Non-perturbative renormalization of the axial current in $N_\mathrm{f} = 3$ lattice QCD with Wilson fermions and tree-level improved gauge action

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Motivation

Axial current

$$A^a_\mu(x) = \bar{\psi}(x)\, \frac{1}{2}\tau^a\, \gamma_\mu\gamma_5\, \psi(x)$$

Applications

- PCAC masses
- decay constants F_{PS} (in particular for scale setting with f_{K})
- matching of HQET currents

Improvement:	$(A_{\mathrm{I}})^a_\mu(x) = A^a_\mu(x) + ac_A \cdot \tilde{\partial}_\mu P^a(x)$
Renormalization:	$(A_{\mathrm{R}})^a_\mu(x) = Z_{\mathrm{A}} \cdot (1 + b_{\mathrm{A}} am_{\mathrm{q}}) \cdot (A_{\mathrm{I}})^a_\mu(x)$

- leading coefficient, sensitive to errors
- non-perturbative Z_{A} needed at $g_0^2 \approx 1$

Strategy

Strategy from $N_f = 2$

- references:
  - arxiv:hep-lat/0503003 for c_{A} ALPHA Collaboration
  - arxiv:hep-lat/0505026, arxiv:0807.1120 for Z_{A} ALPHA Collaboration
- Schrödinger functional
- pseudoscalar sources with wave function $\omega_\pi(\mathbf{x} - \mathbf{y})$ approximating ground state
- line of constant physics (LCP)
- renormalization condition based on continuum chiral Ward identity

Renormalization Condition

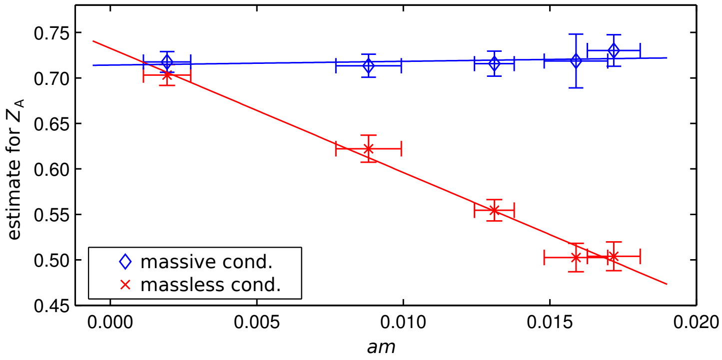
- based on chiral Ward identity, similar to PCAC
- insertions of two axial currents A_0 and external sources O_{ext}

$$\int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\left\langle A_0^a(x)\,A_0^b(y)\,O_{\text{ext}}^c\right\rangle$$
$$-2m\int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\int_{y_0}^{x_0}\mathrm{d}x_0'\left\langle P^a(x_0',\mathbf{x})\,A_0^b(y)\,O_{\text{ext}}^c\right\rangle$$
$$= \mathrm{i}\int \mathrm{d}^3\mathbf{y}\,\langle V_0^c(y)\,O_{\text{ext}}^c\rangle$$

- RHS due to variation of second A_0 insertion
- non-vanishing PCAC mass is explicitly taken into account to facilitate extrapolation to $m = 0$

Renormalization Condition

comparison of the chiral extrapolation at $\beta = 5.2$, taken from $N_\mathrm{f} = 2$:



`arxiv:hep-lat/0505026`, fig. 2

Setup

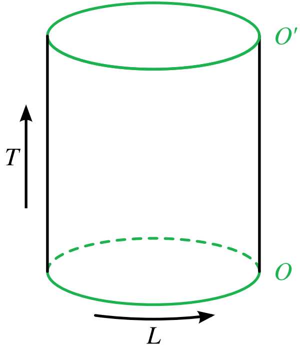
Schrödinger functional

- periodic in space, Dirichlet in time
- boundary fields ζ, ζ' to build sources

Dimensions

$$T/L = 3/2 \qquad\qquad L \approx 1.2\,\mathrm{fm}$$

- trade-off between large infrared cutoff and small $\mathcal{O}(a^2)$ effects arxiv:0807.1120
- big $\mathcal{O}(a^2)$ ambig. @ $N_\mathrm{f} = 2$, $L = 0.8\,\mathrm{fm}$



Pseudoscalar Sources at Top and Bottom

$$O^c_\mathrm{ext} = -\frac{1}{6L^6}\,\epsilon^{cde}\,O'^d\,O^e$$

$$O^e = \sum_{\mathbf{uv}} \bar{\zeta}(\mathbf{u})\,\frac{1}{2}\tau^e\,\gamma_5\,\omega(\mathbf{u}-\mathbf{v})\,\zeta(\mathbf{v})$$

Wave Functions

choose WF ω_π that couples only to the ground state

- (periodic) basis functions

$$\bar{\omega}_1(r) = \mathrm{e}^{-r/r_0} \qquad \bar{\omega}_2(r) = r \cdot \mathrm{e}^{-r/r_0} \qquad \bar{\omega}_3 = \mathrm{e}^{-r/(2r_0)}$$

$$\omega_i(x) = N_i \sum_{\mathbf{n}\in\mathbb{Z}^3} \bar{\omega}_i(|x - \mathbf{n}L|)$$

