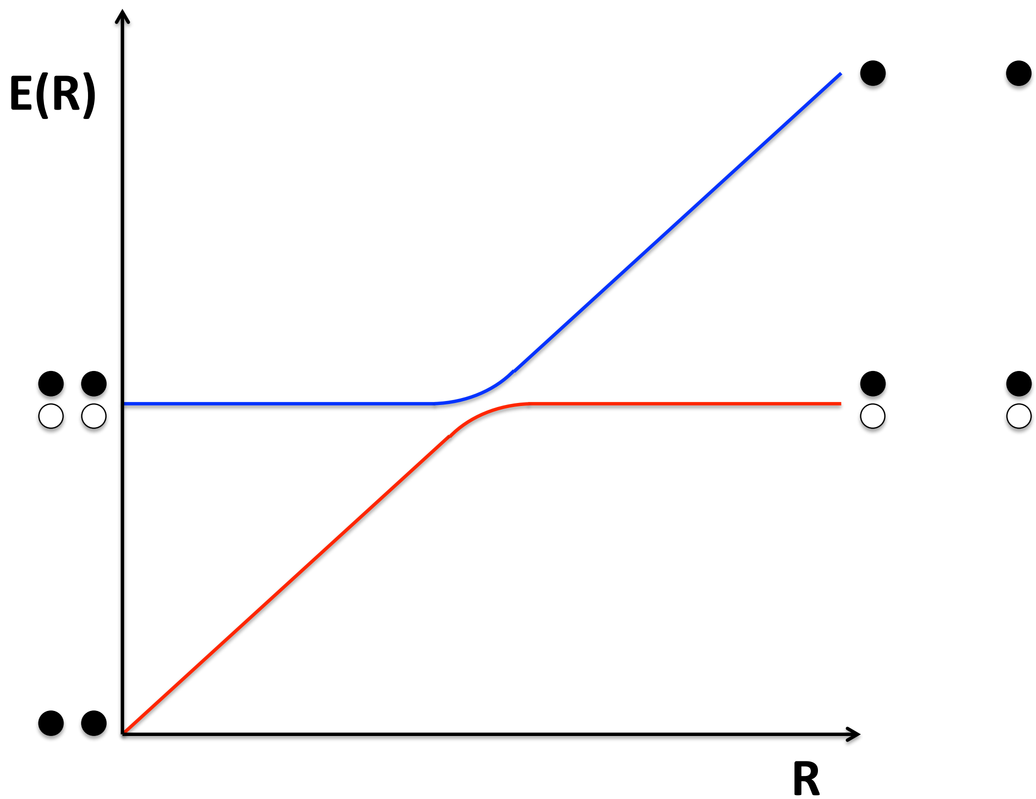


Figure 1:

II. Nontrivial spectra of trivial gauge theories

- **Two dimensional gauge theories are trivial - no transverse degrees of freedom.**
- **True only if we neglect boundary conditions.**

Quantum Maxwell Dynamics in 1+1 dimensions (QMD_2) on a circle

$$E_n = \frac{e^2}{2} L n^2, \qquad n = 0, \pm 1, \pm 2, ... \qquad\qquad [Manton,' 84]$$

An effective 1DOF hamiltonian

$$H = -\frac{e^2}{2L} \frac{d^2}{dA^2}, \qquad 0 \le A \; < \; L_A = \frac{2\pi}{L} \tag{1}$$

The spectrum

$$\psi_n(A) = e^{inAL} = e^{ip_n A}, \qquad p_n = n\frac{2\pi}{L_A} = nL, \qquad E_n = \frac{e^2}{2} L n^2 \tag{2}$$

What is A ?

$$A_x(x,t) = A(x,t), \qquad \xrightarrow{\partial_x A(x,t) = 0} A(x,t) = A(t) \neq 0$$

In a periodic (in x) world one cannot set a constant A to 0 by a gauge transformation – 1 DOF left

Why periodicity in A ?

