UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

The in-medium heavy quark potential from quenched and dynamical lattice QCD

Alexander Rothkopf
Insitute for Theoretical Physics
Heidelberg University

in collaboration with:
Y. Burnier and O.Kaczmarek

XXXII International Symposium on Lattice Field Theory 2014
Columbia University, NY, USA 2014-06-25 12:50h

Universität Bielefeld

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Deutsche Forschungsgemeinschaft

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

- Connection to Euclidean lattice QCD via **spectral functions**:

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho(r,\omega) \quad \Longleftrightarrow \quad W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho(r,\omega)$$

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

- Connection to Euclidean lattice QCD via **spectral functions**:

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho(r,\omega) \quad \Longleftrightarrow \quad W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \boxed{\rho(r,\omega)}$$

$$V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{i\omega t} \rho(r,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{i\omega t} \rho(r,\omega)}$$

Bayesian spectral reconstruction

A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

- Connection to Euclidean lattice QCD via **spectral functions**:

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho(r,\omega) \quad \Longleftrightarrow \quad W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\rho(r,\omega)$$

$$V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{i\omega t}\rho(r,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\rho(r,\omega)}$$

Bayesian spectral reconstruction

A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

- Potential from fit of **lowest lying peak** (skewed Lorentzian) position = Re[V] width = Im[V]

Y. Burnier, A.R. PRD86 (2012) 051503

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

- Connection to Euclidean lattice QCD via **spectral functions**:

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho(r,\omega) \quad \Longleftrightarrow \quad W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\rho(r,\omega)$$

$$V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{i\omega t}\rho(r,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\rho(r,\omega)}$$

Bayesian spectral reconstruction

A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

  - Potential from fit of **lowest lying peak** (skewed Lorentzian) position = Re[V] width = Im[V]

Y. Burnier, A.R. PRD86 (2012) 051503

- Bayesian reconstruction challenging: Need prior information to regularize ill-defined χ^2 fit

The T>0 $Q\bar{Q}$ potential from lattice QCD

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Complex in-medium heavy $Q\bar{Q}$ potential from effective field theory in **real-time**: NRQCD

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1 \qquad V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{i\partial_t W(r,t)}{W(r,t)}$$

for a brief review see
A.R. MPLA 28 (2013) 133000
and references therein

- Connection to Euclidean lattice QCD via **spectral functions**:

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho(r,\omega) \quad \Longleftrightarrow \quad W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\rho(r,\omega)$$

$$V^{Q\bar{Q}}(r) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{i\omega t}\rho(r,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\rho(r,\omega)}$$

Bayesian spectral reconstruction

A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

  - Potential from fit of **lowest lying peak** (skewed Lorentzian) position = Re[V] width = Im[V]

Y. Burnier, A.R. PRD86 (2012) 051503

- Bayesian reconstruction challenging: Need prior information to regularize ill-defined χ^2 fit
  - **Recent improvement** over Maximum Entropy Method: **new prior**, analytic treatment of α

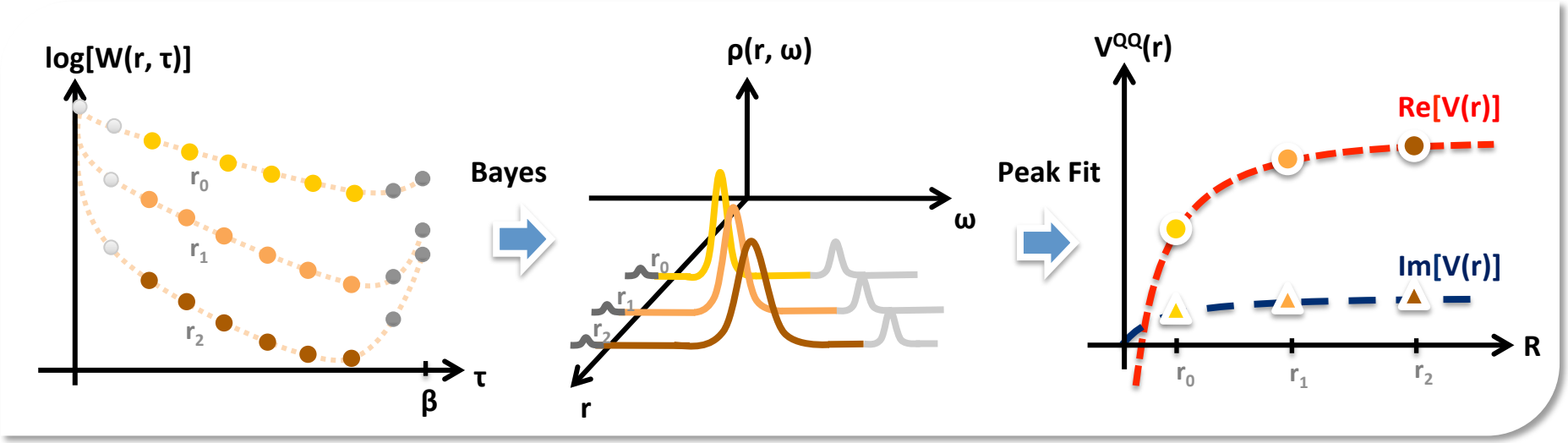
$$S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

for more details see
Y.Burnier, A.R. PRL 111 (2013) 18, 182003

Extraction strategy summary

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- From Euclidean lattice QCD correlators to the complex heavy quark potential



A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)

- Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
  Practical reason: Absence of cusp divergences, hence less suppression along τ

Two projects for V^{QQ} from the lattice

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 x N_\tau$
  with Y. Burnier
  - Fixed scale approach: $\beta=7.0$ $\xi=a_s/a_\tau=4$ $a_s=0.039$fm

N_τ	24	32	40	48	56	64	72	80	96
T/T_C	3.11	2.33	1.86	1.55	1.33	1.17	1.04	0.93	0.78
N_{meas}	2750	1570	1680	1110	760	1110	700	940	690

  - **Focus**: Achieve a large number of time steps for accurate spectral width reconstruction

Two projects for V^{QQ} from the lattice

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 x N_\tau$

  with Y. Burnier

  - Fixed scale approach: $\beta=7.0$ $\xi=a_s/a_\tau=4$ $a_s=0.039$fm

N_τ	24	32	40	48	56	64	72	80	96
T/T_C	3.11	2.33	1.86	1.55	1.33	1.17	1.04	0.93	0.78
N_{meas}	2750	1570	1680	1110	760	1110	700	940	690

  - **Focus**: Achieve a large number of time steps for accurate spectral width reconstruction

- Dynamical lattice QCD: isotropic lattices with asqtad action $48^3 x 12$ (HotQCD)

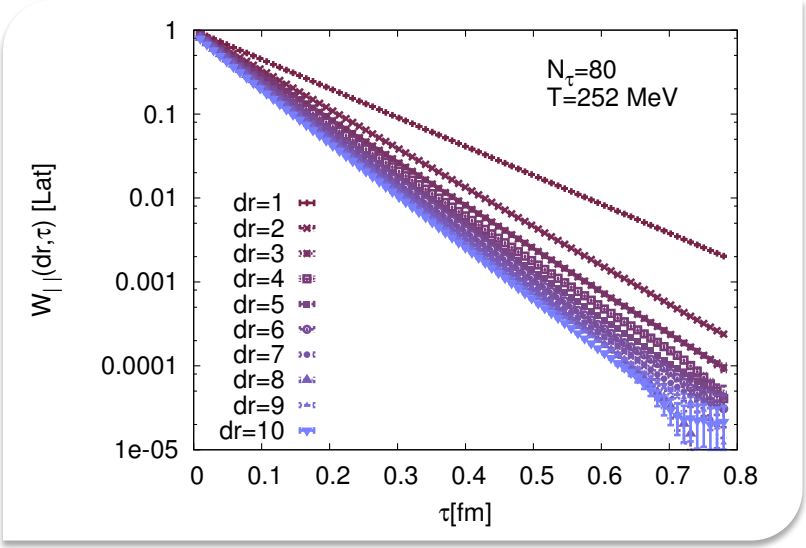
  with O. Kaczmarek

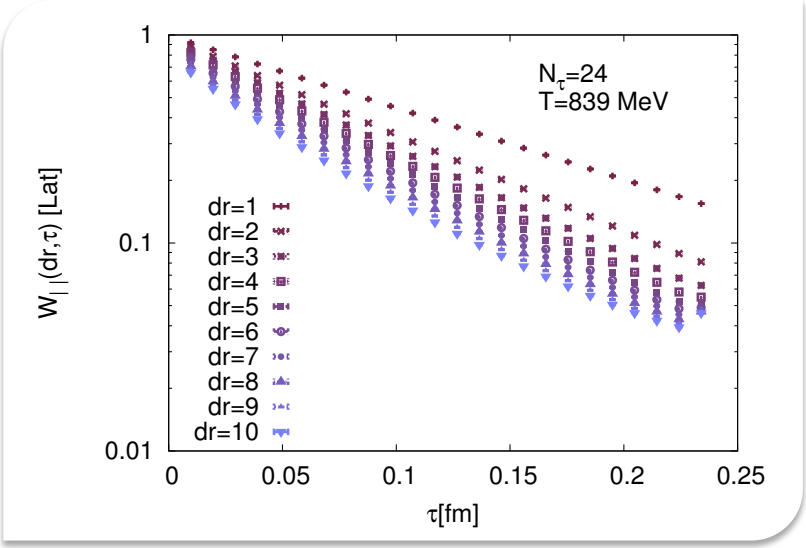
A. Bazavov et. al. PRD 85 (2012) 054503

β	6.80	6.90	7.00	7.125	7.25	7.30	7.48
T/T_C	0.85	0.94	1.04	1.18	1.33	1.39	1.64
a [fm]	0.111	0.100	0.090	0.080	0.071	0.068	0.057
N_{meas}	1295	1340	1015	840	1620	1150	1130

  - **Focus**: Effect of light fermion on in-medium QQ interactions i.e. Re[V]

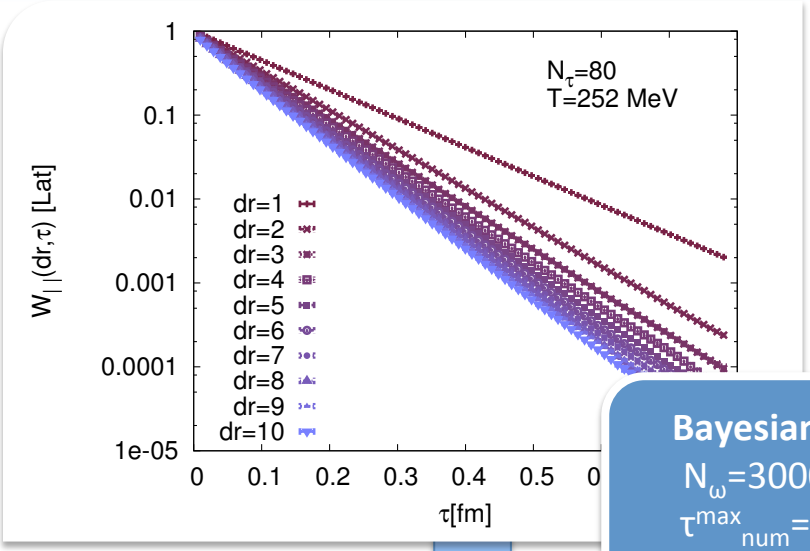
Towards $V^{QQ}(r)$ on quenched lattices

