Phase structure and Higgs boson mass in a Higgs-Yukawa model with a dimension-6 operator

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Outline

Higgs boson and vacuum stability

- Higgs boson mass: 126 GeV
- Electroweak vacuum meta stable for $m_H \lesssim 129$ GeV [Degrassi et al. 2013]
  - Only standard model
  - Evolution of all SM parameters up to the Planck scale
  - Meta stability: λ turns negative (at a scale of $10^{8\ldots14}$ GeV)

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  - Only standard model
  - Evolution of all SM parameters up to the Planck scale
  - Meta stability: λ turns negative (at a scale of $10^{8\ldots14}$ GeV)
- Triviality $\rightarrow$ EW sector *just* an effective theory
- New physics could appear anywhere between a few TeV or above the Planck scale

Adding higher order operators

- $\lambda_6 \phi^6$ term in the action is allowed
- $\lambda_6 > 0 \quad \rightarrow \quad$ the EW vacuum is stable even with negative λ
- Could emerge as a low energy effect of some higher scale physics
- Very easy extension of the SM

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  - Influence the Higgs boson mass - New lower bound?
- Investigate the effect of this term for small cutoffs ($\mathcal{O}$(TeV))
  - Compatibility with 126 GeV Higgs / Bounds to λ_6?
  - Numerically by means of lattice simulations
  - Perturbatively via the constraint effective potential (CEP)

Higgs-Yukawa model

$$S^{\text{cont}}[\bar{\psi},\psi,\varphi] = \int d^4x \left\{ \bar{t}\partial\!\!\!/ t + \bar{b}\partial\!\!\!/ b + y_b \bar{\psi}_L \varphi\, b_R + y_t \bar{\psi}_L \tilde{\varphi}\, t_R + h.c. \right\}$$
$$+ \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi\right)^\dagger \left(\partial^\mu \varphi\right) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2 + \left[\lambda_6 \left(\varphi^\dagger \varphi\right)^3 \right] \right\}$$

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$$S_B^{\text{lat}}[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\mu} + \Phi_{x-\mu}\right] + \sum_x \Phi_x^\dagger \Phi_x$$
$$+ \hat{\lambda} \sum_x \left[\Phi_x^\dagger \Phi_x - N_f\right]^2 + \hat{\lambda}_6 \sum_x \left[\Phi_x^\dagger \Phi_x\right]^3$$

with:

$$\varphi = \sqrt{2\kappa} \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad \lambda_6 = \frac{\hat{\lambda}_6}{8\kappa^3}, \quad m_0^2 = \frac{1 - 2N_f \hat{\lambda} - 8\kappa}{\kappa}$$

Implementation

- Polynomial Hybrid Monte Carlo algorithm [Frezzotti & Jansen 1997-1999]
- Overlap fermions ($N_f = 1, y_t = y_b$)
- Details of the code [Gerhold 2010, PhD Thesis]
- Scale setting: renormalized vacuum expectation value of the scalar field: $\frac{v_r}{a} = 246$ GeV
- Definition of the cutoff: $\Lambda = \frac{1}{a} = \frac{246 \text{ GeV}}{v_r}$
- Higgs boson mass: Pole of the real part of the propagator

Constraint effective potential in the broken phase

[O'Raifeartaigh, et al. 2007; Gerhold et al. 2009]

- Scalar doublet can be decomposed into Higgs and Goldstone modes
- In the broken phase, the CEP explicitly only depends on the zero mode of the Higgs field $\tilde{h}_0 = \sqrt{V}\breve{v}$
- The *global* minimum of the CEP determines the vev
- The Higgs boson mass is given by the curvature

$$\frac{dU}{d\breve{v}} = 0\Big|_{\breve{v}=vev} \qquad \frac{d^2U}{d\breve{v}^2} = m_H^2\Big|_{\breve{v}=vev}$$

- Keep explicitly the latttice structure
- Perturbative derivation of the CEP not unique

CEP I

$$U_1(\breve{v}) = U_f(\hat{v}) + \frac{m_0^2}{2}\hat{v}^2 + \lambda\hat{v}^4 + \lambda_6\hat{v}^6 + 6\lambda\breve{v}^2(P_H + P_G)$$
$$+ \lambda_6\breve{v}^4(15P_H + 9P_G) + \lambda_6\breve{v}^2(45P_H^2 + 54P_HP_G + 45P_G^2)$$

With the propagator sums $P_{G/H}$ given by:

$$P_G = \sum_{p\neq 0}\frac{1}{\hat{p}^2} \qquad P_H = \sum_{p\neq 0}\frac{1}{\hat{p}^2 + m_H^2}$$

- Explicit appearance of m_H: self consistent solution

CEP II

$$
\begin{aligned}
U_2(\hat{v}) = U_f(\hat{v}) &+ \frac{m_0^2}{2}\hat{v}^2 + \lambda\hat{v}^4 + \lambda_6\hat{v}^6 \\
&+ \frac{1}{2V}\sum_{p\neq 0}\Big[\log\left(\hat{p}^2 + m_0^2 + 12\lambda\hat{v}^2 + 30\lambda_6\hat{v}^4\right) \\
&\qquad\qquad + 3\left(\hat{p}^2 + m_0^2 + 12\lambda\hat{v}^2 + 30\lambda_6\hat{v}^4\right)\Big] \\
&+ \lambda\left(3\,\tilde{P}_H^2 + 6\,\tilde{P}_H\tilde{P}_G + 15\,\tilde{P}_G^2\right) + \lambda_6\hat{v}^2\left(45\,\tilde{P}_H^2 + 54\,\tilde{P}_H\tilde{P}_G + 45\,\tilde{P}_G^2\right) \\
&+ \lambda_6\left(15\,\tilde{P}_H^3 + 27\,\tilde{P}_H^2\tilde{P}_G + 45\,\tilde{P}_H\tilde{P}_G^2 + 105\,\tilde{P}_G^3\right)
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\tilde{P}_H = \frac{1}{V}\sum_{p\neq 0}\frac{1}{\hat{p}^2 + m_0^2 + 12\hat{v}^2\lambda + 30\hat{v}^4\lambda_6}
$$

