# Rare kaon decays from lattice QCD

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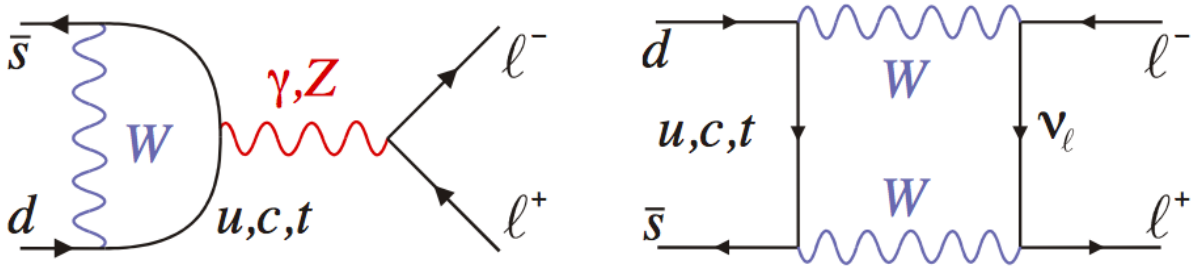
# Motivation

- rare kaon decays: $K \to \pi X$, $X = \nu\bar{\nu}$ or $l^+l^-$
- at quark level, $s \to dX$, includes $W$-box, $Z$,$\gamma$-penguin contributions




- FCNC process, $2_{\rm nd}$-order weak inter. $\to$ rare experimentally observed

SM effects suppressed by higher order $\to$ ideal for probe of NP

# Phenomelogical background

# Status for $K^+ \to \pi^+ \nu\bar{\nu}$

- $\mathcal{A}(K^+ \to \pi^+ \nu\bar{\nu})$ is known to be dominated by $t$-quark

$$X_t : P_c : \delta P_{c,u} = 1 : 0.25 : 0.03 \text{ [Brod, 1009.0947]}$$

- why lattice QCD?
- NA62@CERN aims at 80 $K^+ \to \pi^+ \nu\bar{\nu}$ events with 10% precision
- SM error for $\mathrm{Br}[K^+ \to \pi^+ \nu\bar{\nu}]$ are 14% [Brod, 1009.0947]
  - 10% from input parameters (CKM, $m_t$, $\alpha_s \cdots$)
  - 4% are theory uncertainty (2% from $\delta P_{c,u}$, 1% from $X_t$, 1% from $P_c$)

$$\delta P_{c,u} = 0.04 \pm 0.02_{\mathrm{NLO\,ChPT}} \text{ [Isidori, hep-ph/0503107]}$$

LQCD impact on $K^+ \to \pi^+ \nu\bar{\nu}$ if $\delta P_{c,u}$ calculated within 50% error

# Status for $K \to \pi l^+ l^-$

$K \to \pi l^+ l^-$ decays: LD contribution is important

most interesting channel is $K_L \to \pi^0 l^+ l^-$, CPV decays

- indirect CPV: $K_L \xrightarrow{\epsilon} K_S \to \pi^0 \gamma^* \to \pi^0 l^+ l^-$
- direct + indirect CPV contribution to branching ratio [1107.6001]

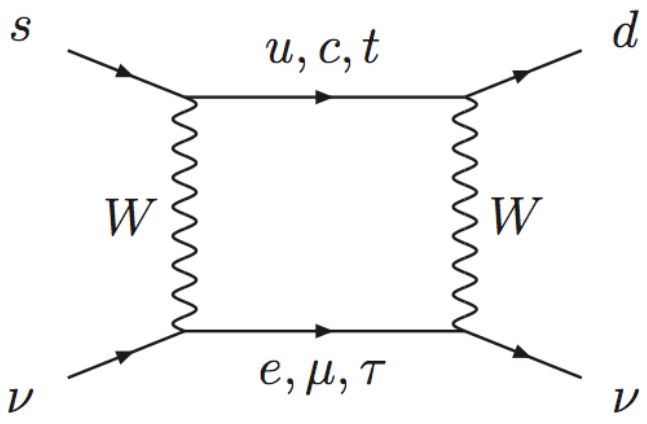
$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{CPV} =$$
$$= 10^{-12} \times \left[ 15.7 |a_S|^2 \pm 6.2 |a_S| \left( \frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + 2.4 \left( \frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 \right]$$

