The charmonium states $X(3872)(1^{++})$ and $Z_c(3900)(1^{+-})$ on HISQ lattices

The Fermilab Lattice and MILC Collaborations

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1. Motivation

- As part of our ongoing study of charmonium levels, we are currently studying the $X(3872)$ and the Z_c(3900) using clover charm quarks (Fermilab interpretation) and HISQ light valence quarks on the MILC configurations with 2+1+1 flavors of HISQ sea quarks. Here, we present results from a preliminary study.
- The $X(3872)$ state with $J^{PC} = 1^{++}$ is one of the better established mysterious charmonium states found in B-meson decays by both Belle [1] and CDF [2] and studied with more precision by CDF [3], D0 [4], BABAR [5, 6], Belle [7] and LHCb [8, 9]. Its mass is remarkably close to the $D^*\bar{D}$ threshold with $M(3872) - M(D^0) - M(D^{0*}) = -0.30 \pm 0.40$ MeV.
- The $Z_c(3900)$ 1^{+-} is a charged, isospin-one charmonium-like structure observed by the BESIII collaboration [10] as an intermediate resonance in an analysis of e^+e^- annihilations into $J/\psi\pi^+\pi^-$ at $\sqrt{s} = 4260$ MeV. This observation has been confirmed by the Belle Collaboration [11] and by Xiao *et al.* using data from the CLEO-c detector [12]. As a charged charmonium-like structure, it must contain at least four quarks, and tetraquark and molecular interpretations have been suggested: see for example [13, 14] and [15].
- Previous lattice calculations with clover up, down, and charm quarks [16] have found evidence for the $X(3872)$. Previous lattice attempts to find a Z_c were unsuccessful [17, 18]. However, recently, with new interpolating operators, Prelovsek *et al.* report evidence for a Z_c^+-like state [19].

2. Technical lattice challenges

- Managing a variational calculation with both open charm and excited closed-charm components.
- Choosing interpolating operators that couple well to these states.
- Extending variational methods to cover systems with staggered fermions.
- Managing hadron correlators that require all-to-all methods.

3. What is new here?

- HISQ fermions with lighter light quarks
- Larger box size. Important for weakly bound states.

4. Ensemble analyzed

- For this preliminary study we work with an ensemble of $16^3 \times 48$ lattices with spacing, approximately 0.15 fm, with 2+1+1 flavors of HISQ sea quarks (physical strange and charm sea quark masses and degenerate up and down quark masses set to 1/5 the strange quark mass). [20].
- The clover (Fermilab) [21] charmed quark mass is tuned approximately to the experimental D_s mass (with the HISQ action for the strange quark.)

5. Staggered variational method

- A variational approach helps to determine multiple eigenvalues of the transfer matrix [22, 23, 24]. We extend the method to staggered fermions [25].
- Define

$$C_{ij}(t) = \langle O_i(t) O_j(0) \rangle \ .$$

- The usual spectral decomposition gives

$$C_{ij}(t) = \sum_n s_n(t) z_{in} z_{jn}^* \frac{\exp(-E_n t)}{2E_n} \ .$$

where $s_n(t) = 1$ or $-(-)^t$ for nonoscillating and oscillating states.
- In terms of a pseudo-transfer matrix T with eigenvalues $\pm\exp(-E_n)$

$$C(t) = Z T^t g (2M)^{-1} Z^\dagger \ ,$$

where g is diagonal and $g_{nn} = 1$ for nonoscillating states and -1 for oscillating states, and M is a diagonal matrix with $M_{nn} = E_n$. Better, still, with $V = Z^{\dagger -1}$, we obtain the generalized eigenvalue problem:

$$C(t)V = C(t_0) V T^{t-t_0} \ ,$$

- With a sufficiently complete interpolating operator basis, we get eigenvalues $\lambda_n(t, t_0) = s_n(t) \exp[-E_n(t - t_0)]$
- In practice $\lambda_n(t, t_0)$ has contributions from higher states and often from opposite-parity states, so we fit to [26].

