NRQCD based S- and P-wave Bottomonium spectra at finite temperature from 48^3x12 lattices with Nf=2+1 light HISQ flavors

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Physics motivation

Heavy Quarkonium provides a unique window into the physics of the quark-gluon plasma (QGP) created in relativistic heavy-ion collisions at RHIC and LHC. I.e. its suppression pattern in nucleus-nucleus collisions, compared to proton-proton collisions encodes vital information about the medium it traverses. Bottomonium is of particular interest, as its suppression is believed to be dominated by thermal effects of the QGP on the bound state. Lattice QCD can provide first principles insight into such in-medium binding properties, especially around the deconfinement phase transition, where QCD is truly strongly interacting. Our study confirms earlier reports on the survival of the $^3S_1$Upsilon state up to 286MeV but in contrast hints at survival of the $^3P_1$ χ_{b1} state up to the same T.

- **Challenge 1**: The large rest mass makes simulations based on relativistic fermion actions prohibitively expensive a^{-1}>>M_b≈4.5GeV. **Strategy:** Deploy effective field theory NRQCD
- **Challenge 2**: Binding properties are encoded in spectral functions, which are not directly accessible in Euclidean time simulations. **Strategy:** Bayesian spectral reconstruction

Lattice Non-Relativistic QCD (NRQCD)

- **NRQCD**: separation of scales $\mathbf{M_b >> T,\Lambda_{QCD}}$ allows to treat bottom quarks as non-relativistic Pauli spinors (ψ,χ) that propagate in the background of a relativistic medium.
- **NRQCD on the lattice:** Systematic expansion of the heavy quark part of the QCD action in $(M_b a)^{-1} < 1$, i.e. up to order $O(v^4)$ in velocity power counting Lepage et.al, Phys.Rev. D46 (1992) 4052-4067
- Initial value problem for b-quark propagator Euclidean time evolution G(τ):

$$G(\mathbf{x},\tau+a) = \left(1-\frac{H_0}{2n}\right)^n U_4^\dagger(\mathbf{x},\tau)\left(1-\frac{H_0}{2n}\right)^n (1-\delta H)\, G(\mathbf{x},\tau) \qquad H_0 = -\frac{\Delta^{(2)}}{2M_b} \qquad \Delta^{(2n)} = \sum_i (\Delta_i^+\Delta_i^-)^n$$

$$\delta H = -\frac{(\Delta^{(2)})^2}{8M_b^3} + \frac{ig}{8M_b^2}(\Delta^\pm\cdot\mathbf{E}-\mathbf{E}\cdot\Delta^\pm) - \frac{g}{8M_b^2}\boldsymbol{\sigma}\cdot(\Delta^\pm\times\mathbf{E}-\mathbf{E}\times\Delta^\pm) - \frac{g}{2M_b}\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{a^2\Delta^{(4)}}{24M_b} - \frac{a(\Delta^{(2)})^2}{16nM_b^2}$$

- S-wave and P-wave Bottomonium propagator D(τ) from appropriate vertex operators

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x},\tau) G_{x\tau} O^\dagger(\mathbf{x}_0,\tau_0) G^\dagger_{x\tau}\rangle_{med} \qquad O(^3S_1;x,\tau) = \sigma_i, \qquad O(^3P_1;x,\tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

- **Light medium degrees** from fully relativistic dynamical QCD simulations

HotQCD HISQ/tree action $48^3 \times N_\tau$ $m_{u,d}/m_s = 0.05$ $T_C = 154(9)MeV$

β	6.664	6.700	6.740	6.770	6.800	6.840	6.880
a[fm]	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau=12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
a[fm]	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
$M_b a$	2.187	2.107	2.030	1.956	1.835	1.753	1.559
$T/T_C(N_\tau=12)$	1.151	1.194	1.240	1.286	1.371	1.436	1.614