- $\mathrm{Im}\,\lambda_t$-term: direct CPV; $|a_S|$-term: indirect CPV
- $\pm$ arises due to the unknown sign of $a_S$

even determination of the sign of $a_S$ is desirable

# Lattice methodology

# $W$-box diagram




- $\mathcal{A}_W^q(K^+ \to \pi^+ \nu\bar{\nu})$ $(q = u, c)$ induced from $W$-box diagram is

$$\mathcal{A}_W^q = -i8G_F^2\lambda_q \int d^4x \, \langle \pi, \bar{\nu}, \nu | T[O^{\bar{s}q,\bar{\nu}l}(x) O^{\bar{q}d,\bar{l}\nu}(0)] | K \rangle$$

- 4-fermion operators $O^{\bar{s}q,\bar{\nu}l}$ and $O^{\bar{q}d,\bar{l}\nu}$ are defined by

$$O^{\bar{s}q,\bar{\nu}l} = \bar{s}_L \gamma^\alpha q_L \otimes \bar{\nu}_L \gamma_\alpha l_L, \quad O^{\bar{q}d,\bar{l}\nu} = \bar{q}_L \gamma^\alpha d_L \otimes \bar{l}_L \gamma_\alpha \nu_L$$

- the leptonic part is given by

$$L_{\alpha\beta}(p_{\bar{\nu}}, p_\nu, p_l) = \bar{u}_L(p_\nu)\gamma_\alpha \frac{i p_l^\mu \gamma_\mu}{p_l^2 - m_l^2} \gamma_\beta v_L(p_{\bar{\nu}})$$

# $W$-box diagram

- 3 gamma matrix multiplication $\Rightarrow$ 1 gamma matrix

$$\gamma_\alpha\gamma_\mu\gamma_\beta = g_{\alpha\mu}\gamma_\beta + g_{\mu\beta}\gamma_\alpha - g_{\alpha\beta}\gamma_\mu - i\epsilon_{\lambda\alpha\mu\beta}\gamma^\lambda\gamma_5$$

- leptonic part is reduced to

$$L_{\alpha\beta}(p_{\bar\nu}, p_\nu, p_l) = \frac{iT_{\alpha\beta,\mu}(p_l)}{p_l^2 - m_l^2}\,\bar u_L(p_\nu)\gamma^\mu v_L(p_{\bar\nu})$$

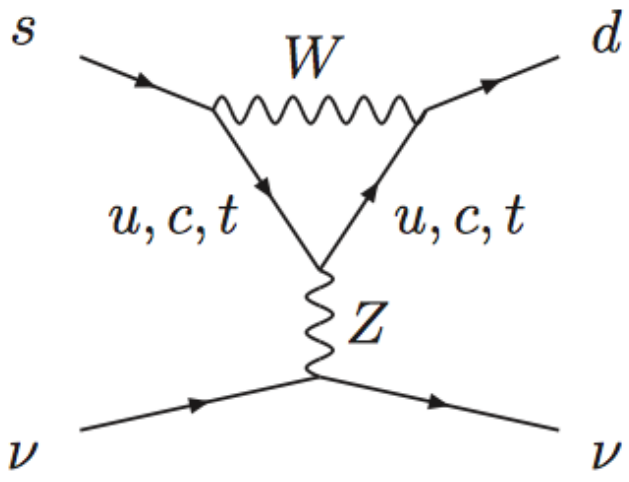
- in position space, $\mathcal{A}_W^q \propto \mathcal{W}_\mu^q(p_K, p_\pi, p_\nu)\,[\bar u_L(p_\nu)\gamma^\mu v_L(p_{\bar\nu})]$