$$
\tilde{P}_G = \frac{1}{V}\sum_{p\neq 0}\frac{1}{\hat{p}^2 + m_0^2 + 4\hat{v}^2\lambda + 6\hat{v}^4\lambda_6}
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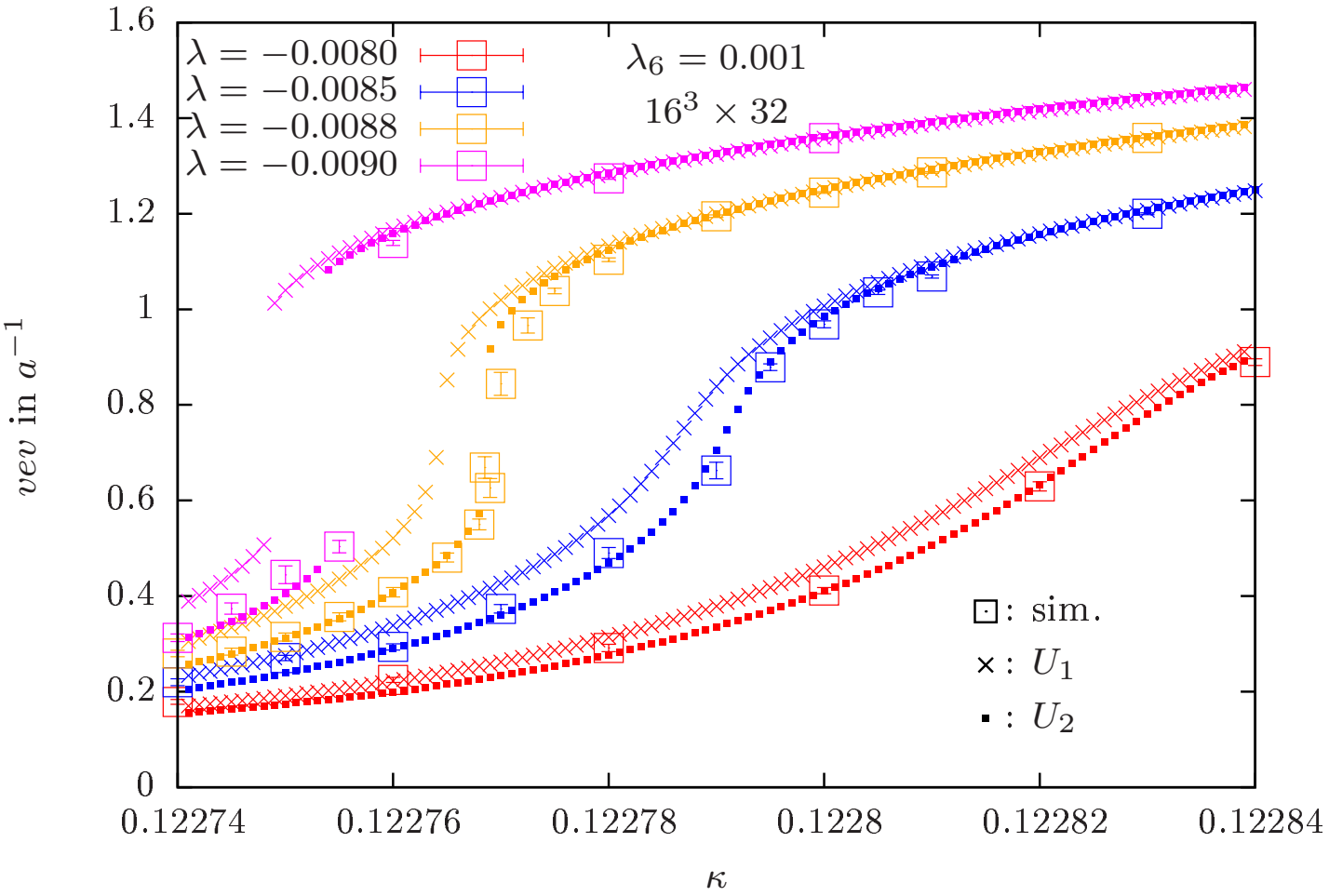
- Limited range of validity

Setup for scans of the phase structure

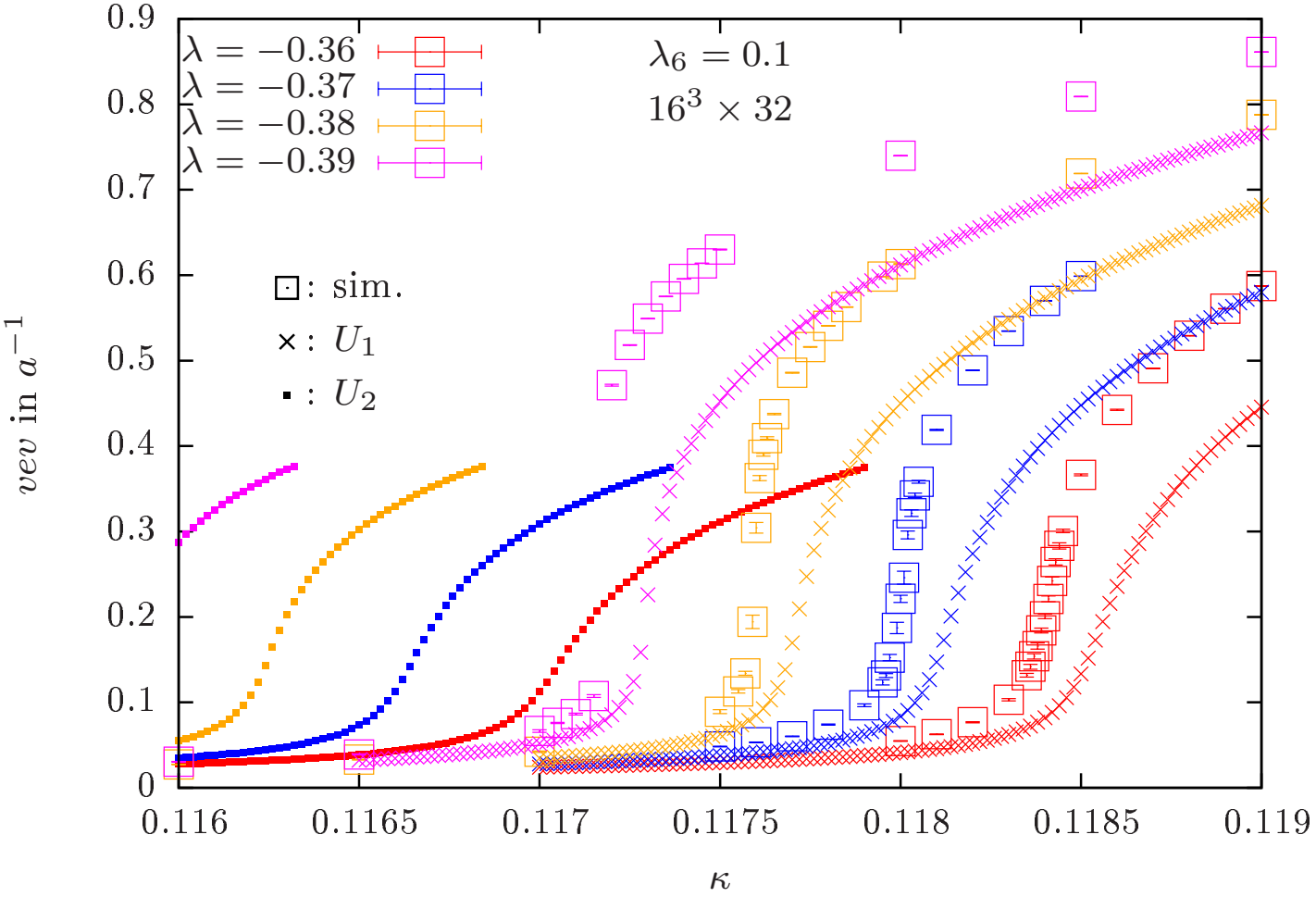
- Set the Yukawa coupling to generate 175 GeV quarks ($m_t = y_t \cdot vev$)
- We fix λ_6 - Two setups: $\lambda_6 = 0.001$ and $\lambda_6 = 0.1$
- A set of negative values λ each
- Perform scans in κ
- Order parameter: vev

