Non-perturbative renormalization of the axial current in $N_f = 3$ lattice QCD with Wilson fermions and tree-level improved gauge action

John Bulava    Michele Della Morte    Jochen Heitger
Christian Wittemeier (speaker)

Lattice 2014, Columbia University, New York City

24 June 2014
Motivation

Axial current

\[ A^a_\mu(x) = \bar{\psi}(x) \frac{1}{2} \gamma^a \gamma^5 \psi(x) \]

Applications

- PCAC masses
- Decay constants \( F_{PS} \) (in particular for scale setting with \( f_K \))
- Matching of HQET currents

Improvement: \( (A_I)^a_\mu(x) = A^a_\mu(x) + ac_A \cdot \tilde{\partial}_\mu P^a(x) \)

Renormalization: \( (A_R)^a_\mu(x) = Z_A \cdot (1 + b_A am_q) \cdot (A_I)^a_\mu(x) \)

- Leading coefficient, sensitive to errors
- Non-perturbative \( Z_A \) needed at \( g_0^2 \approx 1 \)
Strategy

Strategy from $N_f = 2$

- references:
  - arxiv:hep-lat/0503003 for $c_A$
  - arxiv:hep-lat/0505026, arxiv:0807.1120 for $Z_A$

- Schrödinger functional
- pseudoscalar sources with wave function $\omega_\pi(x - y)$ approximating ground state
- line of constant physics (LCP)
- renormalization condition based on continuum chiral Ward identity
Renormalization Condition

- based on chiral Ward identity, similar to PCAC
- insertions of two axial currents $A_0$ and external sources $O_{\text{ext}}$

\[
\int d^3x d^3y \, \epsilon^{abc} \left\langle A_0^a(x) A_0^b(y) O_{\text{ext}}^c \right\rangle \\
-2m \int d^3x d^3y \, \epsilon^{abc} \int_{x_0}^{x_0'} dx_0' \left\langle P^a(x_0', x) A_0^b(y) O_{\text{ext}}^c \right\rangle \\
= i \int d^3y \left\langle V_0^c(y) O_{\text{ext}}^c \right\rangle
\]

- RHS due to variation of second $A_0$ insertion
- non-vanishing PCAC mass is explicitly taken into account to facilitate extrapolation to $m = 0$
comparison of the chiral extrapolation at $\beta = 5.2$, taken from $N_f = 2$:
Setup
Schrödinger functional
- periodic in space, Dirichlet in time
- boundary fields $\zeta$, $\zeta'$ to build sources

Dimensions

$$\frac{T}{L} = \frac{3}{2} \quad L \approx 1.2 \text{ fm}$$
- trade-off between large infrared cutoff and small $O(a^2)$ effects [arxiv:0807.1120]
- big $O(a^2)$ ambig. @ $N_f = 2$, $L = 0.8 \text{ fm}$

Pseudoscalar Sources at Top and Bottom

\[ O_{\text{ext}} = -\frac{1}{6L^6} \epsilon^{cde} O'^d O^e \]

\[ O^e = \sum_{uv} \bar{\zeta}(u) \frac{1}{2} \gamma^e \gamma_5 \omega(u - v) \zeta(v) \]
choose WF $\omega_\pi$ that couples only to the ground state

- (periodic) basis functions
  \[
  \bar{\omega}_1(r) = e^{-r/r_0} \quad \bar{\omega}_2(r) = r \cdot e^{-r/r_0} \quad \bar{\omega}_3 = e^{-r/(2r_0)}
  \]
  \[
  \omega_i(x) = N_i \sum_{n \in \mathbb{Z}^3} \bar{\omega}_i(|x - nL|)
  \]
  ($r_0$: some physical length scale)

- determine eigenvalues $\lambda^{(0)} > \lambda^{(1)} > \lambda^{(2)}$ and eigenvectors $\eta^{(0)}$, $\eta^{(1)}$, $\eta^{(2)}$ of $3 \times 3$ matrix $F_1(\omega_i, \omega_j)$

$$\eta^{(0)} = (0.53176, 0.59773, 0.59996)$$

- approximate $\omega_\pi$ by
  \[
  \omega_\pi \approx \sum_i \eta_i^{(0)} \omega_i
  \]
Correlators

- basic Ward identity:

\[
\int d^3 x d^3 y \, \epsilon^{abc} \left\langle A^a_0(x) A^b_0(y) O^c_{\text{ext}} \right\rangle \\
- 2m \int d^3 x d^3 y \, \epsilon^{abc} \int_{y_0}^{x_0} dx' \left\langle P^a(x', x) A^b_0(y) O^c_{\text{ext}} \right\rangle \\
= i \int d^3 y \left\langle V^c_0(y) O^c_{\text{ext}} \right\rangle
\]