If space is periodic, gauge transformations also have to be periodic (up to $2\pi n$)

$$g(x) = e^{i\Lambda(x)} = g(x+L), \quad \longrightarrow \quad \Lambda(x+L) = \Lambda(x) + 2\pi n$$

Take $\Lambda(x) = 2\pi\frac{x}{L}$**, then**

$$A \longrightarrow A + \partial_x \Lambda(x) = A + \frac{2\pi}{L}, \quad \textit{are gauge equivalent} \quad \Longrightarrow \quad A \in (0, \frac{2\pi}{L}]$$

Interpretation

- **a string with n units of electric flux winding around a circle**
- **Gauss's law satisfied thanks to the nontrivial topology - topological strings**
- **electric charge even without electrons/sources !**

A generalization: Θ parameter

a)

$$H = -\frac{e^2}{2L}\left(\frac{d}{dA} + i\Theta L\right)^2,$$

$$E_n = \frac{e^2}{2}L(n+\Theta)^2, \quad \psi_n(A) = e^{inAL}$$

b)

$$\tilde{H} = -\frac{e^2}{2L}\frac{d^2}{dA^2},$$

$$E_n = \frac{e^2}{2}L(n+\Theta)^2, \quad \tilde{\psi}_n(A) = e^{i(n+\Theta)AL},$$

$$\tilde{\psi}_n(A) = e^{i\Theta AL}\psi_n(A)$$

Interpretation: $e\Theta$ – classic, constant electric field

II. QMD_2 on a lattice

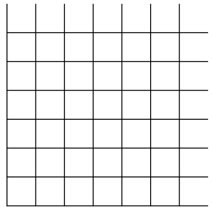


Figure 2:

Partition function on a 2x2 lattice

$$\begin{aligned} Z \;=\; & \int_0^{2\pi} B(\theta_{12} + \vartheta_{22} - \theta_{11} - \vartheta_{12}) B(\theta_{22} + \vartheta_{12} - \theta_{21} - \upsilon_{22}) \\ & \quad B(\theta_{11} + \vartheta_{21} - \theta_{12} - \vartheta_{11}) B(\theta_{21} + \vartheta_{11} - \theta_{22} - \vartheta_{21}) \\ & d(links) \end{aligned}$$

$$B(\phi_P) = e^{\beta \cos(\phi_P)}, \quad d(links) = \Pi_l \frac{d\alpha_l}{2\pi}$$

A character expansion (Fourier analysis on a group)

$$B(\phi) = \Sigma_{n=-\infty}^{\infty} I_n(\beta) \exp{(in\phi)},$$

The partition function ”almost” factorizes

$$Z = \Sigma_n I_n(\beta)^4 \;\; \longrightarrow \;\; \Sigma_n I_n(\beta)^{N_V}, \quad N_V = N_t * N_x.$$

The continuum limit

$$Z \;=\; \#\, \Sigma_n \left(\frac{I_n(\beta)}{I_0(\beta)}\right)^{N_x * N_t},$$

$$aN_t = T, \quad aN_x = L, \quad \beta = \frac{1}{e^2 a^2}, \quad a \to 0.$$

Asymptotic expansion of modified Bessel function

$$I_n(\beta) \to \frac{e^\beta}{\sqrt{2\pi\beta}} \left(1 - \frac{4n^2 - 1}{8\beta} + ...\right)$$

gives

$$Z_{LQMD_2} \;\to\; \#\; \Sigma_n \left(1 - \frac{e^2}{2} n^2 a^2\right)^{N_x N_t} = \Sigma_n e^{-E_n T}, \quad E_n = \frac{1}{2} e^2 n^2 L,$$

$\longrightarrow$ **Manton fluxes result in the continuum limit of lattice QMD_2**

Emergence of a constant mode - Coulomb gauge on a lattice

A single row of $N_x = 3$ horizontal links $\theta_1, \theta_2, \theta_3$

A local gauge transformation specified by $\alpha_1, \alpha_2, \alpha_3$

$$
\begin{aligned}
\theta_1 &\rightarrow {}^g\theta_1 = \theta_1 + \alpha_1 - \alpha_2 \\
\theta_2 &\rightarrow {}^g\theta_2 = \theta_2 + \alpha_2 - \alpha_3 \\
\theta_3 &\rightarrow {}^g\theta_3 = \theta_3 + \alpha_3 - \alpha_1
\end{aligned}
$$

or

$$
{}^g\theta_i = \theta_i + \beta_i, \quad \Sigma_{i=1}^{3}\beta_i = 0
$$

If we choose

$$
\begin{aligned}
\beta_1 &= \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) - \theta_1 \\
\beta_2 &= \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) - \theta_2 \\
\beta_3 &= \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) - \theta_3
\end{aligned}
$$

then all new link angles are equal

$$^g\theta_1 =^g \theta_2 =^g \theta_3 = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) \equiv \theta_{row}.$$

