Leading order hadronic contribution to $(g_\mu - 2)$

Rehan Malak
Budapest-Marseille-Wuppertal Collaboration

Special thanks to Eric Gregory, Laurent Lellouch, Craig McNeile, Alfonso Sastre

Lattice 2014
1. Anomalous magnetic moment of the muon in the Standard Model

2. $a_{\mu}^{HVP,LO}$ from Lattice QCD

3. Momentum derivatives of correlation functions in Fourier space

4. Results

5. Conclusion
1) Anomalous magnetic moment of the muon in the Standard Model
Standard model prediction and current deviation:

<table>
<thead>
<tr>
<th></th>
<th>$a_\mu \cdot 10^{-10}$</th>
<th>$\delta a_\mu \cdot 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EXPERIMENT:</strong></td>
<td>11659208.9</td>
<td>6.3</td>
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<tr>
<td><strong>STANDARD MODEL:</strong></td>
<td>11659180.2</td>
<td>4.9</td>
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<tr>
<td>QED</td>
<td>11658471.8951</td>
<td>0.0080</td>
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<tr>
<td>EW</td>
<td>15.4</td>
<td>0.1</td>
</tr>
<tr>
<td>QCD HVP,LO</td>
<td>692.3</td>
<td>4.2 (e^+e^- decay datas)</td>
</tr>
<tr>
<td>QCD HVP,H0</td>
<td>-9.84</td>
<td>0.07</td>
</tr>
<tr>
<td>QCD LbL</td>
<td>10.5</td>
<td>2.6 (estimation)</td>
</tr>
<tr>
<td><strong>DEVIATION</strong></td>
<td>28.7</td>
<td>8.0 $\Rightarrow$ 3.6$\sigma$</td>
</tr>
</tbody>
</table>

⇒ Can Lattice QCD help?
2) $a_{\mu}^{HVP,LO}$ from Lattice QCD
Previous work (only the last 12 months !)

- The muon anomalous magnetic moment, a view from the lattice Christopher Aubin, Thomas Blum, Maarten Golterman, Kim Maltman, Santiago Peris arXiv:1311.5504
- Leading-order hadronic contributions to $(g-2)_\mu$ Eric B. Gregory, Zoltan Fodor, Christian Hoelbling, Stefan Krieg, Laurent Lellouch, Rehan Malak, Craig McNeile, Kalman Szabo arXiv:1311.4446
- Leading-order hadronic contribution to the anomalous magnetic moment of the muon from Nf=2+1+1 twisted mass fermions Florian Burger, Xu Feng, Grit Hotzel, Karl Jansen, Marcus Petschlies, Dru B. Renner arXiv:1311.3885
- The hadronic vacuum polarization with twisted boundary conditions Christopher Aubin, Thomas Blum, Maarten Golterman, Santiago Peris arXiv:1311.1078
- Using analytic continuation for the hadronic vacuum polarization computation Xu Feng, Shoji Hashimoto, Grit Hotzel, Karl Jansen, Marcus Petschlies, Dru B. Renner arXiv:1311.0652
- Tests of hadronic vacuum polarization fits for the muon anomalous magnetic moment Maarten Golterman (SFSU and IFAE), Kim Maltman (York U. and U. of Adelaide), Santiago Peris (UAB) arXiv:1310.5928
- The hadronic vacuum polarization with twisted boundary conditions Christopher Aubin, Thomas Blum, Maarten Golterman, Santiago Peris arXiv:1307.470

⇒ . . . sorry to those whose work I missed !
Compute $a_{\mu}^{HVP,LO}$ in Lattice QCD directly with Euclidean momenta:
Compute $a^{HVP, LO}_\mu$ in Lattice QCD directly with Euclidean momenta:

- With $J_\mu :=$ EM-current:

$$\Pi_{\mu\nu}(Q) = \sum_x \langle J_\mu(x) J_\nu(0) \rangle e^{iQx}$$

If (Euclidean) Lorentz violations neglected:

$$\Pi(Q^2) = \Pi_{\mu\nu}(Q)$$

Subtraction:

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$$

Integration with known QED kernel $\Pi$:

$$a^{HVP, LO}_\mu = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dQ^2 \Pi(Q^2) \hat{\Pi}(Q^2)$$
Compute $a_{\mu}^{HVP,LO}$ in Lattice QCD directly with Euclidean momenta:

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- Integration with known QED kernel $w_\Pi$:

$$a_{\mu}^{HVP, LO} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dQ^2 w_\Pi(Q^2) \hat{\Pi}(Q^2)$$
The small $Q^2$ challenge

Compute $a_{\mu}^{HVP,LO}$ in Lattice QCD directly with Euclidean momenta:

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\[ \hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) \]

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The small $Q^2$ challenge

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- Integration with QED kernel $w_\Pi$:

\[
a_{\mu}^{HVP,LO} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dQ^2 w_\Pi(Q^2) \hat{\Pi}(Q^2) \quad \Leftarrow \quad \text{peak at } Q^2 \sim Q^2_{\text{min}}/10
\]
Small $Q^2$ solutions

- Extrapolate from $Q^2_{\text{min}}$ with pole dominance type model and hope for the best (most early work)
- Implement twisted boundary conditions to go to lower $Q^2_{\text{min}}$ and extrapolate (Sachrajda et al 2004, Jäger et al 2010)
- Compute $\Pi(0)$ directly using momentum expansion of quark propagators (de Divitiis et al 2012)
- Use Padés to extrapolate $\Pi(Q^2)$ to small $Q^2$ (Golterman et al 2013)
- $E = 0$ time moments of $\Pi_{\mu\nu}(t, \tilde{p} = 0)$ to fix coefficients of Padés (Davies et al 2014)
- Moment analysis of $\Pi(Q^2)$ (de Rafael 2014)
- Here consider momentum derivatives of correlation functions in Fourier space (Lellouch et al 1995)

All solutions have specific systematic errors which have yet to be understood.
3) Momentum derivatives of correlation functions in Fourier space
The benefits of derivatives

\[ \Pi_{\mu\nu}(Q) = (Q_{\mu} Q_{\nu} - \delta_{\mu\nu} Q^2) \Pi(Q^2) \]
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\[ \Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\Pi(Q^2) \]

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\[ \Pi_{\mu \nu}(Q) = (Q_\mu Q_\nu - \delta_{\mu \nu} Q^2) \Pi(Q^2) \]

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\[ \Rightarrow \text{Direct computation of } \Pi(Q^2) : \]
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\[ \mu \neq \nu \ , \ \partial_\mu \partial_\nu \Pi_{\mu\nu}(Q_\mu = 0 \ , \ Q_\nu = 0) = \Pi(Q^2) \]
The benefits of derivatives

\[ \Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2) \Pi(Q^2) \]

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The benefits of derivatives

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\[ \Rightarrow \Pi(0) = \left. \frac{\partial^2 \Pi_{12}(Q)}{\partial Q_1 \partial Q_2} \right|_{Q=0} \]

Can be generalized to all \( Q^2 \Rightarrow \) consider \( \partial_\sigma \partial_\rho \Pi_{\mu\nu}(Q) \).

\[ \Rightarrow \text{Direct computation of } \Pi(Q^2) : \]

\[ \mu \neq \nu, \quad \partial_\mu \partial_\nu \Pi_{\mu\nu}(Q_\mu = 0 , Q_\nu = 0) = \Pi(Q^2) \]

\[ \Rightarrow \text{Direct computation of the Adler function :} \]

\[ \partial_\mu \partial_\mu \Pi_{\mu\mu}(Q_\mu = 0) = -2Q^2 \frac{\partial}{\partial Q^2} \Pi(Q^2) = -2A(Q^2) \]
Derivative can be computed as moments in Fourier space, e.g.:

$$\partial_\sigma \partial_\rho \Pi_{\mu\nu}(Q) = - \sum_x x_\sigma x_\rho \Pi_{\mu\nu}(x) e^{iQx}$$

⇒ because higher moments emphasize large distances more, expect enhanced FV effects.

