Leading order hadronic contribution to $(g_\mu - 2)$

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Budapest-Marseille-Wuppertal Collaboration

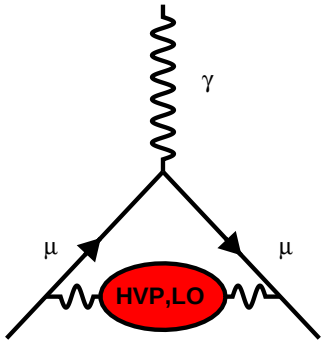
Special thanks to Eric Gregory, Laurent Lellouch, Craig McNeile, Alfonso Sastre

Lattice 2014

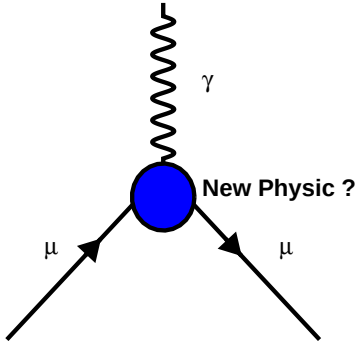
1) Anomalous magnetic moment of the muon in the Standard Model

- Standard model prediction and current deviation :

	$a_\mu.10^{-10}$	$\delta a_\mu.10^{-10}$
EXPERIMENT :	11659208.9	6.3
STANDARD MODEL :	11659180.2	4.9
QED	11658471.8951	0.0080
EW	15.4	0.1
QCD { QCD HVP,LO	692.3	4.2 (e^+e^- decay datas)
QCD { QCD HVP,HO	-9.84	0.07
QCD { QCD LbL	10.5	2.6 (estimation)
DEVIATION	28.7	8.0 $\Rightarrow 3.6\sigma$



$\Rightarrow$



$\Rightarrow$ Can Lattice QCD help?

2) $a_\mu^{HVP,LO}$ from Lattice QCD

Previous work (only the last 12 months !)

- A New Strategy for the Lattice Evaluation of the Leading Order Hadronic Contribution to $(g-2)_\mu$ Maarten Golterman, Kim Maltman, Santiago Peris arXiv:1405.2389
- Strange and charm quark contributions to the anomalous magnetic moment of the muon Bipasha Chakraborty, C. T. H. Davies, G. C. Donald, R. J. Dowdall, J. Koponen, G. P. Lepage, T. Teubner arXiv:1403.1778
- The muon anomalous magnetic moment, a view from the lattice Christopher Aubin, Thomas Blum, Maarten Golterman, Kim Maltman, Santiago Peris arXiv:1311.5504
- Leading-order hadronic contributions to $(g-2)_\mu$ Eric B. Gregory, Zoltan Fodor, Christian Hoelbling, Stefan Krieg, Laurent Lellouch, Rehan Malak, Craig McNeile, Kalman Szabo arXiv:1311.4446
- Leading-order hadronic contribution to the anomalous magnetic moment of the muon from Nf=2+1+1 twisted mass fermions Florian Burger, Xu Feng, Grit Hotzel, Karl Jansen, Marcus Petschlies, Dru B. Renner arXiv:1311.3885
- The hadronic vacuum polarization with twisted boundary conditions Christopher Aubin, Thomas Blum, Maarten Golterman, Santiago Peris arXiv:1311.1078
- Using analytic continuation for the hadronic vacuum polarization computation Xu Feng, Shoji Hashimoto, Grit Hotzel, Karl Jansen, Marcus Petschlies, Dru B. Renner arXiv:1311.0652
- Tests of hadronic vacuum polarization fits for the muon anomalous magnetic moment Maarten Golterman (SFSU and IFAE), Kim Maltman (York U. and U. of Adelaide), Santiago Peris (UAB) arXiv:1310.5928
- Four-Flavour Leading-Order Hadronic Contribution To The Muon Anomalous Magnetic Moment Florian Burger, Xu Feng, Grit Hotzel, Karl Jansen, Marcus Petschlies, Dru B. Renner arXiv:1308.4327
- The hadronic vacuum polarization with twisted boundary conditions Christopher Aubin, Thomas Blum, Maarten Golterman, Santiago Peris arXiv:1307.470

$\Rightarrow$ *. . . sorry to those whose work I missed !*

a_μ on the lattice: general approach (T.Blum 2003)

Compute $a_\mu^{HVP,LO}$ in Lattice QCD directly with Euclidean momenta :

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- Integration with known QED kernel w_Π :

$$a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 w_\Pi(Q^2) \hat{\Pi}(Q^2)$$

The small Q^2 challenge

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- Integration with QED kernel w_Π : $\Leftarrow$ *divergent* in $Q^2 = 0$

$$a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 w_\Pi(Q^2) \hat{\Pi}(Q^2) \Leftarrow \textit{peak at } Q^2 \sim Q^2_{min}/10$$

Small Q^2 solutions

- Extrapolate from $Q^2_{\rm min}$ with pole dominance type model and hope for the best (most early work)
- Implement twisted boundary conditions to go to lower $Q^2_{\rm min}$ and extrapolate (Sachrajda et al 2004, Jäger et al 2010)
- Compute $\Pi(0)$ directly using momentum expansion of quark propagators (de Divitiis et al 2012)
- Use Padés to extrapolate $\Pi(Q^2)$ to small Q^2 (Golterman et al 2013)
- $E = 0$ time moments of $\Pi_{\mu\nu}(t, \vec{p} = 0)$ to fix coefficents of Padés (Davies et al 2014)
- Moment analysis of $\Pi(Q^2)$ (de Rafael 2014)
- Here consider momentum derivatives of correlation functions in Fourier space (Lellouch et al 1995)

All solutions have specific systematic errors which have yet to be understood.

3) Momentum derivatives of correlation functions in Fourier space

The benefits of derivatives

$$\Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\Pi(Q^2)$$

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$$\mu \neq \nu \ , \ \partial_\mu \partial_\nu \Pi_{\mu\nu}(Q_\mu = 0 \ , \ Q_\nu = 0) = \Pi(Q^2)$$

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$\Rightarrow$ Direct computation of the Adler function :

$$\partial_\mu \partial_\mu \Pi_{\mu\mu}(Q_\mu = 0) = -2Q^2 \frac{\partial}{\partial Q^2} \Pi(Q^2) = -2\,\mathcal{A}(Q^2)$$

Derivative from moments in Fourier space

Derivative can be computed as moments in Fourier space, e.g.:

$$\partial_\sigma \partial_\rho \Pi_{\mu\nu}(Q) = -\sum_x x_\sigma x_\rho \Pi_{\mu\nu}(x) e^{iQx}$$

⇒ because higher moments emphasize large distances more, expect enhanced FV effects.

Implementation :

⇒ spatial derivatives in Fourier space in simulation code

⇒ big outputs(NetCDF binary format) :

$$3^2 \text{indices combinations} \times \#momenta$$

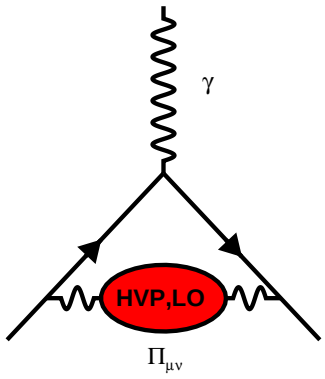
⇒ temporal Fourier derivatives transform in analysis code

4) Results

BMWc implementation

Hadronic vacuum polarization tensor :

$$\Pi_{\mu\nu}(\underbrace{\hat{Q}}_{\frac{2}{a}sin(\frac{aQ}{2})}) = Z_V \sum_x \langle J_\mu^{cvc}(x) J_\nu^{loc}(y) \rangle e^{iQ(x+\frac{a\hat{\mu}}{2}-y)}$$



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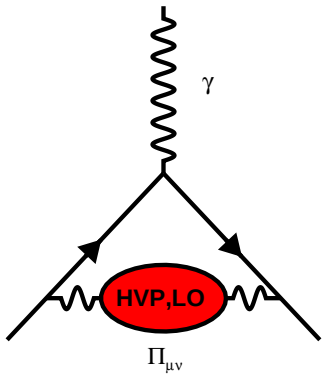

