The in-medium heavy quark potential from quenched and dynamical lattice QCD

Alexander Rothkopf
Institute for Theoretical Physics
Heidelberg University

in collaboration with:
Y. Burnier and O. Kaczmarek

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The $T>0$ $Q\bar{Q}$ potential from lattice QCD

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in \textbf{real-time}: NRQCD

\[ \frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad V_{Q\bar{Q}}(r) = \lim_{t \to \infty} \frac{i \partial_t W(r, t)}{W(r, t)} \]

for a brief review see A.R. MPLA 28 (2013) 133000 and references therein
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- Connection to Euclidean lattice QCD via spectral functions:

\[ W(r, t) = \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \rho(r, \omega) \quad \leftrightarrow \quad W(r, \tau) = \int_{-\infty}^{\infty} d\omega \, e^{-\omega \tau} \rho(r, \omega) \]
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Bayesian spectral reconstruction


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- Potential from fit of lowest lying peak (skewed Lorentzian) position = Re[V] width = Im[V]
  
  Y. Burnier, A.R.  PRD86 (2012) 051503
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  \frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \quad \text{and} \quad V^{Q\bar{Q}}(r) = \lim_{t \to \infty} \frac{i \partial_t W(r, t)}{W(r, t)}
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  - Bayesian spectral reconstruction

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- Bayesian reconstruction challenging: Need prior information to regularize ill-defined $\chi^2$ fit
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- Bayesian reconstruction challenging: Need prior information to regularize ill-defined $\chi^2$ fit

- Recent improvement over Maximum Entropy Method: new prior, analytic treatment of $\alpha$
  \[
  S = \alpha \sum_{l=1}^{N_\omega} \Delta \omega_l \left( 1 - \frac{\rho_l}{m_l} + \log \left[ \frac{\rho_l}{m_l} \right] \right)
  \]
  for more details see
  Y.Burnier, A.R. PRL 111 (2013) 18, 182003
Extraction strategy summary

- From Euclidean lattice QCD correlators to the complex heavy quark potential

Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
Practical reason: Absence of cusp divergences, hence less suppression along $\tau$
Two projects for $V^{QQ}$ from the lattice

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 \times N_\tau$
  - Fixed scale approach: $\beta=7.0$, $\xi=a_s/a_\tau=4$, $a_s=0.039\text{fm}$

<table>
<thead>
<tr>
<th>$N_\tau$</th>
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<td>$T/T_C$</td>
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<td>1.55</td>
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<td>1.17</td>
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- **Dynamical lattice QCD**: isotropic lattices with asqtad action $48^3 \times 12$ (HotQCD)

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<tr>
<th>$\beta$</th>
<th>6.80</th>
<th>6.90</th>
<th>7.00</th>
<th>7.125</th>
<th>7.25</th>
<th>7.30</th>
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<td>1.33</td>
<td>1.39</td>
<td>1.64</td>
</tr>
<tr>
<td>$a \text{ [fm]}$</td>
<td>0.111</td>
<td>0.100</td>
<td>0.090</td>
<td>0.080</td>
<td>0.071</td>
<td>0.068</td>
<td>0.057</td>
</tr>
<tr>
<td>$N_{\text{meas}}$</td>
<td>1295</td>
<td>1340</td>
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  **Focus**: Effect of light fermion on in-medium QQ interactions i.e. Re$[V]$
Towards $V^{QQ}(r)$ on quenched lattices

N$_c$=80
T=252 MeV

N$_c$=24
T=839 MeV
Towards $V^{QQ}(r)$ on quenched lattices

Bayesian reconstruction:
$N_\omega = 3000$, $I_\omega^{\num} = [-12, 25]$
$\tau_{\num}^{\max} = 20$, $m(\omega) =$ const.
512 bits precision, $\Delta_{\min}^{\text{prel}} = 10^{-60}$
Towards $V^{QQ}(r)$ on quenched lattices

- Identify the lowest lying peak and fit its shape over the Full-Width at Half Maximum
Re[V] in quenched lattice QCD

Preliminary

N_f=0

1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

Re[V] [GeV]  

r [fm]

209.7 MeV  251.6 MeV  279.6 MeV  314.6 MeV  419.4 MeV  359.5 MeV  629.1 MeV  503.3 MeV  838.8 MeV
Re[V] in quenched lattice QCD

PRELIMINARY

Transition from a confining to a Debye screened behavior
Re[V] in quenched lattice QCD

- Transition from a confining to a Debye screened behavior
- Comparison to color singlet free energies $F^{(1)}(r)$: agreement within errorbars

$$F^{(1)}(r) = -\frac{1}{\beta} \log [W_{\parallel}(r, \tau = \beta)]_{CG}$$
- Transition from a confining to a Debye screened behavior
- Comparison to color singlet free energies $F_1(r)$: agreement within errorbars
  
  $$F_1(r) = -\frac{1}{\beta} \log [W_{||}(r, \tau = \beta)]_{CG}$$

- At $T \approx T_c$ extraction $V^{QQ}$ benefits from using all datapoints instead of just $W_{||}(\tau = \beta)$
Debye-Hückel Fit of the Debye mass

\[ r \text{[fm]} \]

\[ \text{Re}[V] \text{[GeV]} \]

\[ N_c=3, N_f=0, \beta=7, \xi=3.5 \]
\[ a=0.039 \text{fm} \]

\[ m_D=0.77\pm0.13 \text{ GeV} \]
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- Since close to continuum ($T_C=270\text{MeV}$) attempt extraction of Debye mass
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Phenomenological fit form for Coulomb and string screening

\[
F^{DH}(r, T) = \frac{\sigma}{m_D(T)} \left[ \frac{\Gamma(1/4)}{2^{3/2} \Gamma(3/4)} - \frac{\sqrt{m_D(T)} r}{2^{3/4} \Gamma(3/4)} K_{1/4}(m_D^2(T)r^2) \right] - \frac{\alpha}{r} \left[ e^{-m_D(T)r} + m_D(T)r \right]
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  \]
- Within the error bars, reasonable agreement with 1-loop HTL

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S. Digal et al. EPJ C43 (2005) 71-75
Imaginary part at finite temperature

- \text{Im}[V^{QQ}] \text{ related to width: need large # of datapoints and high signal/noise}
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To improve the width reconstruction: better default model \( m(\omega) \neq \text{const.} \)
Towards $V_{QQ}^Q(r)$ on dynamical lattices

\[ W/(dr, \tau) \text{ [Lat]} \]

$\beta = 6.800$
$T = 147.8 \text{ MeV}$

$\beta = 7.480$
$T = 286.1 \text{ MeV}$
Towards $V^{QQ}(r)$ on dynamical lattices

Bayesian reconstruction:
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A brief look at $\text{Im}[V^{QQ}]$ in dynamical LQCD
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At r>0.5fm decrease in signal to noise deteriorates width determination
A brief look at $\text{Im}[V^{QQ}]$ in dynamical LQCD

- $N_\tau=12$ leads to underestimation of $\text{Im}[V]$, where signal to noise is otherwise ok.
- At $r>0.5\text{fm}$ decrease in signal to noise deteriorates width determination.
- Still: obtained values are of the same order of magnitude as the HTL prediction.
Conclusion

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\text{Re}[V^{QQ}]$ and $\text{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
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**Thank you for your attention**