Simulations vs. CEP $\lambda_6 = 0.001$



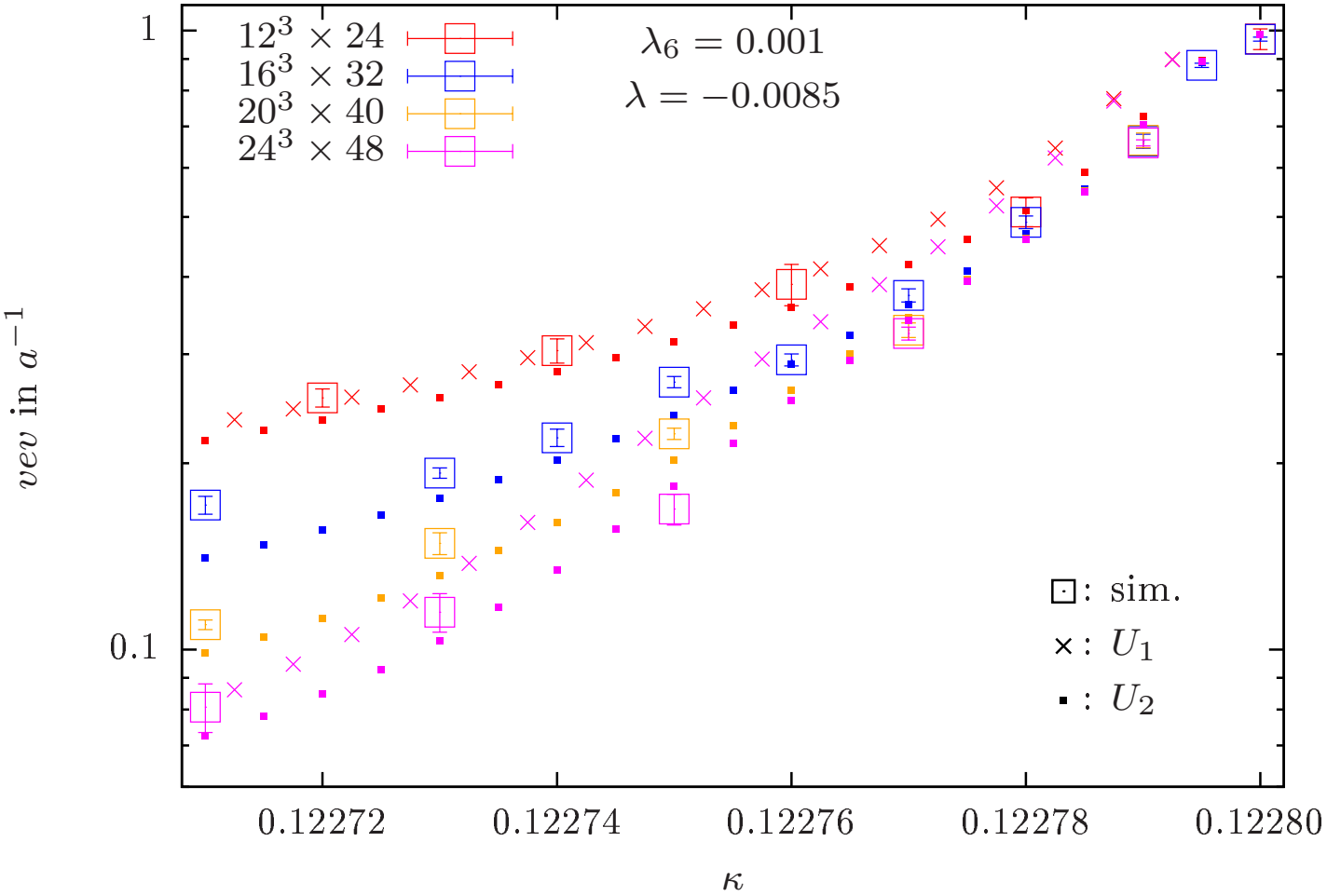
- Good agreement for both potentials

Simulations vs. CEP $\lambda_6 = 0.1$

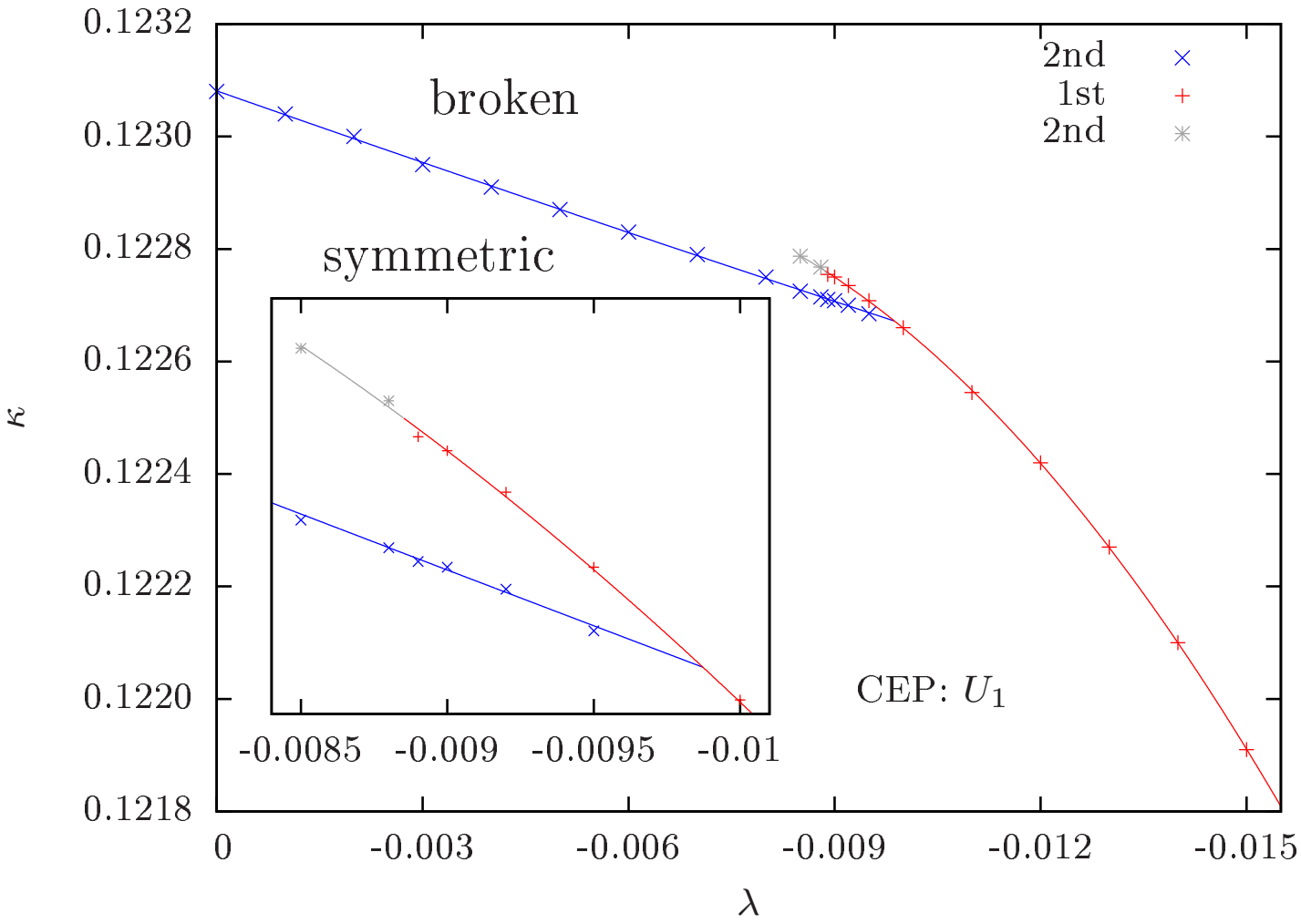


