Phase structure and Higgs boson mass in a Higgs-Yukawa model with a dimension-6 operator

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Outline

1. Introduction
2. The constraint effective potential
3. Phase structure
4. Mass bounds
5. Summary
Higgs boson and vacuum stability

- Higgs boson mass: 126 GeV

- Electroweak vacuum meta stable for $m_H \lesssim 129$ GeV
  - Only standard model
  - Evolution of all SM parameters up to the Planck scale
  - Meta stability: $\lambda$ turns negative (at a scale of $10^{8...14}$ GeV)

[Degrassi et al. 2013]
Higgs boson and vacuum stability

- Higgs boson mass: 126 GeV

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  - Only standard model
  - Evolution of all SM parameters up to the Planck scale
  - Meta stability: $\lambda$ turns negative (at a scale of $10^{8\ldots14}$ GeV)

- Triviality $\rightarrow$ EW sector just an effective theory

- New physics could appear anywhere between a few TeV or above the Planck scale
Adding higher order operators

- $\lambda_6 \phi^6$ term in the action is allowed

- $\lambda_6 > 0 \rightarrow$ the EW vacuum is stable even with negative $\lambda$

- Could emerge as a low energy effect of some higher scale physics

- Very easy extension of the SM
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  - Influence the Higgs boson mass - New lower bound?
- Investigate the effect of this term for small cutoffs ($\mathcal{O}(\text{TeV})$)
  - Compatibility with 126 GeV Higgs / Bounds to $\lambda_6$?
  - Numerically by means of lattice simulations
  - Perturbatively via the constraint effective potential (CEP)
\[ S^{\text{cont}}[\bar{\psi}, \psi, \varphi] = \int d^4x \left\{ \bar{t} \dot{t} + \bar{b} \dot{b} + y_b \bar{\psi}_L \varphi \, b_R + y_t \bar{\psi}_L \tilde{\varphi} \, t_R + \text{h.c.} \right\} \\
+ \int d^4x \left\{ \frac{1}{2} (\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 + \left[ \lambda_6 (\varphi^\dagger \varphi)^3 \right] \right\} \]
Higgs-Yukawa model

\[ S^{\text{cont}}[\tilde{\psi}, \psi, \varphi] = \int d^4x \left\{ i\bar{\psi}t + b\bar{\phi}b + y_b \bar{\psi}_L \varphi b_R + y_t \bar{\psi}_L \tilde{\varphi} t_R + h.c. \right\} \]

\[ + \int d^4x \left\{ \frac{1}{2} \left( \partial_\mu \varphi \right)^\dagger \left( \partial_\mu \varphi \right) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda \left( \varphi^\dagger \varphi \right)^2 + \left( \lambda_6 \left( \varphi^\dagger \varphi \right)^3 \right) \right\} \]

\[ S^{\text{lat}}[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[ \Phi_{x+\mu} + \Phi_{x-\mu} \right] + \sum_x \Phi_x^\dagger \Phi_x \]

\[ + \hat{\lambda} \sum_x \left[ \Phi_x^\dagger \Phi_x - N_f \right]^2 + \hat{\lambda}_6 \sum_x \left[ \Phi_x^\dagger \Phi_x \right]^3 \]

with:

\[ \varphi = \sqrt{2\kappa} \left( \frac{\Phi^2 + i\Phi^1}{\Phi^0 - i\Phi^3} \right), \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad \lambda_6 = \frac{\hat{\lambda}_6}{8\kappa^3}, \quad m_0^2 = \frac{1 - 2N_f \hat{\lambda} - 8\kappa}{\kappa} \]
Implementation

- Polynomial Hybrid Monte Carlo algorithm
  
- Overlap fermions \( (N_f = 1, \, y_t = y_b) \)

- Details of the code

- Scale setting: renormalized vacuum expectation value of the scalar field:
  \[
  \frac{v_r}{a} = 246 \text{ GeV}
  \]

- Definition of the cutoff: \( \Lambda = \frac{1}{a} = \frac{246 \text{ GeV}}{v_r} \)

- Higgs boson mass: Pole of the real part of the propagator
Introduction

The constraint effective potential

Phase structure

Mass bounds

Summary
Constraint effective potential in the broken phase

[O’Raifeartaigh, et al. 2007; Gerhold et al. 2009]

- Scalar doublet can be decomposed into Higgs and Goldstone modes
- In the broken phase, the CEP explicitly only depends on the zero mode of the Higgs field \( \tilde{h}_0 = \sqrt{V} \tilde{v} \)
- The global minimum of the CEP determines the vev
- The Higgs boson mass is given by the curvature

\[
\frac{dU}{d\tilde{v}} = 0 \bigg|_{\tilde{v} = vev} \quad \frac{d^2U}{d\tilde{v}^2} = m_H^2 \bigg|_{\tilde{v} = vev}
\]
- Keep explicitly the lattice structure
- Perturbative derivation of the CEP not unique
\[ U_1(\hat{v}) = U_f(\hat{v}) + \frac{m_0^2}{2} \hat{v}^2 + \lambda \hat{v}^4 + \lambda_6 \hat{v}^6 + 6 \lambda \hat{v}^2 (P_H + P_G) \\
+ \lambda_6 \hat{v}^4 (15P_H + 9P_G) + \lambda_6 \hat{v}^2 (45P_H^2 + 54P_H P_G + 45P_G^2) \]