$\Longrightarrow$**Only one degree of freedom remains**

- Volume reduction

III. Adding external charges

Wilson loops

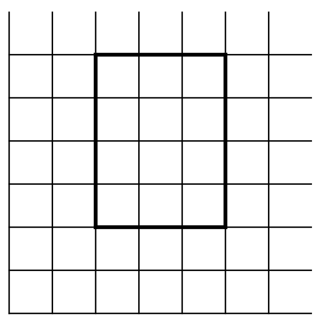


Figure 3:

$$W[\Gamma] = \Pi_{l \in \Gamma} e^{i\theta_l} \tag{3}$$

$$Z\langle W \rangle = \Sigma_n I_n(\beta)^{N_x * N_t - n_x * n_t} I_{n+1}(\beta)^{n_x * n_t}. \tag{4}$$

Time like Polyakov loops

$$Z < P^\dagger(1) P(1+n_x) >= \Sigma_n I_n(\beta)^{N_t * (N_x - n_x)} I_{n+1}(\beta)^{N_t * n_x}, \tag{5}$$

Continuum limit

$$aN_t = T, \quad aN_x = L, \quad \beta = \frac{1}{e^2 a^2}, \quad a \to 0.$$

$$Z < P(0)^\dagger P(R) >= \Sigma_n e^{-E_n^{PP} T}, \tag{6}$$

with

$$E_n^{PP} = \frac{e^2}{2} \left(n^2 (L-R) + (n+1)^2 R \right), \quad n = 0, \pm 1, \pm 2, ... \tag{7}$$

A straightforward interpretation:

$$E_n^{PP} = \frac{e^2}{2}\left(n^2(L-R) + (n+1)^2 R\right), \quad n = 0, \pm 1, \pm 2, ... \tag{8}$$

- Time like Polyakov lines modify Gauss's low at spatial points 0 and R - they introduce external unit charges at these positions.
- Such charges cause additional unit of flux extending over distance R.
- Hence the two contributions to the eigenenergies: an "old" flux over the distance $L-R$ and the new one, bigger by one unit (fluxes are additive !) , over R.
- Interesting special cases:

  → at large T the lowest, $n = 0$ and $n = -1$, states dominate. Then we just have standard (unit flux) strings of length R and L-R ,

  → $R = 0$ – old topological flux with charge n.

  → $R = L$ – when external charges meet at the "end point" of a circle, they annihilate ($e^+\delta_P(0) + e^-\delta_P(L) = 0$)and leave behind a topological string with length L and charge bigger by one unit.
- Varying R interpolates between integer valued topological fluxes.

Equivalent form

$$E_n^{PP} = \frac{e^2}{2}L(n+\rho)^2 + const.(L,R), \quad \rho = \frac{R}{L}, \quad const. = \frac{e^2}{2}L\rho(1-\rho) \tag{9}$$

- Indeed $e\frac{R}{L}$ is the electric field, generated by two sources, *averaged* over the whole volume.
- The system does not see any distances, $A_x(x) = const.$, hence averaging over the volume.
- Changing R allows to mimic arbitrary real charge $q = e(n+\rho)$.
- Only $[\rho]$ is relevant.

- **Θ parameter acquires now a straightforward interpretation**

$$\Theta_{Manton} = \rho = \frac{R}{L},$$

- **A new constant term.**

Θ-vacua

- The transformation $A \longrightarrow A + \frac{2\pi}{L}$ is a large gauge transformation, $\Lambda(x) = \frac{2\pi x}{L}$, $\Lambda(x + L) = \Lambda(x) + 2\pi$
- Full analogy 4D YM and/or the crystal : many classical configurations around which we can quantize
- Θ vacua: $|\Theta\rangle = \Sigma_m e^{i\Theta m}|m\rangle$
- The wave function of a Θ-state $\psi_\Theta(x) = \langle x|\Theta\rangle$ satisfies $\psi_\Theta(x-d) = e^{i\Theta}\psi_\Theta(x)$
- The solution (Bloch theorem) : $\psi_\Theta(x) = e^{i\Theta x/d}u_\Theta(x)$, with periodic $u_\Theta(x)$
- Our case: $\psi_n(A) = e^{i(n+\rho)AL} = e^{i\rho AL}e^{inAL}$ is exactly of Bloch type upon identification $x \to A$, $d \to 2\pi/L$, $\Theta \to 2\pi\rho$

- Introducing external charges fixes the Θ-vacuum in QMD_2.

