Lattice 2014
Columbia University
New York

Effects of near-zero Dirac eigenmodes on axial U(1) symmetry at finite temperature

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for JLQCD collaboration

1.Introduction

Chiral symmetry breaking in QCD (N_f=2, m_{ud}=0)

$T = 0$

$$\underbrace{SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}}_{\text{SSB}} \times U(1)_{\mathrm{V}} \times \underbrace{U(1)_{\mathrm{A}}}_{\text{Anomaly}}$$

$$\longrightarrow SU(2)_{\mathrm{V}} \times U(1)_{\mathrm{V}}$$ Remains

$T > T_c$

$$SU(2)_{\mathrm{V}} \longrightarrow SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$$ Restored

$$U(1)_{\mathrm{A}} \longrightarrow$$ **??**

Susceptibilities, <u>Dirac Spectrum</u>

Cossu's talk

This Talk

Dirac Spectrum and Symmetry

$SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$

Banks-Casher Relation

$$|\rho(0)| = \frac{\Sigma}{\pi}$$

$U(1)_{\mathrm{A}}$

Atiyah-Singer Index Theorem

$$n_+ - n_- = \nu$$

$n_\pm$: # of chiral zero-modes

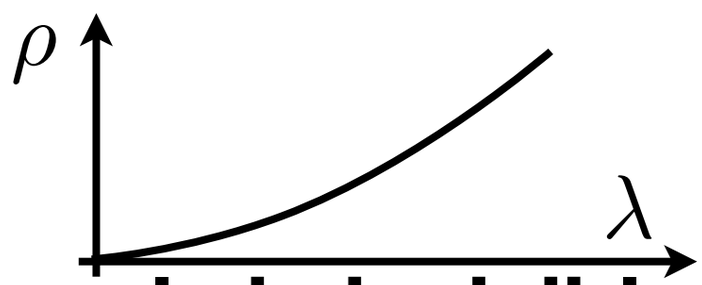
Dirac low modes are important
for both symmetries

Dirac Spectrum and Symmetry

Aoki-Fukaya-Taniguchi (2012) argued that, if we assume

- SU(2) x SU(2) is restored ($T > T_c$)
- Ginsparg-Wilson relation is satisfied
- Analyticity in mass

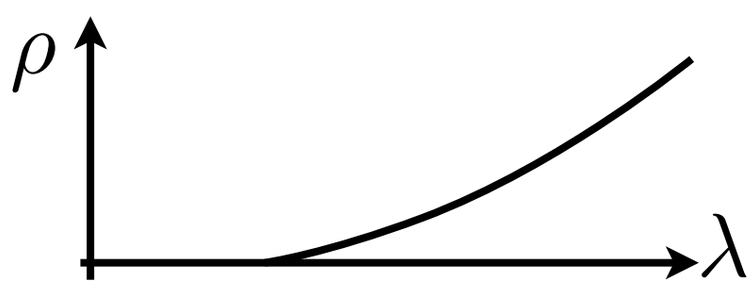
➡ **Spectrum starts from cubic power*** **at least**



$$\rho = c_3 \lambda^3 + \cdots$$

➡ **U(1)$_A$ anomaly is invisible in the (pseudo) scaler correlators** $\begin{pmatrix} \mathrm{Vol} \to \infty \\ m_{\mathrm{ud}} \to 0 \end{pmatrix}$

*G.Cossu et al (JLQCD 2013) reported a gap in the Dirac spectrum



Cohen(1996) argued that:

If the chiral zero-mode's effect is ignored,

and if there is a **gap in the Dirac spectrum**

-> $U(1)_A$ breaking susceptibility

$$= \chi_\pi - \chi_\delta$$

$$= \int_0^\infty d\lambda \frac{4m^2 \rho(\lambda)}{(m^2+\lambda^2)^2} = 0$$

(Controversial) Previous lattice studies

Group	Action	Vol.	Gap	U(1)A
JLQCD(2013)	Overlap Fixed Topology	L=16	**Yes**	Restored
Chiu et al (2013)	Optimized Domain-wall	L=16	**Yes?** $\rho \sim \lambda^3 + \cdots$	Restored
Ohno et al (2011)	HISQ	L=32	**No**	Violated
LLNL/RBC (2013)	Domain-wall	L=16, 32	**No**	Violated

What makes the difference:
Finite V effects ?
Fixed topology ?
Chiral symmetry ?

This Work

Finite volume	→	Larger volume
Fixed Topology	→	Tunneling Allowed
Chiral symmetry	→	OV/DW reweighting

Whats' New in This work?

	G.Cossu et al (2013)	**This Work**
Fermion	Overlap	Mobius Domain-wall
GW-rel.	Exact	m_{res}~1MeV or lower
Cost		
Lat. Size	16	16, 32
Topology tunneling	Frozen	Allowed
Comment		**We also try reweighting to OV**

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2. Mobius DW
3. Domain-wall Dirac spectrum
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5. (Reweighted) overlap Dirac spectrum
6. Summary

2.Mobius DW

Mobius Domain Wall

Edwards-Heller (2000)

Overlap: $$D_N(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H_K)$$

(Satisfy Ginsparg-Wilson relation)

	Domain Wall	Mobius DW
4-dim eff. operator	$D^4 = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{T^{-L_s}-1}{T^{-L_s}+1}$ $T^{-1} = \frac{1+H_T}{1-H_T}$ $\quad H_T = \gamma_5 \frac{D_W}{2+D_W}.$	$D^4 = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{\prod_s^{Ls} T_s^{-1}-1}{\prod_s^{Ls} T_s^{-1}+1}$ $T_s^{-1} = \frac{1+\omega_s H_M}{1-\omega_s H_M}.$ $\quad H_M = \gamma_5 \frac{bD_W}{2+cD_W},$ **New parameter b, c**
Parameter	L_s (Ls→∞ : OV)	L_s, b, c (Ls→∞ : OV)

b and c make m_{res} small

(b=2, c=1, 10^{-1}-10^{-3} smaller m_{res} for L_s=12)

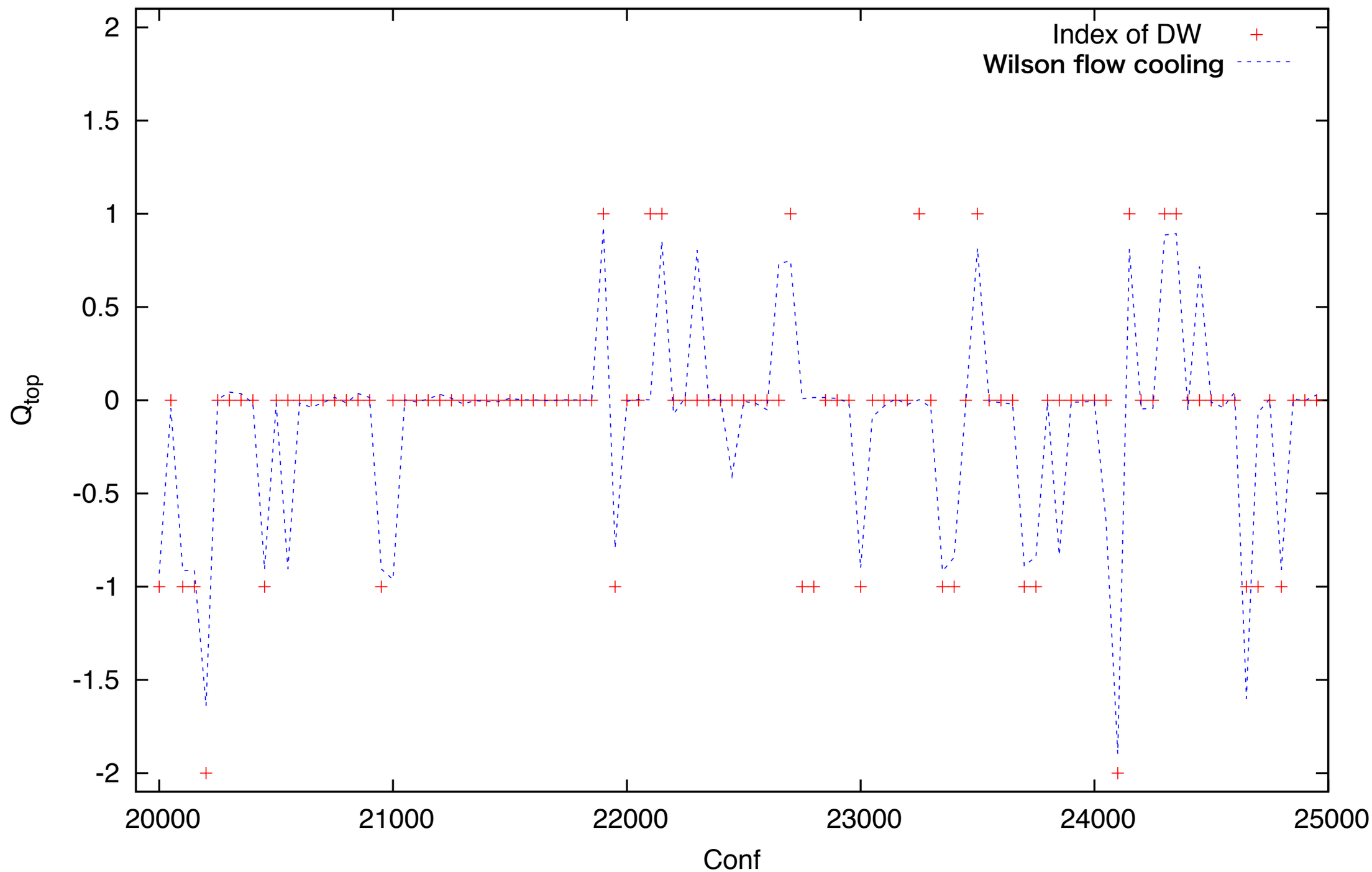
Lattice set up

Gauge action:tree level **Symanzik**
Fermion :Mobius DW(b=2, c=1, Scaled Shamir + Tanh)
w/ **Stout** smearing(3)
code :Irolro++(G. Cossu et al.)
Resource :BG/Q(KEK)