- Qualitative agreement for U_1

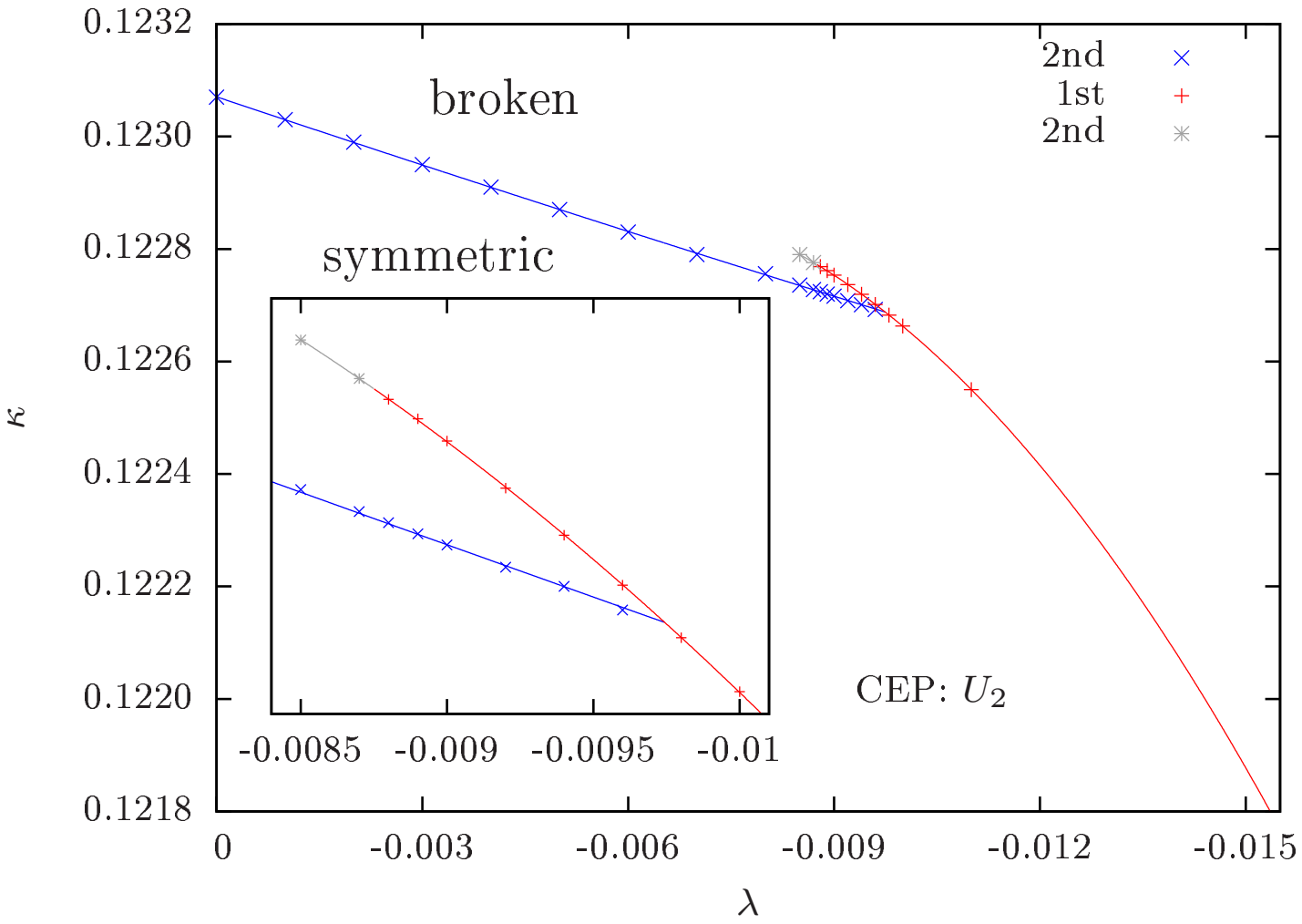
Volume dependence



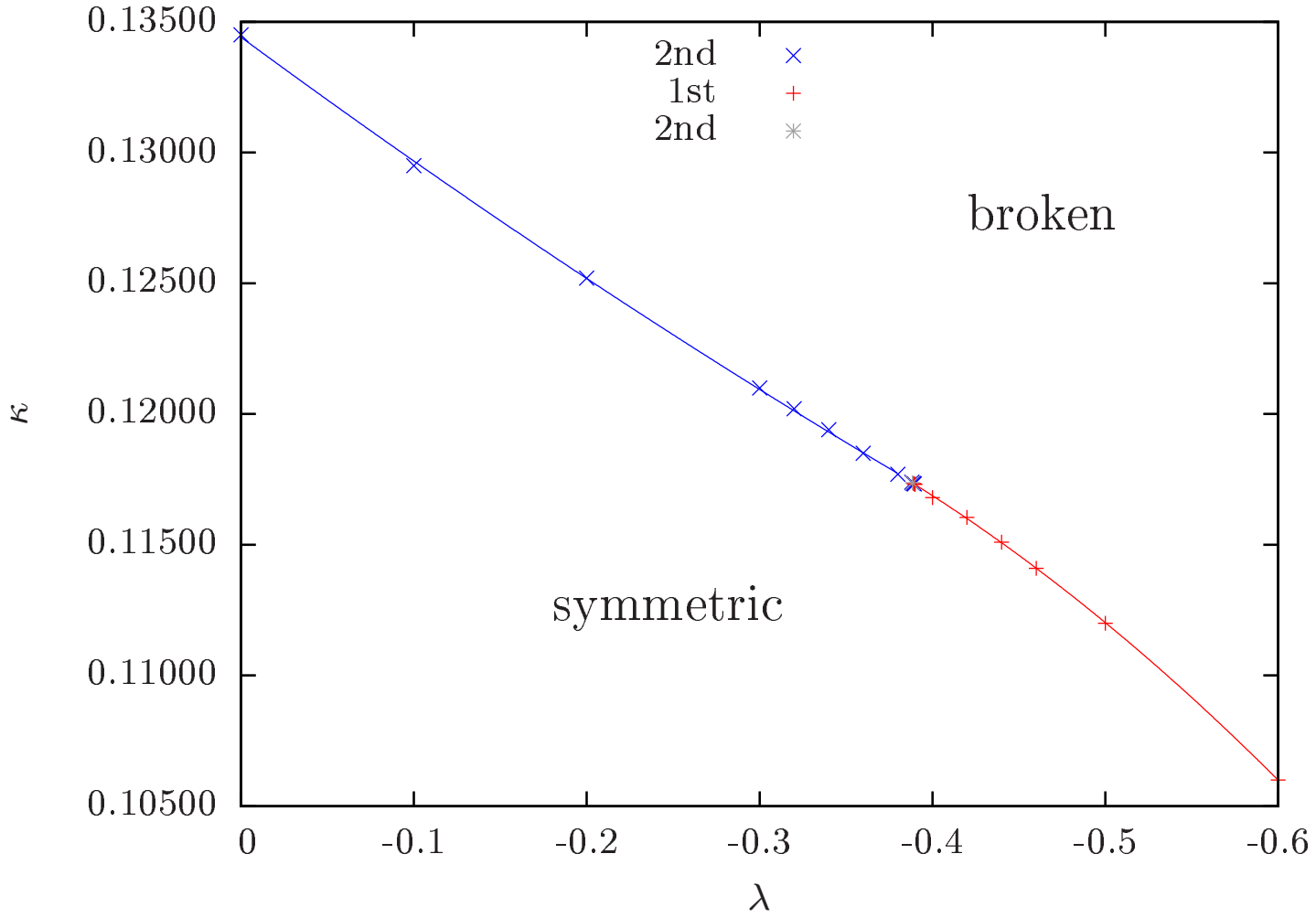
Phase structure $\lambda_6 = 0.001$, U_1