$$\begin{aligned} \lambda_n(t, t_0) &= a \exp[-E_n(t-t_0)] + b \exp[-E'_n(t-t_0)] \\ &- (-)^t c \exp[-\overline{E_n}(t-t_0)] - (-)^t d \exp[-\overline{E'_n}(t-t_0)] \dots \ . \end{aligned}$$

6. $X(3872)$ interpolating operators

- cc Interpolating operators ($J^{PC} = 1^{++}$)

$$\begin{matrix} \bar{c}\gamma_5\gamma_i c & , & \bar{c}\Delta\gamma_5\gamma_i\Delta c & , & \bar{c}\nabla_k\gamma_5\gamma_i\nabla_k c \\ \bar{c}\varepsilon_{ijk}\gamma_j\nabla_k c & , & \bar{c}\varepsilon_{ijk}\gamma_4\gamma_j\nabla_k c & , & \bar{c}\left|\varepsilon_{ijk}\right|\gamma_5\gamma_j\mathbb{D}_k c \end{matrix}$$

- DD^* interpolating operators ($J^{PC} = 1^{++}$)

$$\begin{aligned} (DD)(t, \boldsymbol{p}=0) &= \left[D^*(t,\mathbf{0})\bar{D}(t,\mathbf{0}) - \bar{D}^*(t,\mathbf{0})D(t,\mathbf{0})\right] + f_I\{u \leftrightarrow d\} \\ (DD)(t, \boldsymbol{p}=\mathbf{1}) &= \left[D^*(t,-\mathbf{1})\bar{D}(t,\mathbf{1}) - \bar{D}^*(t,\mathbf{1})D(t,-\mathbf{1})\right. \\ &\quad \left. + D^*(t,\mathbf{1})\bar{D}(t,-\mathbf{1}) - \bar{D}^*(t,-\mathbf{1})D(t,\mathbf{1})\right] \\ &\quad + f_I\{u \leftrightarrow d\} \end{aligned}$$

where, $f_I = +1$ for $I = 0$ and $f_I = -1$ for $I = 1$.
Each charmed meson interpolating operator is given by
$D(t, \boldsymbol{p}) = \sum_x e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \overline{q}(\boldsymbol{x}, t)\gamma_5 c(\boldsymbol{x}, t)$, $D^*(t, \boldsymbol{p}) = \sum_x e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \overline{q}(\boldsymbol{x}, t)\gamma_i c(\boldsymbol{x}, t)$
Stochastic and smeared-stochastic sources are used throughout.

7. $X(3872)$ Diagrams

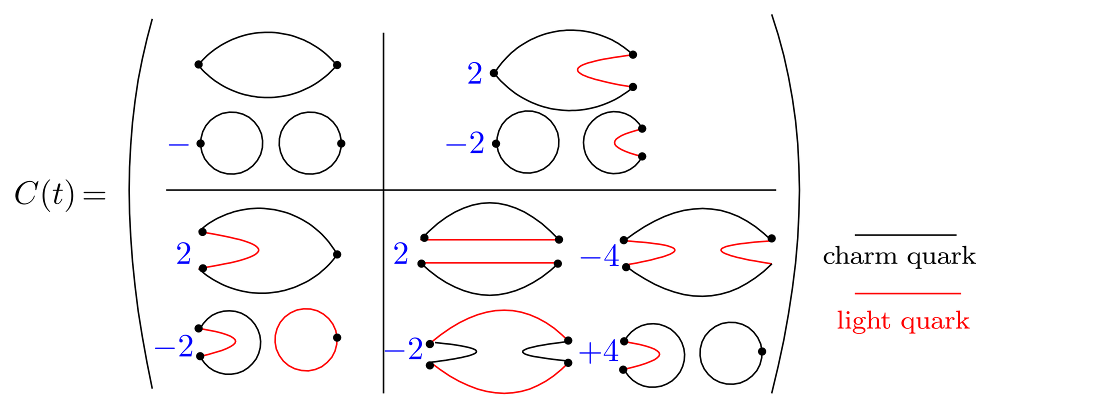

Figure 1. $X(3872)$ Quark-line diagrams for the hadronic correlator matrix in this calculation. We are not including charm quark annihilation, because it is expected to be negligible at our level of precision, so we omit the second row in each panel above.