A. Bazavov et. al. Phys. Rev. D 85 (2012) 054503

- Lattice NRQCD contains an implicit renormalization dependent energy shift: need to fix the absolute energy scale from additional T≈0 runs. (S-wave ground state mass ≡ m_Υ^{PDG})
- Energy shift simplifies relation to spectra at T>0: $D(\tau) = \int_{-2M_b}^{\infty} d\omega\, e^{-\omega\tau}\rho(\omega)$

Bayesian Spectral Reconstruction

- An **ill-posed problem**: Extract a spectral function $\rho(\omega_l)=\rho_l$ along N_ω frequencies from N_τ<<N_ω noisy datapoints $D(\tau_i)=D_i$ (**likelihood L** fit alone is underdetermined)

$$D_i^\rho = \sum_{l=1}^{N_\omega} \Delta\omega_l e^{-\omega_l\tau_i}\rho_l \qquad L = \frac{1}{2}\sum_{i,j}^{N_\tau}\left(D_i - D_i^\rho\right)C_{ij}^{-1}\left(D_j - D_j^\rho\right)$$

C_{ij} covariance matrix

- Bayes Theorem: Incorporation of **prior information (I) regularizes the χ^2 fit**

$$P[\rho|D,I] \propto P[D|\rho,I]\,P[\rho|I] \qquad P[D|\rho,I] = \exp[-L-\gamma(L-N_\tau)^2]$$

Since the correct ρ leads to L~N_τ

- **Improved prior functional:** enforces (1) positive definiteness of ρ (2) independence of the result from dimension of ρ (3) smoothness of ρ, where data does not imprint peaks

$$P[\rho|I] = e^S \qquad S = \alpha\sum_{l=1}^{N_\omega}\Delta\omega_l\left(1-\frac{\rho_l}{m_l}+\log\left[\frac{\rho_l}{m_l}\right]\right)$$

m_l **default model**: correct spectrum in the absence of data

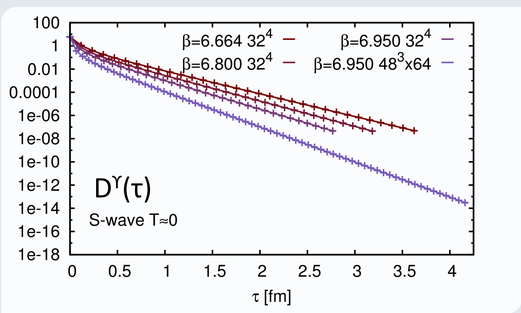
Y.Burnier, A. Rothkopf Phys.Rev.Lett. 111 (2013) 18, 182003

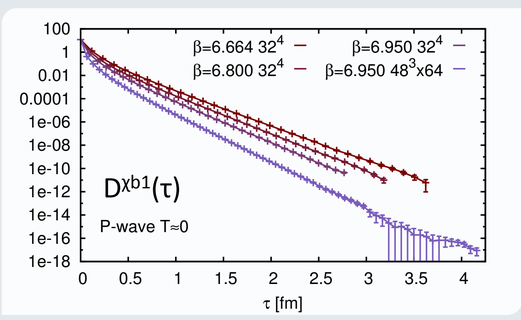
- α is integrated out analytically P[α]=1, Bayesian solution as maximum of the posterior

$$P[\rho|D,I] \propto P[D|\rho,I]\int_0^\infty d\alpha\, P[\rho|I,\alpha] \qquad \frac{\delta}{\delta\rho}P[\rho|D,I]\Big|_{\rho=\rho^{BR}} = 0$$

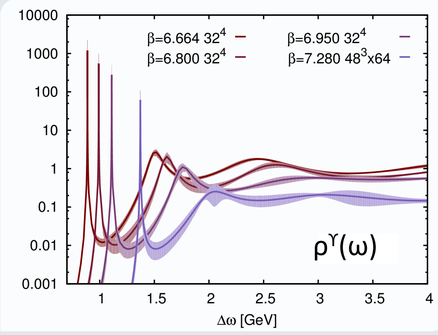
Bottomonium at T=0

- **T=0 correlators** from 100 measurements at each β, P-wave suffers from larger mass