- D=4 : in a Θ-vacuum some field configurations acquire electric charge [Witten '76].

More, different charges

R_2 - distance between doubly charged sources
R_1 - distance between singly charged ones

$$Z < P(i)^\dagger P(j)^{2\dagger} P^2(j+n_2) P(i+n_1) >=$$

$$\Sigma_n I_n(\beta)^{N_t(N_x-n_1)} I_{n+1}(\beta)^{N_t(n_1-n_2)} I_{n+3}(\beta)^{N_t n_2},$$

- eigenenergies in the continuum limit

$$\begin{aligned} E_n^{PPPP} &= \frac{e^2}{2}\left(n^2(L-R_1)+(n+1)^2(R_1-R_2)+(n+3)^2R_2\right) \\ &= \frac{e^2}{2}L\left((n+\rho_1+2\rho_2)^2+\rho_1(1-\rho_1)+4\rho_2(2-\rho_1-\rho_2)\right) \end{aligned}$$

etc. 1 DOF quantum mechanical systems can be also readily constructed.

- This time $\Theta = (R_1+2R_2)/L$, i.e. it is again equal to the external field averaged over the whole volume.

IV. Arbitrary charges on a lattice

Why? To learn about screening

Massive Schwinger model

$$\sigma_q = m\, e \left(1 - \cos\left(2\pi \frac{q}{e}\right)\right) \qquad m/e << 1, \qquad [Coleman\ et\ al.,\ '75]$$

$\Rightarrow$ generalizations for large N QCD_2.

$\Rightarrow$ How to put arbitrary (noncongruent with e) charges on a lattice?

- One way: as above $q = e(n + R/L)$
- Another way: new observables

Wilson loops with arbitrary charge

$$Z\langle W_Q\rangle = \int (W[\Gamma])^Q \, e^{-S}, \qquad Q = q/e$$

Contras:

gauge invariance – not if you carefully/consistently deal with multivaluedness

dependence on the boundaries in angular variables – not if you do loops

Pros:

Results are consistent ($MC \leftrightarrow TH$)

New structure appears QMD_2

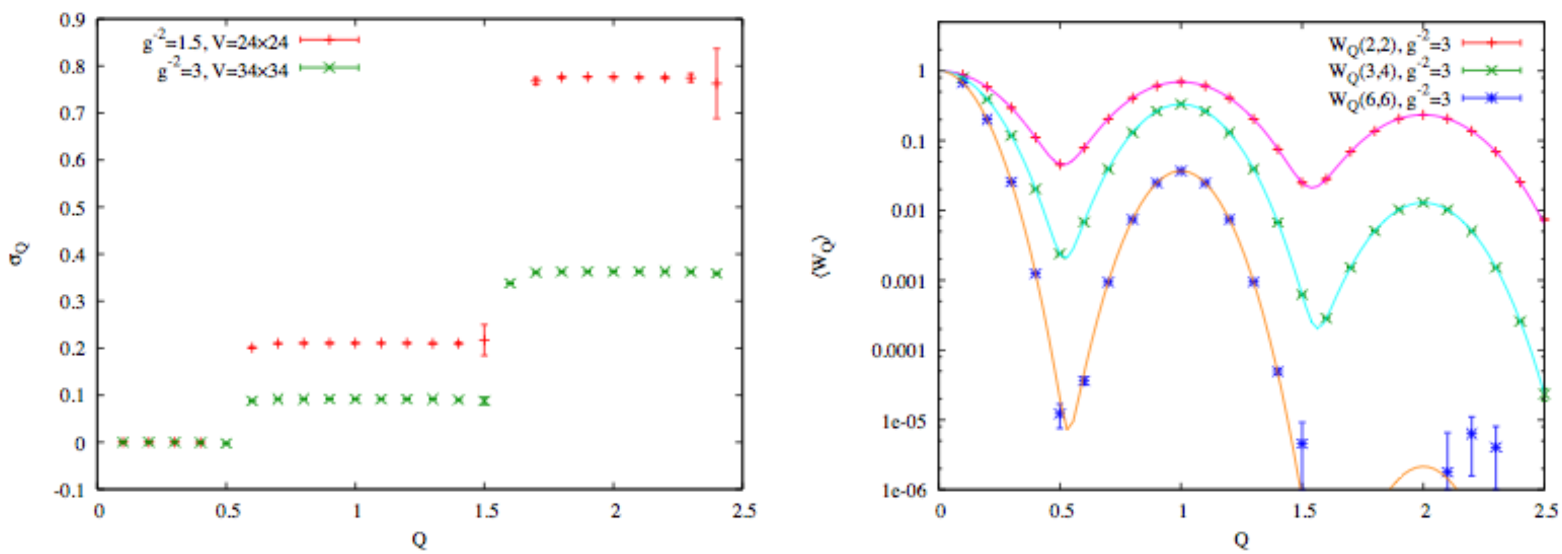
Why not !