8. $X(3872)$ channel effective mass

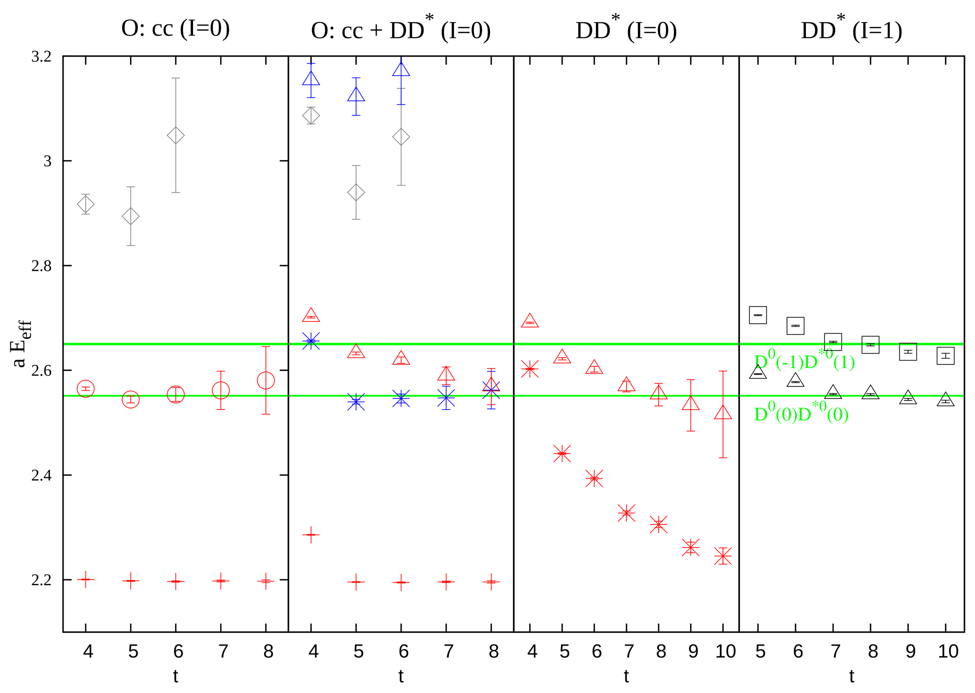

Figure 2. Effective masses from the lowest few eigenvalues in the style of Ref. [16]. Each panel shows the result of including a different set of interpolating operators. The green lines correpond to the energies of non-interacting $\bar{D}(\boldsymbol{p})D^*(-\boldsymbol{p})$ scattering states. The lower one represents $\bar{D}(\mathbf{0})D^*(\mathbf{0})$ and upper, $\bar{D}(\mathbf{1})D^*(-\mathbf{1})$. The symbols represent effective masses for different sets of interpolating operators: panel (a): cc only, (b): mixing cc and $\bar{D}D^*$, (c): $\bar{D}D^*$ with isospin 0 and (d) $\bar{D}D^*$ with isospin 1 and $J^{PC} = 1^{++}$. It is likely that the state represented by blue stars is the $X(3872)$