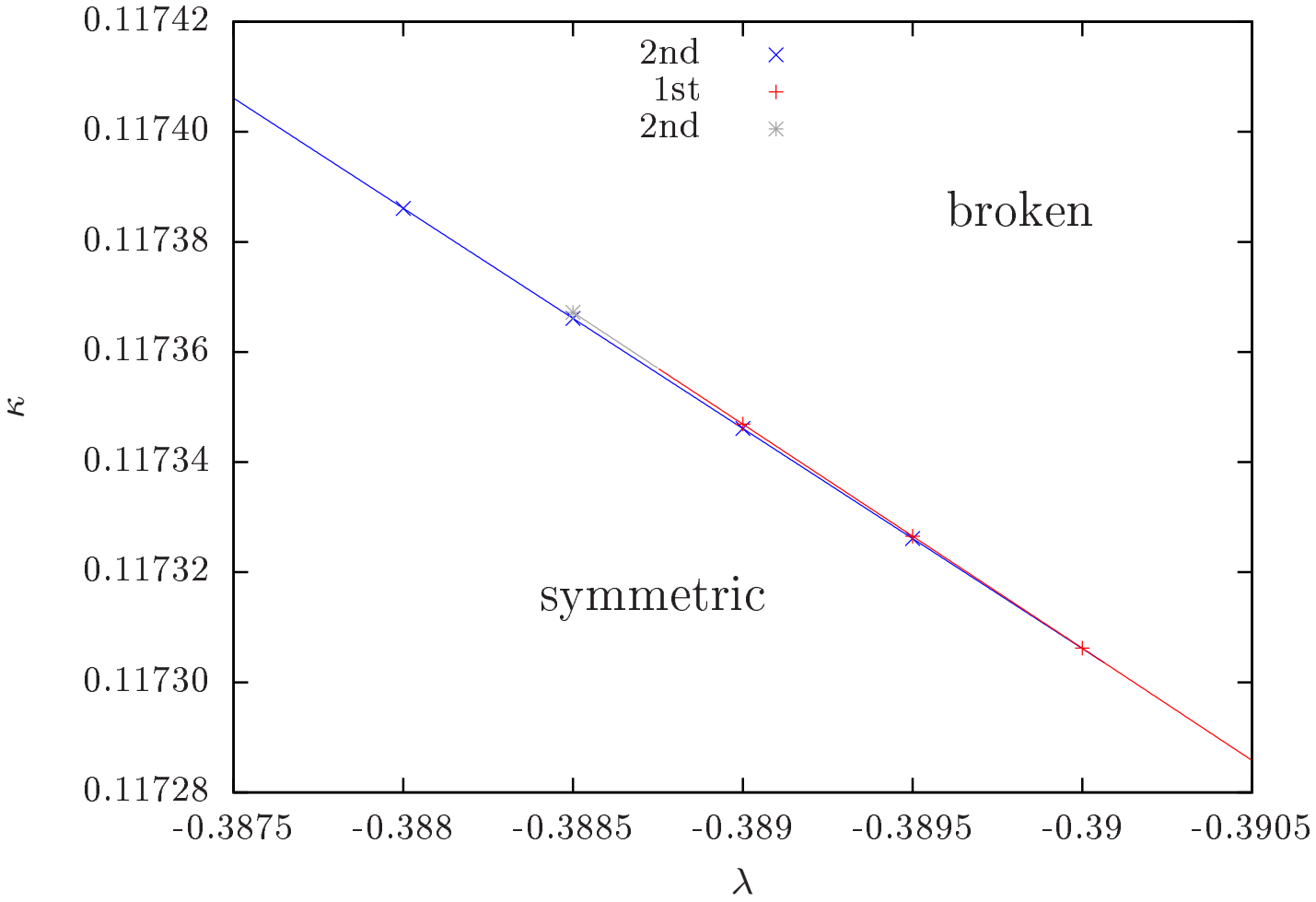
Phase structure $\lambda_6 = 0.001$, U_2