Q-loops theoretically

$$Z\langle W_Q\rangle \;=\; \Sigma_{m,n} I_n^{N_x N_t - n_x n_t} I_m^{n_x n_t} S(Q-m+n)^{n_x+n_t},$$

$$S(x) = \left(\frac{\sin \pi x}{\pi x}\right)^2$$

and "experimentally" [P. Korcyl, M. Koren]



(a) String tension as a function of Q

(b) Ansatz vs. data (not a fit)

Figure 4:

- Q-loops can be defined on a lattice - MC agrees with TH
- They *do not* create states with arbitrary charge
  – they excite the only existing quantum states with integer charges

Continuum limit

$$Z\langle W_Q\rangle \longrightarrow \Sigma_{m,n} \exp\left(-\frac{e^2}{2}n^2 L(T-t)\right)\exp\left(-\frac{e^2}{2}\left(n^2(L-R)+m^2R\right)t\right)$$
$$S(Q-(n-m))^{(t+R)/a}$$

does not exist at fixed, not integer Q.

$\Longrightarrow$ However the *classical* limit:

$Q \to \infty$, with $q = Qe - fixed$, on a fixed lattice $(a, N's, const.)$

does exist!

Then $\beta \equiv b^2 = 1/e^2 a^2 \to \infty$, but not because $a \to 0$,
but because $e \to 0$.
The spectrum of fluxes becomes continuous: $n \to u = n/b, m \to v = n/b$

Therefore $(Q = q/e = \sqrt{\beta/\kappa} = b/g, g = 1/qa)$

$$ZK_{\Pi QQ} = \beta \int du dv \exp\left(-\frac{1}{2}(u^2(N_x - n_x) + v^2 n_x)\right)$$
$$S\left(b(g^{-1} - (u - v))\right)^2 e^{ibu(\Theta_{L-R} - \Theta'_{L-R})} e^{ibv(\Theta_R - \Theta'_R)}$$

using

$$S(b\Delta) \xrightarrow{b\to\infty} \frac{1}{b}\delta(\Delta)$$

gives

$$ZK_{\Pi QQ} = \sqrt{\beta} \int du \exp\left(-\frac{1}{2}(u^2(N_x - n_x) + (u - g^{-1})^2 n_x)\right)$$
$$e^{ibu(\Theta_{L-R} - \Theta'_{L-R})} e^{ib(u-g^{-1})(\Theta_R - \Theta'_R)}$$