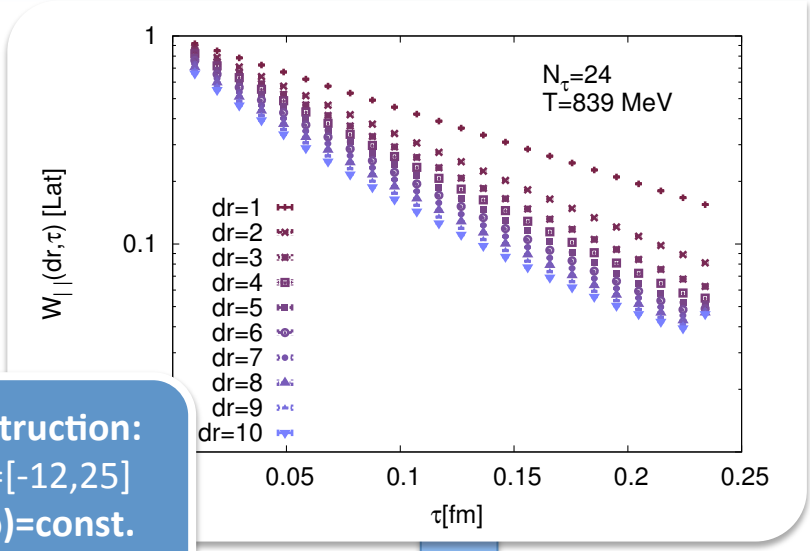




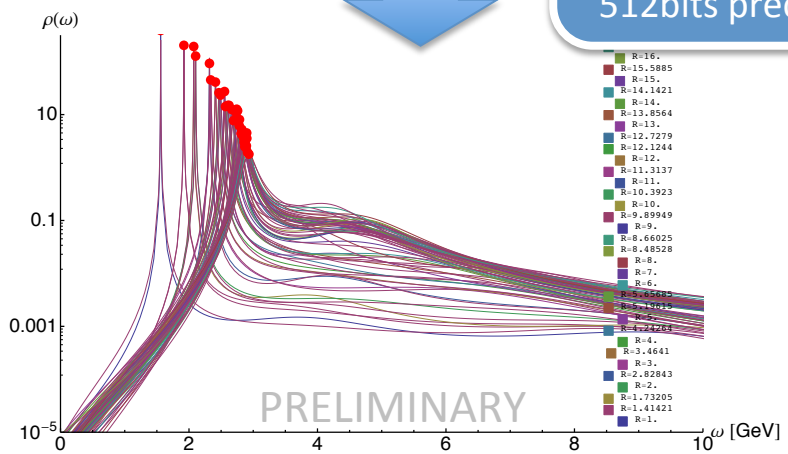
Towards $V^{QQ}(r)$ on quenched lattices

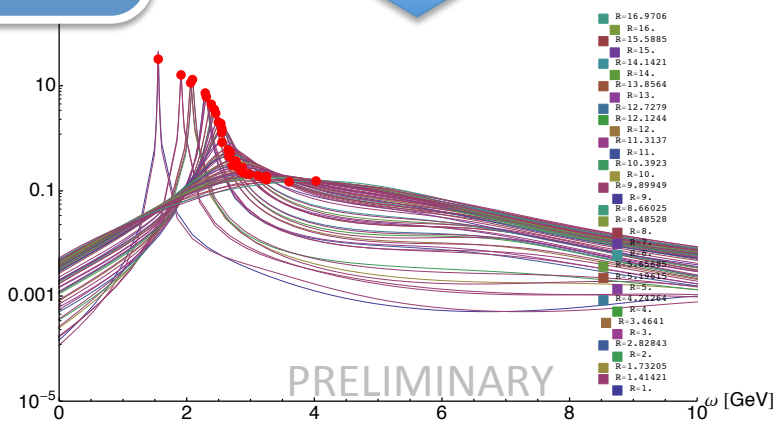
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





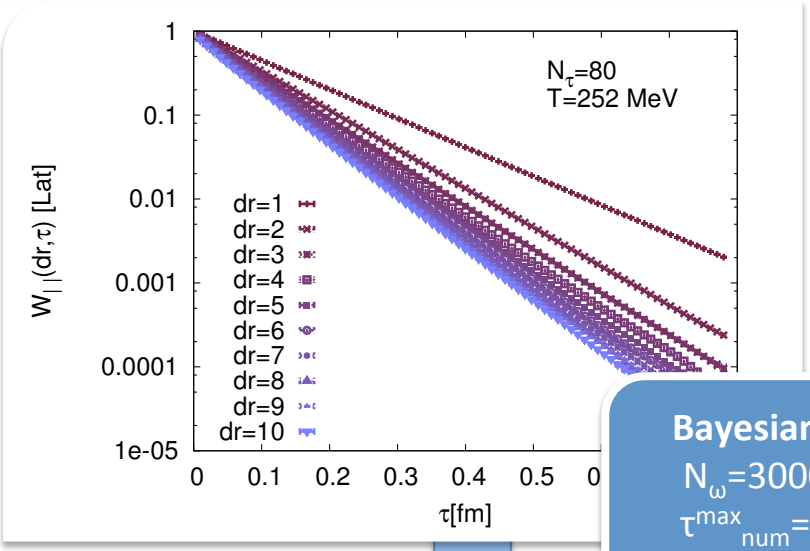
Bayesian reconstruction:
N_ω=3000, I_ω^{num}=[-12,25]
τ^{max}_{num}=20, **m(ω)=const.**
512bits precision, $\Delta^{min}=10^{-60}$

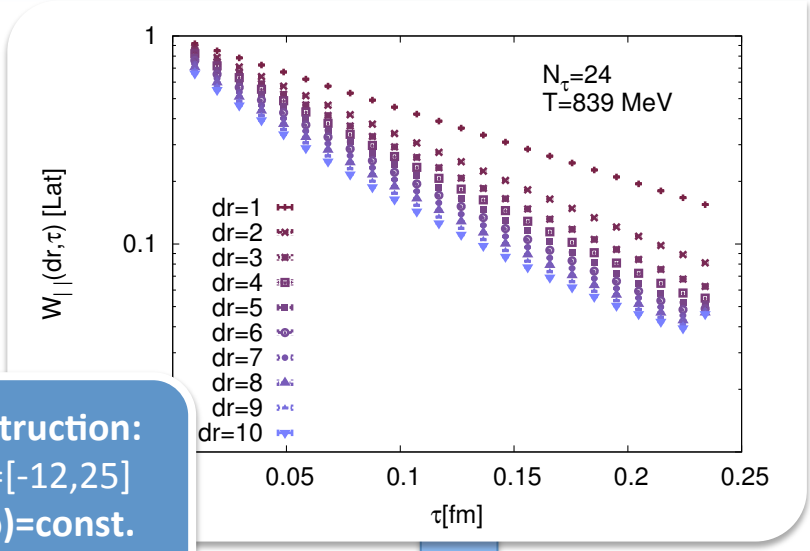




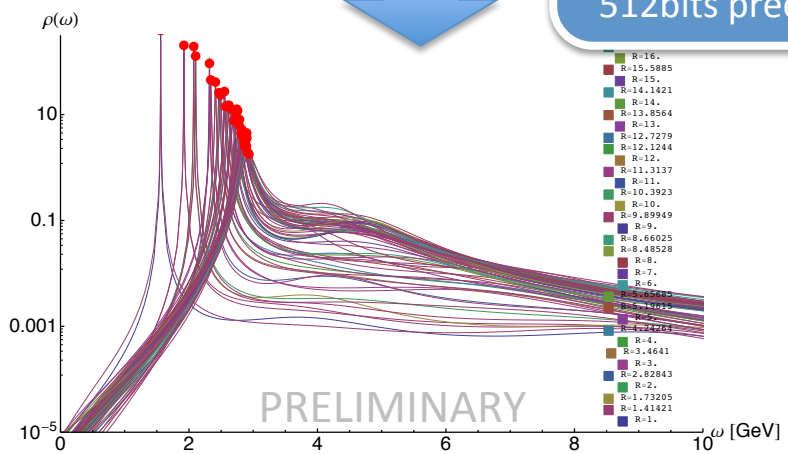
Towards $V^{QQ}(r)$ on quenched lattices

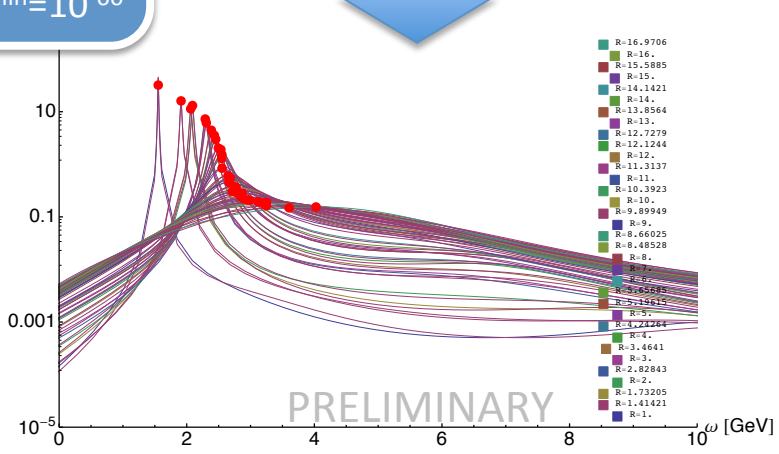
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





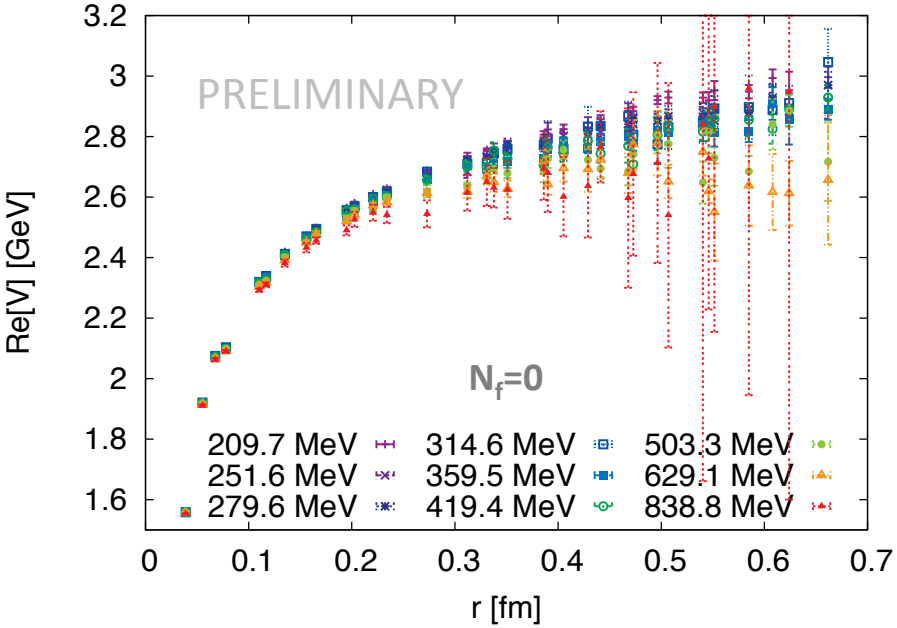
Bayesian reconstruction:
N_ω=3000, I_ω^{num}=[-12,25]
τ^{max}_{num}=20, **m(ω)=const.**
512bits precision, $\Delta^{min}=10^{-60}$