- in terms of renormalized Schrödinger-functional correlation functions:

\[
Z_A^2 \cdot \left[ F^I_{AA}(x_0, y_0) - 2m \cdot \tilde{F}^I_{PA}(x_0, y_0) \right] = F_1
\]

\( b_A \) term is neglected, \( O(am) \) effect

\[
Z_A(g_0^2) = \lim_{m \to 0} \sqrt{F_1} \left[ F^I_{AA}(x_0, y_0) - 2m \cdot \tilde{F}^I_{PA}(x_0, y_0) \right]^{-1/2}
\]
Correlators

\[ f_{XY}(x_0, y_0) = -\frac{a^6}{6L^6} \sum_{x,y} \varepsilon^{abc} \varepsilon^{cde} \left\langle O^d \cdot X^a \cdot Y^b \cdot O^e \right\rangle \]

with insertions of

\[ A_0^a(x_0), \quad \partial_0 P^a(x_0), \quad \tilde{P}^a(x, y_0) = \sum_{t=y_0}^{x_0} w(t) \cdot P^a(t, x) \]

Connected and Disconnected Contributions:

- standard choice: \( x_0 = 2/3 \cdot T \) and \( y_0 = 1/3 \cdot T \)
- implemented in SFCF code and checked against old results
- alternative definition \( Z_{A,\text{con}} \) with connected only

Christian Wittemeier (Lattice 2014)

\( Z_A \) with \( N_f = 3 \) and TLI action
Simulation Parameters and Status of Results

- possible re-use of configurations from $c_A$ determination
- previous talk by J. Heitger
- openQCD code
- Lüscher, Schaefer (arxiv:1206.2809)
- $N_f = 3$ and tree-level-improved (Lüscher–Weisz) action
- $T = 3/2 \cdot L$
- $\theta = 0$, vanishing background field
- $\beta$ tuned to keep $L$ constant ($\approx 1.2$ fm)
- $\kappa$ tuned towards vanishing (PCAC) quark mass
### First Results

<table>
<thead>
<tr>
<th>$L/a$</th>
<th>$T/a$</th>
<th>$\beta$</th>
<th>$\kappa$</th>
<th>$am_{\text{PCAC}}$</th>
<th>$Z_{A,\text{con}}$</th>
<th>$Z_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>17</td>
<td>3.3</td>
<td>0.13652</td>
<td>$-0.00096(71)$</td>
<td>0.80(10)</td>
<td>0.65(10)</td>
</tr>
<tr>
<td>12</td>
<td>17</td>
<td>3.3</td>
<td>0.13660</td>
<td>$-0.0086(6)$</td>
<td>0.82(10)</td>
<td>0.63(10)</td>
</tr>
<tr>
<td>16</td>
<td>23</td>
<td>3.512</td>
<td>0.13700</td>
<td>$+0.0064(2)$</td>
<td>0.78(5)</td>
<td>0.76(5)</td>
</tr>
<tr>
<td>16</td>
<td>23</td>
<td>3.512</td>
<td>0.13703</td>
<td>$+0.0056(3)$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>16</td>
<td>23</td>
<td>3.512</td>
<td>0.13710</td>
<td>$+0.0024(2)$</td>
<td>0.80(5)</td>
<td>0.74(5)</td>
</tr>
<tr>
<td>20</td>
<td>29</td>
<td>3.676</td>
<td>0.13680</td>
<td>$+0.0139(2)$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>20</td>
<td>29</td>
<td>3.676</td>
<td>0.13700</td>
<td>$+0.0066(1)$</td>
<td>0.79(5)</td>
<td>0.79(5)</td>
</tr>
<tr>
<td>24</td>
<td>35</td>
<td>3.810</td>
<td>0.13712</td>
<td>$-0.00269(8)$</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

- only $\mathcal{O}(1000)$ MDU analyzed so far
- $Z_{A,\text{con}}$ not yet conclusive (need more statistics)
- $Z_A$: no strong mass dependence observed
Summary

- renormalization condition based on PCAC relation with non-vanishing quark mass
- evaluation in Schrödinger-functional setup
- reuse of configurations from $c_A$ determination

Outlook

- most measurements yet to be done...
- maybe some new simulations at smaller masses
- crosscheck analysis
- determination of $Z_V$

Thank you!