Now, do the gaussian integral, take the continuum limit to obtain

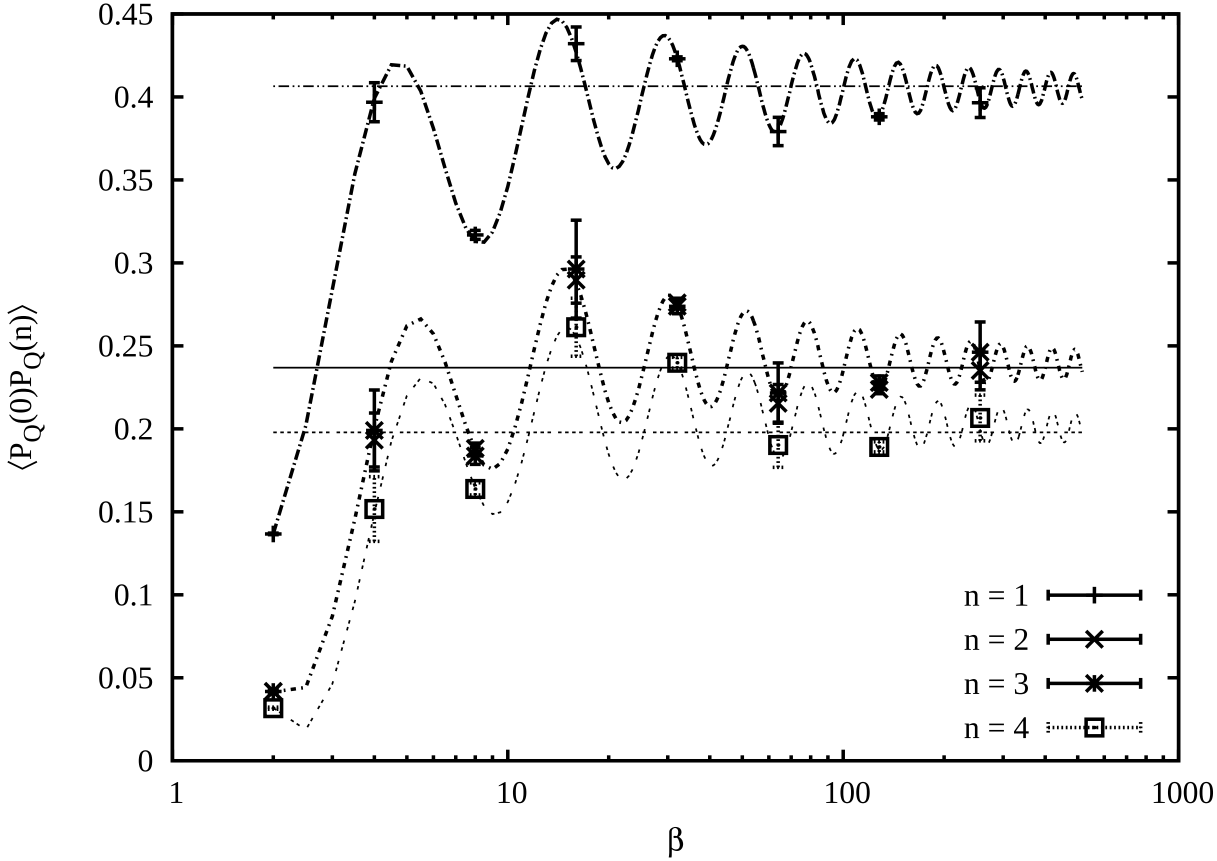
$$ZK_{\Pi QQ} = \sqrt{\beta}\sqrt{\frac{2\pi a}{L}}\exp\left(-\frac{L}{2}\frac{(A-A')^2}{a}\right)\exp\left(-\frac{q^2}{2}\rho(1-\rho)La\right)$$

$\Longrightarrow$ a free particle propagating over a time a, but in a constant background potential

$$V = \frac{q^2}{2}\rho(1-\rho)L$$

with arbitrary, real value of a classical charge q.

- The classical energy with a continuous charge q results from the contribution of many microscopic states with discrete charges.
- the structure (zeroes of the string tension)



0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
10
100
1000
β
⟨P_Q(0)P_Q(n)⟩
n = 1
n = 2
n = 3
n = 4

V. Charge quantization

The connection between universality of a charge and compactness of a gauge group is general.

Also valid in our world (i.e. 3+1 QED) .

The same charge of e, μ, p, ... $\longleftrightarrow$ U(1) gauge transformations are compact.

C. N. Yang, PRD **1** (1970) 2360.

V. Nonabelian case: YM_2 on a circle

• Continuum: problem reduces to N constant in space, but constrained, angles θ_i, $\Sigma_i \theta_i = 0$.

Hamiltnian is again quadratic and the spectrum is known explicitly [Hetrick and Hosotani '89]

$$E_{\{n\}} = \frac{g^2 L}{4}\left(\Sigma_i n_i^2 - \frac{1}{N}\left(\Sigma_i n_i\right)^2\right), \quad i = 1, ..., N-1$$

• Continuum: different spectrum was obtained by Rajeev: $E_R = \frac{g^2 L}{2} C_2(R)$
• Discrepancy comes from the Casimir energy due to the curvature of the group manifold [Hetrick '93, Witten '91,'92]
• External charges in YM_2 – studied by many [Semenoff et al. '97] but above interpretation in terms of screening was not.

EU grant (via Foundation for Polish Science)

Jagellonian University International PhD Studies in Physics of Complex Systems

- 1 M Euro
- 4 years
- 14 PhD students (1/2 - 2 years abroad)
- 9 Local Supervisors
- 17 Foreign Partners: J. Ambjorn, J.P. Blaizot, H. Nicolai, S. Sharpe, ...