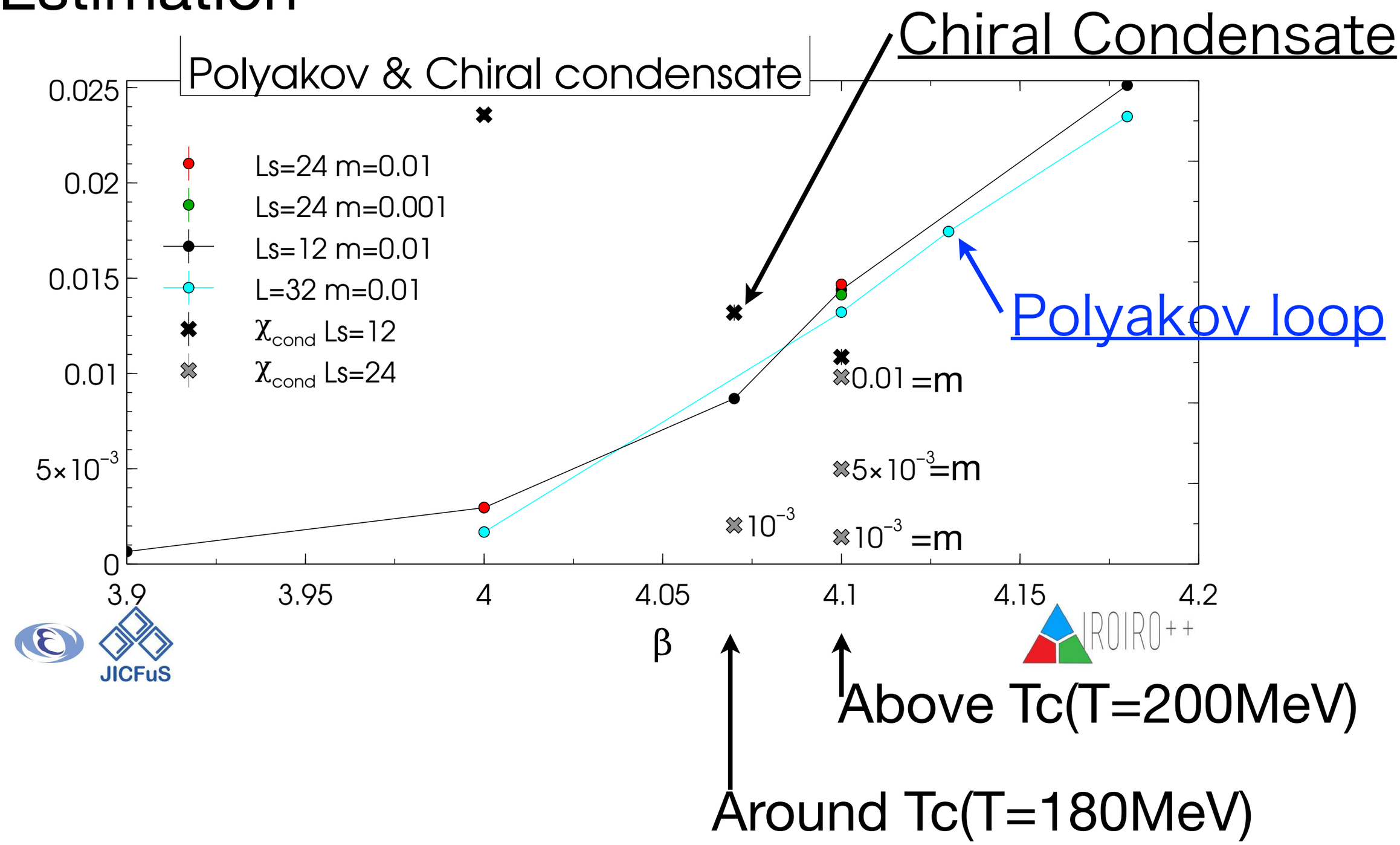
$L^3 \times L_t$	β	m_{ud}(MeV)	L_s	$m_{\rm res}$(MeV)	Temp.(MeV)	Note
$16^3 \times 8$	4.07	30	12	2.5	180	488 Conf. every 50 Trj.
$16^3 \times 8$	4.07	3.0	24	1.4	180	319 Conf. every 20 Trj.
$16^3 \times 8$	4.10	32	12	1.2	200	480 Conf. every 50 Trj.
$16^3 \times 8$	4.10	3.2	24	0.8	200	538 Conf. every 50 Trj.
$32^3 \times 8$	4.10	32	12	1.7	200	175 Conf. every 20 Trj.
$32^3 \times 8$	4.10	16	24	1.7	200	294 Conf. every 20 Trj.
$32^3 \times 8$	4.10	3.2	24	-	200	88 Conf. every 10 Trj.

Topological charge changes along HMC

$$L = 16,\ \beta = 4.10,\ m = 0.01,\ L_s = 12$$



Tc Estimation



Vol. dependence of Polyakov loop
Decreasing of Chiral condensate

3.Domain-wall Dirac spectrum

Observable

Histogram of Dirac operator

$$H_m \psi_i = \lambda_i^m \psi_i$$

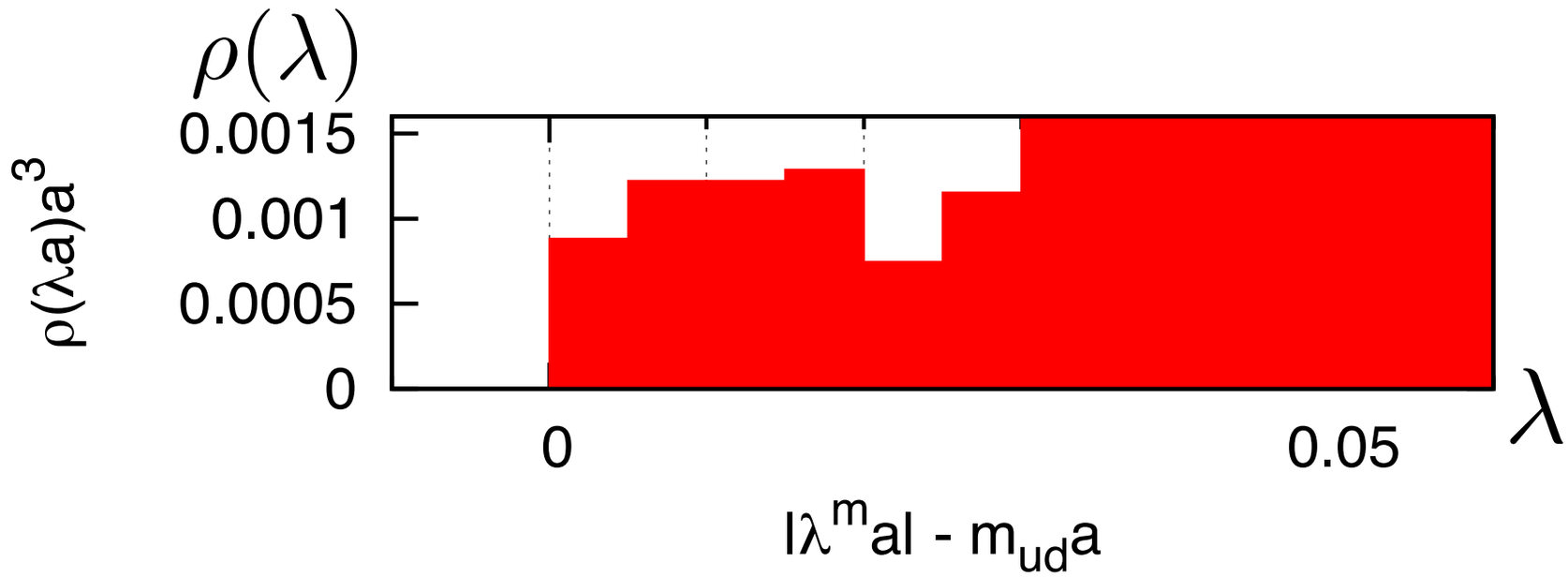
$$H_m = \gamma_5 [(1 - m_{ud}) D^4 + m_{ud}]$$

$$D^4 = [\mathcal{P}^{-1} (D^5_{\mathrm{DW}}(m = 1))^{-1} D^5_{\mathrm{DW}}(m_{ud}) \mathcal{P}]_{11}$$

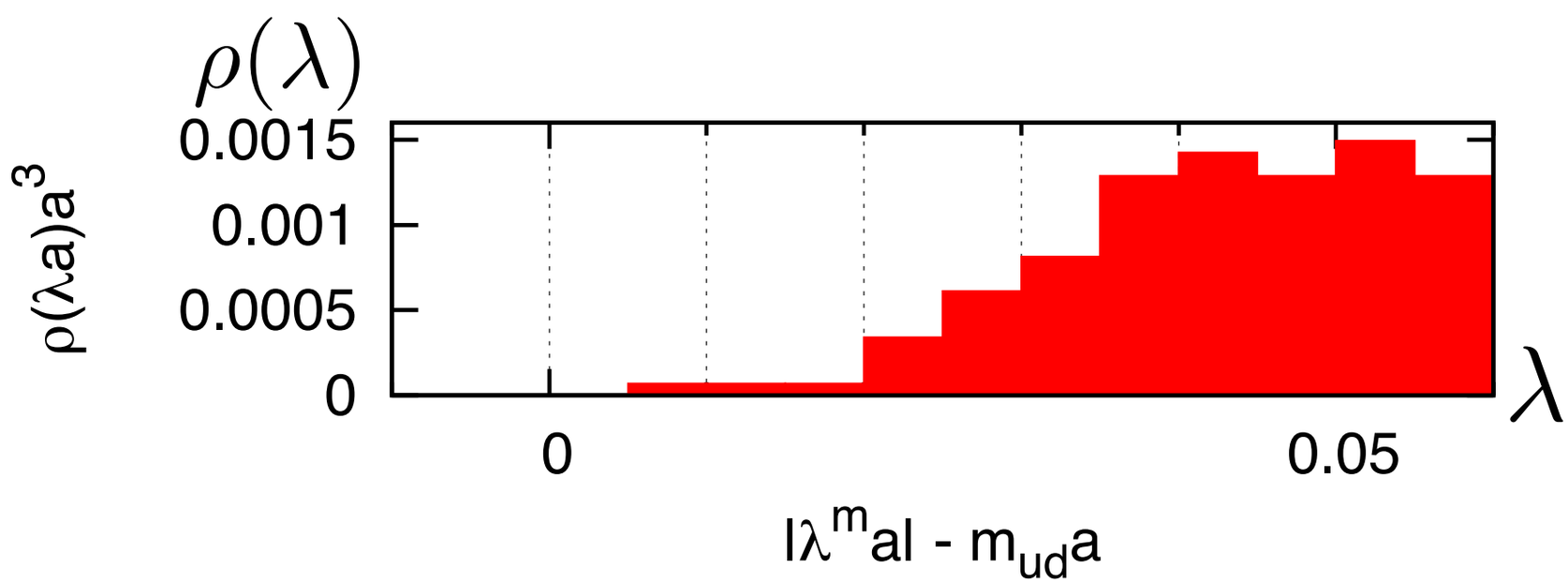
3.Histogram for DW(T~ Tc)

Akio Tomiya(Osaka Univ.)

T=180MeV~Tc(L=16)



m_{ud}=30MeV
m_{res}=2.5MeV



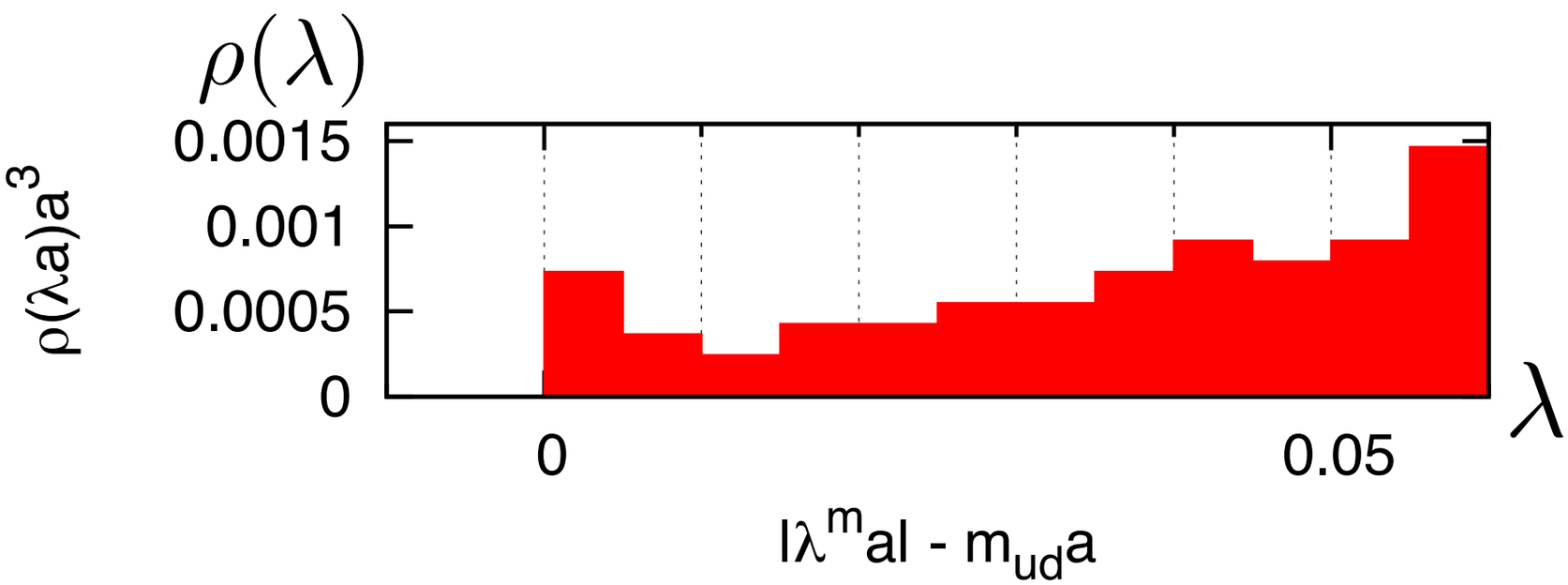
m_{ud}=3.0MeV
m_{res}=1.4MeV

Gap? Finite V effect?

