Correlation functions with Karsten-Wilczek fermions

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Introduction

Minimally doubled fermions are a category of lattice fermion actions that retains chiral anomaly and a chiral symmetry while having the minimal number of degenerate fermions — two — which are combined as a degenerate doublet of quarks. Karsten-Wilczek fermions [5] are a type of minimally doubled fermions that retains two-original pairs of the naive Dirac operator on exactly one Euclidean site. Since the poles of minimally doubled Dirac operators generally lie at different points in the Dirac mass, the intermediate lattice symmetry is necessarily broken.

Karsten-Wilczek action at tree level

The Karsten-Wilczek fermion action reads (with Wilson parameter $\xi > 1$)

$$S[N, \psi, \bar{\psi}] = \sum_{x} \sum_{\alpha} \left\{ -\bar{\psi}_x \left( 2 \mathcal{D}_x - \left( \mathcal{D}_x \right)^{\dagger} \right) \psi_x + \frac{\xi}{2} \bar{\psi}_x \left( \mathcal{D}_x \right)^{\dagger} \mathcal{D}_x \psi_x \right\},$$

and the gauge action is assumed to be Wilson's plaquette action. The standard discrete symmetries of this action were reviewed first in [2] and read

- $\gamma^5$-invariance of the Dirac operator,
- spatial rotations,
- spatial reflections (parity $P$),
- the product of charge conjugation ($C$) and time reflection ($\bar{T}$).

Eq. (1) differs from the original Karsten-Wilczek action [6] by the additional term induced by $\xi$. The continuum limit of the action is dominated by one-particle eigenstates. The additional term is responsible for the breaking of $\gamma^5$-invariance.

Lattice operators

Lattice fermions for $\mathcal{O}_{55}$ can be constructed using $\hat{T}$ as in the chiral limit for Wilson fermions. Various $\mathcal{O}_{55}$ operators require $\mathcal{T}$ to have next-to-next neighbour terms in $\mathcal{O}_{55}$. The two additional independent five-lattice fermion operators are chosen as

$$\mathcal{O}_{55} = \mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{15} = \frac{1}{4} \sum_{n=0}^{N-1} \sum_{i,j} \bar{\psi}_i \left( \mathcal{D}_i \right)^{\dagger} \mathcal{D}_j \psi_j - \frac{1}{4} \sum_{n=0}^{N-1} \sum_{i,j} \bar{\psi}_i \left( \mathcal{D}_j \right)^{\dagger} \mathcal{D}_i \psi_j,$$

where $\mathcal{S}_0$ are the $\mathcal{O}_{55}$-invariant correlation functions. The $\mathcal{O}_{55}$-symmetry of the naive Dirac action (of [6]) and permits two further symmetry transformations $\mathcal{F}C$ and $\mathcal{F}T$. The latter has been pointed out as “mirrored fermion symmetry” in [5]. Hence, there is one further independent non-standard discrete symmetry $\mathcal{O}_{55}$.

Oscillations in correlation functions

For real times and high temperatures, the frequency shift of a quark to antiquark correlation function $\langle \bar{c} c \rangle$ is given by $\omega_{\bar{c} c} = \omega_{c \bar{c}} - 2m$, where $m$ is the mass of the quark. This effect is known as the mass quenching effect and is responsible for the suppression of the quark-antiquark correlation function at high temperatures.

Counterterms

The action in eq. (1) requires inclusion of one relevant and two marginal counterterms that were perturbatively calculated in [3] and one-loop level

$$S_{\text{ct}} = -i g_{\text{ct}} \bar{c} \gamma^\mu (\not\partial + i \not\Lambda) c,$$

where $g_{\text{ct}}$ is another dimension-five field which absorbs the $\alpha$ such that the chiral limit is the continuum limit.

Decomposition of interacting fields

The free field has two components related by $\hat{T}$ of eq. (4).

$$\bar{c}_x = \sum_{n=0}^{N-1} \bar{c}_i \mathcal{T}_i^x, \quad c_x = \sum_{n=0}^{N-1} c_i \mathcal{T}_i^x,$$

where two kernels $\mathcal{T}_i^x$ implement an energy smearing (of [7]) and satisfy $\mathcal{U} \mathcal{T}^x \mathcal{U}^{-1} = \mathcal{T}_i^x$ [8]. These two components lead to new correlation functions with alternating sign similar to cases of naive or staggered fermions. With interactions, the operator of eq. (5) cancels effects that would spoil the continuum limit. In the following, approximate tuning is assumed, where the mismatch of tuning, $\delta c = c - c(\bar{c})$, is considered small. A local field transform

$$\bar{c}_x = \sum_{n=0}^{N-1} \bar{c}_i \mathcal{T}_i^x, \quad c_x = \sum_{n=0}^{N-1} c_i \mathcal{T}_i^x,$$

that modifies the temporal boundary condition (of [2]), absorbs the mismatch $\delta c$ into the definition of the fields $\bar{c}, c$ at leading order.

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Non-perturbative tuning

Various methods for non-perturbative tuning of $\mathcal{O}_{55}$ are compared here. Interpolation uses a Padé fit up to $\mathcal{O}(\bar{c})$ with $\mathcal{O}(c)$, fixed by eq. (8).

Literature