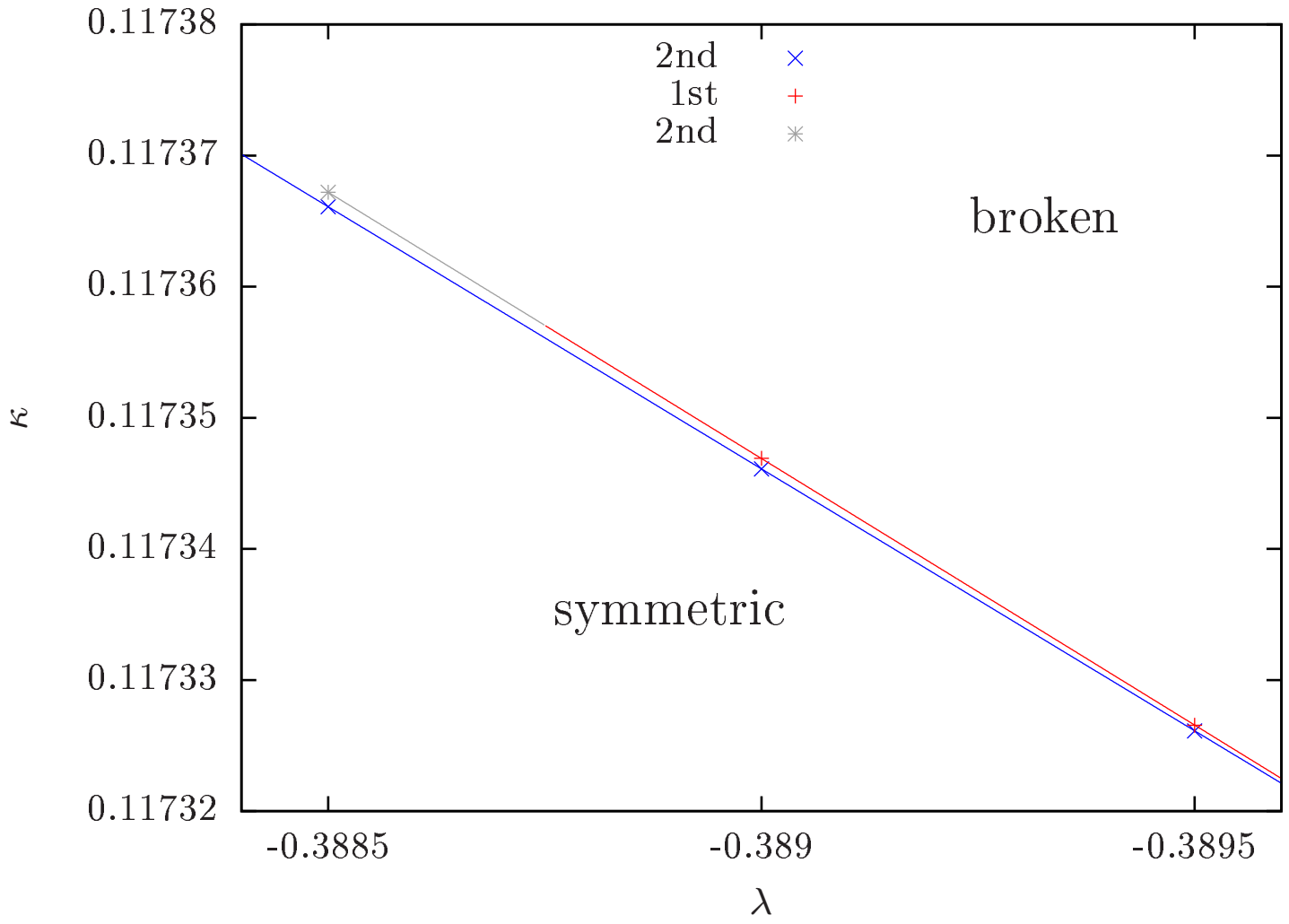
Phase structure $\lambda_6 = 0.1$, U_1



Phase structure $\lambda_6 = 0.1$, U_1



Phase structure $\lambda_6 = 0.1$, U_1

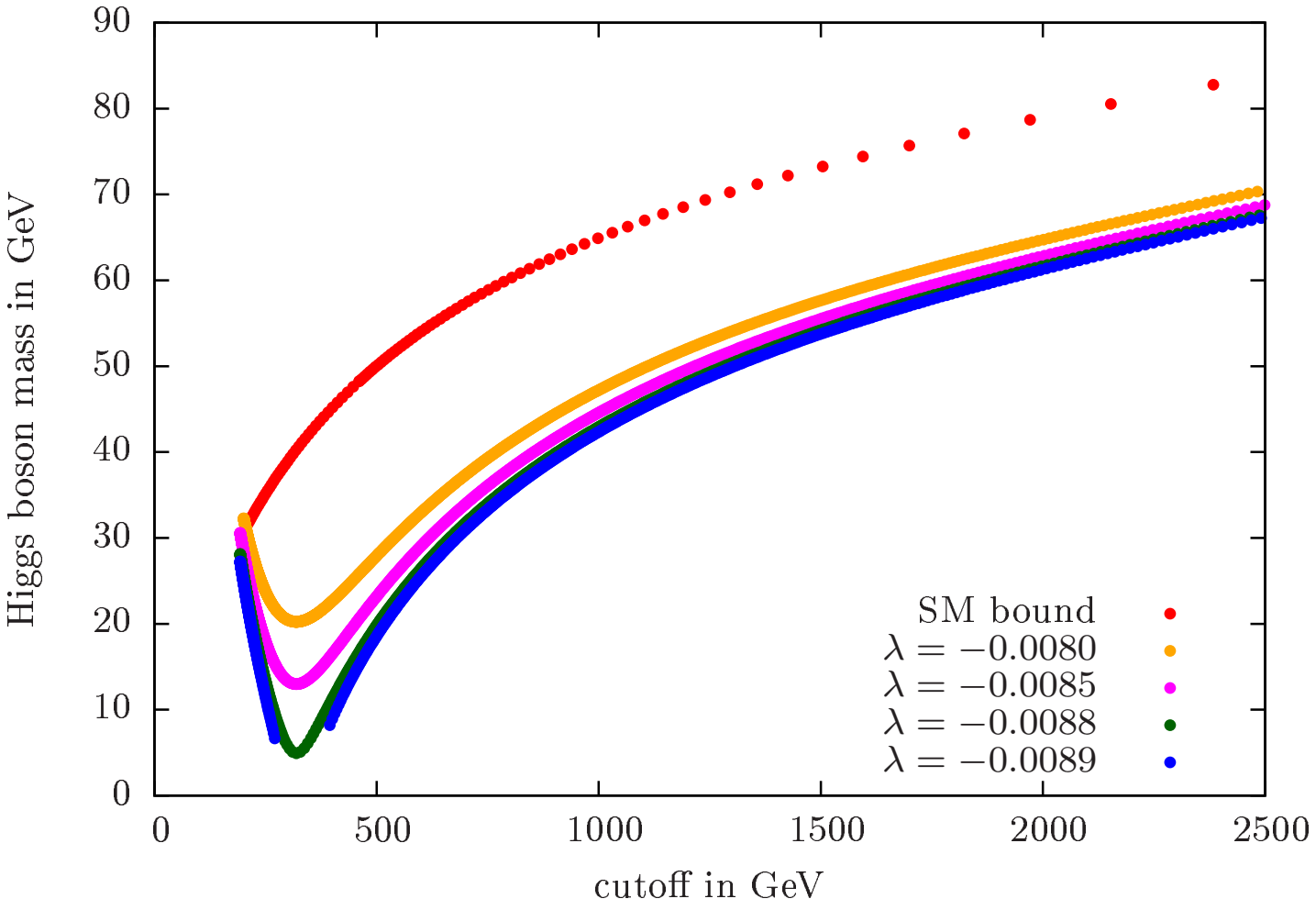


Procedure

- Stay in the regime of second order transition
- Determine the Higgs boson mass
- Perform infinite volume limit
- Compare the masses with the SM lower bound ($\lambda_6 = 0$ and $\lambda = 0$)

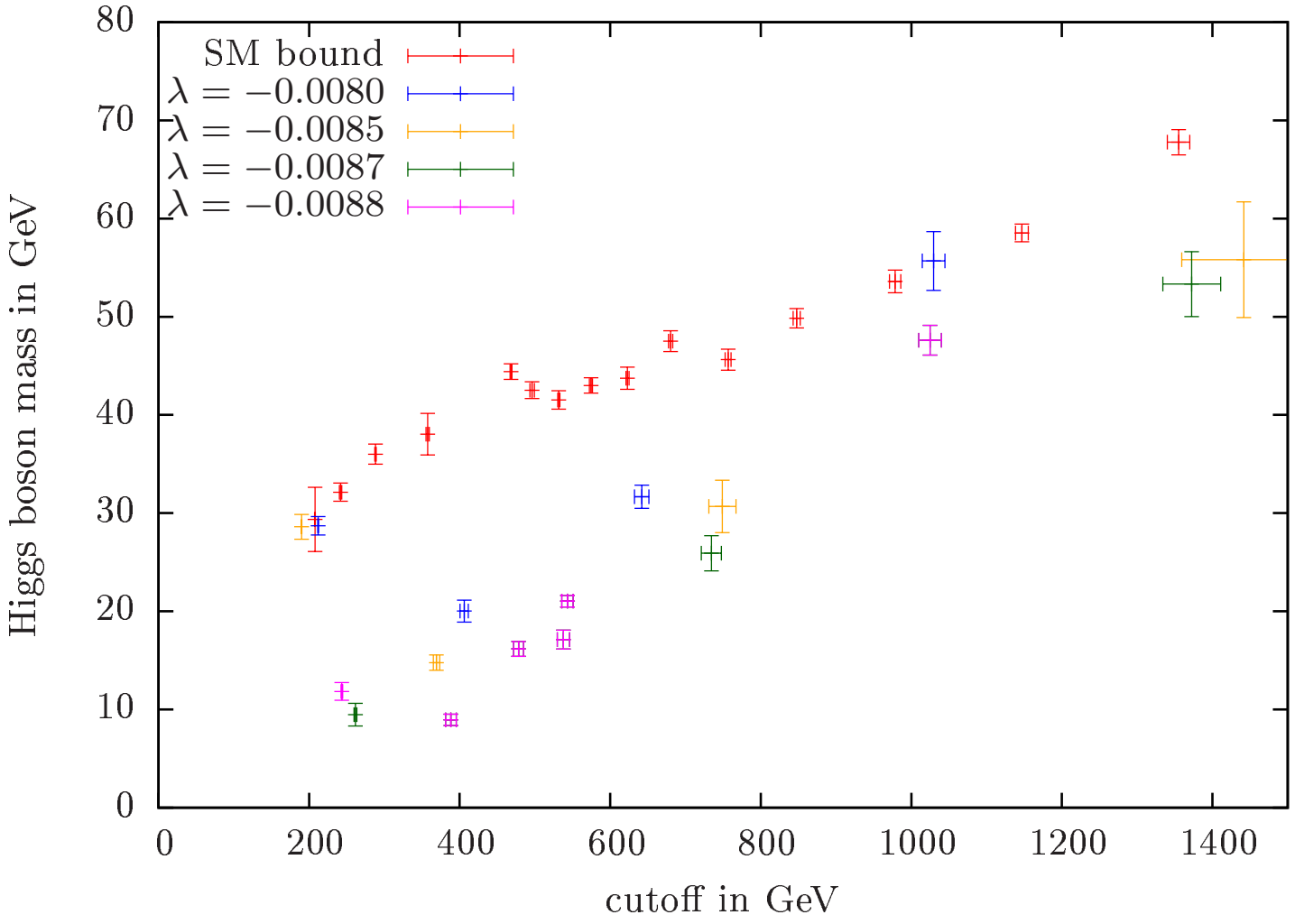
[Gerhold et al. 2009]