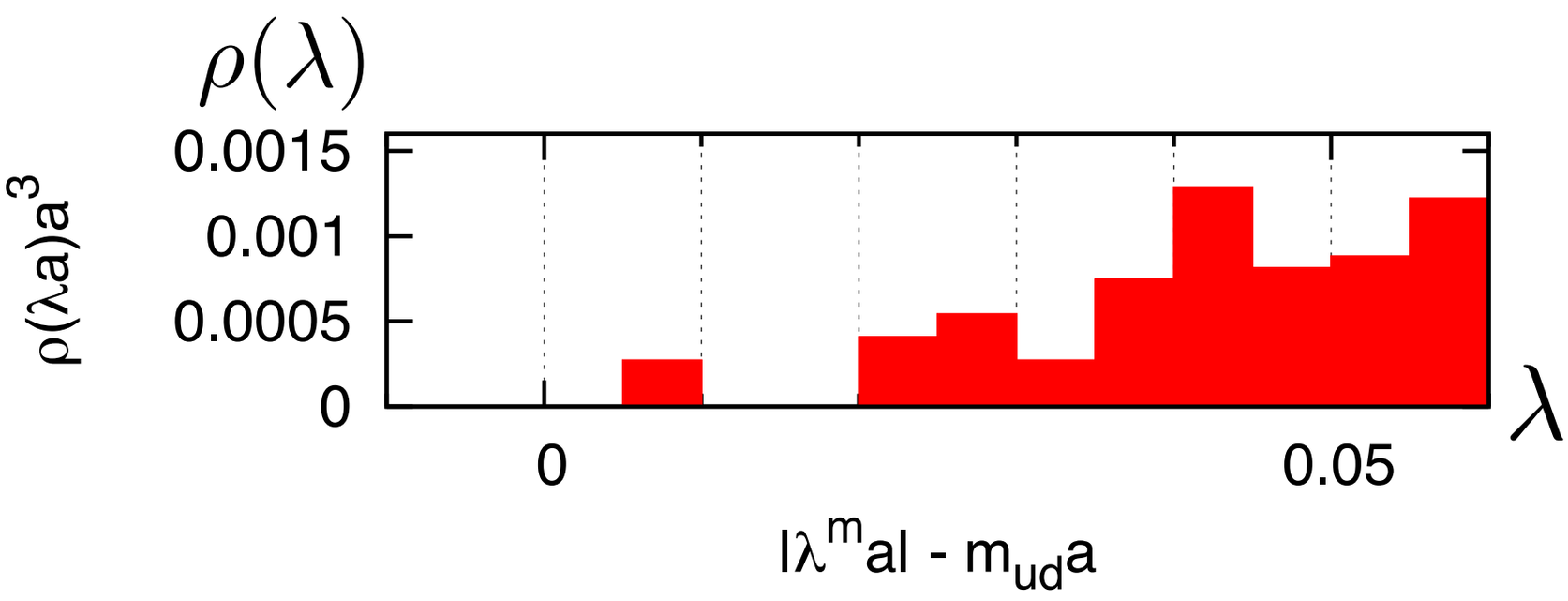
3.Histogram for DW(above Tc)

Akio Tomiya(Osaka Univ.)

T=200MeV>Tc (L=16)



m_{ud}=32MeV
m_{res}=1.2MeV



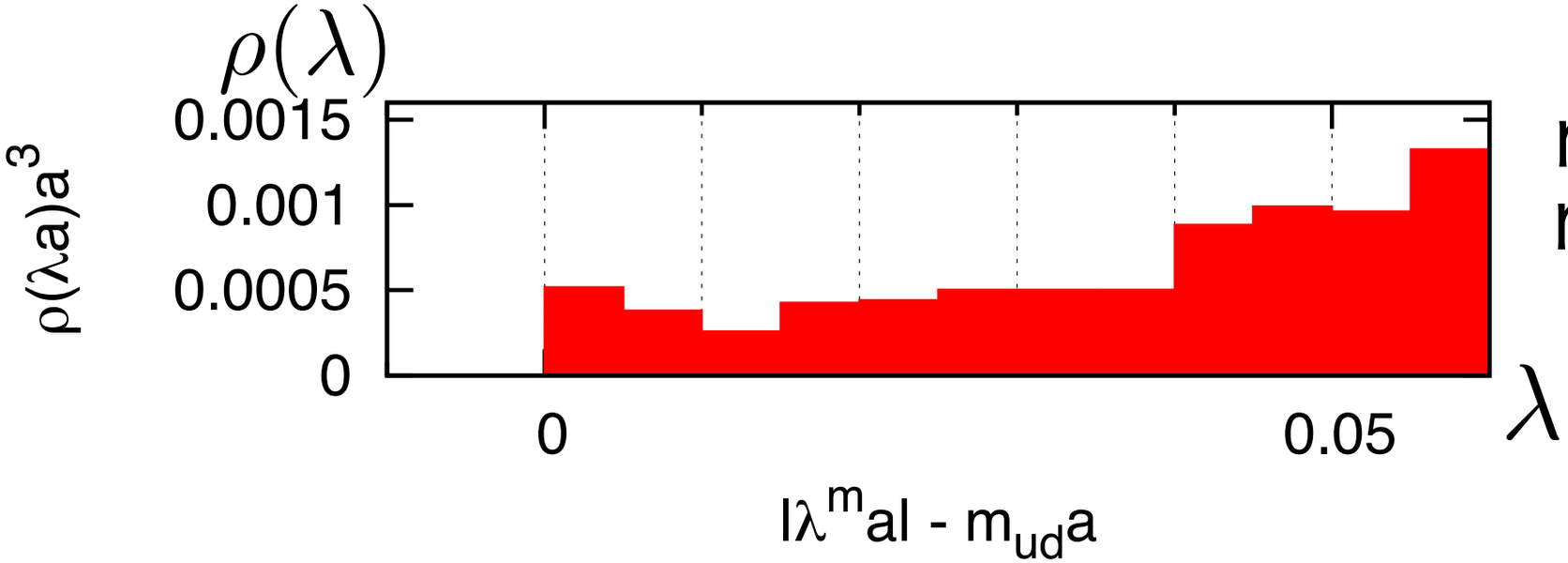
m_{ud}=3.2MeV
m_{res}=0.8MeV

Gap? Finite V effect?

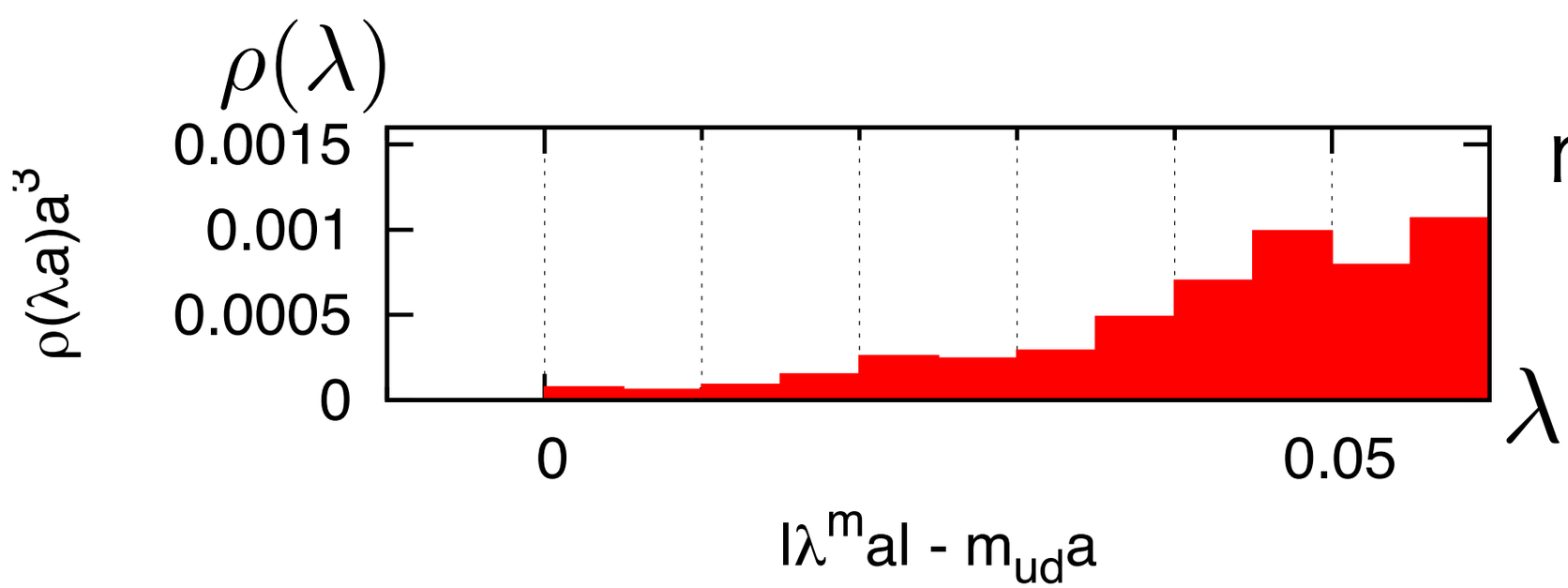
3.Histogram for DW(above Tc)

Akio Tomiya(Osaka Univ.)

T=200MeV>Tc (L=32)



m_{ud}=32MeV
m_{res}=1.7MeV



m_{ud}=3.2MeV

Very small but non-zero =>**Gap is not apparent**

U(1) looks broken

3.Histogram for DW

Short summary

L=32, T=200 MeV m_{ud}=3.2MeV No clear Gap

$U(1)_A$ looks broken

Consistent with LLNL/RBC(2013). Then, What is the difference from OV(JLQCD)?

Finite V?

topology tunneling?

Violation of Ginsparg-Wilson relation?