9. $X(3872)$ Spectrum

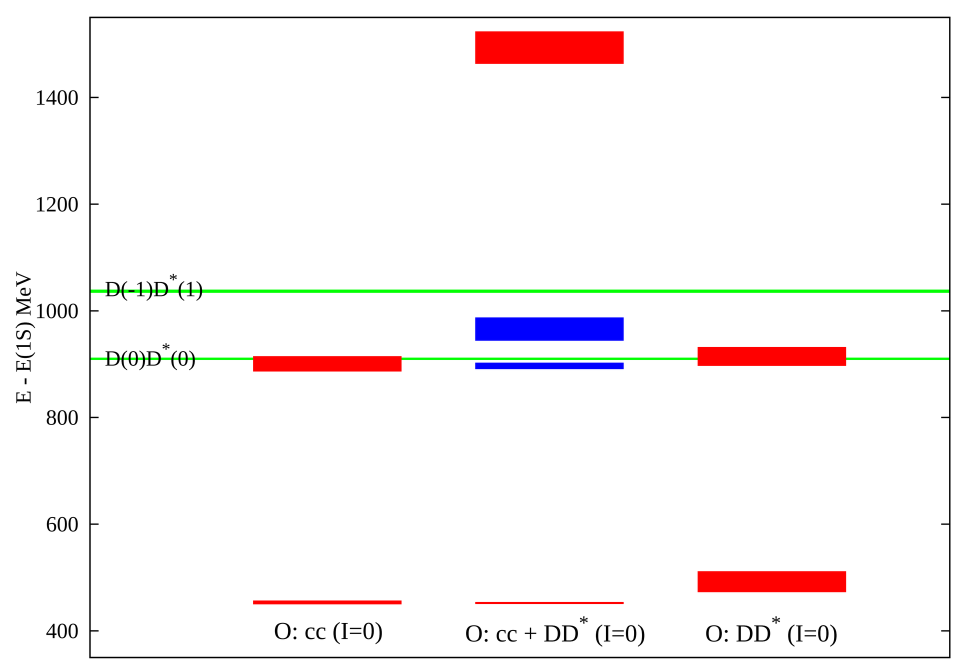

Figure 3. Energy levels from the variational calculation in MeV expressed as a splitting relative to the spin-averaged 1S charmonium levels. Panels have the same meanings as in first three in the previous figure. Left panel: the unmixed $\chi_{c1}(1P)$ and $\chi_{c1}(2P)$ states. Middle panel: mixed cc and DD^* states resulting in the $X(3872)$ and a DD^* scattering states. Right panel: the unmixed $D(\mathbf{0})D^*(\mathbf{0})$ and $D(\mathbf{1})D^*(-\mathbf{1})$ states. The lower blue bar represents the $X(3872)$ with binding energy relative to the $D\bar{D}^*$ threshold of 13(6) MeV with our unphysical lattice parameters.

10. $Z_c(3900)$ diagrams

- Hadronic correlation matrix

$$C(t) = \begin{pmatrix} \left\langle 0 \middle| (J/\psi\pi)(t)(J/\psi\pi)^\dagger(t_s) \middle| 0 \right\rangle & \left\langle 0 \middle| (DD)(t)(J/\psi\pi)^\dagger(t_s) \middle| 0 \right\rangle \\ \left\langle 0 \middle| (J/\psi\pi)(t)(DD)^\dagger(t_s) \middle| 0 \right\rangle & \left\langle 0 \middle| (DD)^\dagger(t)(DD)(t_s) \middle| 0 \right\rangle \end{pmatrix}$$

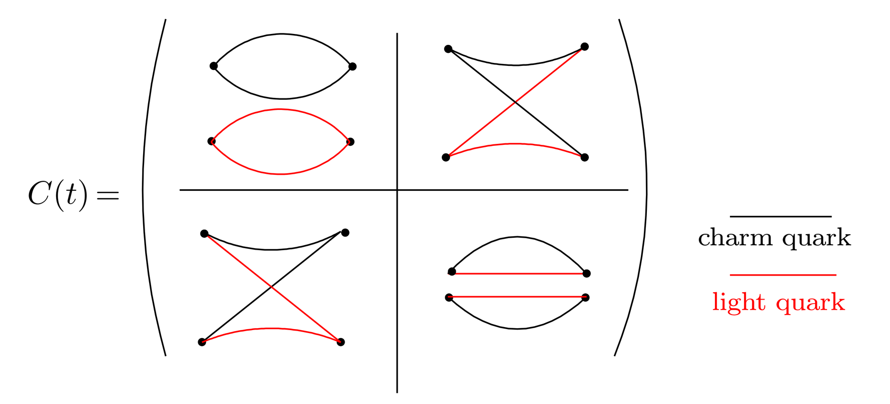

Figure 4. Diagrammatic representation of the hadronic correlation matrix for the $Z_c(3900)$.