With the propagator sums \( P_{G/H} \) given by:

\[ P_G = \sum_{p \neq 0} \frac{1}{\hat{p}^2} \quad P_H = \sum_{p \neq 0} \frac{1}{\hat{p}^2 + m_H^2} \]

- Explicit appearance of \( m_H \): self consistent solution
\[ U_2(\hat{v}) = U_f(\hat{v}) + \frac{m_0^2}{2} \hat{v}^2 + \lambda \hat{v}^4 + \lambda_6 \hat{v}^6 \]
\[ + \frac{1}{2V} \sum_{p \neq 0} \left[ \log \left( \hat{p}^2 + m_0^2 + 12\lambda \hat{v}^2 + 30\lambda_6 \hat{v}^4 \right) \right. \]
\[ + 3 \left( \hat{p}^2 + m_0^2 + 12\lambda \hat{v}^2 + 30\lambda_6 \hat{v}^4 \right) \]
\[ + \lambda \left( 3 \tilde{P}_{H}^2 + 6 \tilde{P}_{H} \tilde{P}_{G} + 15 \tilde{P}_{G}^2 \right) + \lambda_6 \hat{v}^2 \left( 45 \tilde{P}_{H}^2 + 54 \tilde{P}_{H} \tilde{P}_{G} + 45 \tilde{P}_{G}^2 \right) \]
\[ + \lambda_6 \left( 15 \tilde{P}_{H}^3 + 27 \tilde{P}_{H}^2 \tilde{P}_{G} + 45 \tilde{P}_{H} \tilde{P}_{G}^2 + 105 \tilde{P}_{G}^3 \right) \]
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\[ \tilde{P}_H = \frac{1}{V} \sum_{p \neq 0} \frac{1}{\hat{p}^2 + m_0^2 + 12\hat{v}^2 \lambda + 30\hat{v}^4 \lambda_6} \]
\[ \tilde{P}_G = \frac{1}{V} \sum_{p \neq 0} \frac{1}{\hat{p}^2 + m_0^2 + 4\hat{v}^2 \lambda + 6\hat{v}^4 \lambda_6} \]
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\]

- Limited range of validity
1 Introduction

2 The constraint effective potential

3 Phase structure

4 Mass bounds

5 Summary
Set the Yukawa coupling to generate 175 GeV quarks \( m_t = y_t \cdot vev \)

We fix \( \lambda_6 \) - Two setups: \( \lambda_6 = 0.001 \) and \( \lambda_6 = 0.1 \)

A set of negative values \( \lambda \) each

Perform scans in \( \kappa \)

Order parameter: \( vev \)
Simulations vs. CEP $\lambda_6 = 0.001$

- Good agreement for both potentials
Simulations vs. CEP $\lambda_6 = 0.1$

Qualitative agreement for $U_1$
$\lambda_6 = 0.001$

$\lambda = -0.0085$
Phase structure $\lambda_6 = 0.001, U_1$
Phase structure $\lambda_6 = 0.001, U_2$
Phase structure $\lambda_6 = 0.1, \ U_1$

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Phase structure $\lambda_6 = 0.1$, $U_1$
Phase structure $\lambda_6 = 0.1, U_1$
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Procedure

- Stay in the regime of second order transition
- Determine the Higgs boson mass
- Perform infinite volume limit
- Compare the masses with the SM lower bound \( (\lambda_6 = 0 \text{ and } \lambda = 0) \) [Gerhold et al. 2009]
Mass vs. cutoff from CEP, $\lambda_6 = 0.001$
Mass vs cutoff from simulations, $\lambda_6 = 0.001$

Preliminary! (No infinite volume extrapolation. Only $24^3 \times 48$ data!)

![Graph showing Higgs boson mass vs cutoff with different values of $\lambda$]
Mass vs. cutoff from CEP, $\lambda_6 = 0.1$

![Graph showing the relationship between Higgs boson mass and cutoff in GeV for different values of $\lambda$. The graph includes a SM bound and curves for $\lambda = -0.380, -0.385, -0.388, -0.389$.](image-url)
Mass vs cutoff from simulations, $\lambda_6 = 0.1$

Preliminary! (Infinite volume, but still limited statistics)
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Summary
We mapped out the phase space of a HY-model including a $\lambda_6 \phi^6$ term.

Regions of first and second order transitions have been found.

A region in parameter space with a metastable vacuum was located.

$\lambda_6$ is compatible with the standard model Higgs boson mass.

$\lambda_6 = 0.001$ makes even a decrease of the Higgs boson mass possible.
We mapped out the phase space of a HY-model including a $\lambda_6 \phi^6$ term

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**Outlook:**

- Increase the range of $\lambda_6$ (to non-perturbative values)
- Establish the nature of the phase transitions numerically
BACKUP
$\lambda_6 = 0.0010$, $\lambda = -0.0085$

\begin{align*}
\lambda_6 &= 0.0010, \quad \lambda = -0.0085 \\
16^3 \times 32 &\quad \text{(red)} \\
32^3 \times 64 &\quad \text{(blue)} \\
64^3 \times 128 &\quad \text{(yellow)} \\
96^3 \times 192 &\quad \text{(green)}
\end{align*}
CEP phase scan

\[ \lambda_6 = 0.0010, \lambda = -0.0087 \]

\[ v_{eu} \text{ in } a^{-1} \]

\[ m_H^2 \text{ in } a^{-2} \]

\[ 16^3 \times 32 \]
\[ 32^3 \times 64 \]
\[ 64^3 \times 128 \]
\[ 96^3 \times 192 \]
$\lambda_6 = 0.0010, \lambda = -0.0088$

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