Implementation:

⇒ spatial derivatives in Fourier space in simulation code
⇒ big outputs (NetCDF binary format):

$$3^2 \text{indices combinations} \times \# \text{momenta}$$

⇒ temporal Fourier derivatives transform in analysis code
4) Results
Hadronic vacuum polarization tensor:

\[ \Pi_{\mu\nu}(\hat{Q}) = Z_V \sum_x \langle J^{cvc}_\mu(x) J^{loc}_\nu(y) \rangle e^{iQ(x + \frac{a\hat{\mu}}{2} - y)} \]

\[ \frac{2}{a} \sin\left(\frac{aQ}{2}\right) \]
Hadronic vacuum polarization tensor:

\[ \Pi_{\mu\nu}(\hat{Q}) = Z_V \sum_x \langle J^{cvc}_\mu(x) J^{loc}_\nu(y) \rangle e^{iQ(x + \frac{a\hat{\mu}}{2} - y)} \]

- local current at the source:

\[ J^{loc}_\nu(y) = \bar{\psi}(y)\gamma_\mu\psi(y) \]
Hadronic vacuum polarization tensor:

$$\Pi_{\mu\nu}(\hat{Q}) = Z_V \sum_x \langle J_{\mu}^{cvc}(x) J^{loc}_{\nu}(y) \rangle e^{iQ(x + \frac{a\hat{\mu}}{2} - y)} \frac{2}{a} \sin\left(\frac{aQ}{2}\right)$$

- local current at the source:
  $$J^{loc}_{\nu}(y) = \bar{\psi}(y)\gamma_{\mu}\psi(y)$$

- conserved current at the sink:
  $$J_{\mu}^{cvc}(x) = \frac{1}{2}\{\bar{\psi}(x + a\hat{\mu})(1 + \gamma_{\mu}) U_{\mu}^\dagger(x) \psi(x) - \bar{\psi}(x)(1 - \gamma_{\mu}) U_{\mu}(x) \psi(x + a\hat{\mu})\}$$
Hadronic vacuum polarization tensor:

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\[ \frac{2a}{a} \sin\left(\frac{aQ}{2}\right) \]

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\[ J_{\mu}^{cvc}(x) = \frac{1}{2}\{ \bar{\psi}(x+a\hat{\mu})(1+\gamma_{\mu})U_{\mu}^{\dagger}(x)\psi(x) - \bar{\psi}(x)(1-\gamma_{\mu})U_{\mu}(x)\psi(x+a\hat{\mu}) \} \]

- currently only connected components:

\[ (S_{\mu}(x,y) \equiv U_{\mu}(x)S(x+a\hat{\mu},y)) \]

\[ \langle J_{\mu}^{cvc}(x)J_{\nu}^{loc}(y) \rangle \sim \frac{1}{2} \text{Tr}_{sc} \{ S_{\mu}(x,y)\gamma_{\nu}\gamma_{5}S_{\mu}^{\dagger}(x,y)\gamma_{5}(1 - \gamma_{\mu}) \} \]

\[ - \frac{1}{2} \text{Tr}_{sc} \{ S(x,y)\gamma_{\nu}\gamma_{5}S_{\mu}^{\dagger}(x,y)\gamma_{5}(1 + \gamma_{\mu}) \} \]
Traditional method vs derivative method

\[ T \times L^3 = 48 \times 48^3 \quad a = 0.116\text{fm} \quad M_\pi = 150\text{MeV} \quad 260\text{conf.} \times 64\text{meas.} \]

Fitting function: \( f(x) = A - x \cdot \text{Pade}[1,1](x) \) with \( x = Q^2 \)

\[ \Rightarrow \text{traditional}: \quad a_{\mu}^{\text{HVP,LO}} = 6.79(13) \times 10^{-8} \quad \chi^2/dof = 4.5/10 \quad \text{(light)} \]
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\[ \Rightarrow \text{derivative:} \quad a_{\mu}^{\text{HVP,LO}} = 6.58(22) \times 10^{-8} \quad \chi^2/dof = 3.8/6 \quad \text{(light)} \]
Padé fits in light sector, low $Q^2$ and systematics