- local current at the source :

$$J_\nu^{loc}(y) = \bar{\psi}(y)\gamma_\mu\psi(y)$$

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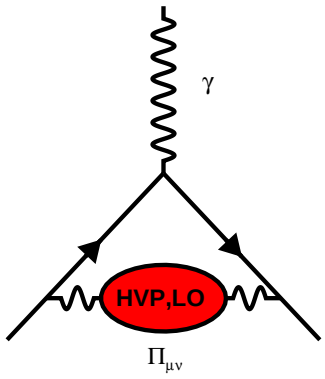

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$$J_\mu^{cvc}(x) = \frac{1}{2}\{\bar{\psi}(x+a\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\psi(x) - \bar{\psi}(x)(1-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu})\}$$

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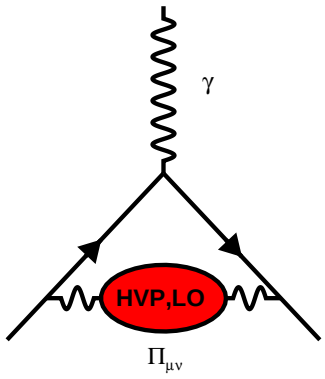

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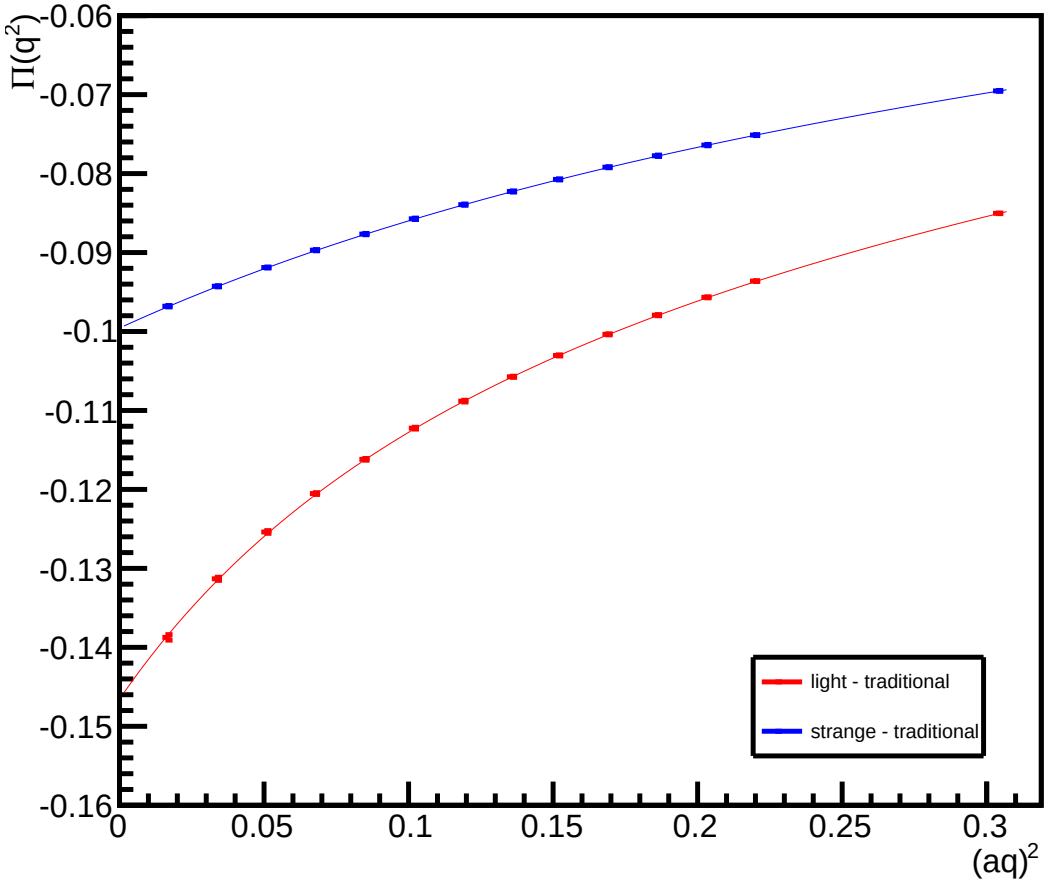
- currently only connected components :
  $(S_\mu(x,y) \equiv U_\mu(x)S(x+a\hat{\mu},y))$

$$\langle J_\mu^{cvc}(x) J_\nu^{loc}(y)\rangle \sim \frac{1}{2}\mathrm{Tr}_{\mathrm{sc}}\{S_\mu(x,y)\gamma_\nu\gamma_5 S^\dagger(x,y)\gamma_5(1-\gamma_\mu)\}$$
$$- \frac{1}{2}\mathrm{Tr}_{\mathrm{sc}}\{S(x,y)\gamma_\nu\gamma_5 S_\mu^\dagger(x,y)\gamma_5(1+\gamma_\mu)\}$$

Traditional method vs derivative method

$T \times L^3 = 48 \times 48^3 \quad a = 0.116 fm \quad M_\pi = 150 MeV \quad$ 260conf. $\times$ 64meas.

Fitting function : f(x) = A-x.Pade[1,1](x) with x=Q^2



$\Rightarrow$ traditional : $a_\mu^{\mathrm{HVP,LO}} = 6.79(13) \times 10^{-8} \quad \chi^2/dof = 4.5/10$ (light)

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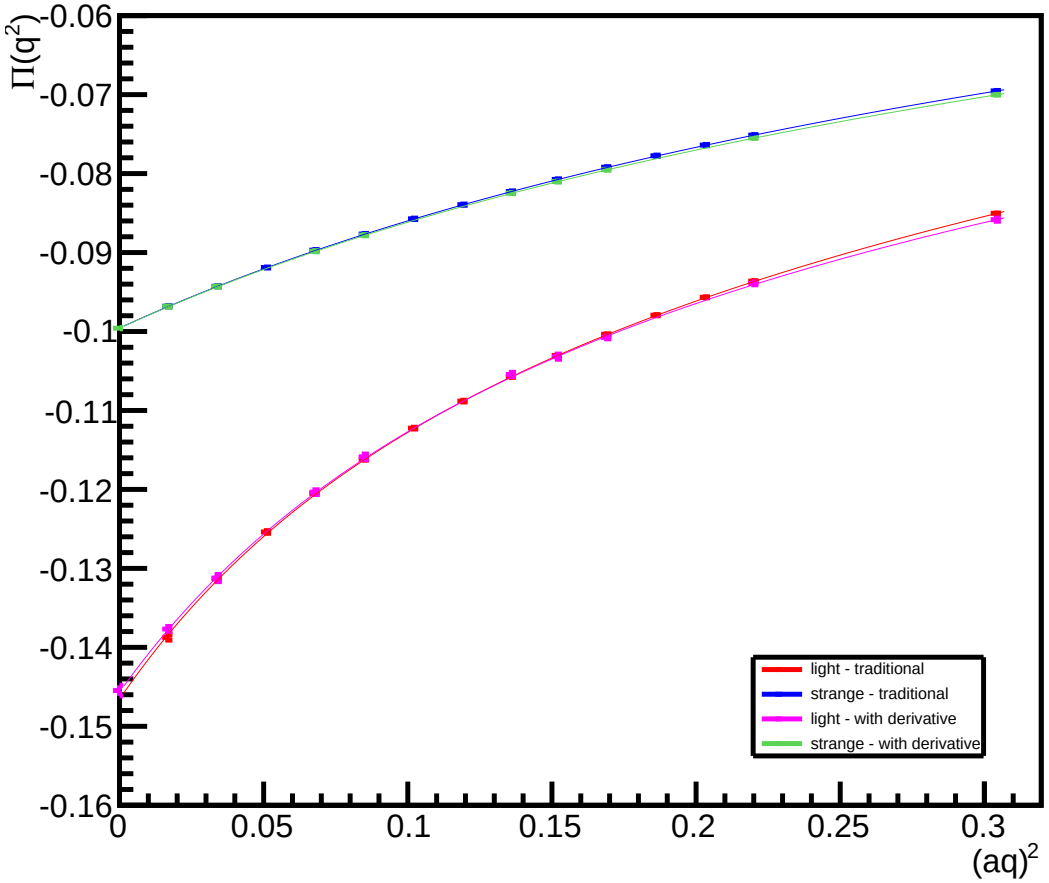