$$\mathcal{W}_\mu^q = \int d^4x\, e^{ip_\nu x}\left[T_{\alpha\beta,\mu}(i\partial)D_0^{-1}(x)\right]\langle\pi|\,T[\bar s_L\gamma^\alpha q_L(x)\;\bar q_L\gamma^\beta d_L(0)]|K\rangle$$

free propagator $D_0^{-1}(x) = \int \frac{d^4p_l}{(2\pi)^4}\,\frac{i}{p_l^2-m_l^2}e^{-ip_l x}$ and $T_{\alpha\beta,\mu}$ are known

- numerical calculation is under investigation

# $Z,\gamma$-penguin diagram




- $\mathcal{A}^q_{Z,\gamma}$ induced from $Z,\gamma$-penguin diagram is

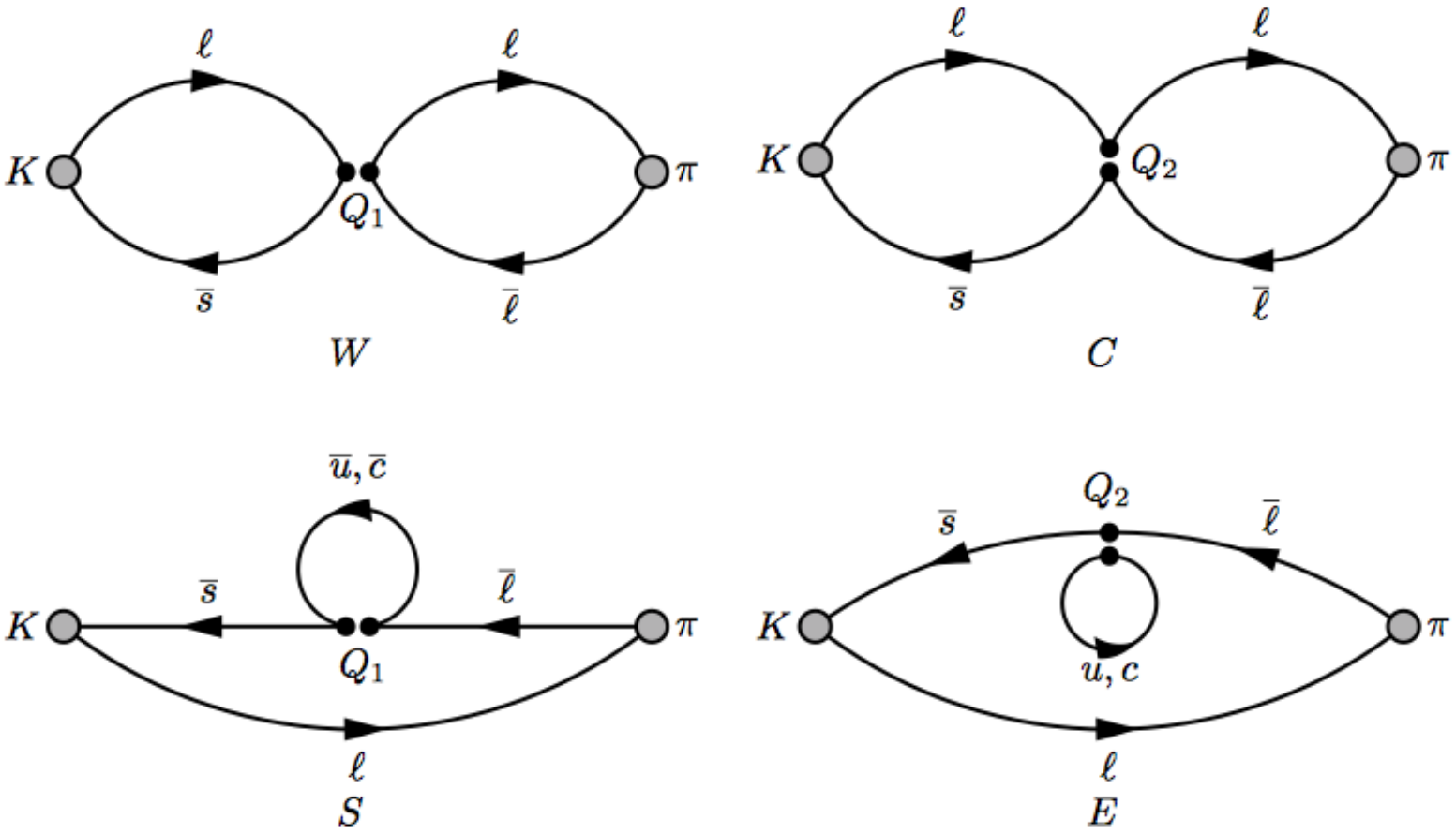
$$\mathcal{A}^q_{Z,\gamma} \propto i \int d^4x \, \langle \pi(p_\pi) | T\{J^{z,\gamma}_\mu(0) H_W(x)\} | K(p_K) \rangle$$