4.Violation of Ginsparg-Wilson relation

Violation of Ginsparg-Wilson relation for each mode

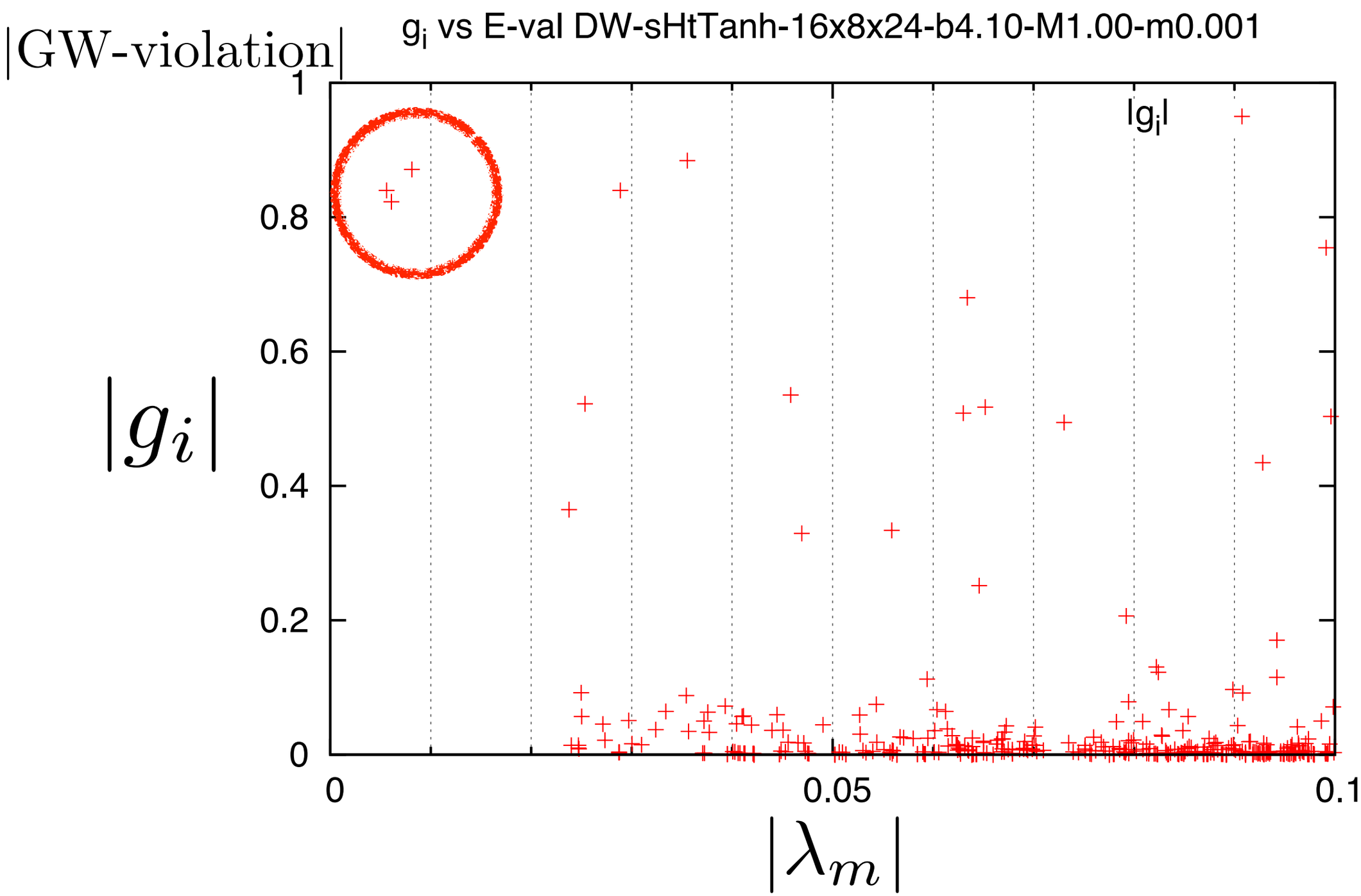
$$g_i \equiv \frac{\psi_i^\dagger \gamma_5 [D\gamma_5 + \gamma_5 D - 2D\gamma_5 D]\psi_i}{\lambda_i^m} \left[\frac{(1-m_{ud})^2}{2(1+m_{ud})}\right]$$

g_i should be zero if GW is satisfied

Cf.

$$m_{\mathrm{res}} = \frac{\sum_i \frac{\lambda_i^m (1+m_{ud})}{(1-m_{ud})^2 (\lambda_i^m)^2} g_i}{\sum_i \frac{1}{(\lambda_i^m)^2}}$$

Low-modes have significant violation of Ginsparg Wilson relation



5.(Reweighted) Overlap Dirac spectrum

Reweighting to OV

$$\langle \mathcal{O} \rangle_{\mathrm{ov}} = \left\langle \mathcal{O} \frac{\det D^2_{\mathrm{ov}}(m_{ud})}{\det D^2_{\mathrm{DW}}(m_{ud})} \frac{\det D^2_{\mathrm{DW}}(1/2a)}{\det D^2_{\mathrm{ov}}(1/2a)} \right\rangle_{\mathrm{DW}}$$

We can measure OV quantity by using DW configuration

$$\begin{cases} \langle \rho(\lambda_{\mathrm{DW}}) \rangle_{\mathrm{DW}} & \\ \langle \rho(\lambda_{\mathrm{ov}}) \rangle_{\mathrm{DW}} & \text{partially quenched OV} \\ \langle \rho(\lambda_{\mathrm{ov}}) \rangle_{\mathrm{ov}} & \text{reweighted overlap} \end{cases}$$

Let's compare them!

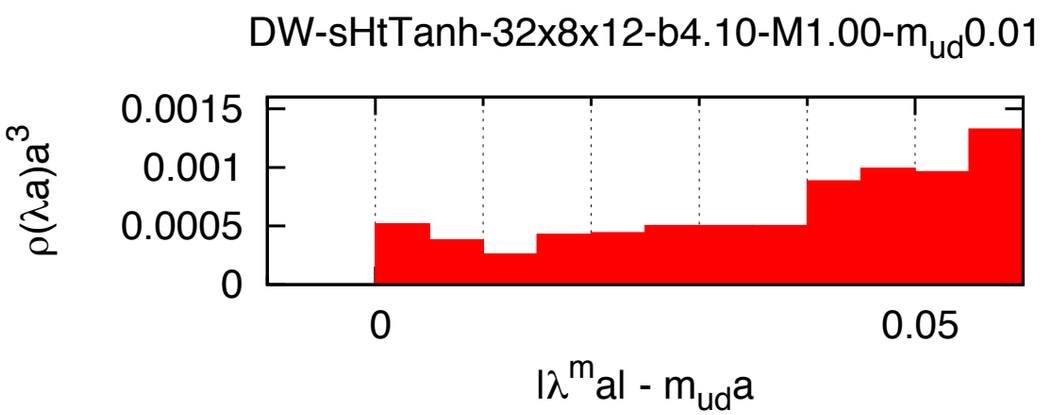
Akio Tomiya(Osaka Univ.)

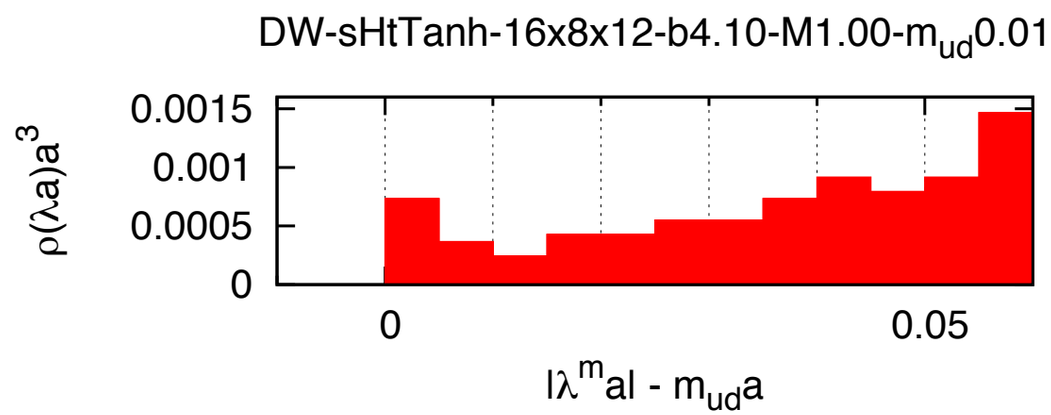
T=200MeV, m_{ud}=32MeV

L32

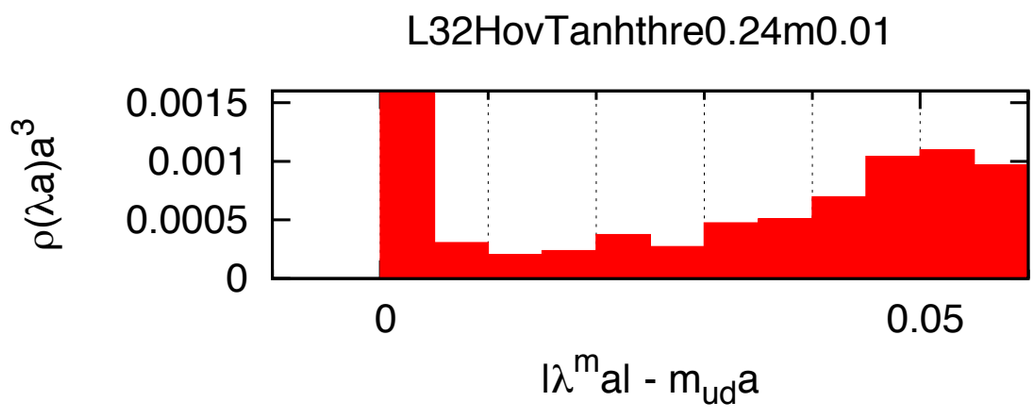
L16

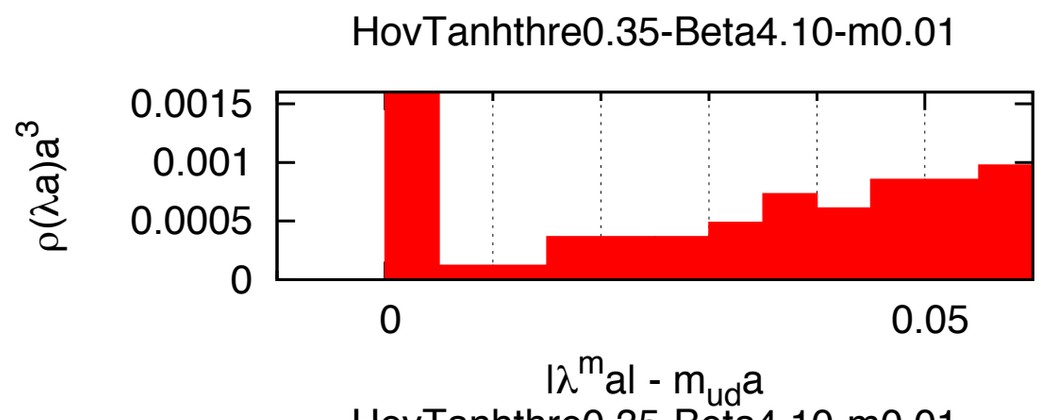
DW





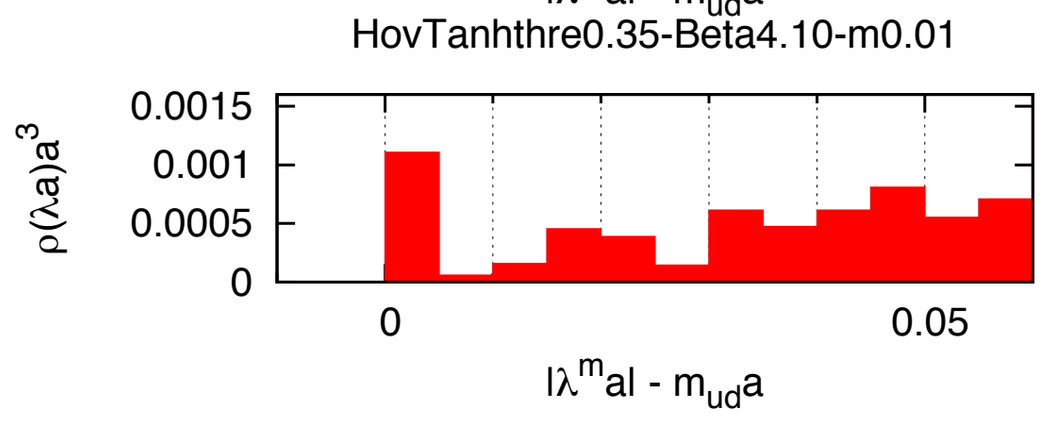
Partially Quenched OV





Reweighted OV

Reweighting not available



Domain-wall and overlap: visible difference.

Akio Tomiya(Osaka Univ.)