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$\Rightarrow$ derivative : $a_\mu^{\mathrm{HVP,LO}} = 6.58(22) \times 10^{-8} \quad \chi^2/dof = 3.8/6$ (light)

Padé fits in light sector, low Q^2 and systematics

$T \times L^3 = 48 \times 48^3 \quad a = 0.116 fm \quad M_\pi = 150 MeV \quad$ 260conf. $\times$ 64meas.
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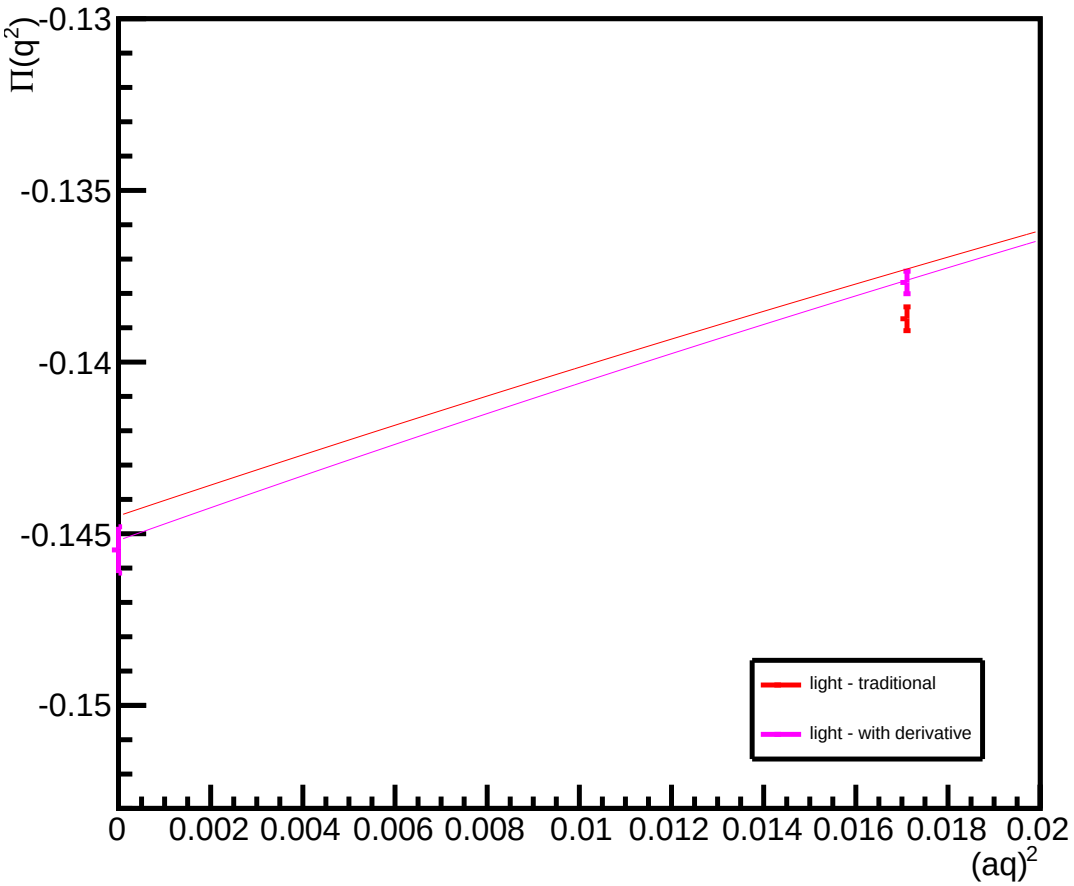

$\Rightarrow$ traditional : $\chi^2/dof = 40.3/11$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 6.7/7$ (light)

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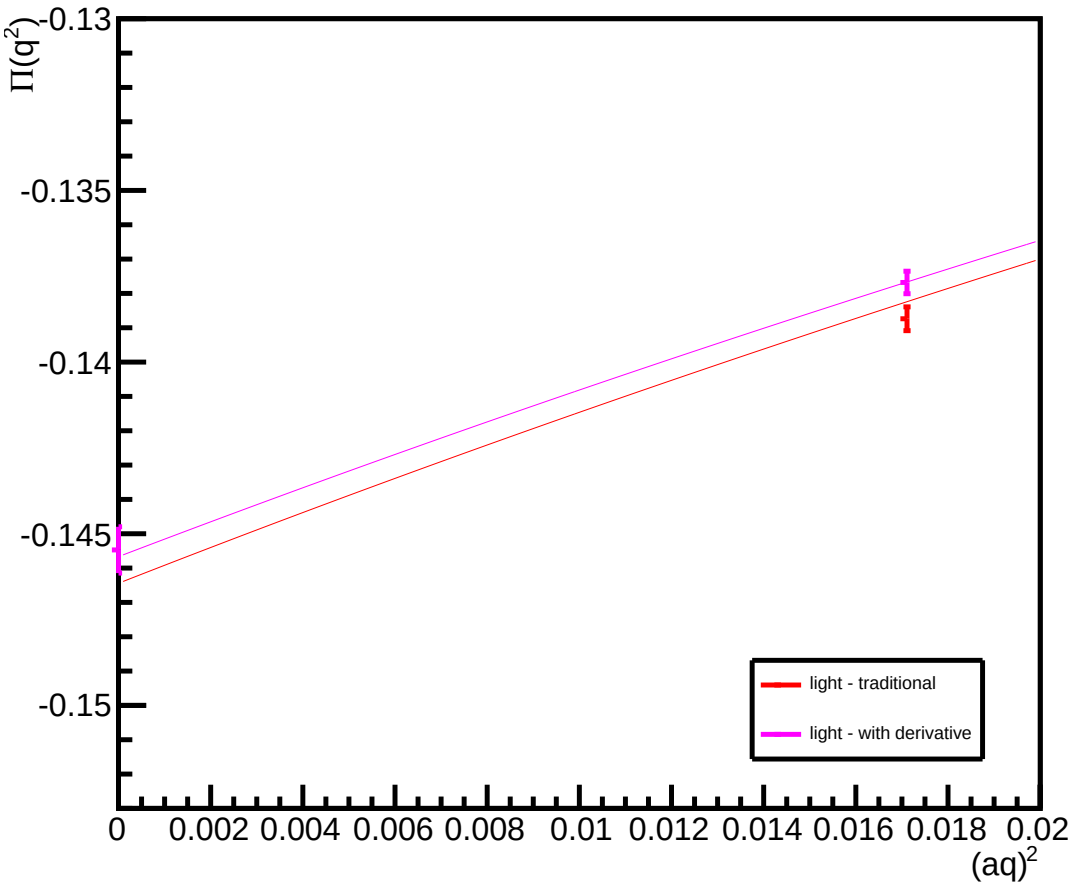
Fitting function : f(x) = A-x.Pade[0,2](x) with x=Q^2