  - $J^\gamma_\mu$ is a vector current; $J^Z_\mu$ also includes axial vector current
  - 4-quark operator $H_W = Q_{1,2}$

$$Q_1 = (\bar{s}_L \gamma^\alpha d_L)(\bar{q}_L \gamma_\alpha q_L) \quad Q_2 = (\bar{s}_L \gamma^\alpha q_L)(\bar{q}_L \gamma_\alpha d_L), \quad q = u, c$$

# Diagrams for correlation function

- $W$: wing, $C$: connected, $S$: saucer, $E$: eye




- difference between $W$ and $C$: two trace in $W$ and one trace in $C$
- there are also disconnected diagrams

# Integral of 4-point function

- Minkowski space

$$X \equiv \int_{-\infty}^{\infty} dt \, \langle \pi(p_\pi) | T[J_\mu(0) H_W(t)] | K(p_K) \rangle$$

integration from $(-\infty, 0) \Rightarrow X_-$; from $(0, \infty) \Rightarrow X_+$

$$X_- = i \sum_n \frac{\langle \pi | J | n \rangle \langle n | H_W | K \rangle}{E_K - E_n + i\epsilon}, \quad X_+ = -i \sum_{n_s} \frac{\langle \pi | H_W | n_s \rangle \langle n_s | J | K \rangle}{E_{n_s} - E_\pi + i\epsilon}$$

- Euclidean space, run the integration from $[-T_a, T_b]$

$$X_{E_-} = -\sum_n \frac{\langle \pi | J | n \rangle \langle n | H_W | K \rangle}{E_K - E_n} \left(1 - e^{(E_K - E_n) T_a}\right)$$

$$X_{E_+} = \sum_{n_s} \frac{\langle \pi | H_W | n_s \rangle \langle n_s | J | K \rangle}{E_{n_s} - E_\pi} \left(1 - e^{(E_\pi - E_{n_s}) T_b}\right)$$

  - exponential divergence if $E_n < E_K$
  - similar situation happens to $K_L$-$K_S$ mass difference (Z. Bai's talk)