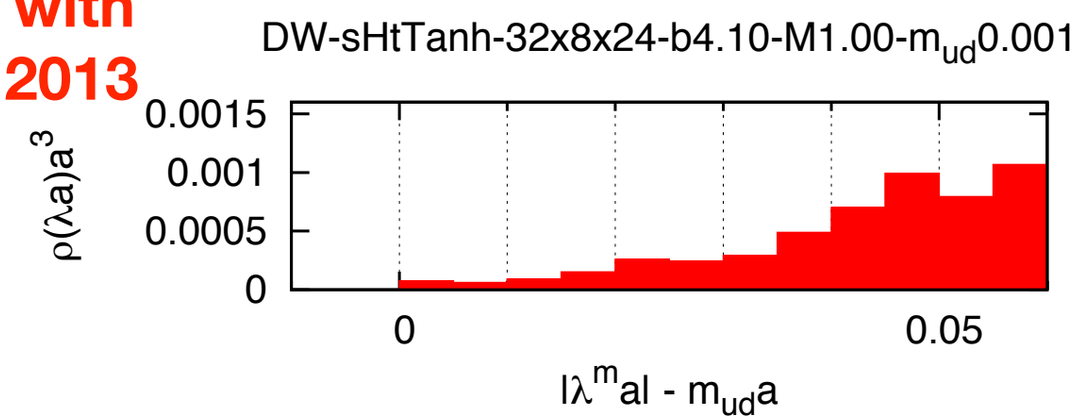
T=200MeV, m_{ud}=3.2MeV

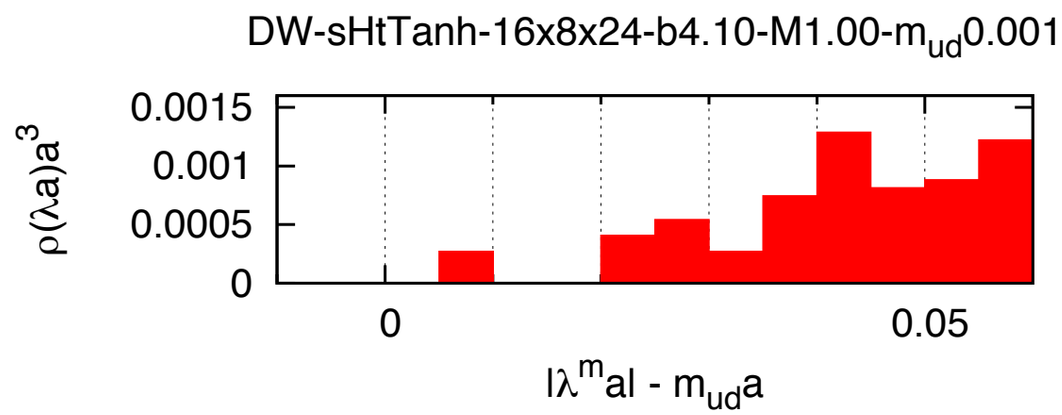
L32

L16

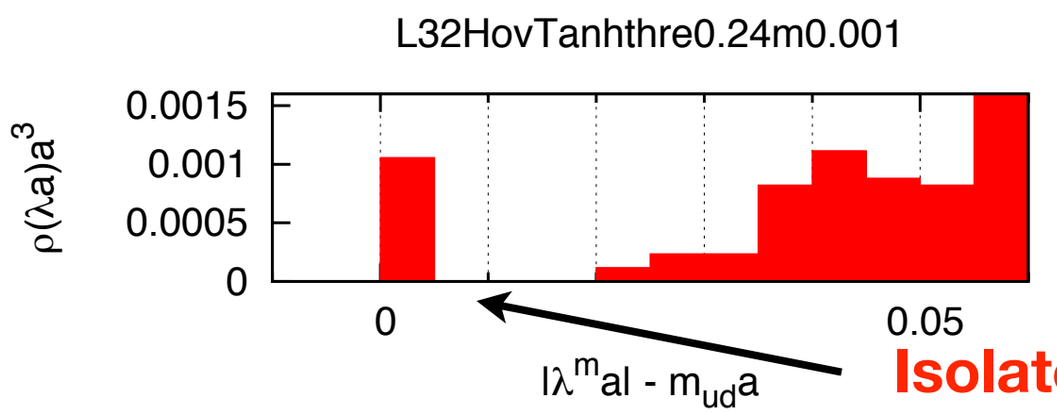
Consistent with LLNL/RBC 2013

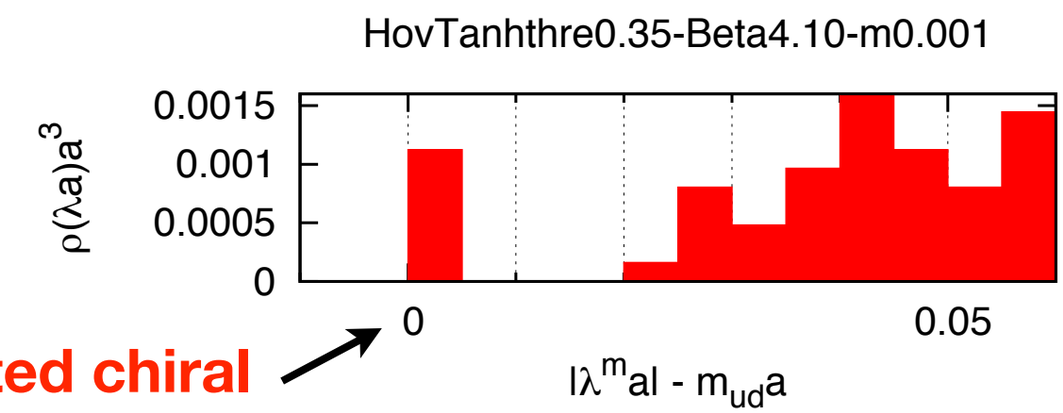
DW





Partially Quenched OV

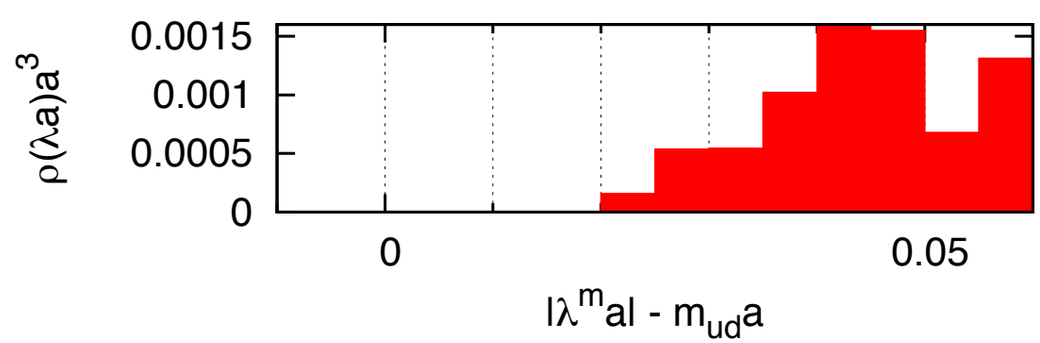




Isolated chiral zero-modes

Reweighted OV

Reweighting not available



Consistent with JLQCD 2013

T=200MeV, m_{ud}=3.2MeV

Observation	
• Strong violation of Ginsparg-Wilson relation in the low lying mode	• the histograms(DW vs OV) look different
• Overlap Dirac operator has isolated chiral zero-modes + gap. (DW vs pqOV)	• Exactly chiral zero-modes should disappear in the large volume limit
• The gap looks stable as Volume increases. (Partially quenched OV L=16 vs L32)	• This gap may suggest $U(1)_A$ symmetry restoration

- We need to confirm this in L=32 overlap (or DW with better chirality) simulations.

6.Summary

Summary

We have studied eigenvalue distribution
of DW and (reweighted)overlap Dirac operators above Tc

1. Mobius Domain-wall spectrum
   => $U(1)_A$ is broken. consistent with LLNL/RBC(2013)
2. We found significant violation of chiral symmetry of low-lying modes even when m_{res} is small.
3. OV/DW reweighting shows gap for lighter mass
   => $U(1)_A$ restoration? consistent with JLQCD(2013)
4. More study of finite volume effect is necessary.
   (OV/DW reweighting works only for smaller lattice)

Backup

Akio Tomiya(Osaka Univ.)

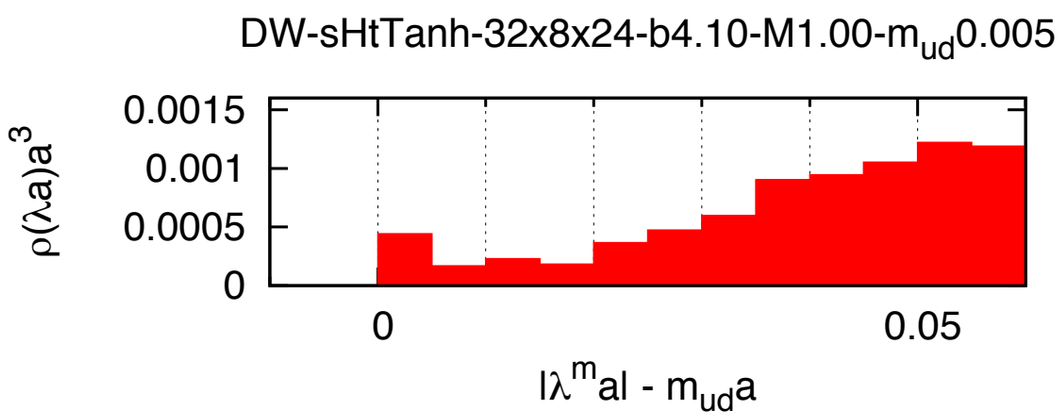
T=200MeV, m_{ud}=16MeV

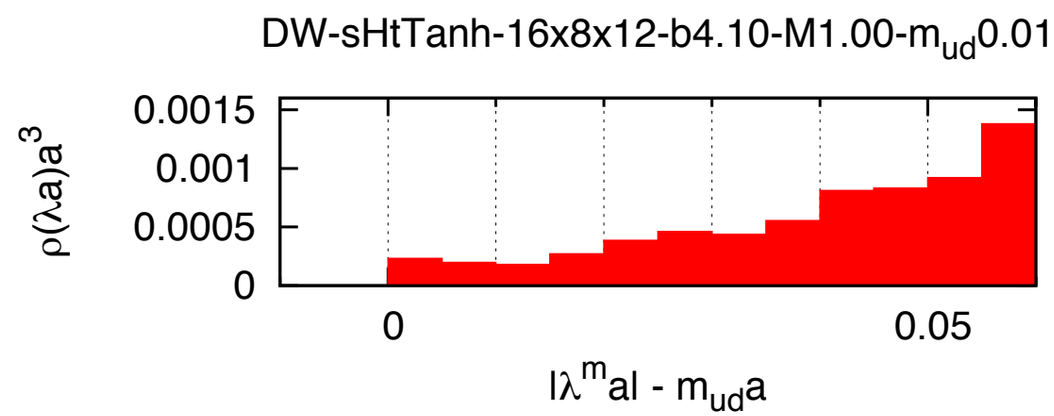
(beta=4.10 m=0.005)