$T \times L^3 = 48 \times 48^3 \ a = 0.116\text{fm} \ M_\pi = 150\text{MeV} \ 260\text{conf.} \times 64\text{meas.}$

Fitting function : $f(x) = A-x.\text{Pade}[0,1](x)$ with $x=Q^2$

$\Rightarrow$ traditional : $\chi^2/dof = 40.3/11 \ (\text{light})$
$\Rightarrow$ derivative : $\chi^2/dof = 6.7/7 \ (\text{light})$
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Fitting function : $f(x) = A - x \cdot \text{Pade}[0,2](x)$ with $x = Q^2$

$\Rightarrow$ traditional : $\chi^2/dof = 5.0/10$ (light)
$\Rightarrow$ derivative : $\chi^2/dof = 3.8/6$ (light)
Padé fits in light sector, low $Q^2$ and systematics

\[ T \times L^3 = 48 \times 48^3 \quad a = 0.116 fm \quad M_\pi = 150 MeV \quad 260 \text{conf.} \times 64 \text{meas.} \]

Fitting function: \( f(x) = A - x \cdot \text{Pade}[1,1](x) \) with \( x = Q^2 \)

\[ \Rightarrow \text{traditional: } \chi^2 / \text{dof} = 4.5 / 10 \text{ (light)} \]

\[ \Rightarrow \text{derivative: } \chi^2 / \text{dof} = 3.8 / 6 \text{ (light)} \]
Padé fits in light sector, low $Q^2$ and systematics

$T \times L^3 = 48 \times 48^3$ \hspace{1em} $a = 0.116\,fm$ \hspace{1em} $M_\pi = 150\,MeV$ \hspace{1em} 260\,conf. $\times$ 64\,meas.

Fitting function: $f(x) = A - x \cdot \text{Pade}[1,2](x)$ with $x = Q^2$

$\Rightarrow$ traditional: $\chi^2/dof = 0.7/9$ (light)
$\Rightarrow$ derivative: $\chi^2/dof = 3.8/5$ (light)
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\[ T \times L^3 = 48 \times 48^3 \quad a = 0.116\text{fm} \quad M_\pi = 150\text{MeV} \quad 260\text{conf.} \times 64\text{meas.} \]

Fitting function: \( f(x) = A - x \cdot \text{Pade}[1,3](x) \) with \( x = Q^2 \)

\[ \Rightarrow \text{traditional: } \chi^2/dof = 0.5/8 \text{ (light)} \]
\[ \Rightarrow \text{derivative: } \chi^2/dof = 3.6/4 \text{ (light)} \]
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Fitting function : \( f(x) = A-x.\text{Pade[2,1]}(x) \) with \( x=Q^2 \)

\[ \Rightarrow \text{traditional : } \chi^2/\text{dof} = 1.4/9 \text{ (light)} \]
\[ \Rightarrow \text{derivative : } \chi^2/\text{dof} = 3.8/5 \text{ (light)} \]
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$\Rightarrow$ derivative : $\chi^2/dof = 0.1/3$ (light)
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Fitting function : $f(x) = A-x.\text{Pade}[3,0](x)$ with $x=Q^2$

$⇒$ traditional : $\chi^2/dof = 7.5/9$ (light)

$⇒$ derivative : $\chi^2/dof = 2.3/5$ (light)
Padé fits in light sector, low $Q^2$ and systematics

$T \times L^3 = 48 \times 48^3 \quad a = 0.116 \text{fm} \quad M_\pi = 150 \text{MeV} \quad 260\text{conf.} \times 64\text{meas.}$