$\Rightarrow$ traditional : $\chi^2/dof = 5.0/10$ (light)
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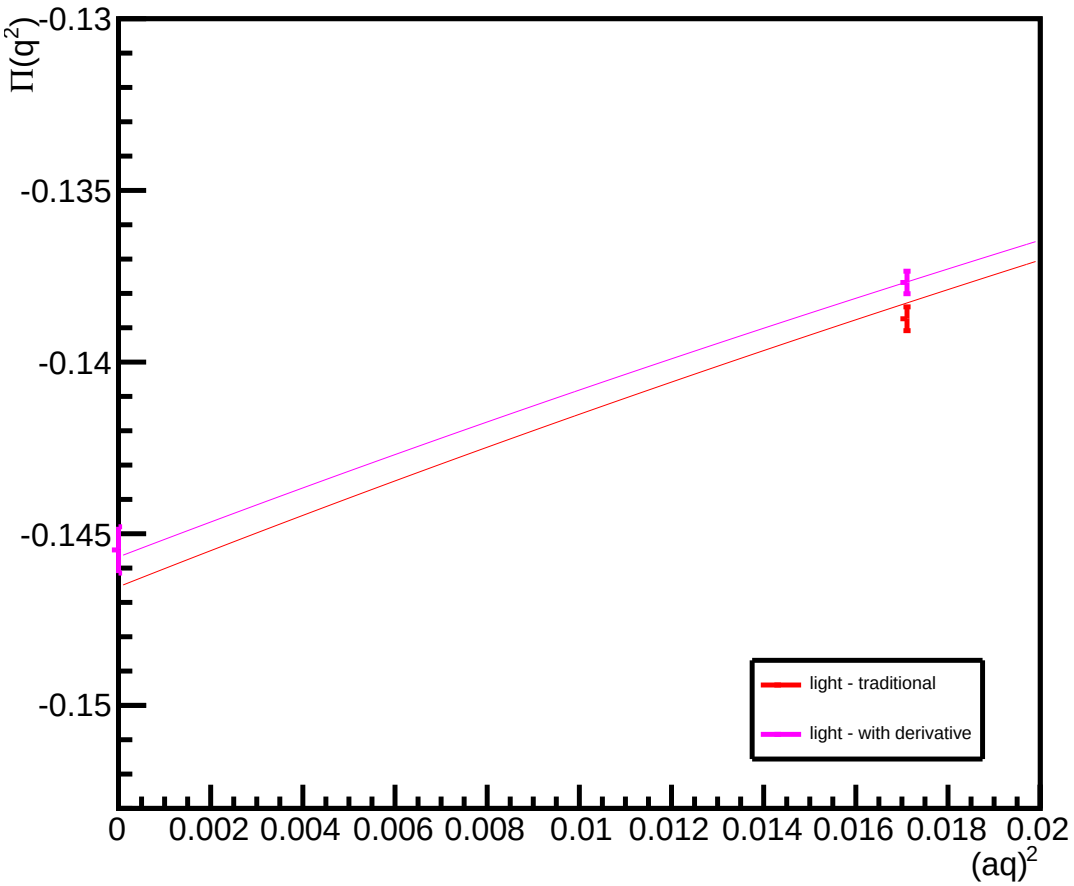

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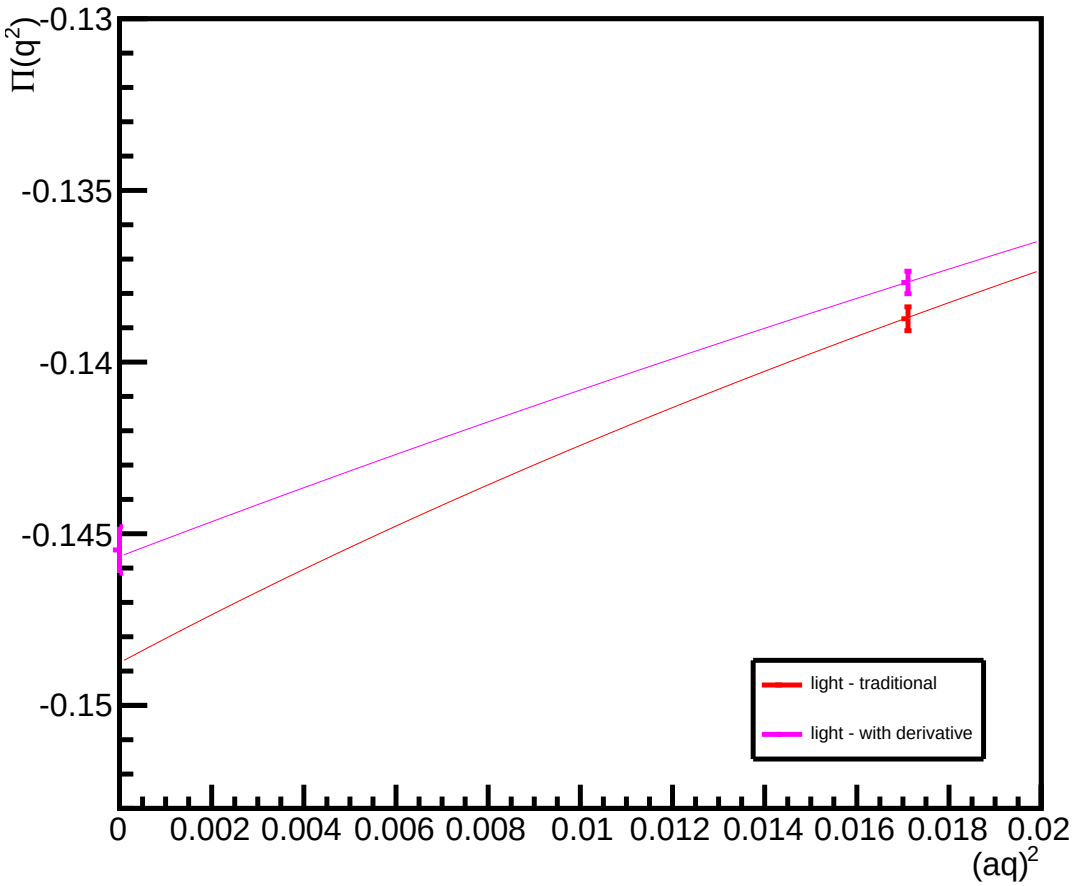

$\Rightarrow$ traditional : $\chi^2/dof = 0.7/9$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 3.8/5$ (light)

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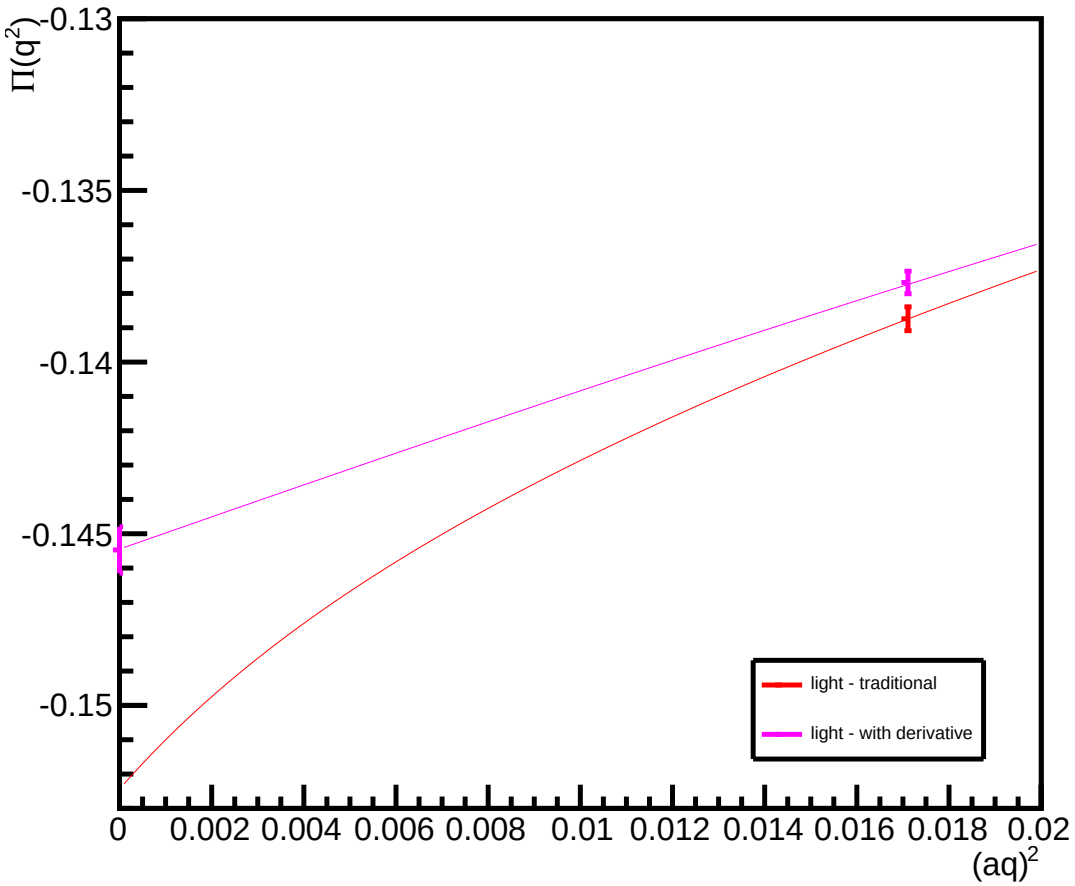