L32

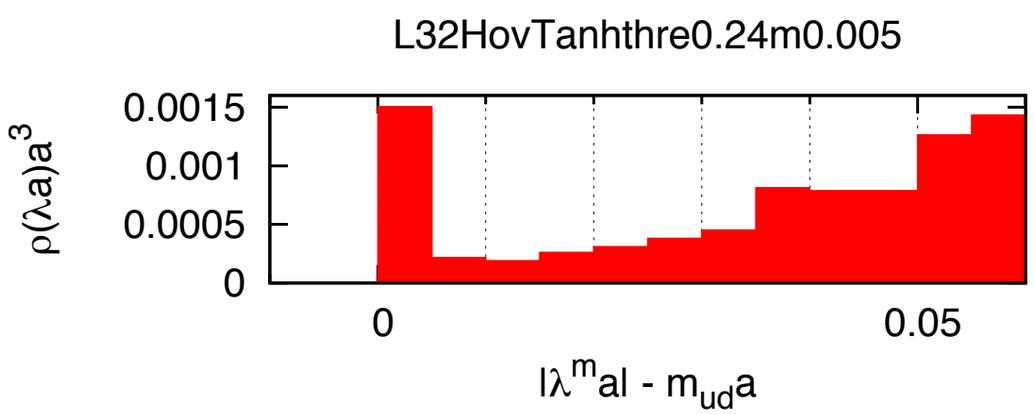
L16

DW



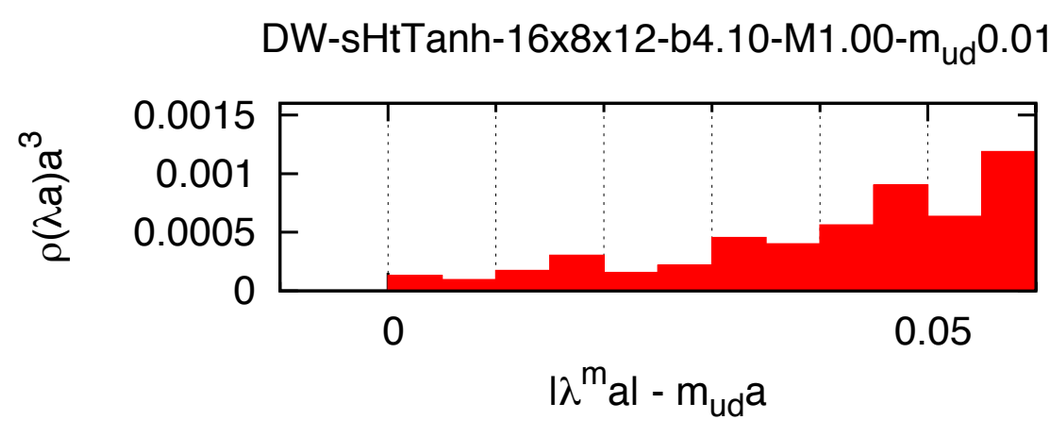


Partially Quenched OV



Reweighted OV

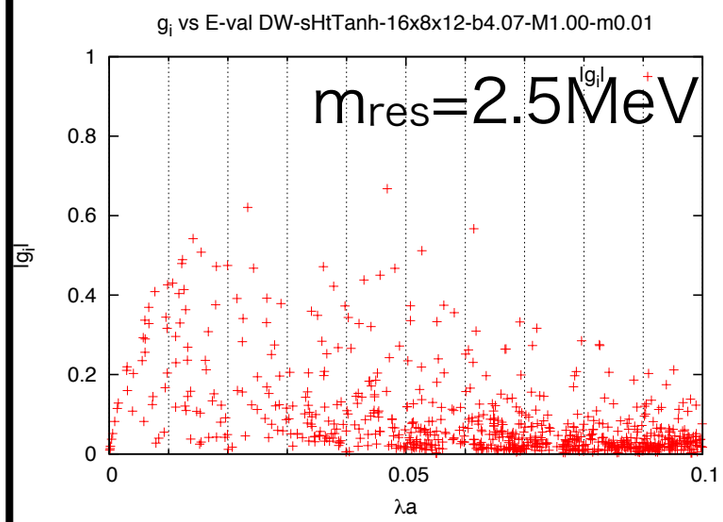
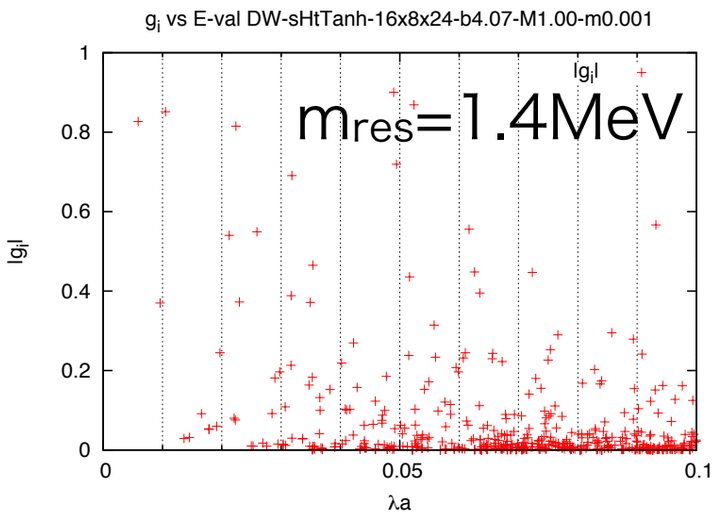
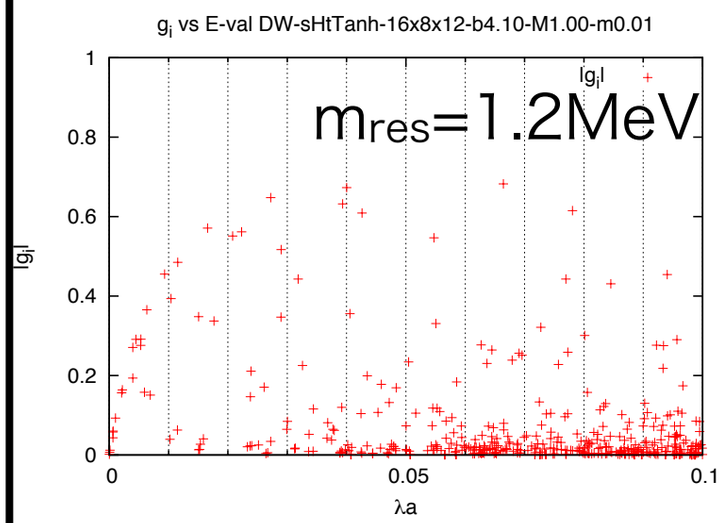
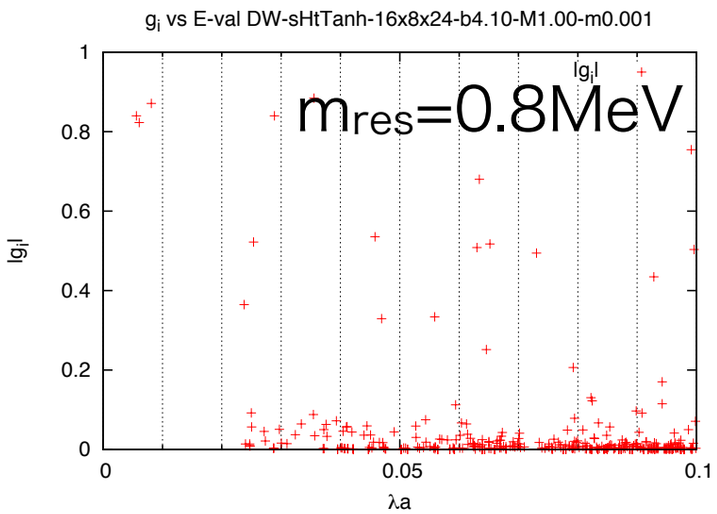
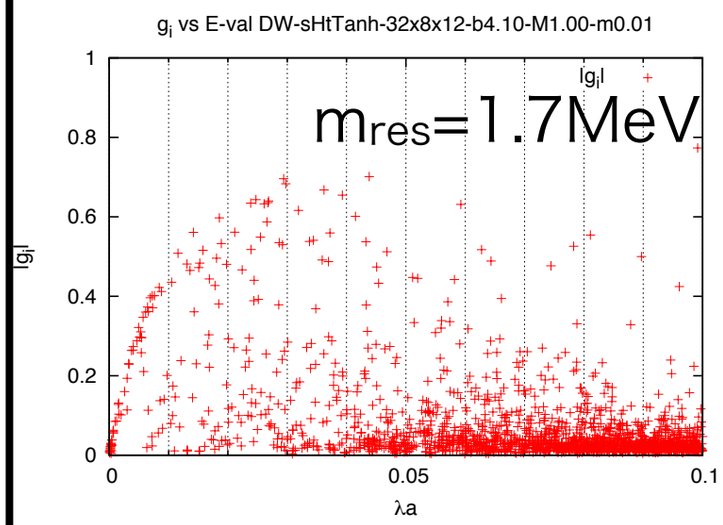
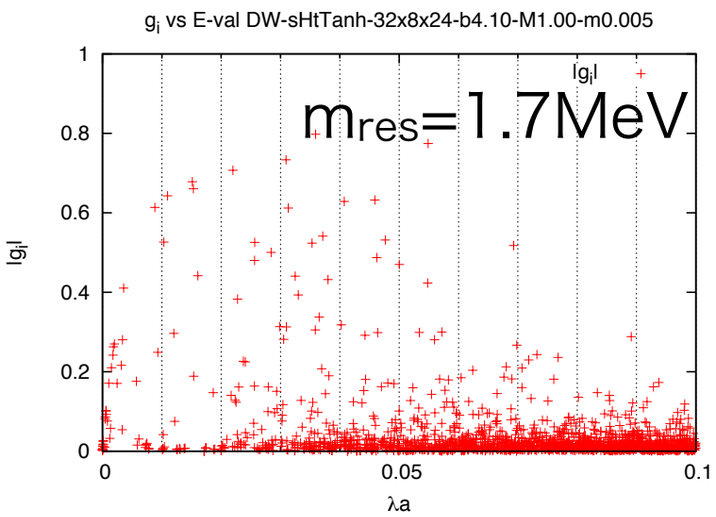
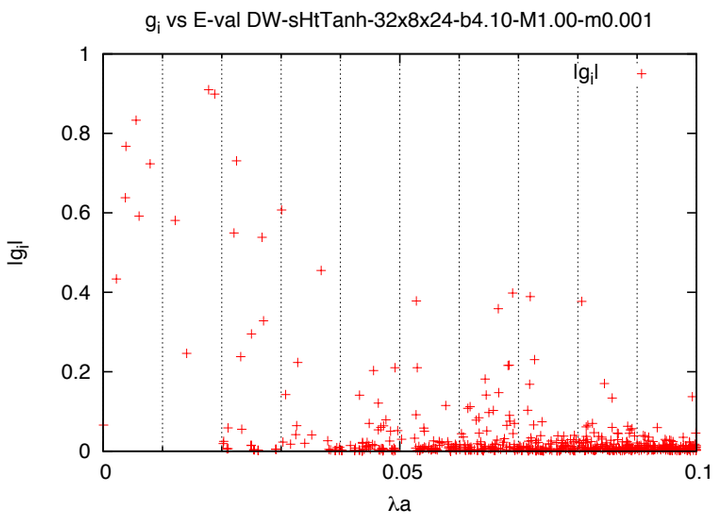
Reweighting not available

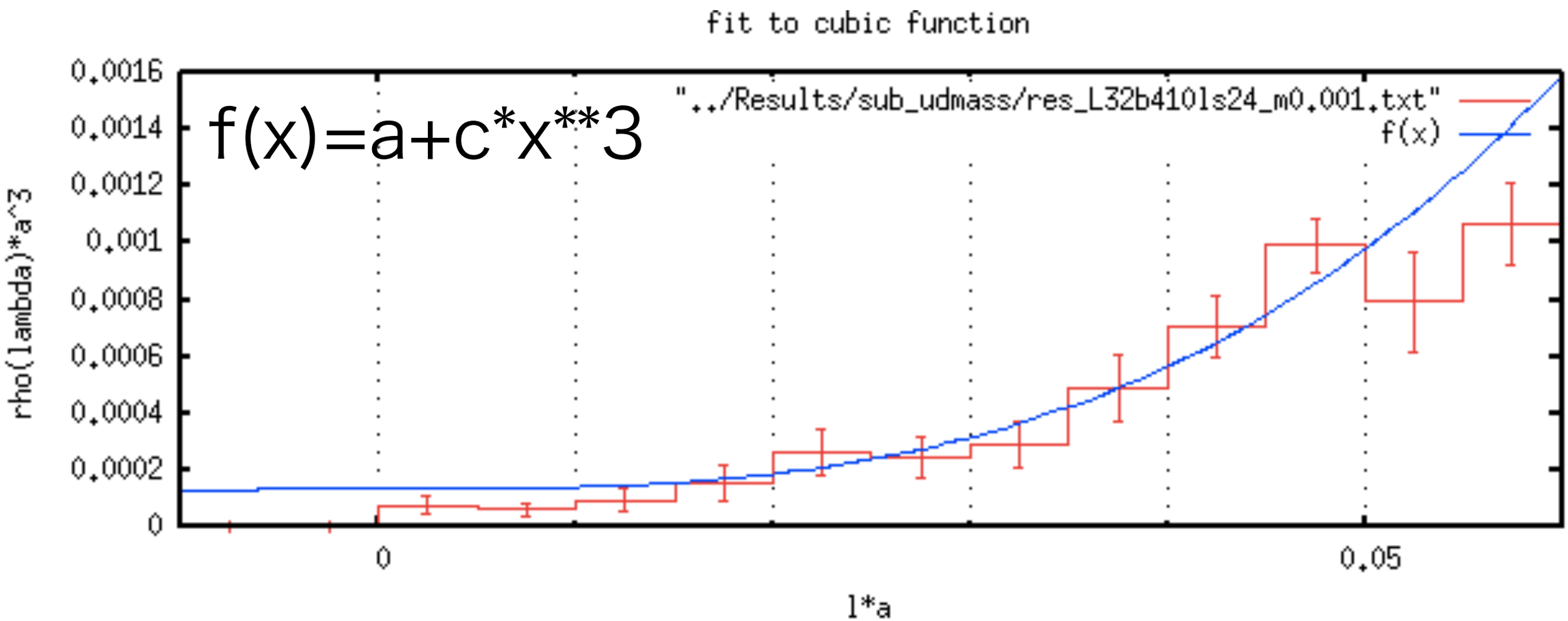


Reweighting to OV with UV suppressing determinant

$$R^{UVS} = \left(\frac{\det \gamma_5 D_{\mathrm{ov}}(m_{ud})}{\det \gamma_5 D_{\mathrm{DW}}(m_{ud})} \right)^2 \left(\frac{\det \gamma_5 D_{\mathrm{DW}}(M)}{\det \gamma_5 D_{\mathrm{ov}}(M)} \right)^2$$