Fitting function: $f(x) = A - x.\text{Pade}[3,1](x)$ with $x=Q^2$

$\Rightarrow$ traditional: $\chi^2/dof = 0.4/8$ (light)
$\Rightarrow$ derivative: $\chi^2/dof = 0.5/4$ (light)
Padé fits in light sector, low $Q^2$ and systematics

$T \times L^3 = 48 \times 48^3 \quad a = 0.116\, fm \quad M_\pi = 150\, MeV \quad 260\, conf. \times 64 \, meas.$

Fitting function : $f(x) = A-x.\text{Pade}[3,1](x)$ with $x=Q^2$

$\Rightarrow$ traditional : extrapolation
$\Rightarrow$ derivative : interpolation $\Rightarrow$ slope in $Q^2 = 0$ independant of the Padé
Padé fits in light sector, low $Q^2$ and systematics

$$T \times L^3 = 48 \times 48^3 \quad a = 0.116 \text{fm} \quad M_\pi = 150 \text{MeV} \quad 260\text{conf.} \times 64\text{meas.}$$
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$T \times L^3 = 48 \times 48^3 \ a = 0.116 fm \ M_\pi = 150 MeV \ 260\text{conf.} \times 64\text{meas.}$

⇒ Restrict fits with bootstrap errors on the parameters less than 50 %.
Padé fits in light sector, low $Q^2$ and systematics

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⇒ Systematics related to the choice of the fit function are smaller
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$\Rightarrow$ Restrict fits with bootstrap errors on the parameters less than 50\%.

$\Rightarrow$ Systematics related to the choice of the fit function are smaller

$\Rightarrow$ Should now try to understand other sources of systematics...
5 ) Conclusion
\[ a^S_M - a^E_P \Rightarrow 3.6\sigma \]
Conclusion

- $a_{\mu}^{SM} - a_{\mu}^{EXP} \Rightarrow 3.6\sigma$
- Fermilab E989 $\Rightarrow \delta^{\exp} a_{\mu} \rightarrow \delta^{\exp} a_{\mu}/4$
Conclusion

- \( a_{\mu}^{SM} - a_{\mu}^{EXP} \Rightarrow 3.6\sigma \)
- Fermilab E989 \( \Rightarrow \delta^{\exp} a_{\mu} \rightarrow \delta^{\exp} a_{\mu}/4 \)
- Lattice QCD HVP computation can be done directly at physical mass.
\[ a_{\mu}^{SM} - a_{\mu}^{\text{EXP}} \Rightarrow 3.6\sigma \]

Fermilab E989 \( \Rightarrow \delta^{\text{exp}} a_{\mu} \rightarrow \delta^{\text{exp}} a_{\mu}/4 \)

Lattice QCD HVP computation can be done directly at physical mass.

Derivatives of \( \Pi_{\mu\nu}(Q) \) can be used to provide information in low-\( Q^2 \) region and reduce the systematics associated to the choice of the fitting function.
Conclusion

- $a_{\mu}^{SM} - a_{\mu}^{EXP} \Rightarrow 3.6\sigma$
- Fermilab E989 $\Rightarrow \delta^{\exp} a_{\mu} \rightarrow \delta^{\exp} a_{\mu} / 4$
- Lattice QCD HVP computation can be done directly at physical mass.
- Derivatives of $\Pi_{\mu\nu}(Q)$ can be used to provide information in low-$Q^2$ region and reduce the systematics associated to the choice of the fitting function.
- Encouraging results . . .
Conclusion

- \( a^{SM}_\mu - a^{\text{EXP}}_\mu \Rightarrow 3.6\sigma \)
- Fermilab E989 \( \Rightarrow \delta^{\text{exp}} a_\mu \rightarrow \delta^{\text{exp}} a_\mu / 4 \)
- Lattice QCD HVP computation can be done directly at physical mass.
- Derivatives of \( \Pi_{\mu\nu}(Q) \) can be used to provide information in low-\( Q^2 \) region and reduce the systematics associated to the choice of the fitting function.
- Encouraging results . . .
- . . . but others sources of systematics have to be understood.
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Thanks for your attention!