$\Rightarrow$ traditional : $\chi^2/dof = 0.5/8$ (light)
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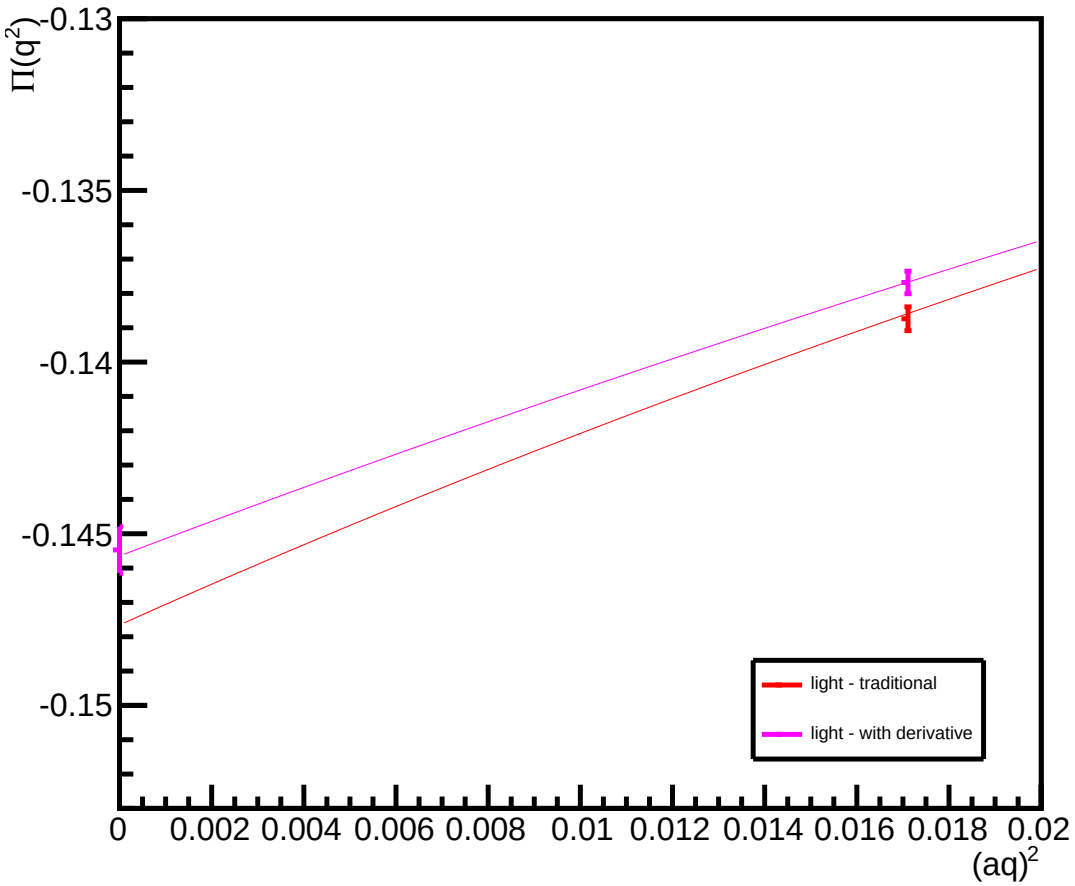

$\Rightarrow$ traditional : $\chi^2/dof = 1.4/9$ (light)
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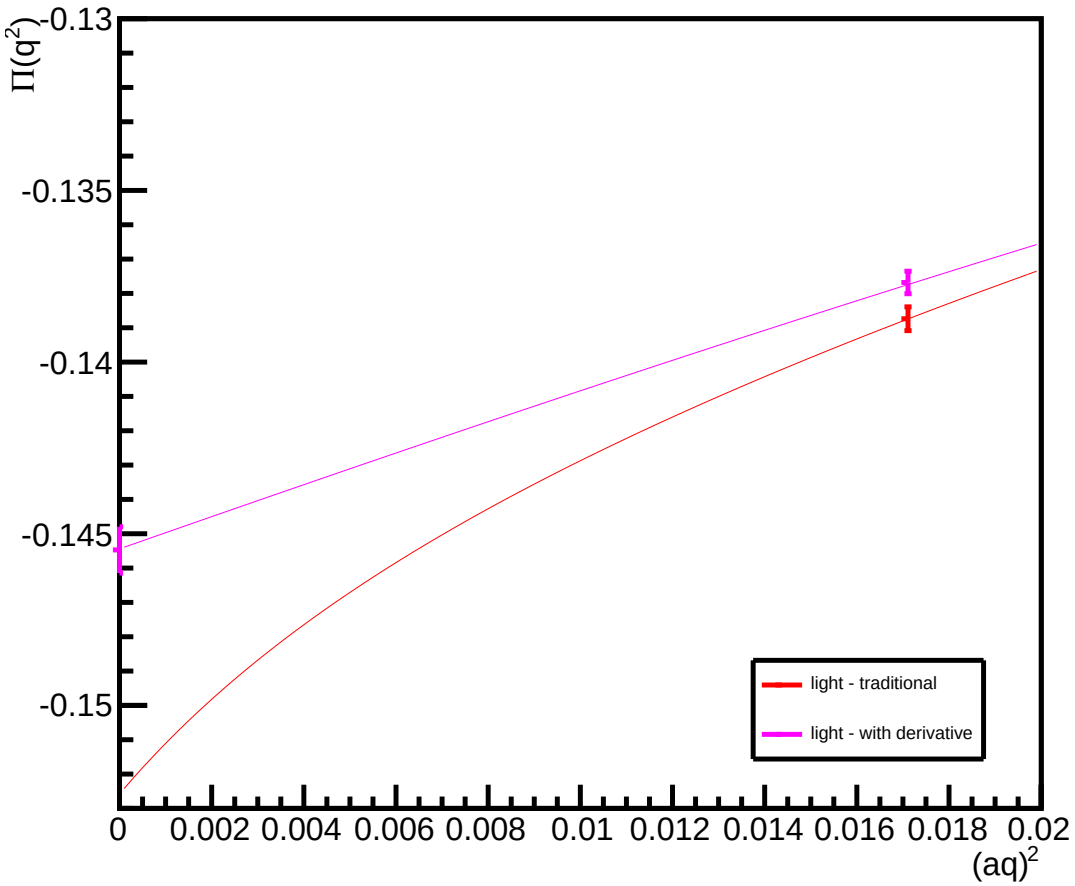