11. $Z_c(3900)$ interpolating operators

- cc Interpolating operators [J/ψ and $\psi(2S)$] ($J^{PC} = 1^{--}$)

$$\begin{matrix} \bar{c}\gamma_i c & , & \bar{c}\gamma_4\gamma_i c & , & \bar{c}\nabla_i c \\ \bar{c}\varepsilon_{ijk}\gamma_5\gamma_j\nabla_k c & , & \gamma_5\mathbb{B}_i & , & \gamma_4\gamma_5\mathbb{B}_i \end{matrix}$$

- cc, π Interpolating operators ($J^{PC} = 1^{+-}$)

$$\begin{aligned} (cc,\pi)(t, \boldsymbol{p}=\mathbf{0}) &= cc(t,\mathbf{0})\pi(t,\mathbf{0}) \\ (cc,\pi)(t, \boldsymbol{p}=\mathbf{1}) &= cc(t,-\mathbf{1})\pi(t,\mathbf{1}) + cc(t,\mathbf{1})\pi(t,-\mathbf{1}) \end{aligned}$$

- DD^* interpolating operators ($J^{PC} = 1^{+-}$)

$$\begin{aligned} (DD)(t, \boldsymbol{p}=\mathbf{0}) &= \left[D^*(t,\mathbf{0})\bar{D}(t,\mathbf{0}) + \bar{D}^*(t,\mathbf{0})D(t,\mathbf{0})\right] - \{u \leftrightarrow d\} \\ (DD)(t, \boldsymbol{p}=\mathbf{1}) &= \left[D^*(t,-\mathbf{1})\bar{D}(t,\mathbf{1}) + \bar{D}^*(t,\mathbf{1})D(t,-\mathbf{1})\right. \\ &\quad \left. + D^*(t,\mathbf{1})\bar{D}(t,-\mathbf{1}) + \bar{D}^*(t,-\mathbf{1})D(t,\mathbf{1})\right] \\ &\quad - \{u \leftrightarrow d\} \end{aligned}$$

12. $Z_c(3900)$ search results

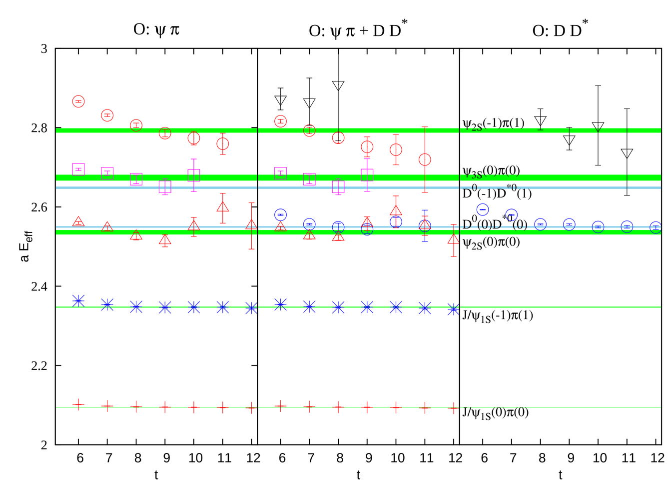

Figure 5. Effective masses in the $I = 1$, $J^{PC} = 1^{+-}$ channel.

- We see no evidence for a bound state here.

13. Conclusions and Outlook

- This preliminary study on a single lattice ensemble with an unphysical light quark mass and box size $L = 2.4$ fm finds a state consistent with the $X(3872)$, but not a Z_c.
- We are enlarging our interpolating operator basis.
- We plan a study at physical light quark masses and larger box size ($L = 4.8$ fm).

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