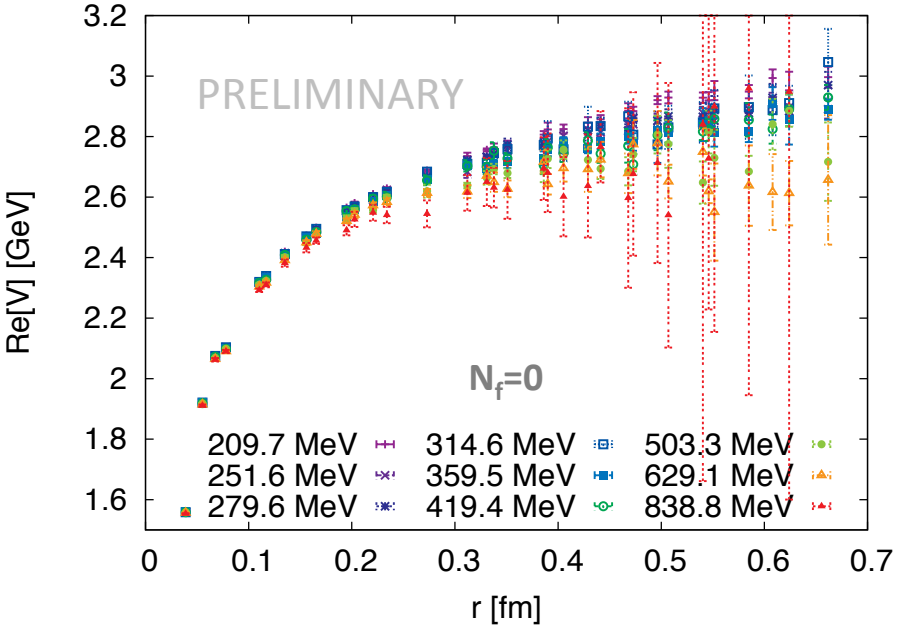


- Identify the lowest lying peak and fit its shape over the Full-Width at Half Maximum

Re[V] in quenched lattice QCD

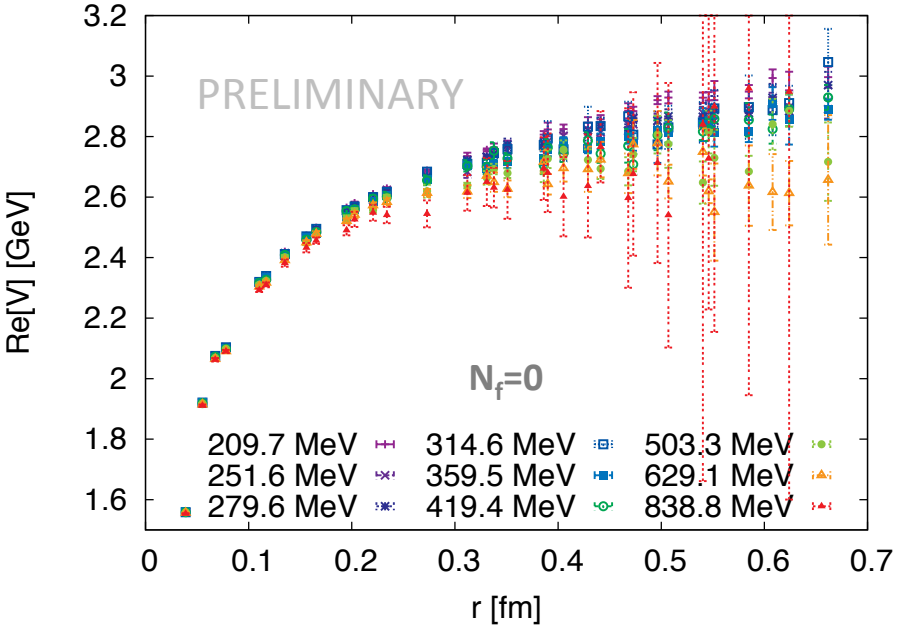


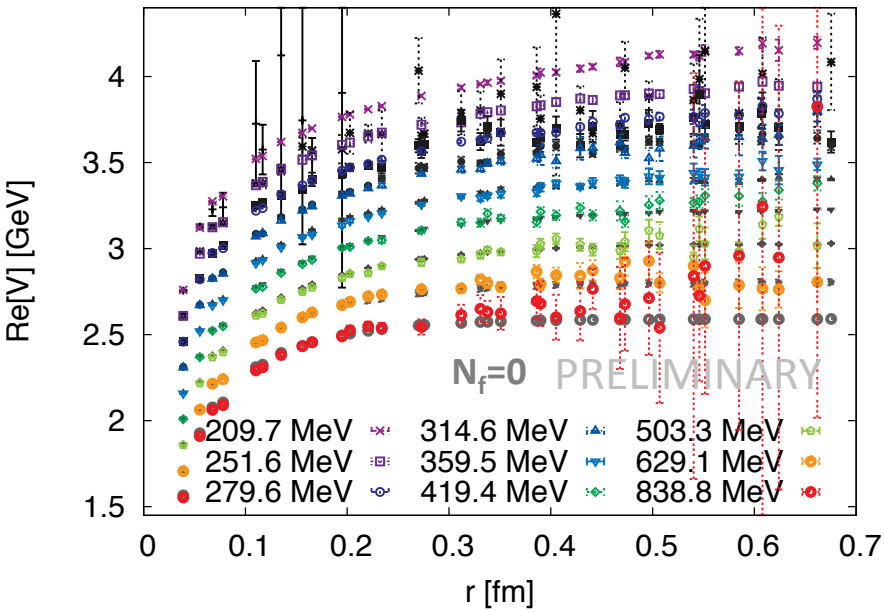
Re[V] in quenched lattice QCD



- Transition from a confining to a Debye screened behavior

Re[V] in quenched lattice QCD



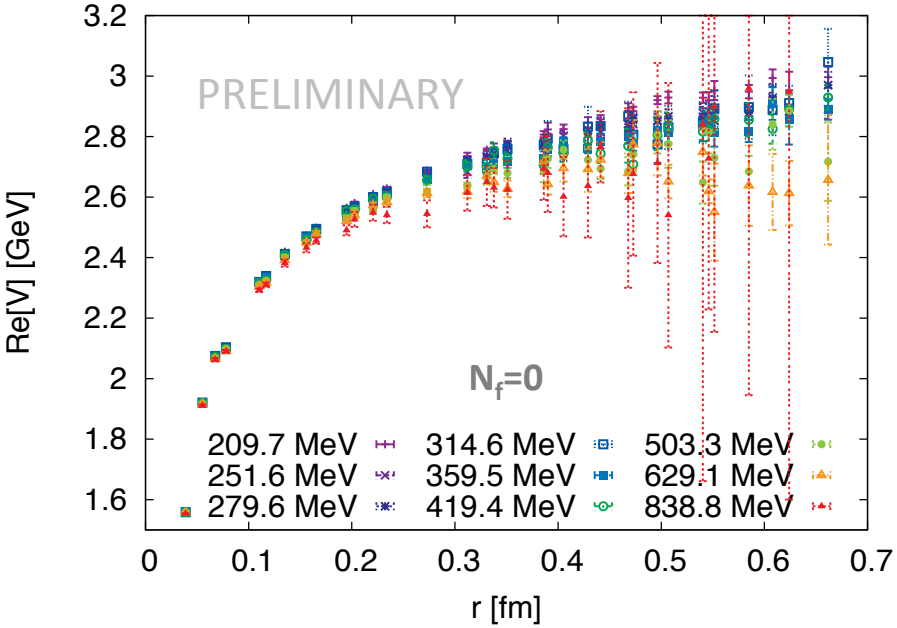


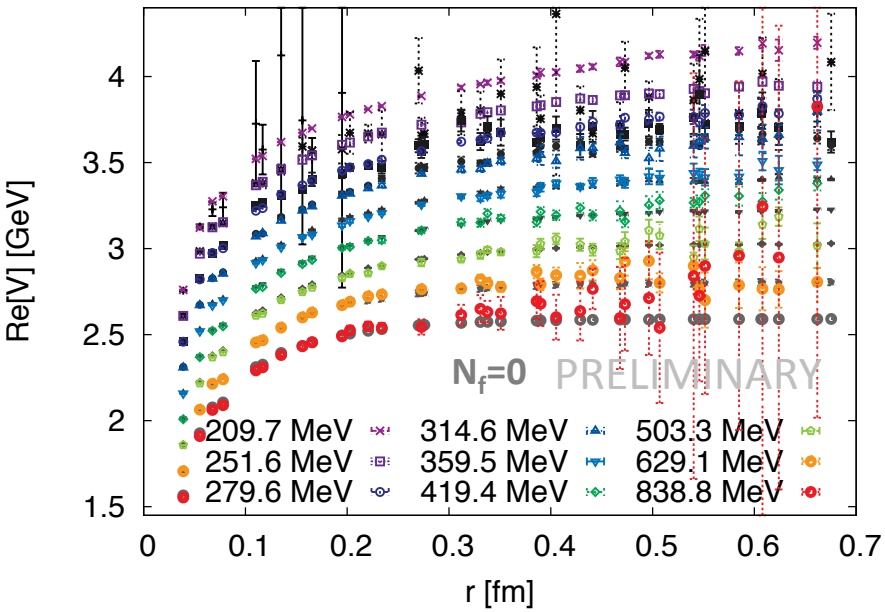
- Transition from a confining to a Debye screened behavior
- Comparison to color singlet free energies $F^1(r)$: agreement within errorbars

$$F^{(1)}(r) = -\frac{1}{\beta}\log\left[W_{||}(r,\tau=\beta)\right]_{CG}$$

Re[V] in quenched lattice QCD

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



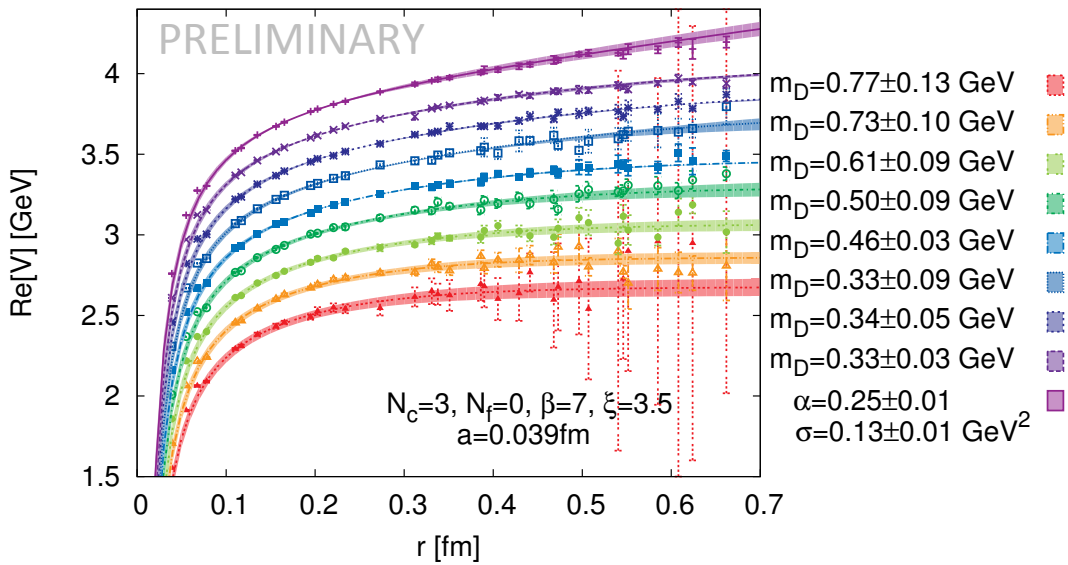


- Transition from a confining to a Debye screened behavior
- Comparison to color singlet free energies $F^1(r)$: agreement within errorbars

