Long-distance contributions to flavour-changing processes

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Introduction

- Our satisfaction at the discovery of the Higgs Boson is (temporarily?) tempered by the absence of a discovery of *new physics* at the LHC.
- Precision Flavour physics is a key tool, complementary to the LHC, in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
  - If the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
  - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.
- It is surprising that no unambiguous inconsistencies have arisen up to now.

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- At this conference we have seen the continued, hugely impressive, improvement in precision for a wide range of quantities.
  - As a community we still have work to do to convince some of our HEP colleagues of the validity of the results:

    Question at EPS2013: Can we trust the lattice?

    CTS@EPS 1993 - $\hat{B}_K = 0.8(2)$,
    CTS@EPS 2013 - $\hat{B}_K = 0.766(10)$. FLAG2, arXiv:1310:8555
- Standard quantities include the spectrum and matrix elements of the form $\langle 0\,|\,O\,|\,h\rangle$ and $\langle h_2\,|\,O\,|\,h_1\rangle$, where the O are local composite operators and h, h_1, h_2 are hadrons.
  - We are seeing the range of O and h, h_1, h_2 extended.
  - We are seeing the extension to two-hadron states (including $K \to \pi\pi$).

    see e.g. talks by R.Briceno & T.Yamazaki.
- In this talk I will discuss 3 topics in which the matrix elements are of non-local operators involving long-distance effects:
  1. $\Delta m_K = m_{K_L} - m_{K_S}$. RBC-UKQCD
  2. Rare kaon decays. RBC-UKQCD
  3. Electromagnetic corrections to leptonic decays.

     N.Carrasco, V.Lubicz, G.Martinelli, CTS, F.Sanfillipo, N.Tantalo, C.Tarantino, M.Testa

1. The $K_L - K_S$ Mass Difference

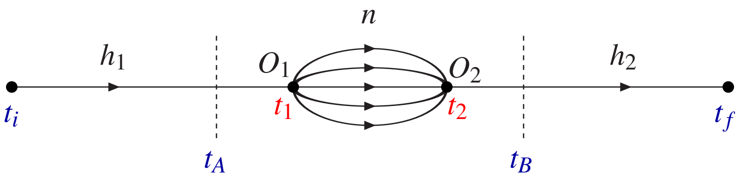
N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931
Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916
Z.Bai (RBC-UKQCD), Session 2G, Monday 17.50

$$\boxed{\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12}\,\mathrm{MeV}.}$$

- Historically led to the prediction of the energy scale of the charm quark.
  Mohapatra, Rao & Marshak (1968); GIM (1970); Gaillard & Lee (1974)
- Tiny quantity $\Rightarrow$ places strong constraints on BSM Physics.
- Within the standard model, Δm_K arises from K^0–$\bar{K}^0$ mixing at second order in the weak interactions:

$$\boxed{\Delta M_K = 2\mathscr{P} \sum_\alpha \frac{\langle \bar{K}^0 \,|\, H_W \,|\, \alpha\rangle \langle \alpha \,|\, H_W \,|\, K^0\rangle}{m_K - E_\alpha}\,,}$$

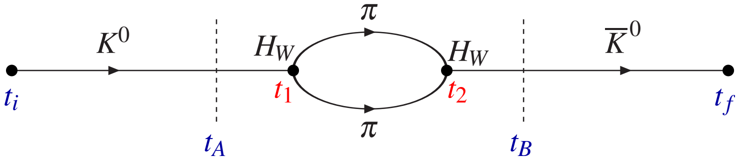
where the sum over $|\alpha\rangle$ includes an energy-momentum integral.



- How do you prepare the states $h_{1,2}$ in the generic integrated correlation function:

$$\int d^4x \int d^4y \,\langle h_2 \,|\, T\{O_1(x)\, O_2(y)\}\,|\, h_1\rangle\,,$$

  when the time of the operators is integrated?
- The practical solution is to integrate over a large subinterval in time $t_A \leq t_{x,y} \leq t_B$, but to create h_1 and to annihilate h_2 well outside of this region.
- This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time.
- This approach has been successfully implemented in the ΔM_K project as explained below.



- Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathscr{P}\sum_\alpha \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,\alpha\rangle\,\langle\alpha\,|\,\mathscr{H}_W\,|\,K^0\rangle}{m_K - E_\alpha} = 3.483(6)\times 10^{-12}\,\mathrm{MeV}.$$

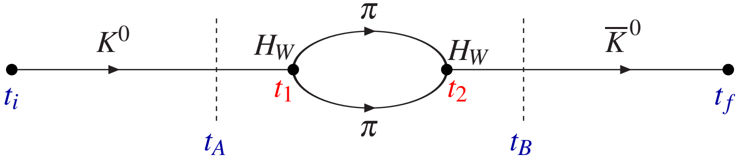
- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A,t_B;t_i,t_f) \;=\; |Z_K|^2 e^{-m_K(t_f-t_i)}\sum_n \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,n\rangle\,\langle n\,|\,\mathscr{H}_W\,|\,K^0\rangle}{(m_K-E_n)^2}\times \left\{e^{(M_K-E_n)T} - (m_K-E_n)T - 1\right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\rm FV} \equiv 2\sum_n \frac{\langle \bar{K}^0\,|\,\mathscr{H}_W\,|\,n\rangle\,\langle n\,|\,\mathscr{H}_W\,|\,K^0\rangle}{(m_K-E_n)}\,.$$