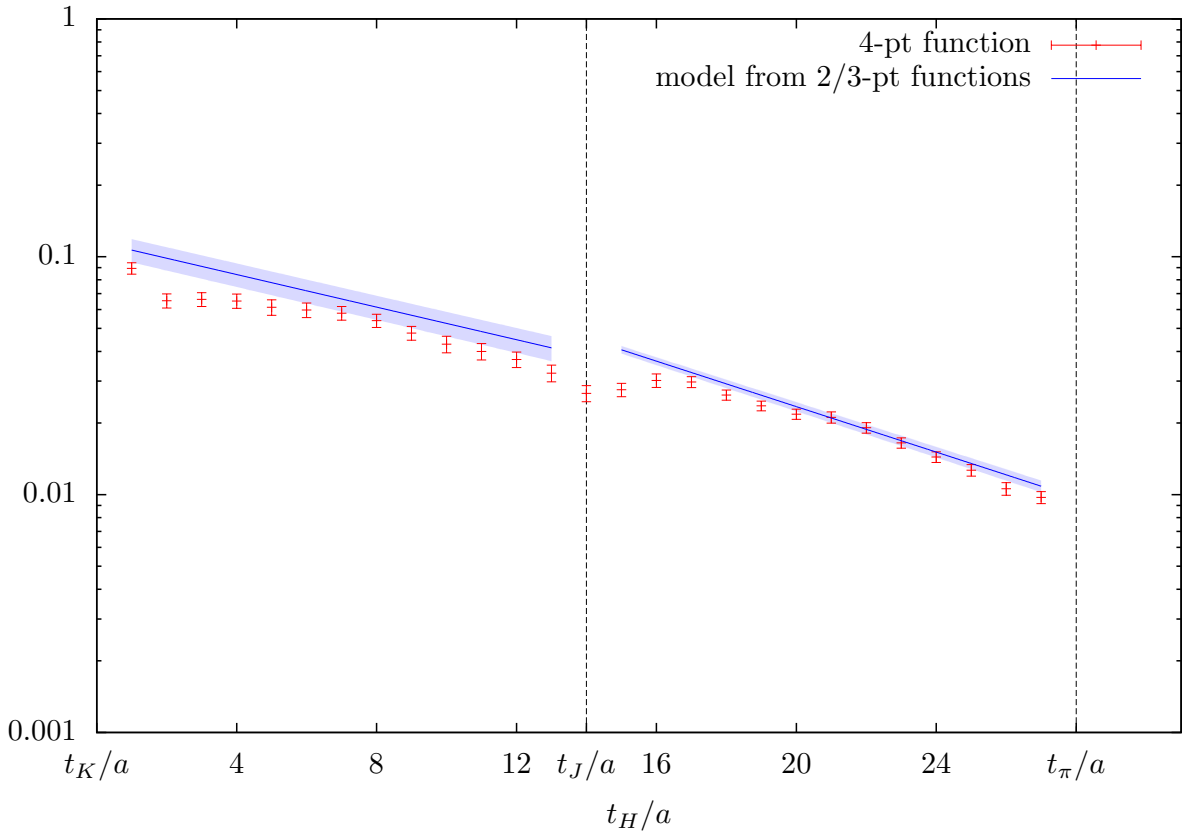
# Preliminary results

# Lattice setup

- $64 \times 24^3$ DWF+Iwasaki lattice generated by RBC-UKQCD
- quark mass $am_l = 0.01$ ($m_\pi \approx 420$ MeV) and $am_s = 0.04$ with a lattice spacing $a \approx 0.12$ fm
- wall source prop. for $K$ and $\pi$, seq. source prop. for conserved vector current $J_\mu$, $H_W$ is the sink
- 127 trajs used in the analysis
- kaon momentum $p_K = \frac{2\pi}{L}(1,0,0)$, pion momentum $p_\pi = (0,0,0)$

# Unintegrated 4-point function

- $\langle \pi(p_\pi)| T[J_\mu(0) H_W(t)] |K(p_K)\rangle$, $t = t_H - t_J$
- large $t$, correlator dominated by ground intermediate state
- blue band gives the ground state contribution




- $W$ and $C$ only, figure compiled by Antonin Portelli

# integrated 4-point function

$$X_{E_-} = -\sum_n \frac{\langle\pi|J|n\rangle\langle n|H_W|K\rangle}{E_K - E_n}\left(1 - e^{(E_K - E_n)T_a}\right)$$