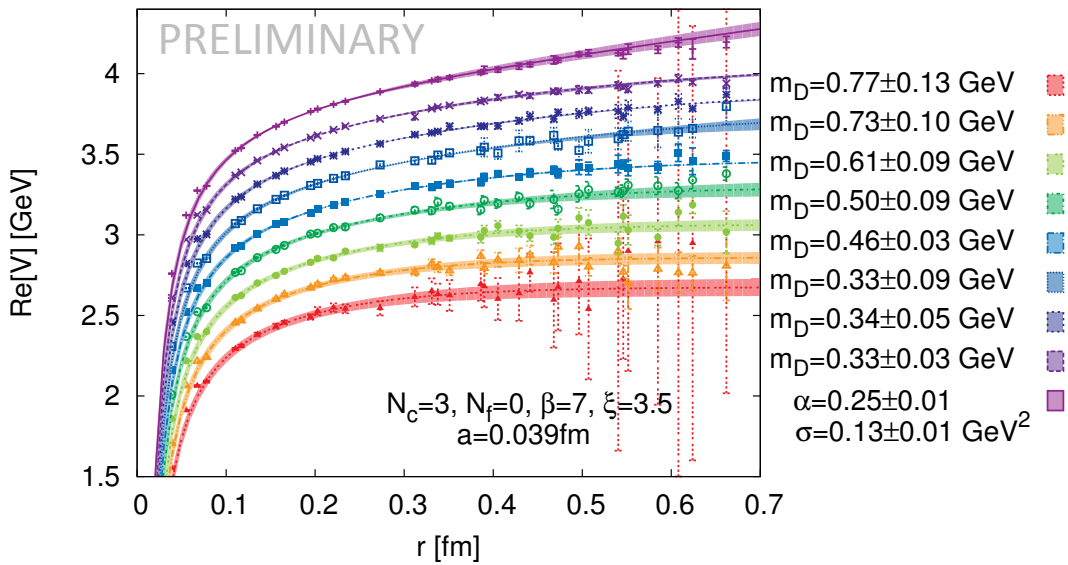
$$F^{(1)}(r) = -\frac{1}{\beta}\log\left[W_{||}(r,\tau=\beta)\right]_{CG}$$

- At $T \approx T_C$ extraction V^{QQ} benefits from using all datapoints instead of just $W_{||}(\tau=\beta)$

Debye-Hückel Fit of the Debye mass

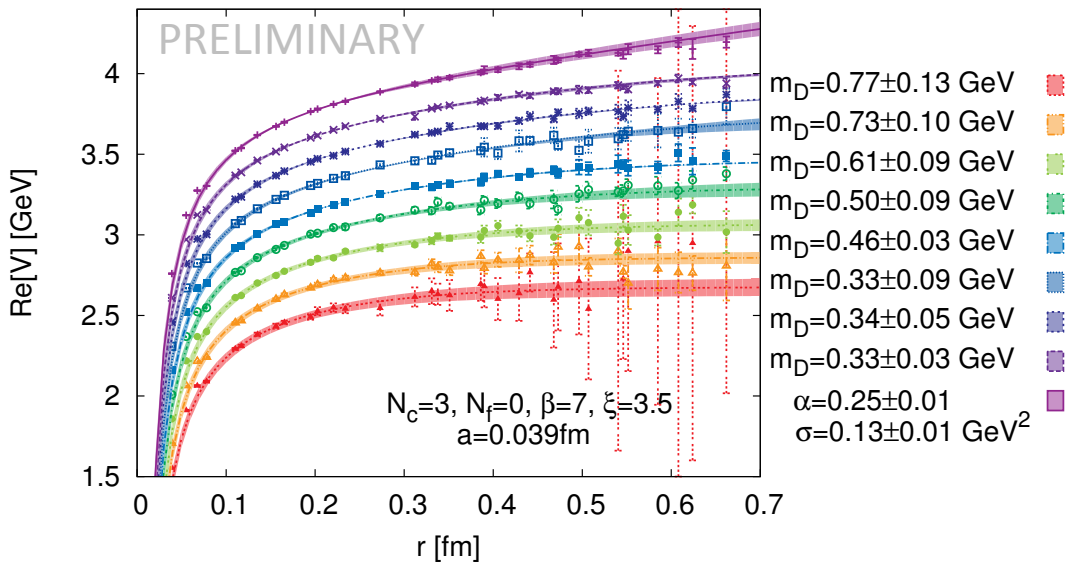


Debye-Hückel Fit of the Debye mass



- Since close to continuum (T_C=270MeV) attempt extraction of Debye mass

Debye-Hückel Fit of the Debye mass

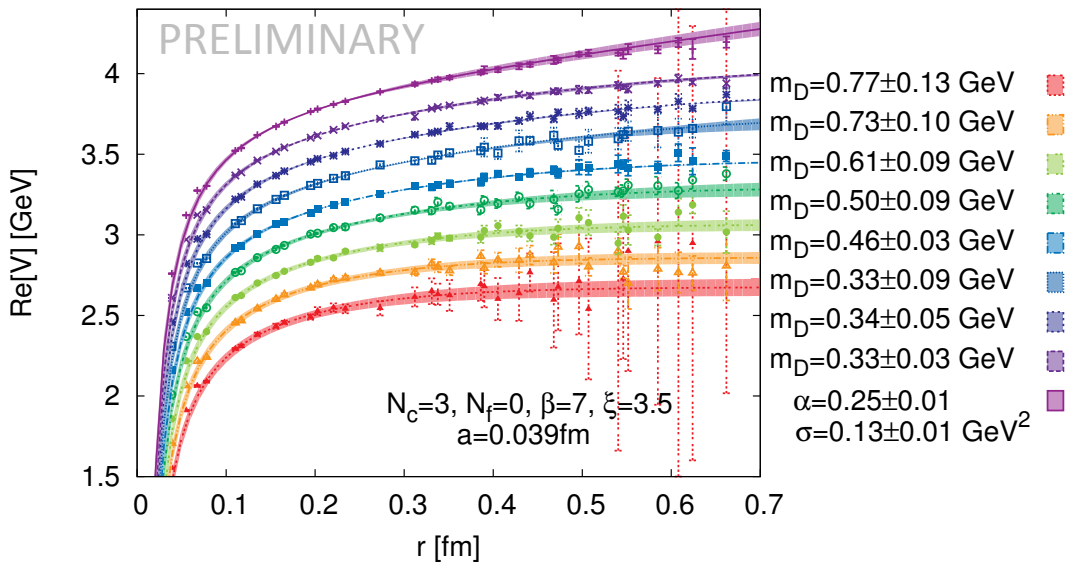


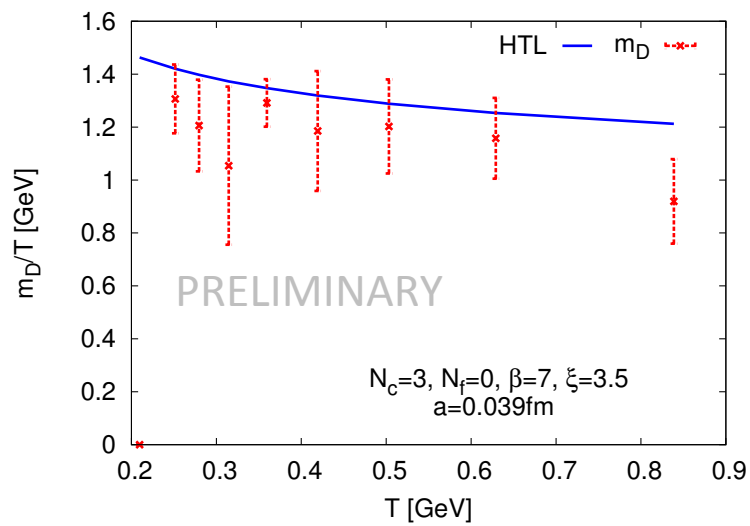
- Since close to continuum (T_C=270MeV) attempt extraction of Debye mass
- Phenomenological fit form for Coulomb and string screening S.Digal et.al. EPJ C43 (2005) 71-75

$$F^{DH}(r,T) = \frac{\sigma}{m_D(T)}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{m_D(T)r}}{2^{3/4}\Gamma(3/4)}K_{1/4}(m_D^2(T)r^2)\right] - \frac{\alpha}{r}\left[e^{-m_D(T)r} + m_D(T)r\right]$$

Debye-Hückel Fit of the Debye mass

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



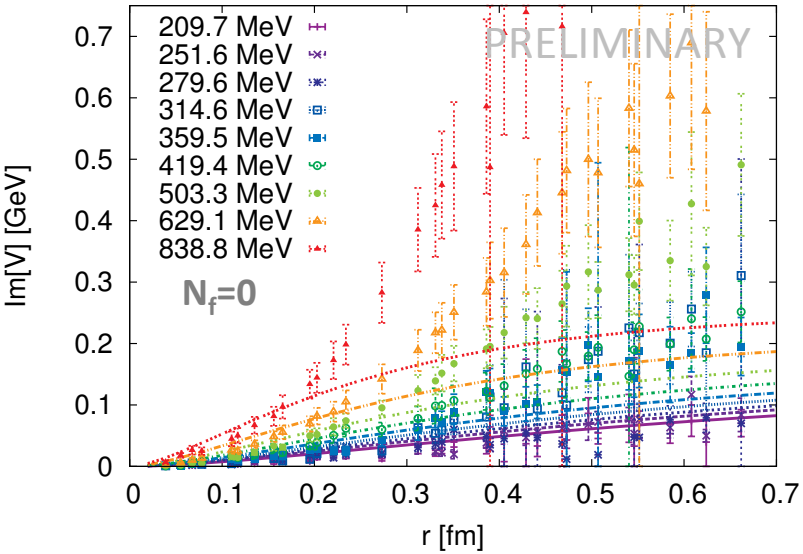


- Since close to continuum (T_C=270MeV) attempt extraction of Debye mass
- Phenomenological fit form for Coulomb and string screening S.Digal et.al. EPJ C43 (2005) 71-75

$$F^{DH}(r,T) = \frac{\sigma}{m_D(T)}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{m_D(T)r}}{2^{3/4}\Gamma(3/4)}K_{1/4}(m_D^2(T)r^2)\right] - \frac{\alpha}{r}\left[e^{-m_D(T)r} + m_D(T)r\right]$$