(r_0: some physical length scale)

- determine eigenvalues $\lambda^{(0)} > \lambda^{(1)} > \lambda^{(2)}$ and eigenvectors $\eta^{(0)}$, $\eta^{(1)}$, $\eta^{(2)}$ of 3×3 matrix $F_1(\omega_i, \omega_j)$

$$\eta^{(0)} = (0.53176, 0.59773, 0.59996)$$

- approximate ω_π by

$$\omega_\pi \approx \sum_i \eta_i^{(0)} \omega_i$$

Correlators

- basic Ward identity:

$$\int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\left\langle A_0^a(x)\,A_0^b(y)\,O_{\mathrm{ext}}^c\right\rangle$$
$$-2m\int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\int_{y_0}^{x_0}\mathrm{d}x_0'\left\langle P^a(x_0',\mathbf{x})\,A_0^b(y)\,O_{\mathrm{ext}}^c\right\rangle$$
$$=\mathrm{i}\int \mathrm{d}^3\mathbf{y}\,\langle V_0^c(y)\,O_{\mathrm{ext}}^c\rangle$$

- in terms of renormalized Schrödinger-functional correlation functions:

$$Z_{\mathrm{A}}^2\cdot\left[F_{\mathrm{AA}}^{\mathrm{I}}(x_0,y_0)-2m\cdot\tilde{F}_{\mathrm{PA}}^{\mathrm{I}}(x_0,y_0)\right]=F_1$$

(b_{A} term is neglected, $\mathcal{O}(am)$ effect)

$$Z_{\mathrm{A}}(g_0^2)=\lim_{m\to 0}\sqrt{F_1}\left[F_{\mathrm{AA}}^{\mathrm{I}}(x_0,y_0)-2m\cdot\tilde{F}_{\mathrm{PA}}^{\mathrm{I}}(x_0,y_0)\right]^{-1/2}$$

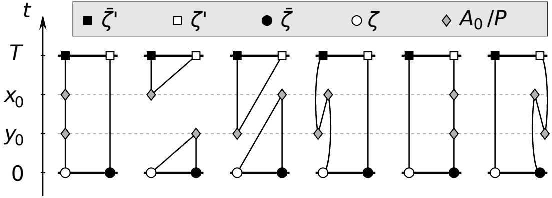
Correlators

$$f_{XY}(x_0, y_0) = -\frac{a^6}{6L^6} \sum_{\mathbf{x},\mathbf{y}} \varepsilon^{abc} \varepsilon^{cde} \left\langle O'^{d} \cdot X^a \cdot Y^b \cdot O^e \right\rangle$$

with insertions of

$$A_0^a(x_0), \qquad \partial_0 P^a(x_0), \qquad \tilde{P}^a(x, y_0) = \sum_{t=y_0}^{x_0} w(t) \cdot P^a(t, \mathbf{x})$$

Connected and Disconnected Contributions:



- standard choice: $x_0 = 2/3 \cdot T$ and $y_0 = 1/3 \cdot T$
- implemented in SFCF code and checked against old results
- alternative definition $Z_{\mathrm{A,con}}$ with connected only

Simulation Parameters and Status of Results

- possible re-use of configurations from c_A determination

  previous talk by J. Heitger
- openQCD code — Lüscher, Schaefer (`arxiv:1206.2809`)
- $N_\mathrm{f} = 3$ and tree-level-improved (Lüscher–Weisz) action
- $T = 3/2 \cdot L$
- $\theta = 0$, vanishing background field
- β tuned to keep L constant ($\approx 1.2\,\mathrm{fm}$)
- κ tuned towards vanishing (PCAC) quark mass

First Results

L/a	T/a	β	κ	am_{PCAC}	$Z_{\mathrm{A,con}}$	Z_{A}
12	17	3.3	0.13652	$-0.00096(71)$	0.80(10)	0.65(10)
12	17	3.3	0.13660	$-0.0086(6)$	0.82(10)	0.63(10)
16	23	3.512	0.13700	$+0.0064(2)$	0.78(5)	0.76(5)
16	23	3.512	0.13703	$+0.0056(3)$	–	–
16	23	3.512	0.13710	$+0.0024(2)$	0.80(5)	0.74(5)
20	29	3.676	0.13680	$+0.0139(2)$	–	–
20	29	3.676	0.13700	$+0.0066(1)$	0.79(5)	0.79(5)
24	35	3.810	0.13712	$-0.00269(8)$	–	–

- only $\mathcal{O}(1000)$ MDU analyzed so far
- $Z_{\mathrm{A,con}}$ not yet conclusive (need more statistics)
- Z_{A}: no strong mass dependence observed

Summary

- renormalization condition based on PCAC relation with non-vanishing quark mass
- evaluation in Schrödinger-functional setup
- reuse of configurations from c_{A} determination

Outlook

- most measurements yet to be done...
- maybe some new simulations at smaller masses
- crosscheck analysis
- determination of Z_{V}

Thank you!