Mass vs. cutoff from CEP, $\lambda_6 = 0.001$

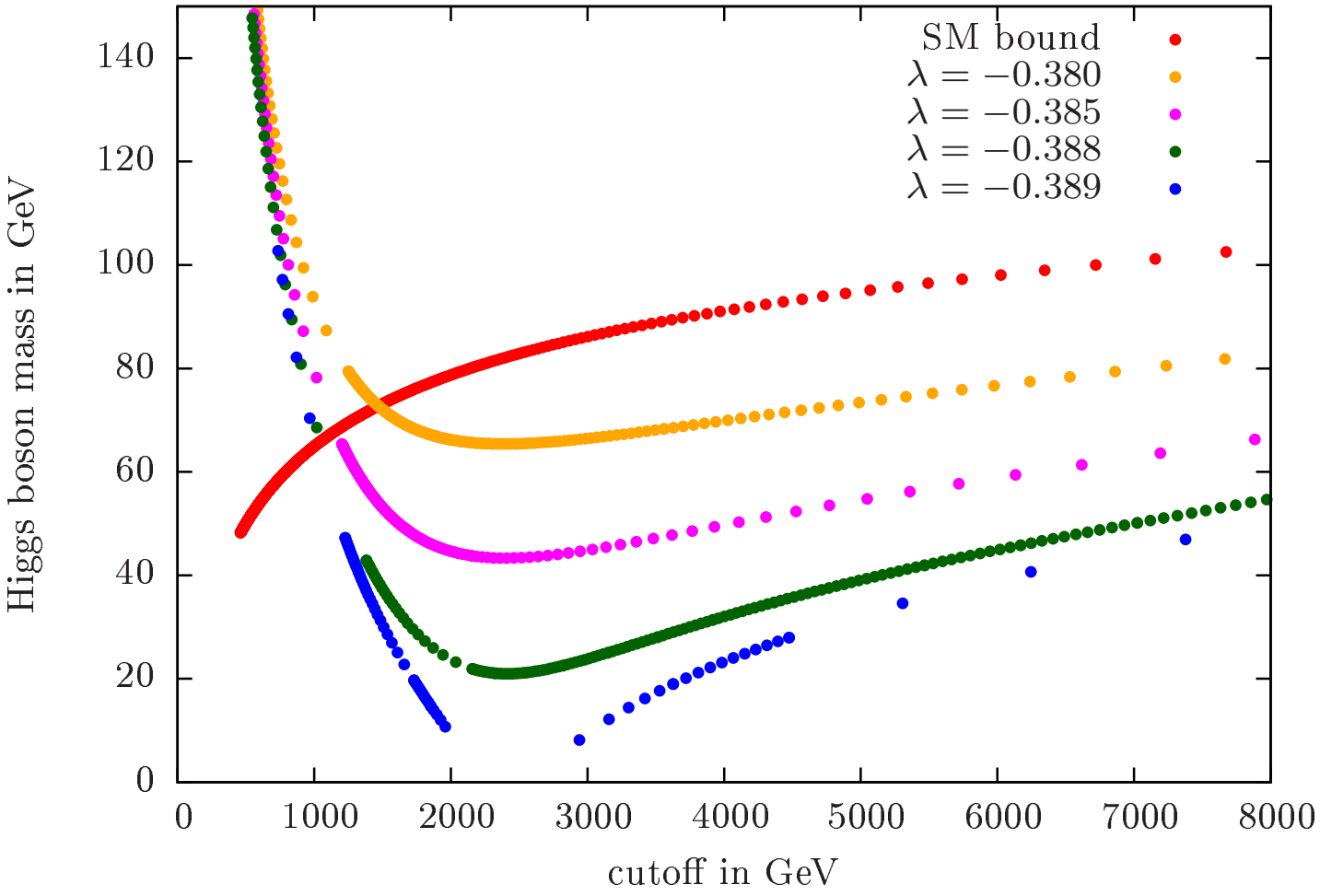


Mass vs cutoff from simulations, $\lambda_6 = 0.001$

Preliminary! (No infinite volume extrapolation. Only $24^3 \times 48$ data!)

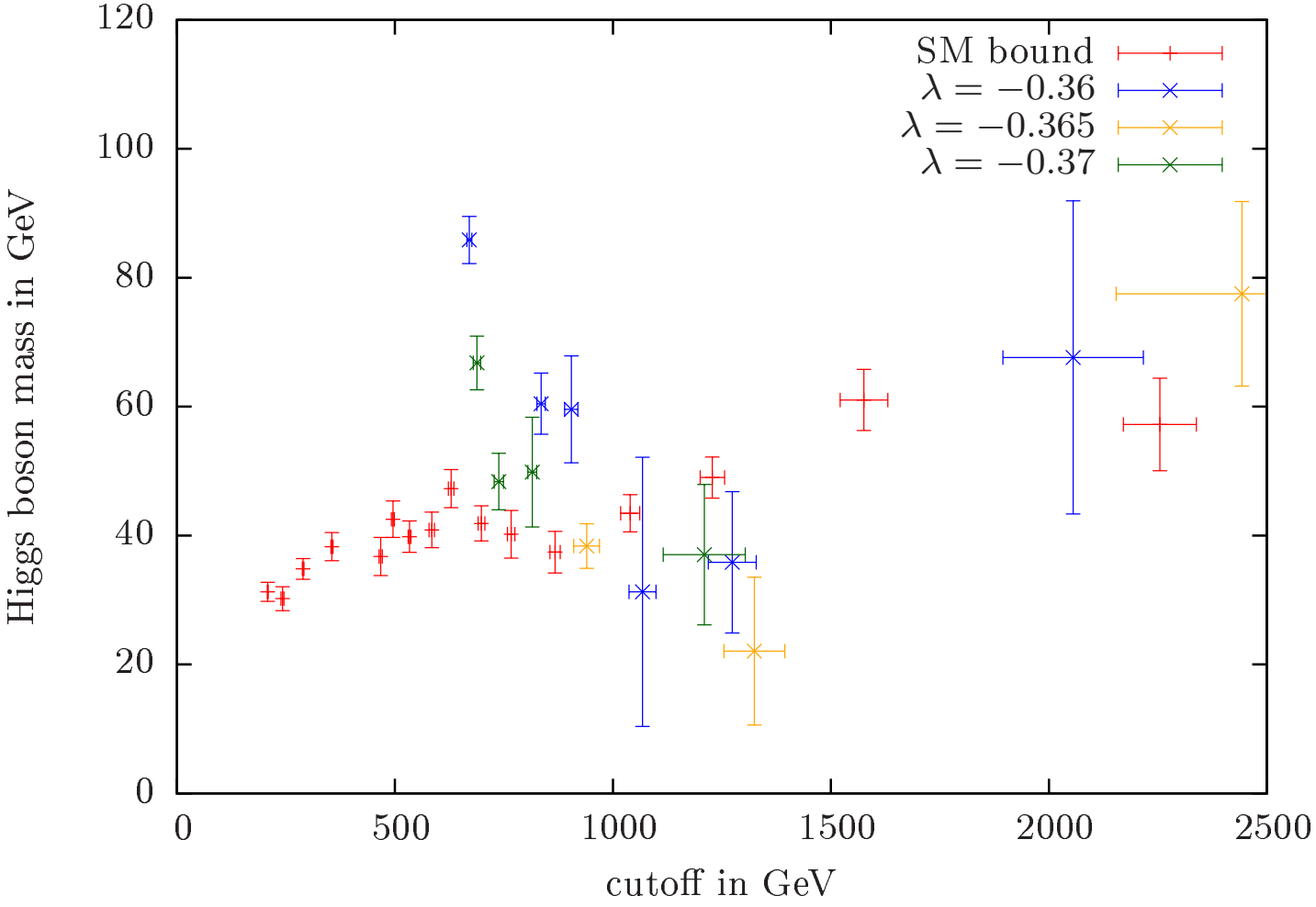


Mass vs. cutoff from CEP, $\lambda_6 = 0.1$



Mass vs cutoff from simulations, $\lambda_6 = 0.1$

Preliminary! (Infinite volume, but still limited statistics)



Conclusion + Outlook

- We mapped out the phase space of a HY-model including a $\lambda_6 \phi^6$ term
- Regions of first and second order transitions have been found
- A region in parameter space with a metastable vacuum was located
- λ_6 is compatible with the standard model Higgs boson mass
- $\lambda_6 = 0.001$ makes even a decrease of the Higgs boson mass possible