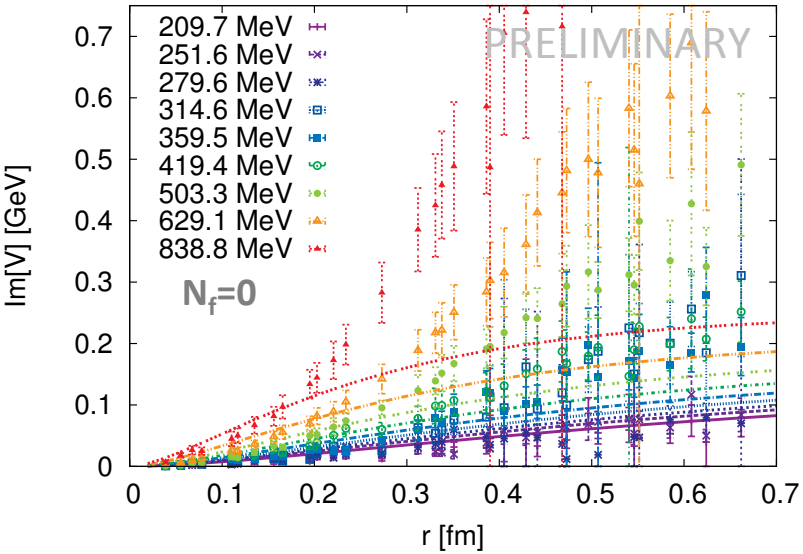
- Within the error bars, reasonable agreement with 1-loop HTL

Imaginary part at finite temperature



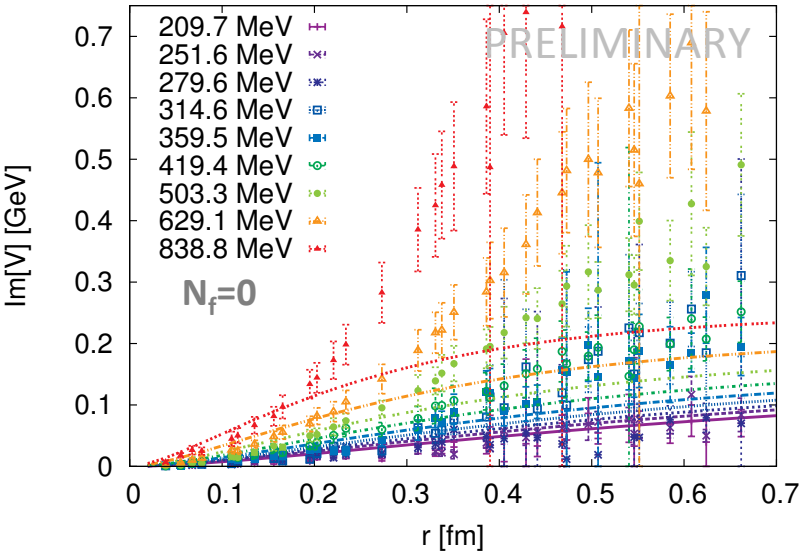
- $\mathrm{Im}[V^{QQ}]$ related to width: need large # of datapoints and high signal/noise

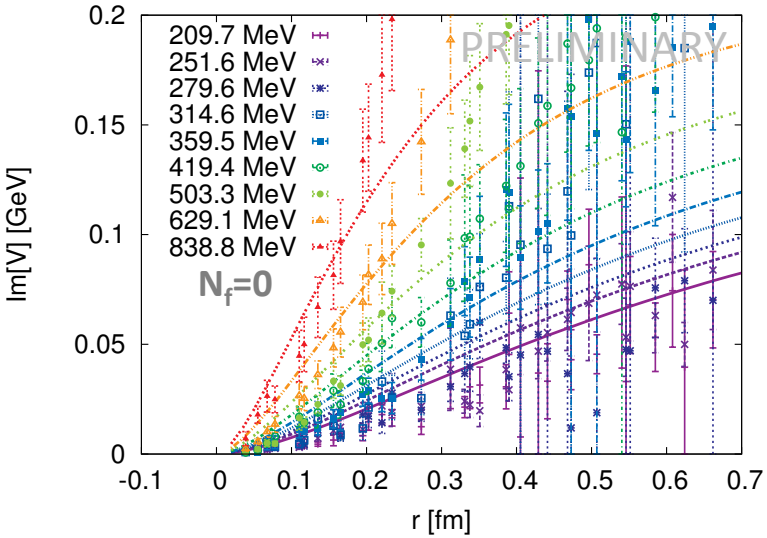
Imaginary part at finite temperature



- $\mathrm{Im}[V^{QQ}]$ related to width: need large # of datapoints and high signal/noise
- The smaller N_τ, the earlier the (artificially) strong rise above HTL sets in

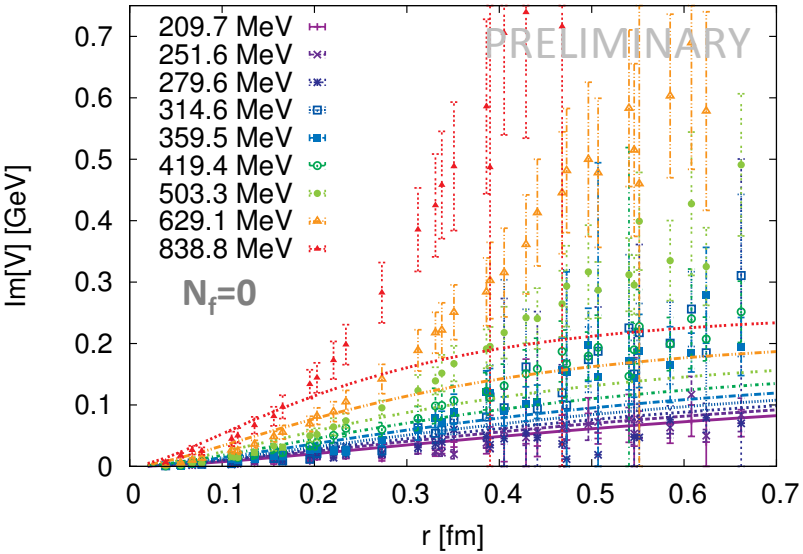
Imaginary part at finite temperature

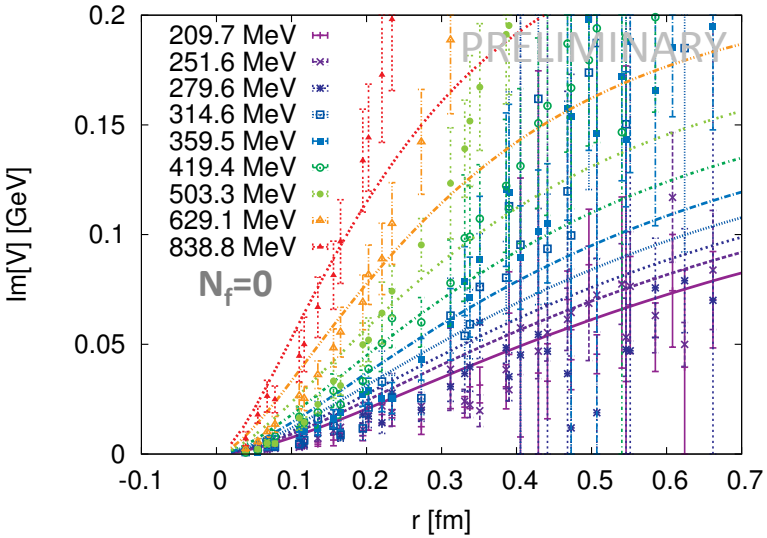




- $\mathrm{Im}[V^{QQ}]$ related to width: need large # of datapoints and high signal/noise
- The smaller N_τ, the earlier the (artificially) strong rise above HTL sets in
- For small r: good agreement with HTL prediction down to $1.17T_C$

Imaginary part at finite temperature

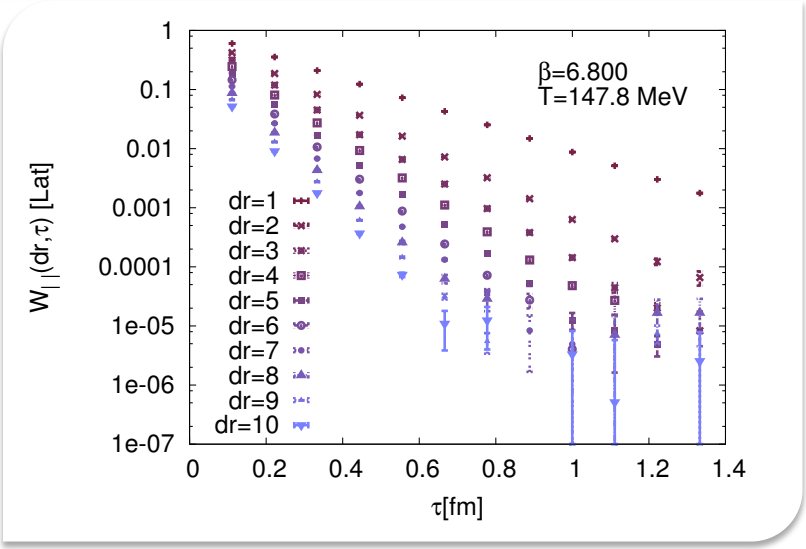


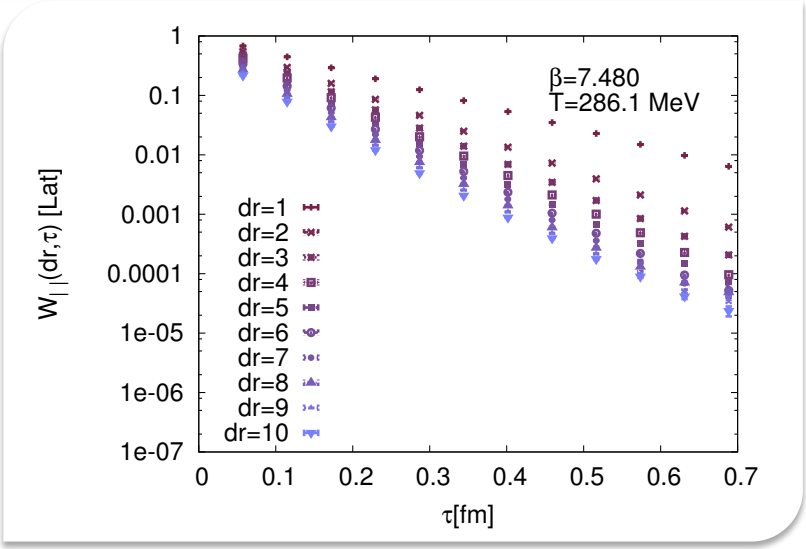


- $\mathrm{Im}[V^{QQ}]$ related to width: need large # of datapoints and high signal/noise
- The smaller N_τ, the earlier the (artificially) strong rise above HTL sets in
- For small r: good agreement with HTL prediction down to $1.17T_C$
- To improve the width reconstruction: better default model $m(\omega)\neq$const.