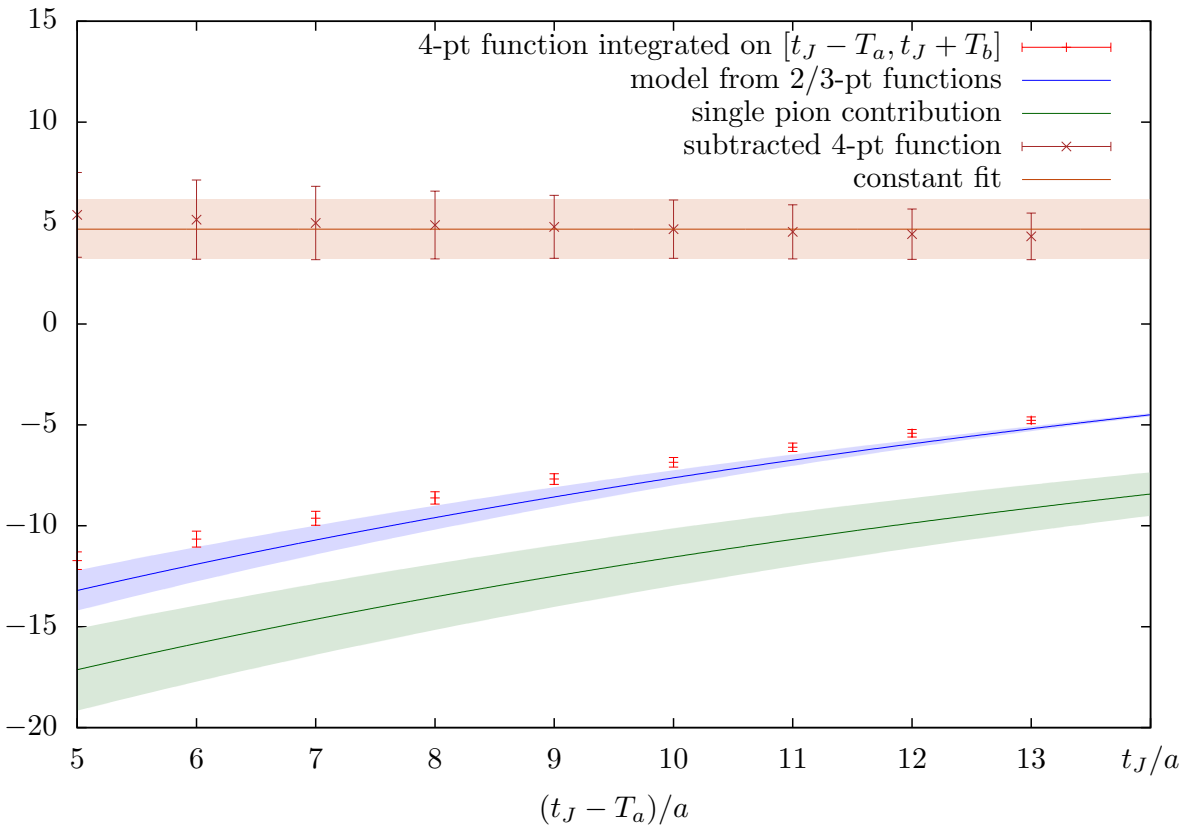
$$X_{E_+} = \sum_{n_s} \frac{\langle\pi|H_W|n_s\rangle\langle n_s|J|K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{(E_\pi - E_{n_s})T_b}\right)$$

$T_b = 9$, $T_a$ runs over $[0, 9]$
(plotted by Antonin Portelli)

model: $|n\rangle = |\pi\rangle$, $|n_s\rangle = |K\rangle$

single pion contamination

$$\frac{\langle\pi|J|\pi\rangle\langle\pi|H_W|K\rangle}{E_K - E_\pi} e^{(E_K - E_\pi)T_a}$$



# Conclusion

- rare kaon decays are important for the probe of NP, determination of CKM matrix element such as $V_{td}$, detect of CPV effects
- both $K^+ \to \pi^+ \nu\bar{\nu}$ and $K \to \pi l^+ l^-$ are well motivated for lattice calculations
  - for $\nu\bar{\nu}$, aim at determining LD contribution $\delta P_{c,u}$ within 50%
  - for $l^+ l^-$, aim at determining the sign and value of $a_S$
- techniques are developed and numerical calculations are running
- although very preliminary, clear signal are oberserved from lattice data