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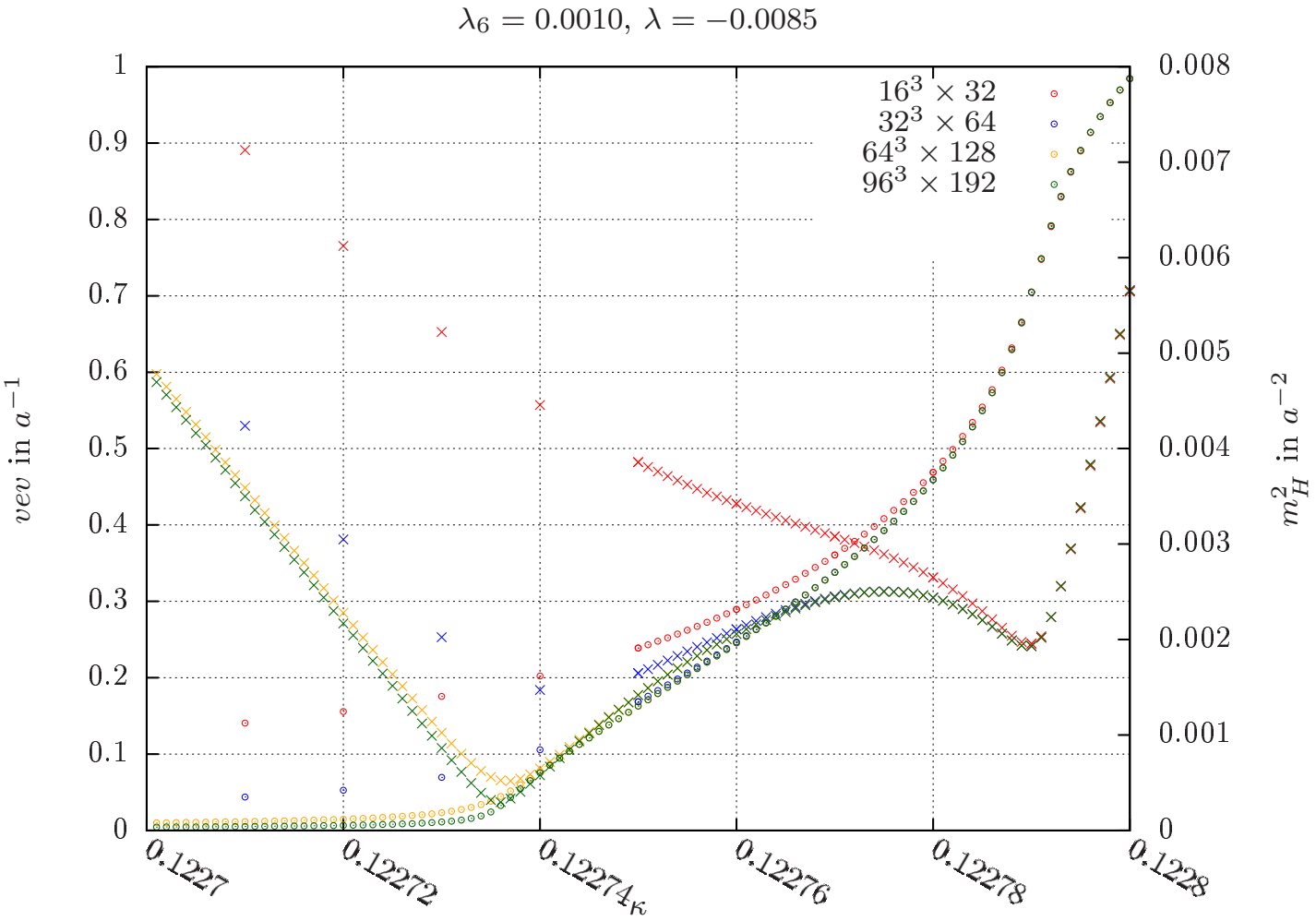
Outlook:

- Increase the range of λ_6 (to non-perturbative values)
- Establish the nature of the phase transitions numrically

BACKUP

CEP phase scan



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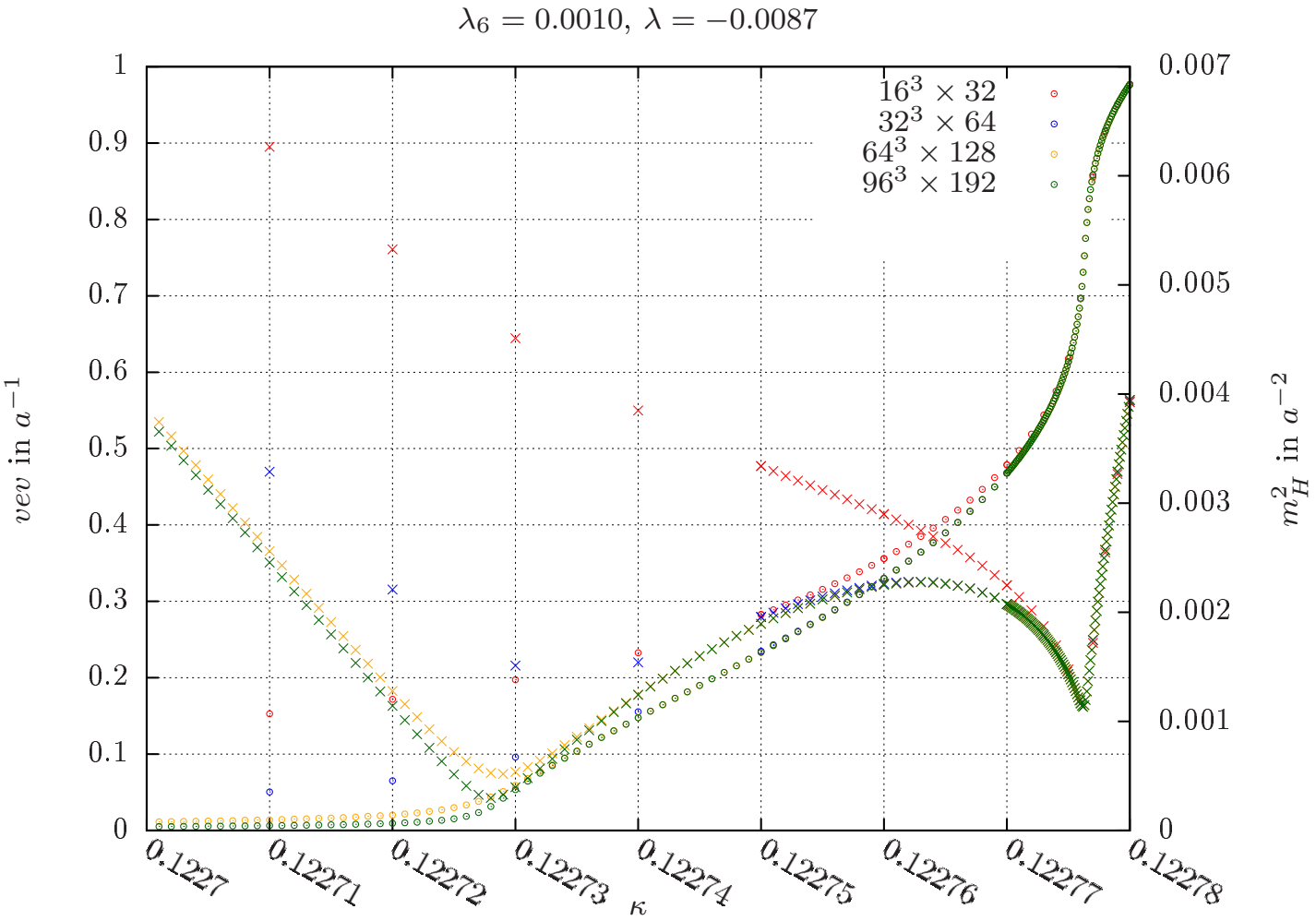

CEP phase scan