Towards $V^{QQ}(r)$ on dynamical lattices

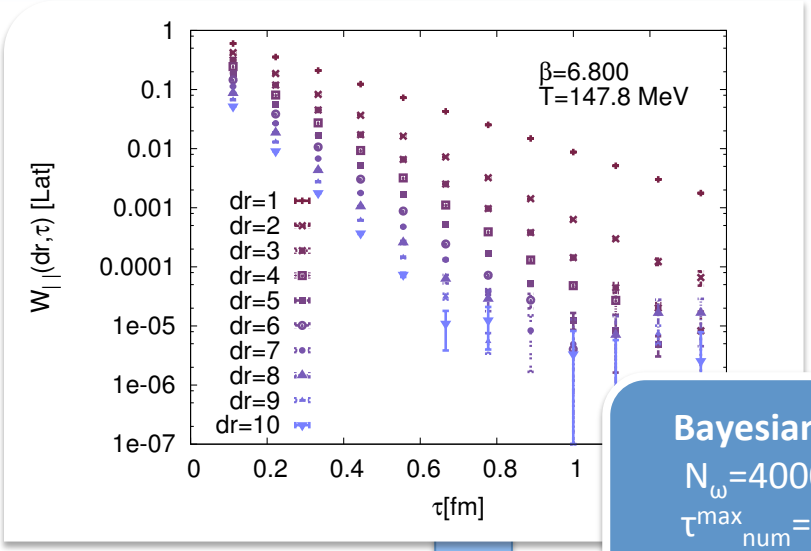
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

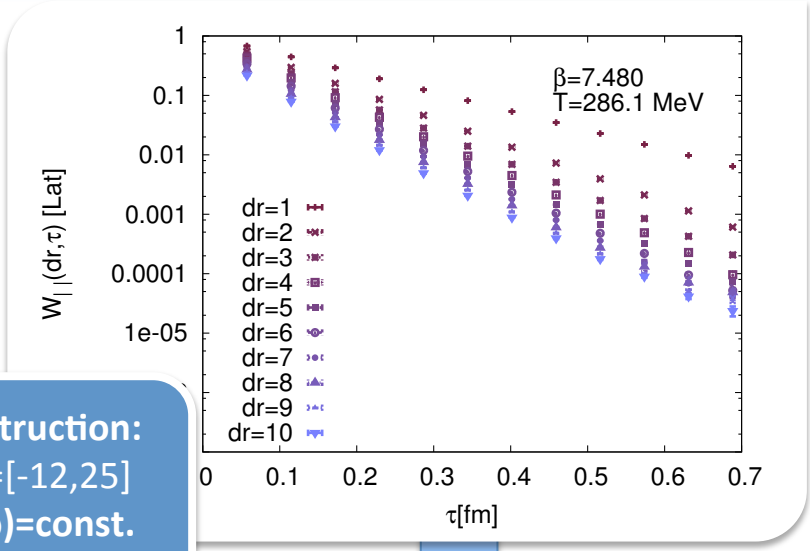




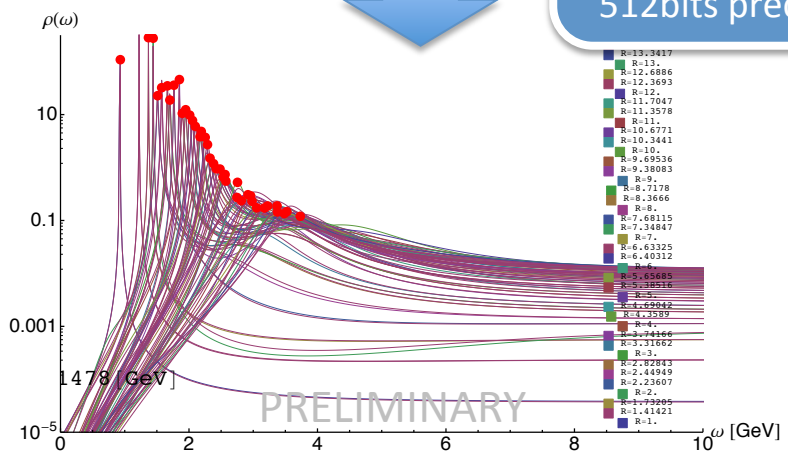
Towards $V^{QQ}(r)$ on dynamical lattices

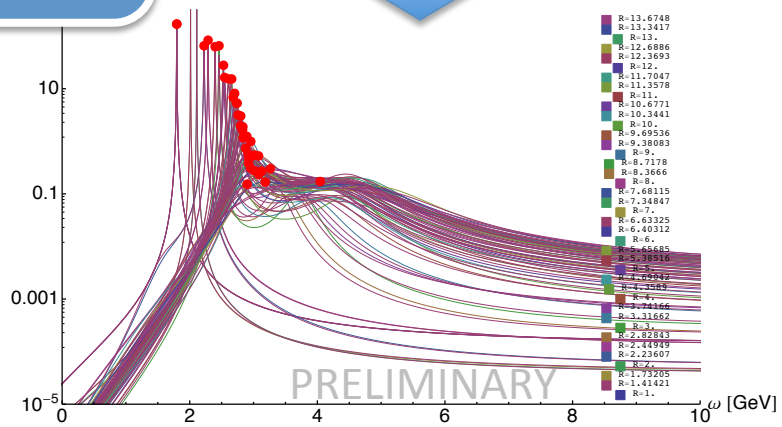
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





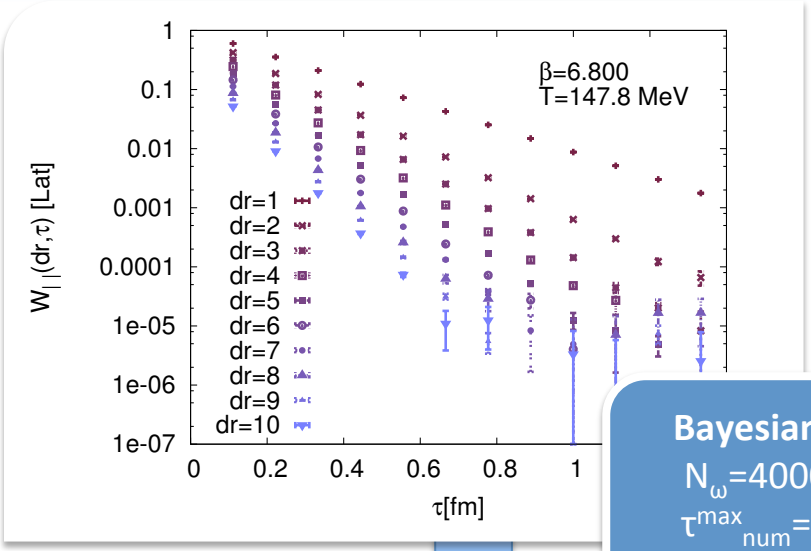
Bayesian reconstruction:
N_ω=4000, I_ω^{num}=[-12,25]
τ^{max}_{num}=20, **m(ω)=const.**
512bits precision, $\Delta^{min}=10^{-60}$

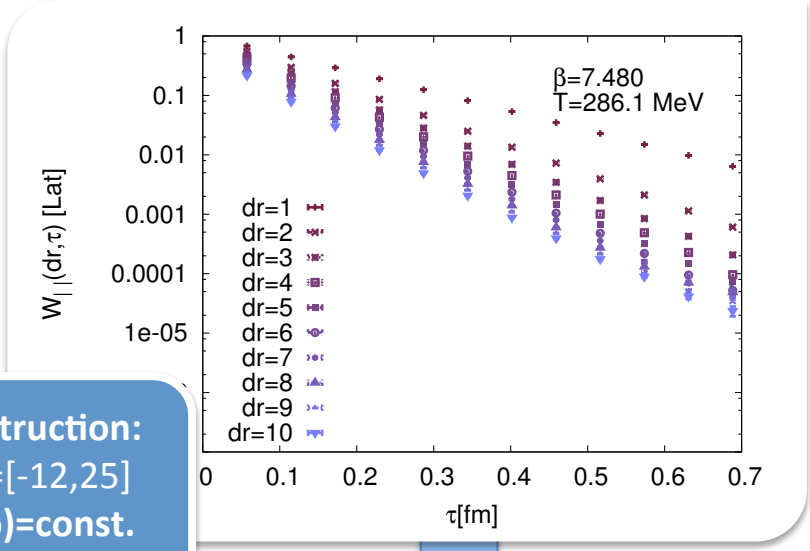




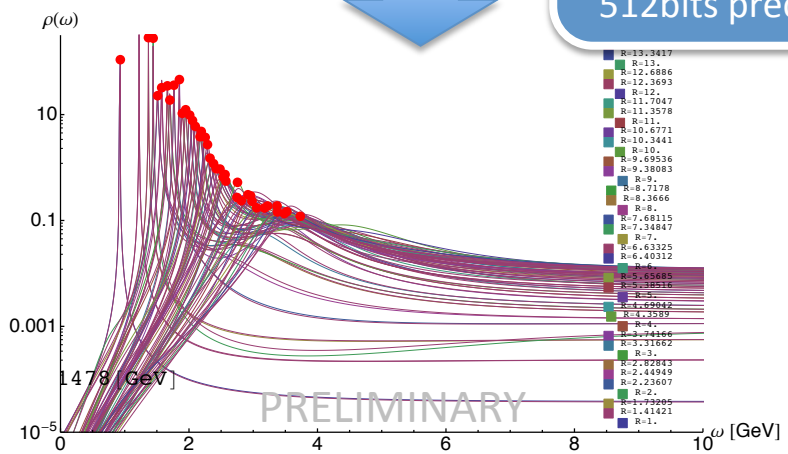
Towards $V^{QQ}(r)$ on dynamical lattices

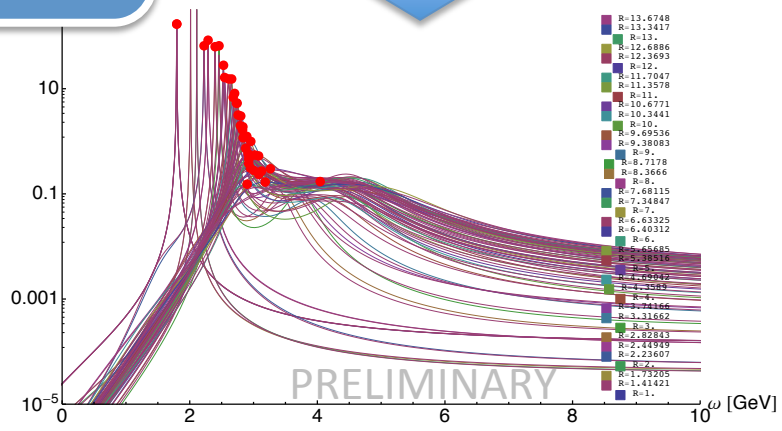
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386





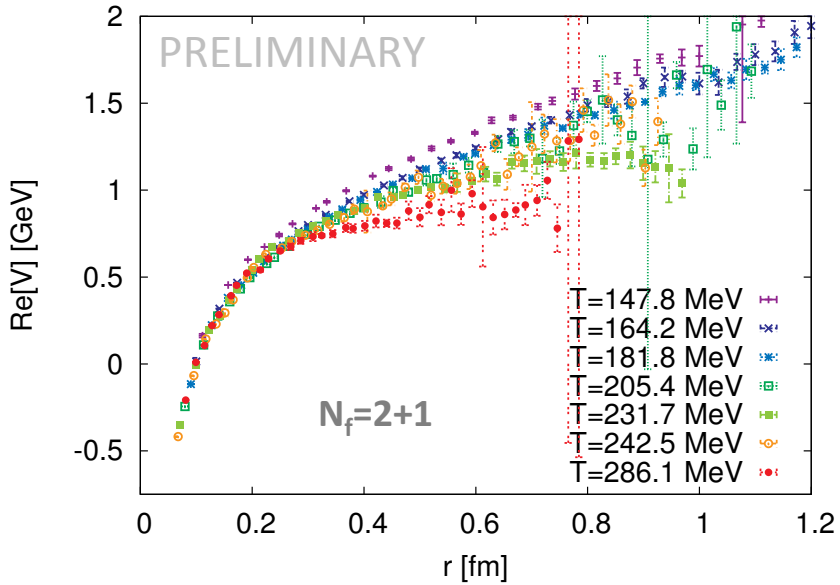
Bayesian reconstruction:
N_ω=4000, I_ω^{num}=[-12,25]
τ^{max}_{num}=20, **m(ω)=const.**
512bits precision, $\Delta^{min}=10^{-60}$