DW/OV rewighting is UV surpassing determinant.
unphysical mode suppressed by
heavy unphysical modes M~O(1/a).

	ma=0.01 m~30MeV	m=0.005	ma=0.001 m~3MeV
L16 b4.07			
L16 b4.10			
L32 b4.10			



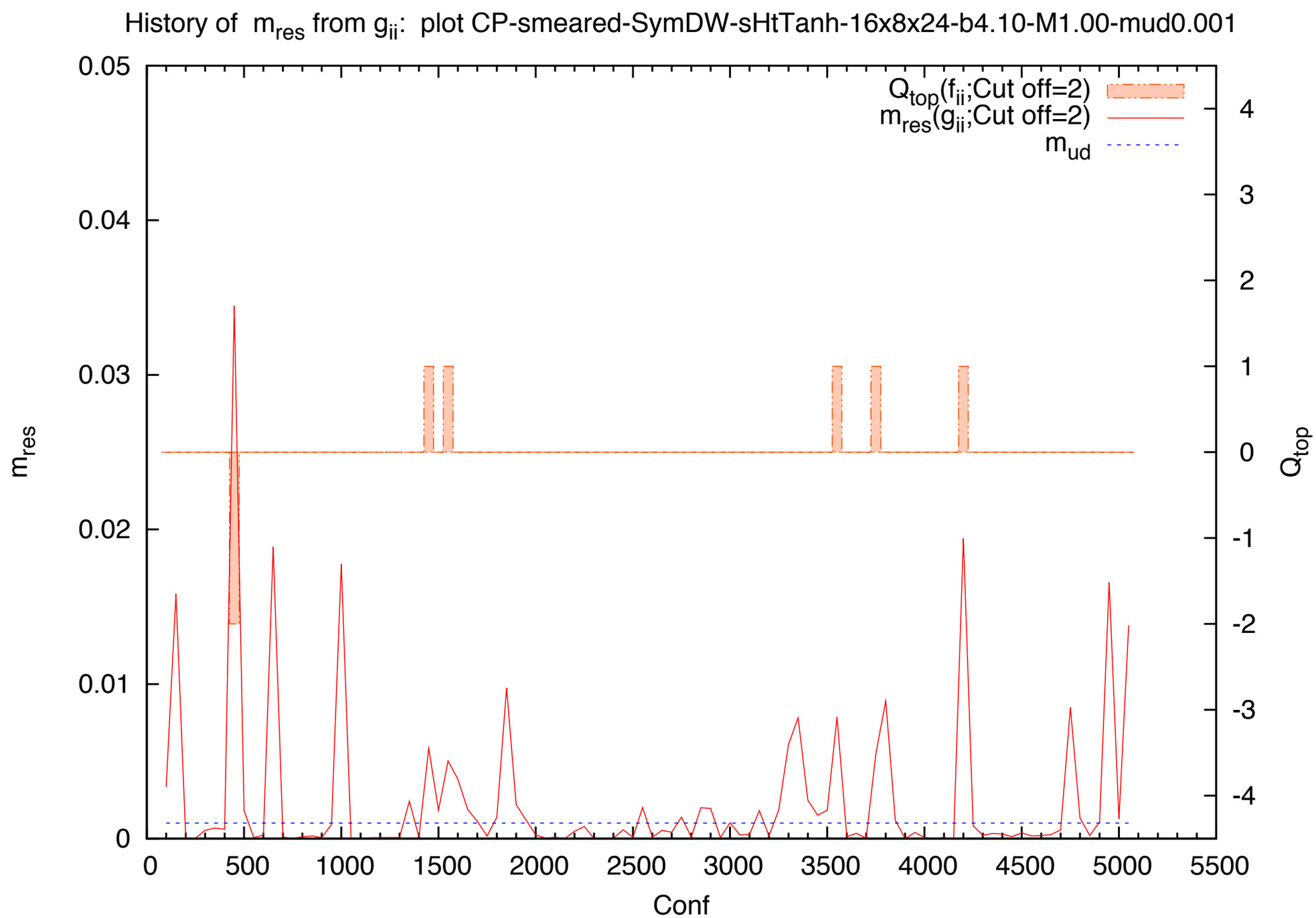
variance of residuals (reduced chisquare) = WSSR/ndf : 1.33016

Final set of parameters	Asymptotic Standard Error
==========	==========

a	= 0.000132414	+/- 6.752e-05	(50.99%)
c	= 6.76224	+/- 1.104	(16.32%)

Large violation of GW-rel

m_{res}(Next to lowest) history



History

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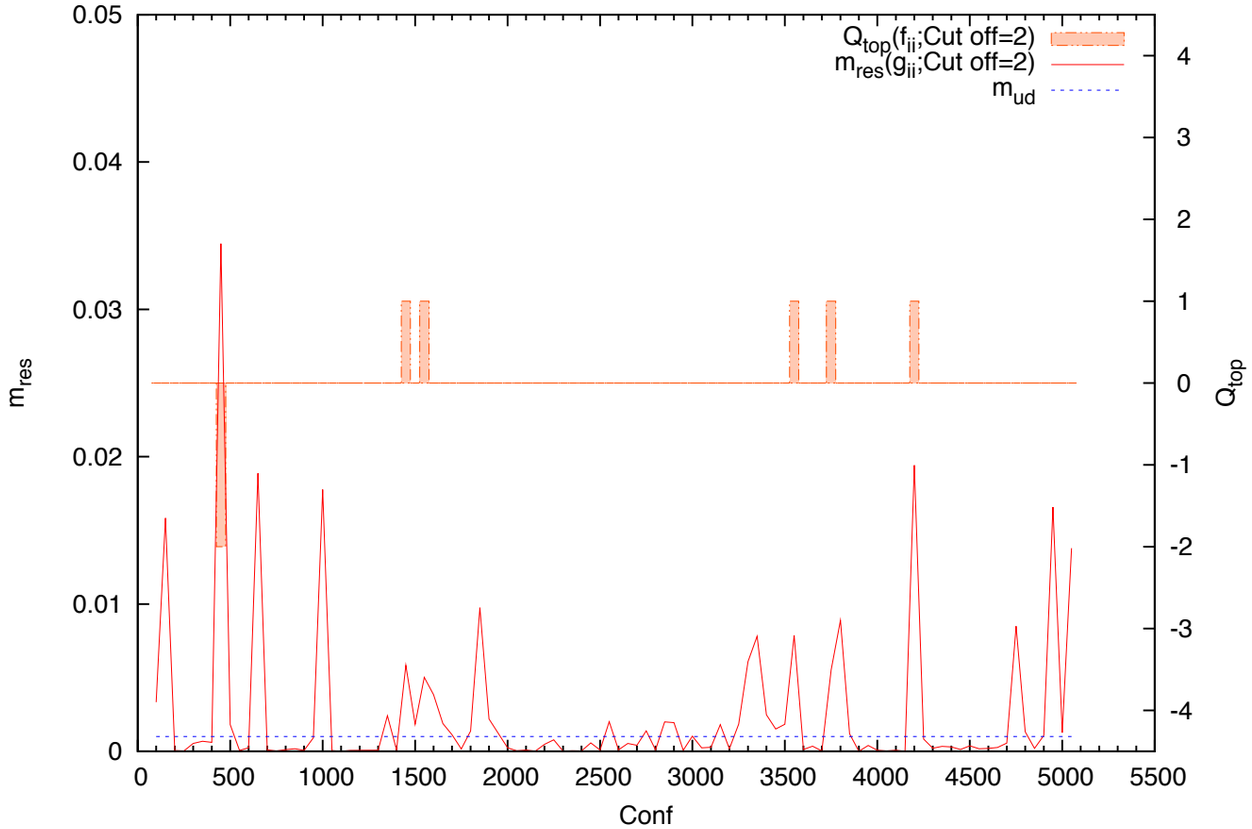
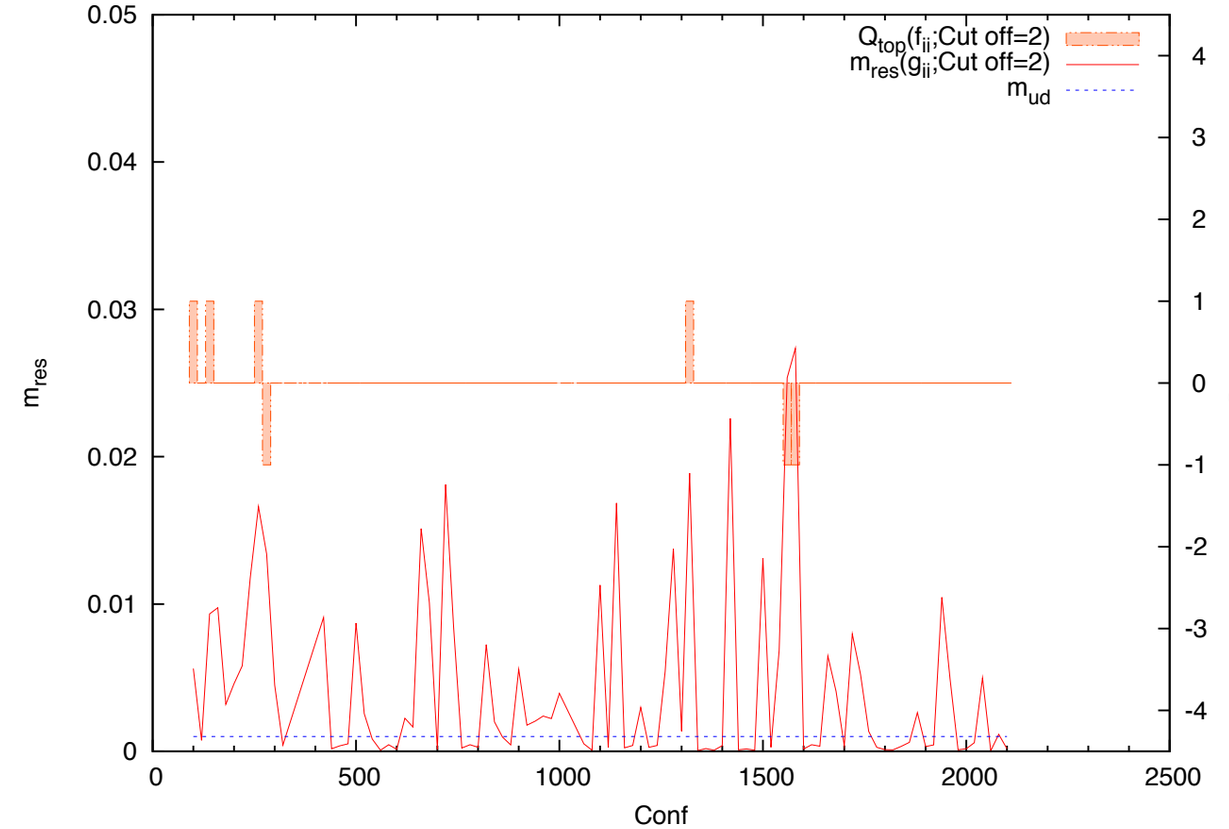
History of m_{res} from g_{ii}: plot CP-smeared-SymDW-sHtTanh-16x8x24-b4.10-M1.00-mud0.001