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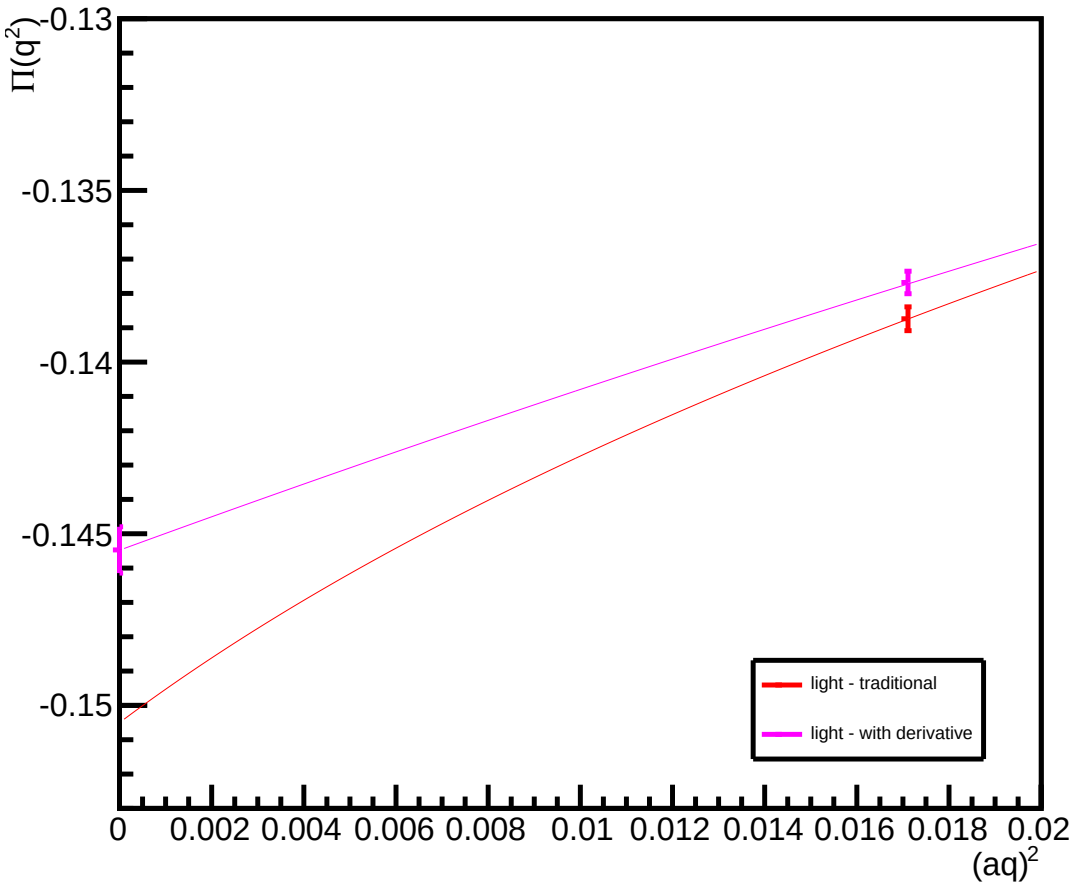

$\Rightarrow$ traditional : $\chi^2/dof = 0.4/7$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 0.1/3$ (light)

Padé fits in light sector, low Q^2 and systematics

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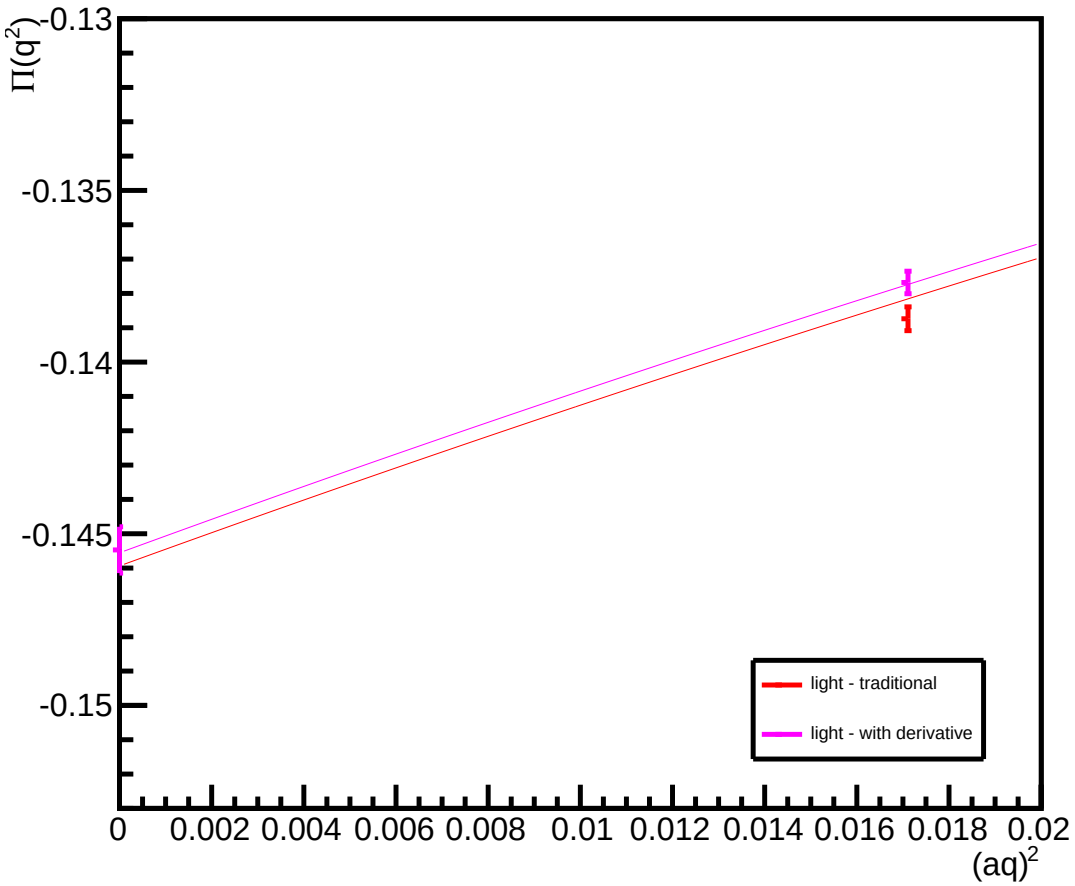
Fitting function : f(x) = A-x.Pade[3,0](x) with x=Q^2



$\Rightarrow$ traditional : $\chi^2/dof = 7.5/9$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 2.3/5$ (light)

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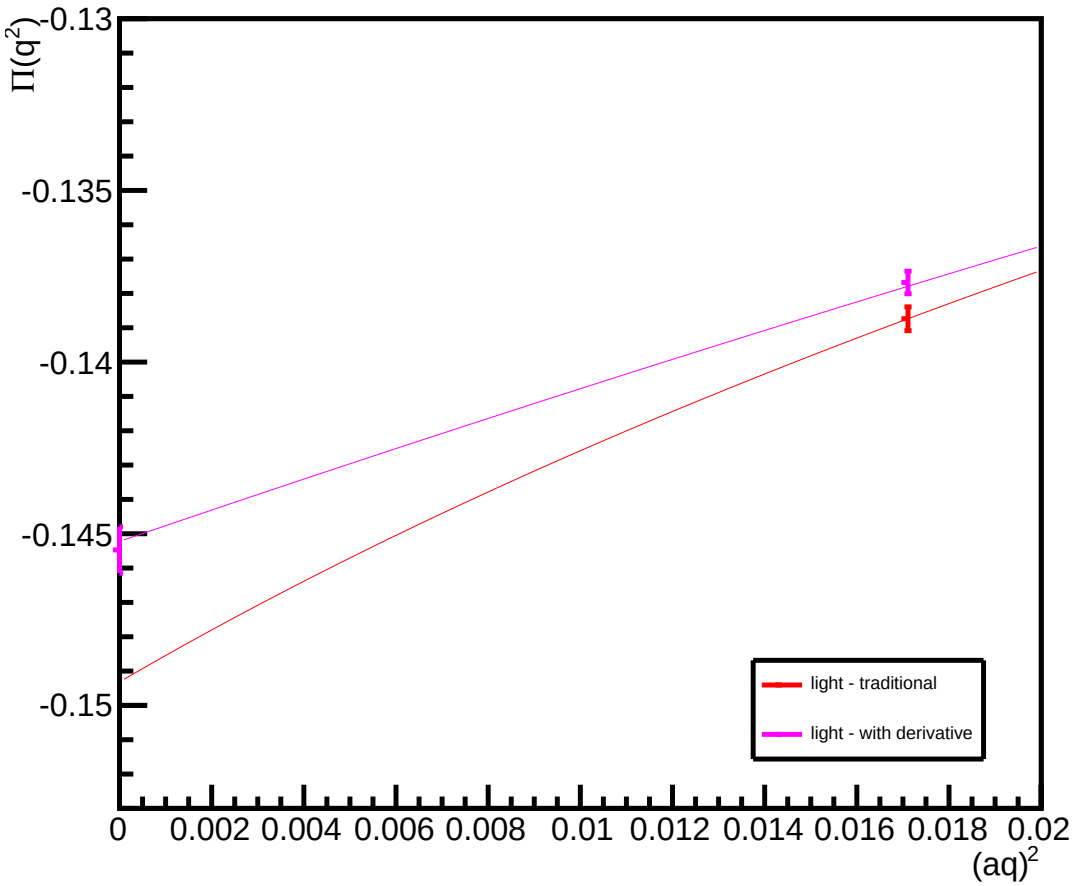