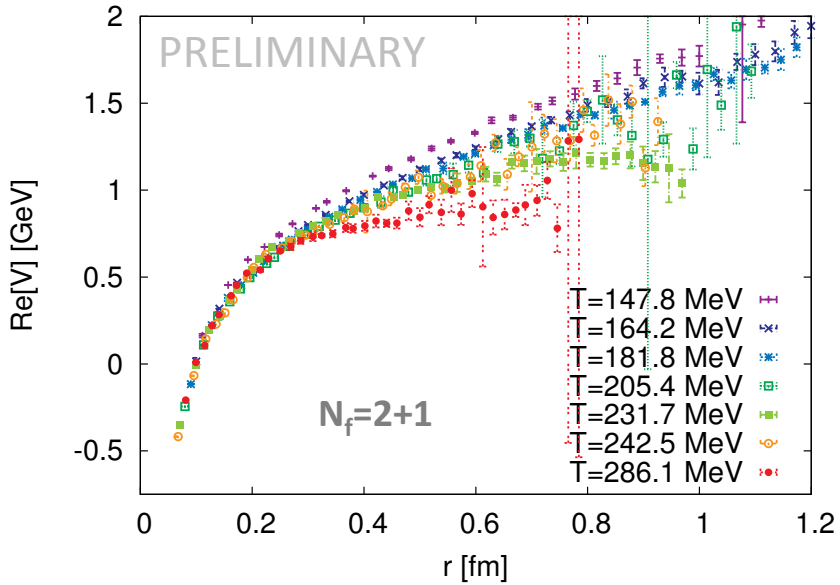


- Identify the lowest lying peak and fit its shape over the Full-Width at Half Maximum

The real part in dynamical lattice QCD

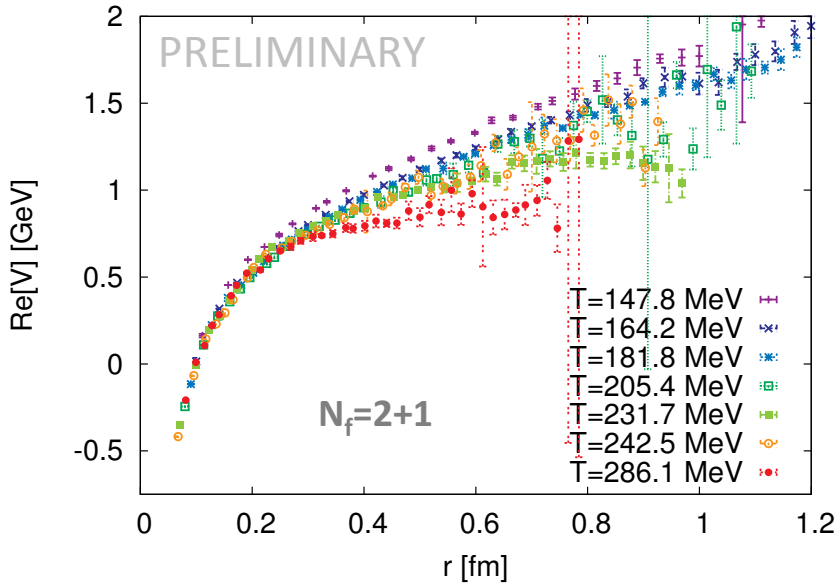


The real part in dynamical lattice QCD



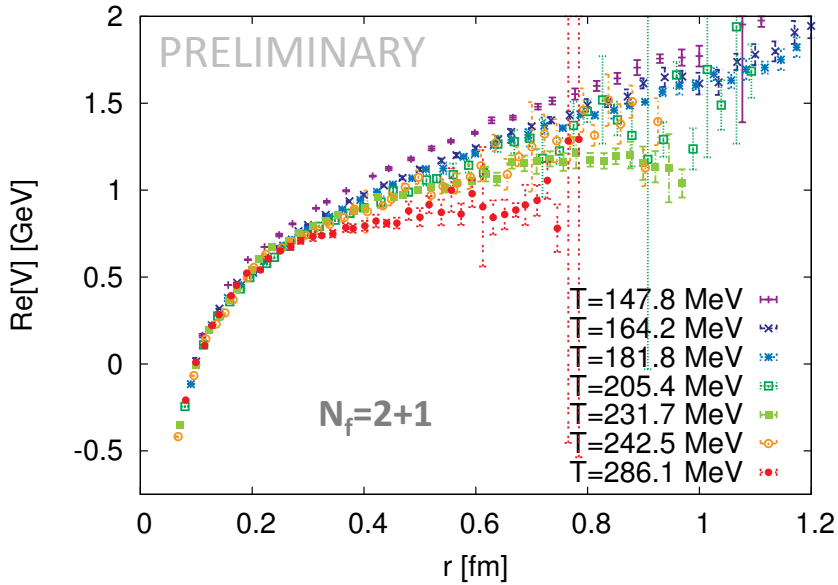
- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)

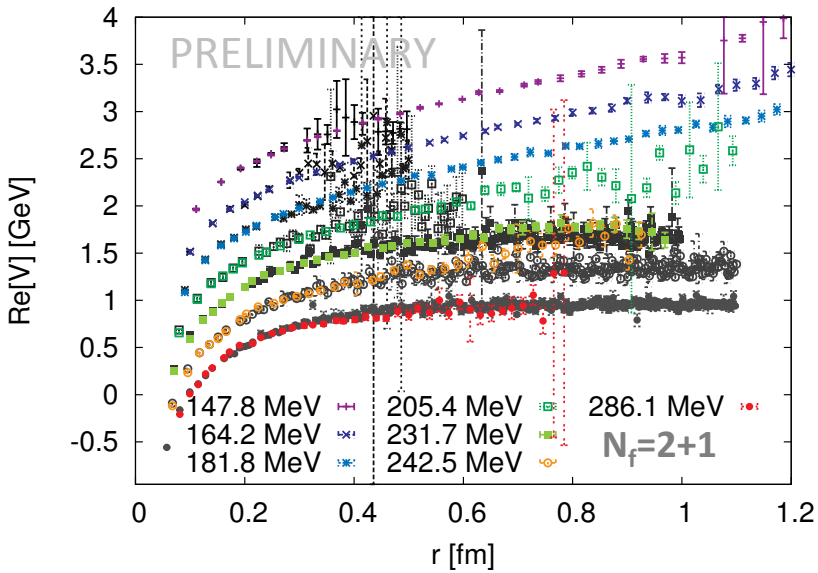
The real part in dynamical lattice QCD



- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)
- Clear transition from confining to Debye screened behavior

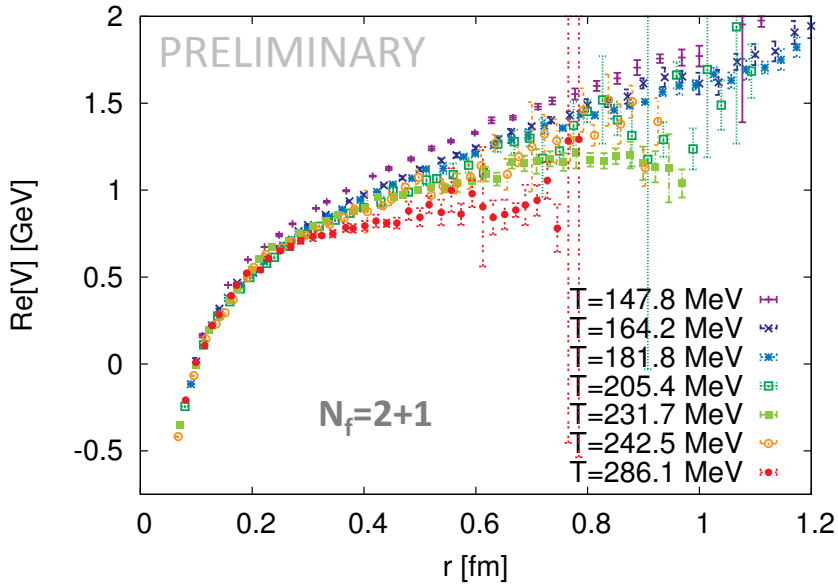
The real part in dynamical lattice QCD

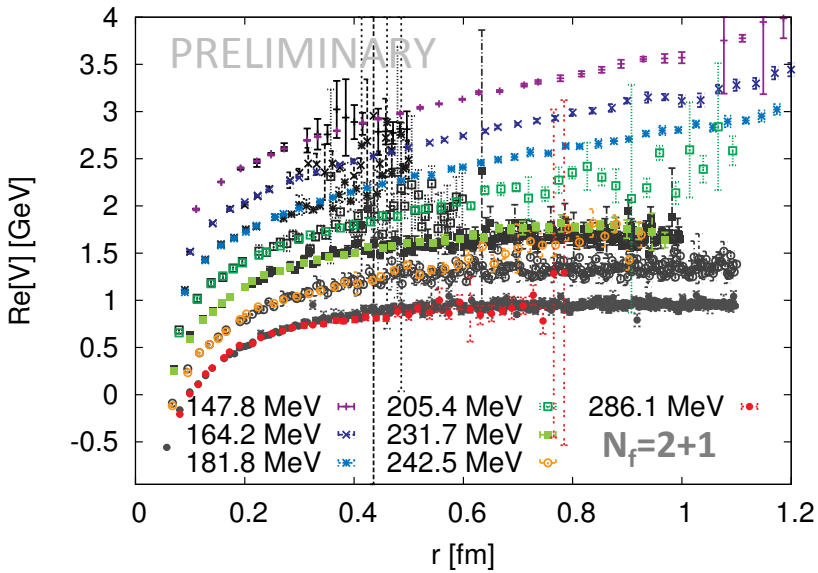




- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)
- Clear transition from confining to Debye screened behavior
- Also here agreement with color singlet free energies (at high T: $V^{QQ}(r) \lesssim F^1(r)$)