- **T=0 Bayesian spectral reconstruction**: $N_\tau = 32, 64(\beta = 7.280), N_\omega = 1000, \tau_{max}^{num} = 20, I_\omega^{num} = [-0.5, 20]$
  - constant default model normalized by D(0), error bars from 10 Jackknife bins



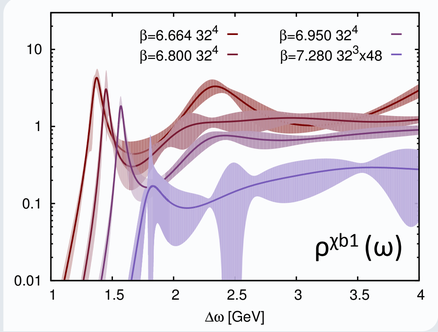
Energy calibration:

$$m^\Upsilon(\beta) = m_{PDG}^\Upsilon + E_{shift}(\beta)$$

$$m_{PDG}^\Upsilon = 9.46030(26)GeV$$

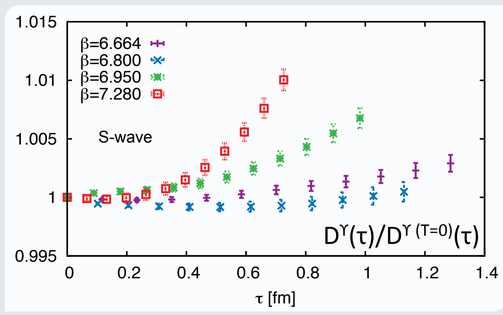
$$m_{NRQCD}^{\chi_{b1}} = 9.917(3)GeV$$

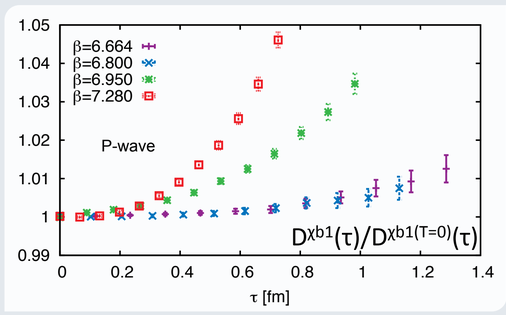
$$m_{PDG}^{\chi_{b1}} = 9.89278(26)(31)GeV$$



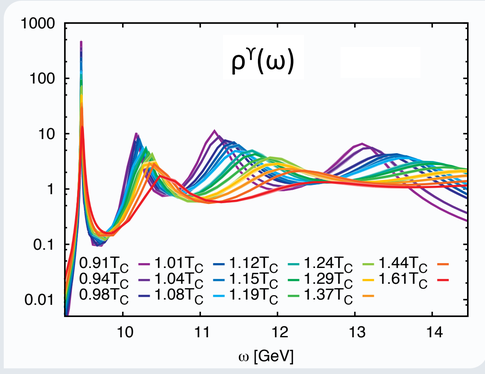
Bottomonium at finite T

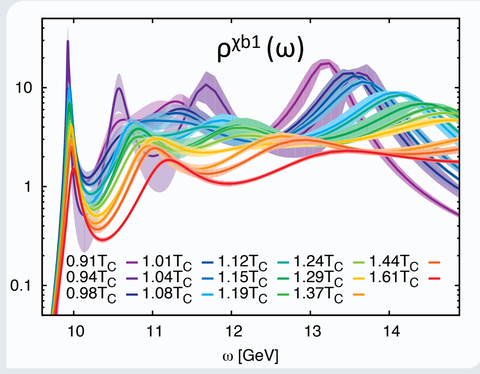
- **T>0 correlators** from 400 measurements on lattices with temporal extend N_τ=12