$\Rightarrow$ traditional : $\chi^2/dof = 0.4/8$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 0.5/4$ (light)

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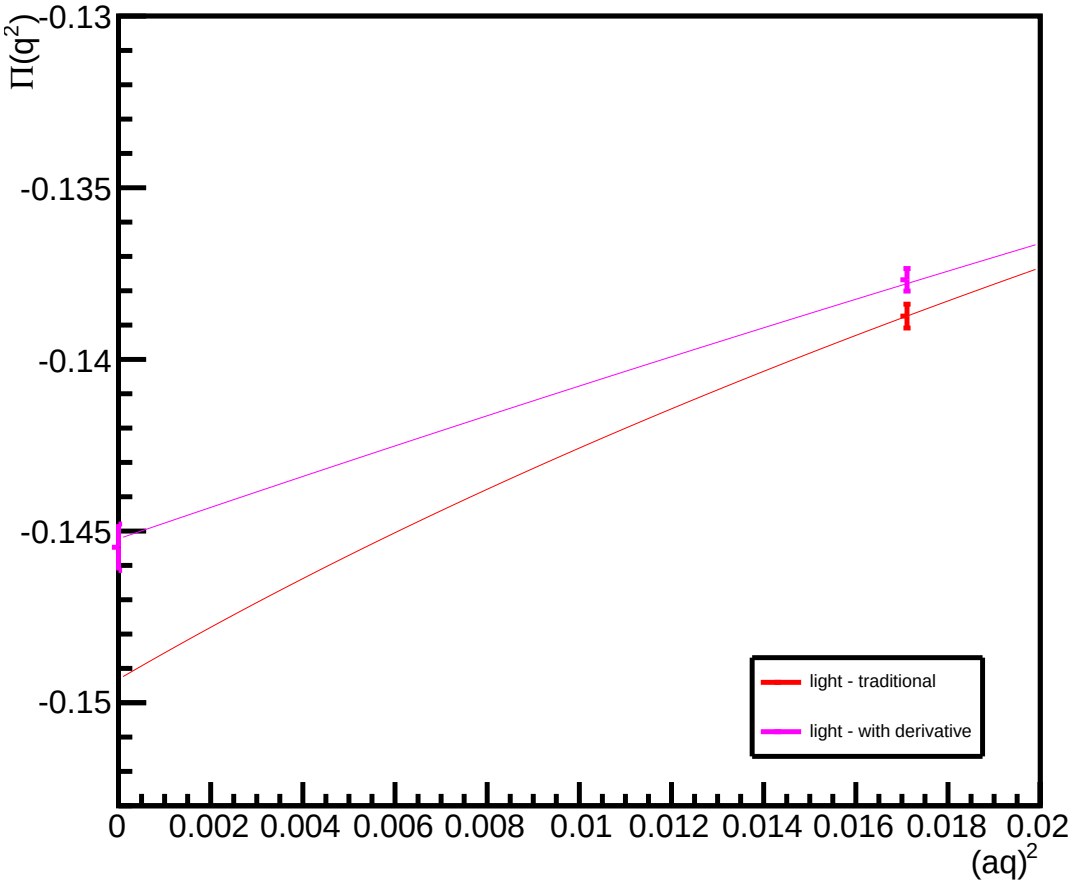

$\Rightarrow$ traditional : extrapolation
$\Rightarrow$ derivative : interpolation $\Rightarrow$ slope in $Q^2 = 0$ independant of the Padé

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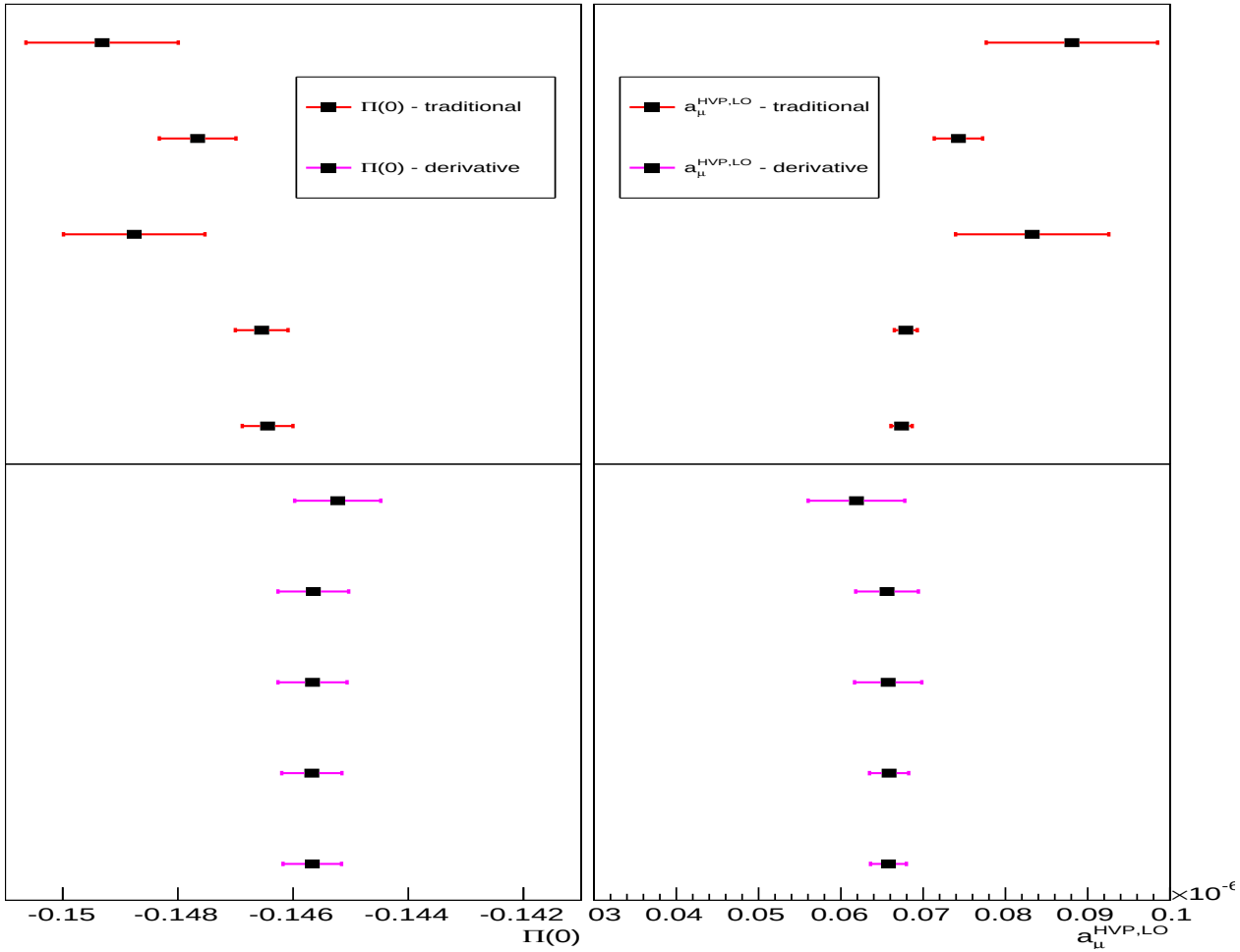