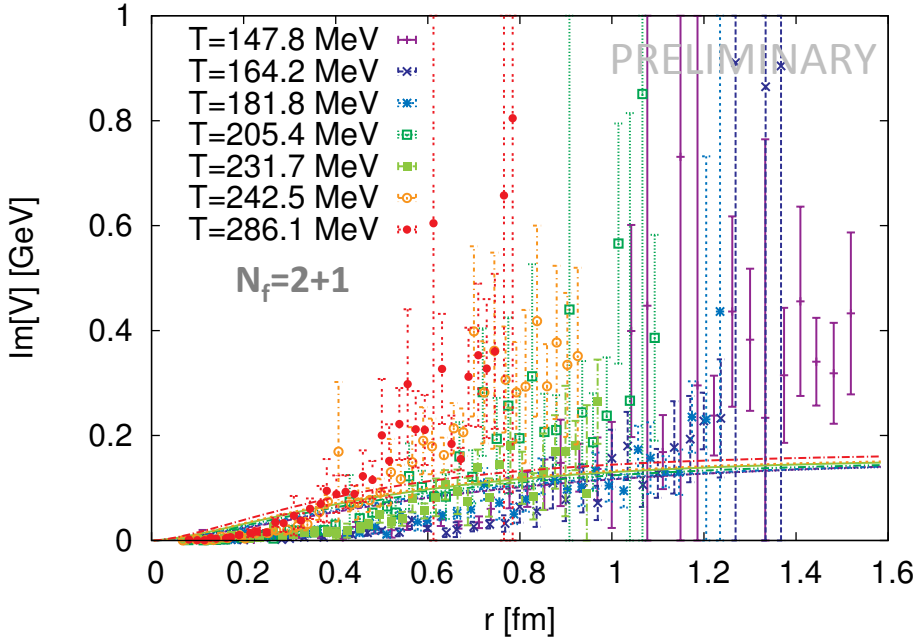
The real part in dynamical lattice QCD



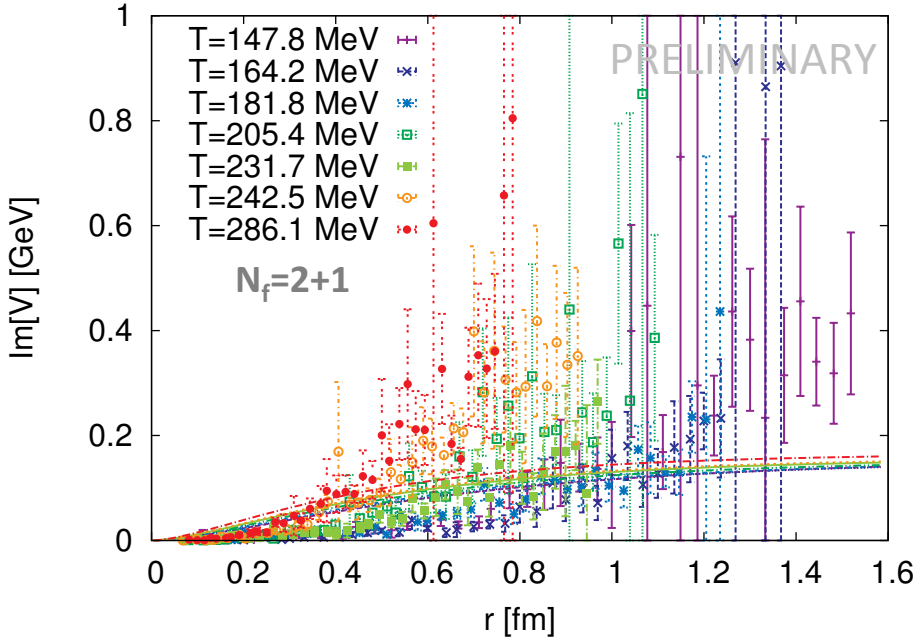


- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)
- Clear transition from confining to Debye screened behavior
- Also here agreement with color singlet free energies (at high T: $V^{QQ}(r) \lesssim F^1(r)$)
- At $T \approx T_C$ Re[V] benefits from using all datapoints instead of just $W_{||}(\tau=\beta)$

A brief look at Im[V^{QQ}] in dynamical LQCD

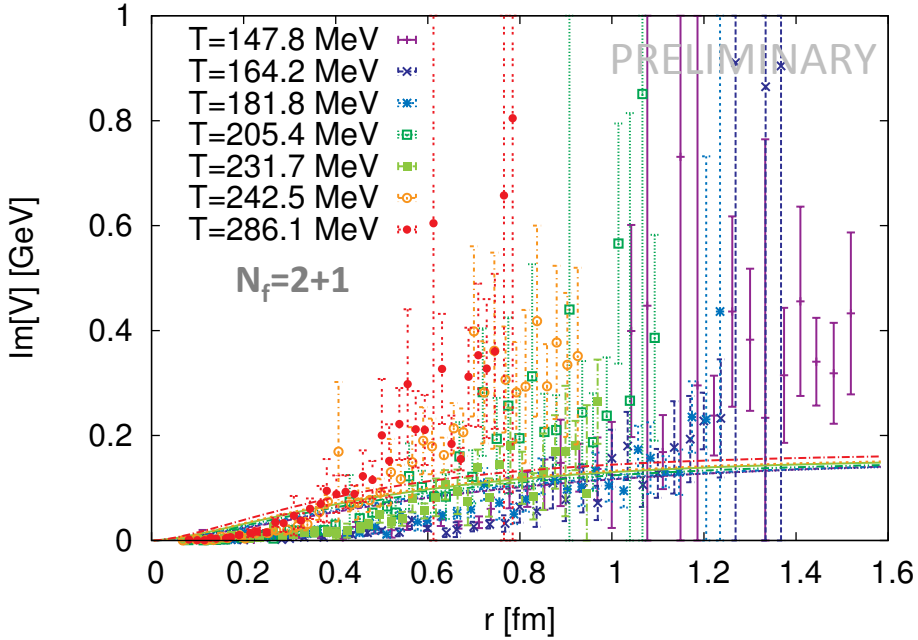


A brief look at Im[V^{QQ}] in dynamical LQCD



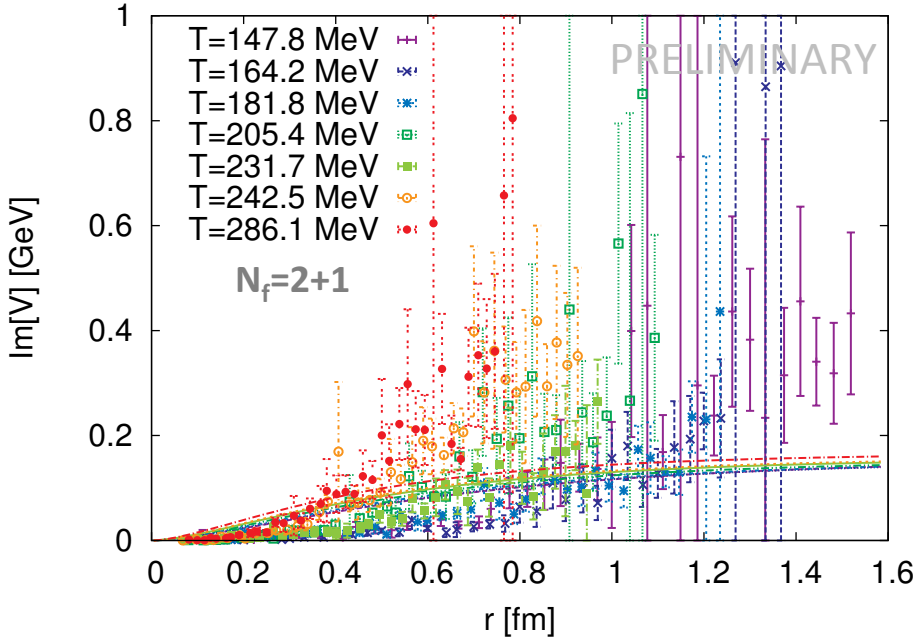
- N_τ=12 leads to underestimation of Im[V], where signal to noise is otherwise ok

A brief look at Im[V^{QQ}] in dynamical LQCD



- N_τ=12 leads to underestimation of Im[V], where signal to noise is otherwise ok
- At r>0.5fm decrease in signal to noise deteriorates width determination

A brief look at Im[V^{QQ}] in dynamical LQCD



- N_τ=12 leads to underestimation of Im[V], where signal to noise is otherwise ok
- At r>0.5fm decrease in signal to noise deteriorates width determination
- Still: obtained values are of the same order of magnitude as the HTL prediction

Conclusion

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\mathrm{Re}[V^{QQ}]$ and $\mathrm{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
    A.R., T. Hatsuda, S.Sasaki PRL 108 (2012) 162001
    A.R. Y. Burnier PRD86 (2012) 051503
  - Spectral functions reliably extracted with new Bayesian reconstruction method.
    A.R. Y. Burnier PRL 111 (2013) 18, 182003

Conclusion

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\mathrm{Re}[V^{QQ}]$ and $\mathrm{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
    A.R., T. Hatsuda, S.Sasaki PRL 108 (2012) 162001
    A.R. Y. Burnier PRD86 (2012) 051503
  - Spectral functions reliably extracted with new Bayesian reconstruction method.
    A.R. Y. Burnier PRL 111 (2013) 18, 182003
- Findings for the real part $\mathrm{Re}[V^{QQ}]$:
  - $N_f=0$ & $N_f=2+1$: $\mathrm{Re}[V^{QQ}]$ agrees with the color singlet free energies F^1 in CG within errors.
  - $N_f=0$: screening masses from Debye-Hückel fit and HTL in good agreement

Conclusion

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\mathrm{Re}[V^{QQ}]$ and $\mathrm{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
    A.R., T. Hatsuda, S.Sasaki PRL 108 (2012) 162001
    A.R. Y. Burnier PRD86 (2012) 051503
  - Spectral functions reliably extracted with new Bayesian reconstruction method.
    A.R. Y. Burnier PRL 111 (2013) 18, 182003
- Findings for the real part $\mathrm{Re}[V^{QQ}]$:
  - N_f=0 & N_f=2+1: $\mathrm{Re}[V^{QQ}]$ agrees with the color singlet free energies F^1 in CG within errors.
  - N_f=0: screening masses from Debye-Hückel fit and HTL in good agreement
- Findings for the imaginary part $\mathrm{Im}[V^{QQ}]$:
  - N_f=0: for r<0.35fm close to HTL values down to shortly above T_C.

Conclusion

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\mathrm{Re}[V^{QQ}]$ and $\mathrm{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
    A.R., T. Hatsuda, S.Sasaki PRL 108 (2012) 162001
    A.R. Y. Burnier PRD86 (2012) 051503
  - Spectral functions reliably extracted with new Bayesian reconstruction method.
    A.R. Y. Burnier PRL 111 (2013) 18, 182003
- Findings for the real part $\mathrm{Re}[V^{QQ}]$:
  - N_f=0 & N_f=2+1: $\mathrm{Re}[V^{QQ}]$ agrees with the color singlet free energies F^1 in CG within errors.
  - N_f=0: screening masses from Debye-Hückel fit and HTL in good agreement
- Findings for the imaginary part $\mathrm{Im}[V^{QQ}]$:
  - N_f=0: for r<0.35fm close to HTL values down to shortly above T_C.
- Current status of **first principles** lattice QCD **input** for **$Q\bar{Q}$ real-time evolution** $V^{QQ}(r)$ in the QGP is complex: $\mathrm{Re}[V^{QQ}]$ close to F^1, $\mathrm{Im}[V^{QQ}]$ close to HTL @ $T>T_C$

Conclusion

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Established approach to the static in-medium heavy quark potential $V^{QQ}(r)$:
  - Definition from QCD via effective field theory NRQCD: Wilson loops/lines at late real-time
  - Connection to lattice QCD: $\mathrm{Re}[V^{QQ}]$ and $\mathrm{Im}[V^{QQ}]$ from the position and width of a skewed Lorentzian in Wilson loop/line spectra
    A.R., T. Hatsuda, S.Sasaki PRL 108 (2012) 162001
    A.R. Y. Burnier PRD86 (2012) 051503
  - Spectral functions reliably extracted with new Bayesian reconstruction method.
    A.R. Y. Burnier PRL 111 (2013) 18, 182003
- Findings for the real part $\mathrm{Re}[V^{QQ}]$:
  - N_f=0 & N_f=2+1: $\mathrm{Re}[V^{QQ}]$ agrees with the color singlet free energies F^1 in CG within errors.
  - N_f=0: screening masses from Debye-Hückel fit and HTL in good agreement
- Findings for the imaginary part $\mathrm{Im}[V^{QQ}]$:
  - N_f=0: for r<0.35fm close to HTL values down to shortly above T_C.
- Current status of **first principles** lattice QCD **input** for **$Q\bar{Q}$ real-time evolution** $V^{QQ}(r)$ in the QGP is complex: $\mathrm{Re}[V^{QQ}]$ close to F^1, $\mathrm{Im}[V^{QQ}]$ close to HTL @ $T>T_C$

Thank you for your attention