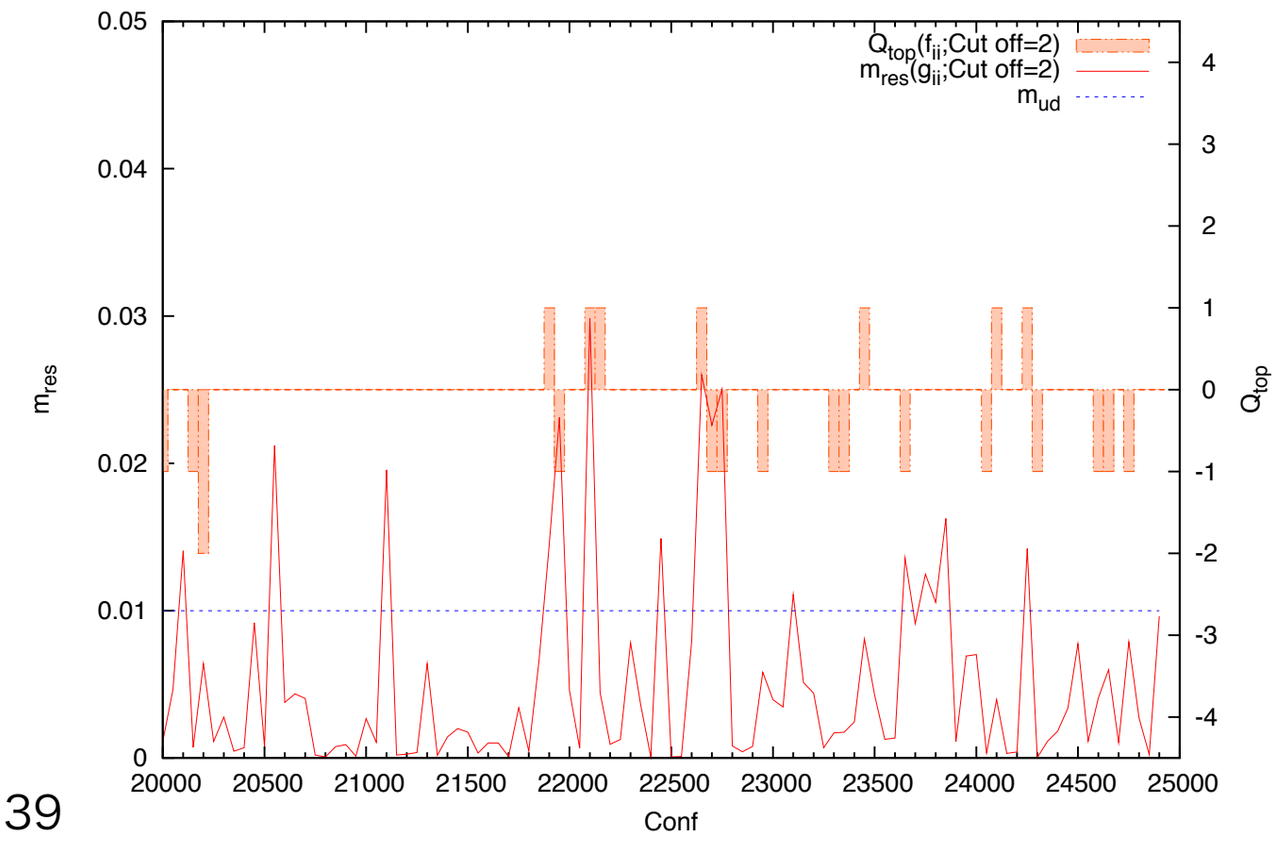
Date:2014/06/04 12:14:59

History of m_{res} from g_{ii}: plot CP-smeared-SymDW-sHtTanh-16x8x24-b4.07-M1.00-mud0.001



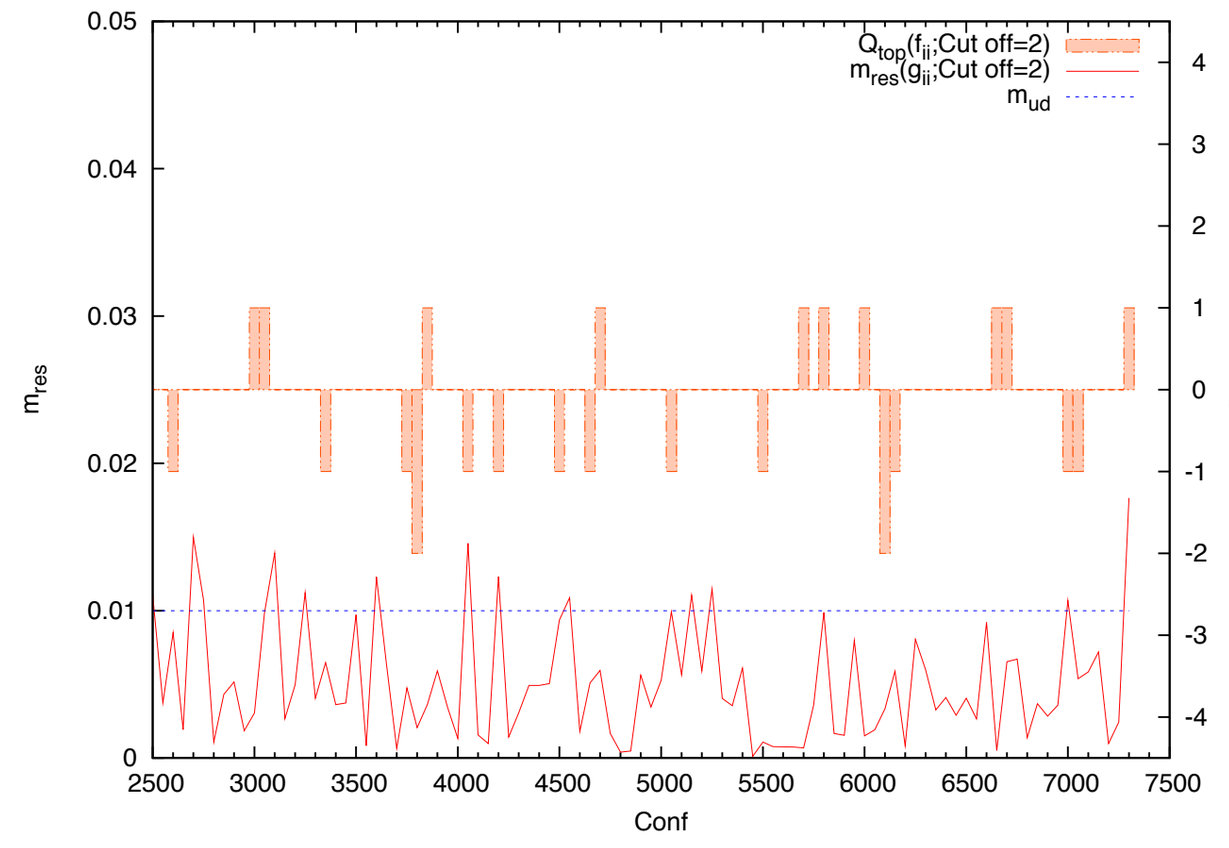
Date:2014/06/04 12:10:28

History of m_{res} from g_{ii}: plot CP-smeared-SymDW-sHtTanh-16x8x12-b4.10-M1.00-mud0.01



Date:2014/06/04 12:13:41

History of m_{res} from g_{ii}: plot CP-smeared-SymDW-sHtTanh-16x8x12-b4.07-M1.00-mud0.01



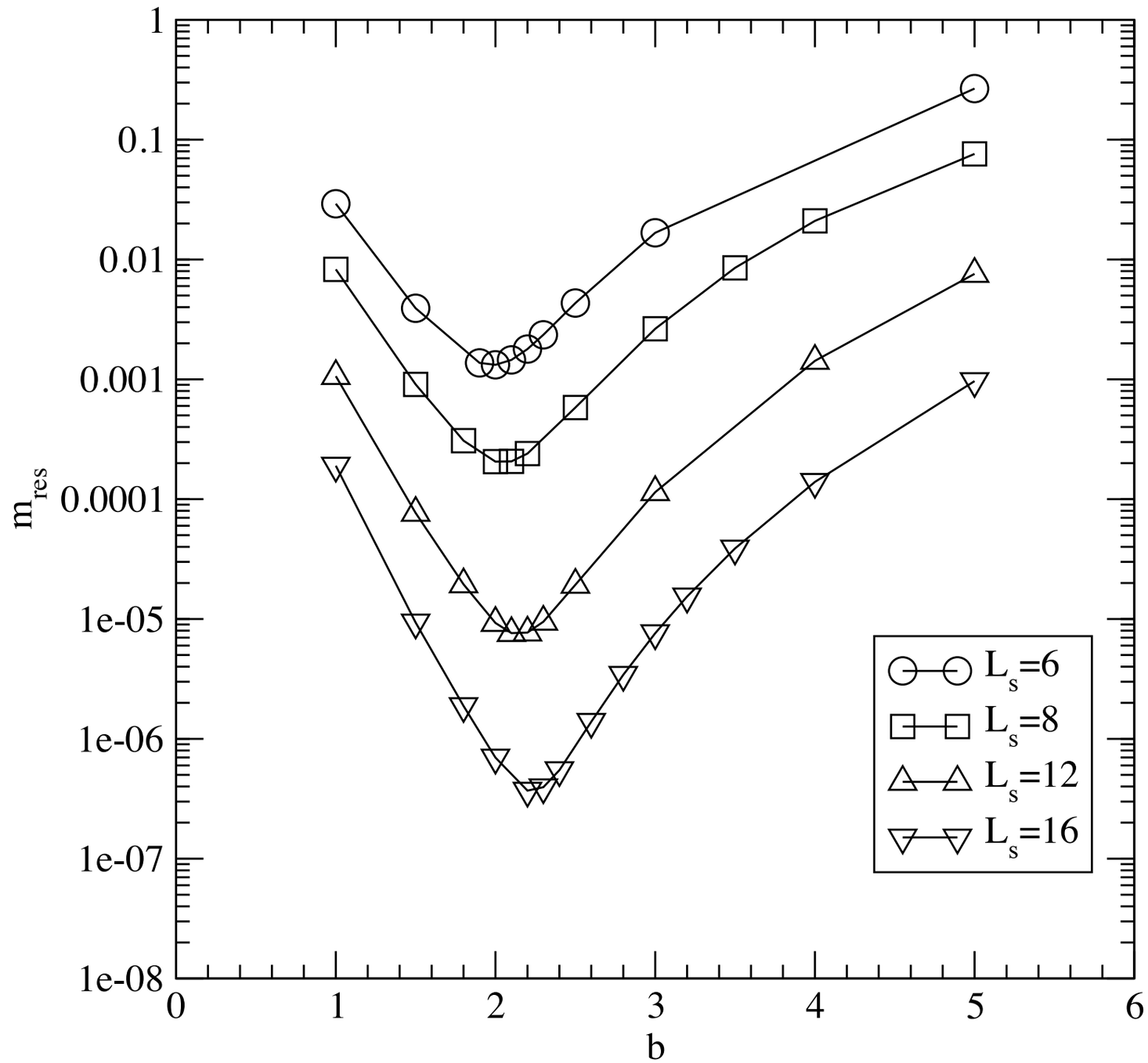


Figure 9: Residual mass with the scaled-Shamir kernel and tanh approximation. The results with $L_s = 6, 8, 12, 16$ are plotted as a function of the scale parameter b. c=1

$$H_M = \gamma_5 \frac{bD_W}{2 + cD_W},$$