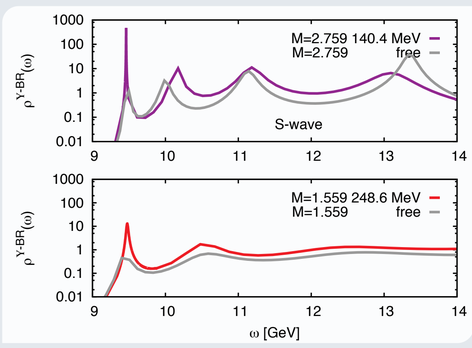


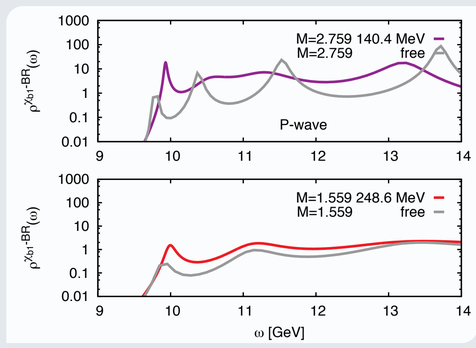
- Clear temperature dependence beyond statistical errors, more pronounced in the P-wave channel.
- **T>0 Bayesian spectral reconstruction**: $N_\omega = 1200, \tau_{max}^{num} = 20, I_\omega^{num} = [-1, 25]$
  - m(ω)=m_0 normalized by D(0), 10 Jackknife bins, high resolution interval (N_{hr}=550) around lowest peak



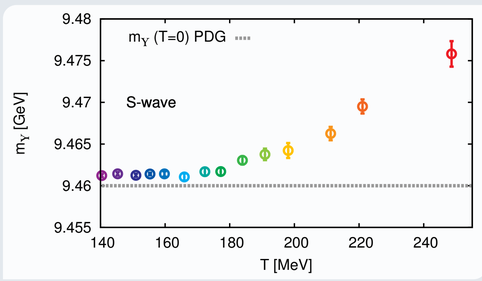


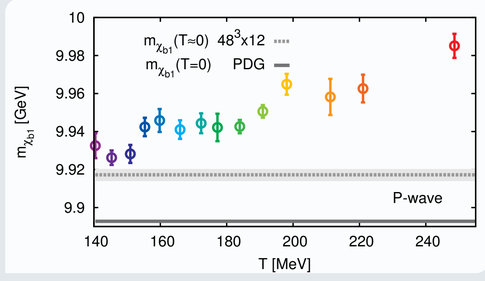
- Both S-wave and P-wave channel show **well defined ground state peaks at all temperatures**. At T>1.44T_C however simple inspection by eye cannot resolve whether χ_{b1} bound state survives.
- **Comparison** to spectra from **free NRQCD** correlators (gray): Due to finite N_τ, reconstruction shows peaked features which are not encoded in the data. However ground state peaks in the full spectrum are always at least a factor 2-3 larger than the artificial ringing in the free spectra





- Fit of the ground state peak with a Lorentzian reveals: Upsilon mass stable up to T=175MeV (> T_C), while χ_{b1} mass appears affected immediately above T_C. Small difference to T=0 mass at lowest T related to reduced number of available datapoints (32 vs. 12).





- Checks of systematic uncertainties (e.g. default model), show that the S-wave reconstruction is reliable. P-wave systematics however are at least twice the statistical error due to worse signal to noise ratio.

Conclusion

- Lattice NRQCD paired with the improved Bayesian method for spectral reconstruction allows a reliable investigation of the spectral features of in-medium Bottomonium states.
- Our study confirms previous findings that the $^3S_1$ S-wave ground state (Upsilon) survives well into the QGP phase up to T=1.61T_C. The comparison to free NRQCD spectra in addition hints at the survival of the $^3P_1$ P-wave ground state (χ_{b1}) up to the same temperature.

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