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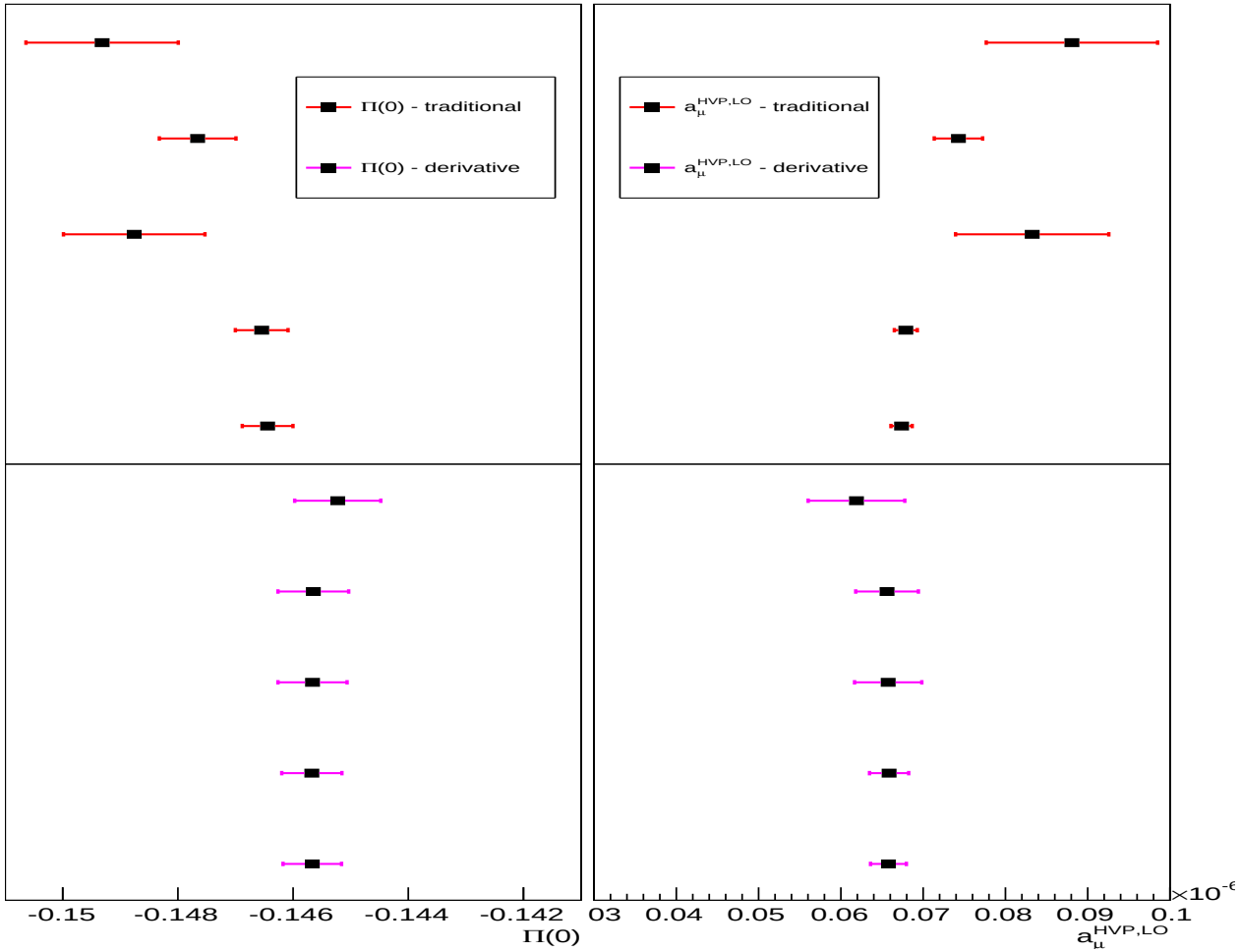

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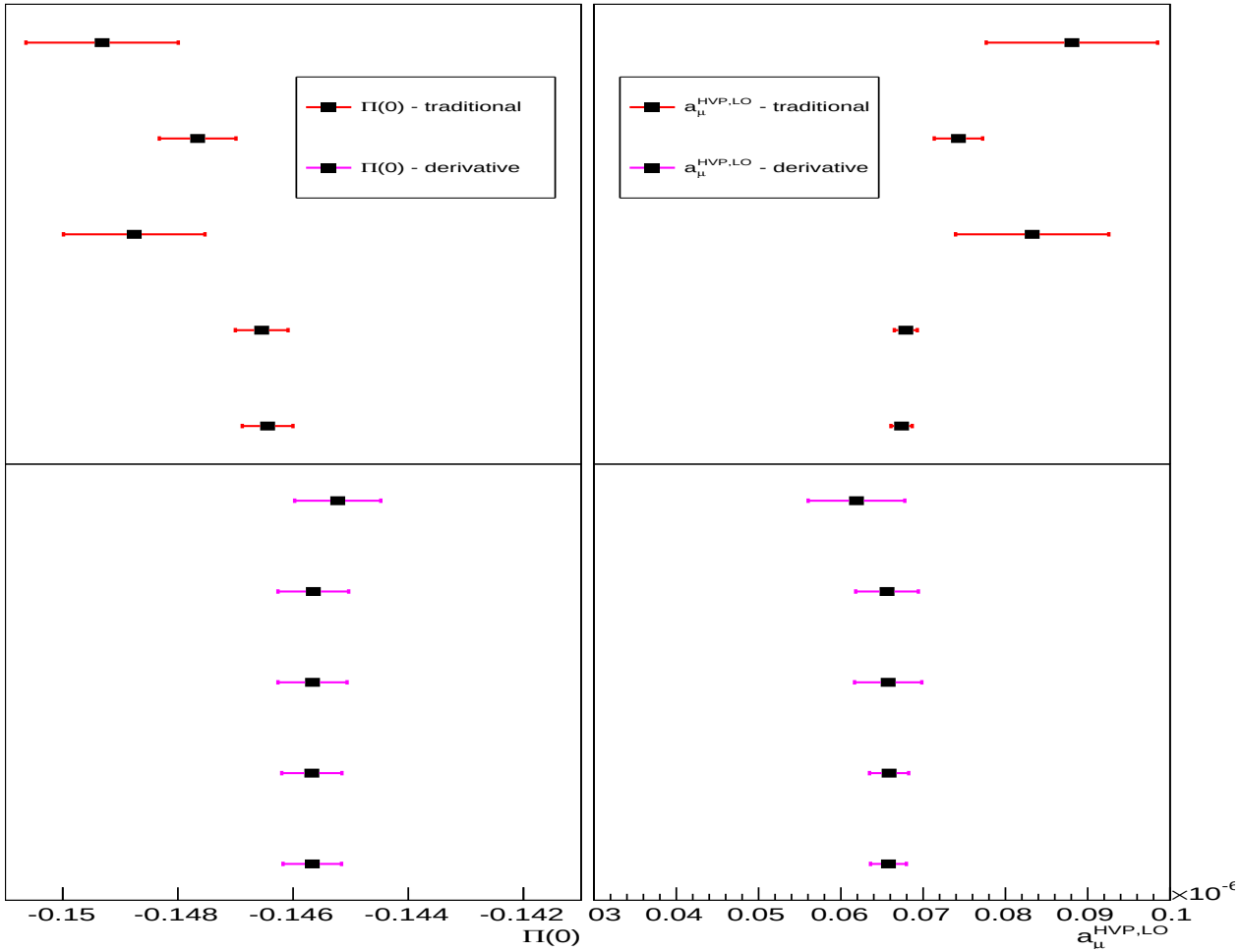

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$\Rightarrow$ Should now try to understand other sources of systematics. . .

5) Conclusion

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Thanks for your attention !