Exponentially growing exponentials



$$C_4(t_A,t_B;t_i,t_f) = |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, n\rangle \langle n \,|\, \mathscr{H}_W \,|\, K^0\rangle}{(m_K-E_n)^2} \times \left\{ e^{(M_K-E_n)T} - (m_K-E_n)T - 1 \right\}.$$

- The presence of terms which (potentially) grow exponentially in T is a generic feature of calculations of matrix elements of bilocal operators.
- There can be π^0 or vacuum intermediate states.
  - The corresponding growing exponentials can be eliminated by adding $c_S\,(\bar{s}d) + c_P\,(\bar{s}\gamma^5 d)$ to H_W, with coefficients c_S and c_P chosen such that $\langle \pi^0 \,|\, H_W \,|\, K\rangle$ and $\langle 0 \,|\, H_W \,|\, K\rangle$ are both zero.
- There are two-pion contributions with $E_{\pi\pi} < m_K$. (Number of such states grows as $L \to \infty$, as in the calculation of $K \to \pi\pi$ decay amplitudes.)

Finite-Volume Corrections

- For s-wave two-pion states, Lüscher's quantization condition is $h(E,L)\pi \equiv \phi(q)+\delta(k) = n\pi$, where $q = kL/2\pi$, ϕ is a kinematical function and δ is the physical s-wave $\pi\pi$ phase shift for the appropriate isospin state.

  M.Lüscher, NPB 354 (1991) 531

- The relation between the physical $K\to\pi\pi$ amplitude A and the finite-volume matrix element M L.Lellouch and M.Lüscher, hep-lat/0003023

$$|A|^2 = 8\pi V^2 \left(\frac{m_K}{k}\right)^3 \left\{k\delta'(k)+q\phi'(q)\right\} |M|^2 .$$

  - In addition to simple factors related to the normalization of states, the LL factor accounts for the non-exponential FV corrections.

- The evaluation of the non-exponential finite-volume corrections in the calculation of Δm_K requires an extension of the LL formalism.

  1. At Lattice 2010, N.Christ, using degenerate perturbation theory, presented the result for the case when the volume is such that there is a state n_0 with $E_{n_0} = m_K$. N.H.Christ, arXiv:1012.6034
  2. At Lattice 2013, I presented the result for the general (s-wave) rescattering case. N.H.Christ, G.Martinelli & CTS, arXiv:1401.1362

     N.H.Christ, X.Feng, G.Martinelli & CTS, in preparation

Finite-Volume Corrections (cont.)

- The general formula can be written: N.H.Christ, G.Martinelli & CTS, arXiv:1401.1362
N.H.Christ, X.Feng, G.Martinelli & CTS, in preparation

$$\Delta m_K = \Delta m_K^{\mathrm{FV}} - 2\pi\, {}_V\langle \bar{K}^0 \,|\, H \,|\, n_0\rangle_V\, {}_V\langle n_0 \,|\, H \,|\, K^0\rangle_V \left[\cot \pi h \, \frac{dh}{dE}\right]_{m_K},$$

where $h(E,L)\pi \equiv \phi(q) + \delta(k)$.

- This formula reproduces the result for the special case when the volume is such that there is a two-pion state with energy $= m_K$. N.H.Christ, arXiv:1012.6034
- Increasing the volumes keeping $h = n/2$ and thus avoiding the power corrections is an intriguing possibility.

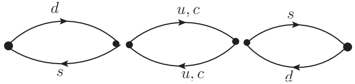
- The $\Delta S = 1$ effective Weak Hamiltonian takes the form:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

  where the $\{Q_i^{qq'}\}_{i=1,2}$ are current-current operators, defined as:

$$\begin{aligned} Q_1^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j) \\ Q_2^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)\,. \end{aligned}$$

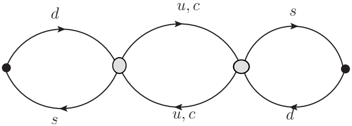
- As the two H_W approach each other, we have the potential of new ultraviolet divergences.
  - Taking the u-quark component of the operators $\Rightarrow$ a quadratic divergence.



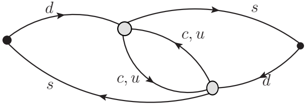
  - GIM mechanism & $V-A$ nature of the currents $\Rightarrow$ elimination of both quadratic and logarithmic divergences.
  - Short distance contributions come from distances of $O(1/m_c)$.

Evaluating Δm_K

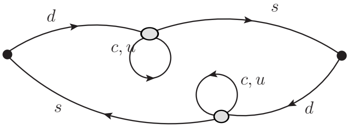
- There are four types of diagram to be evaluated:



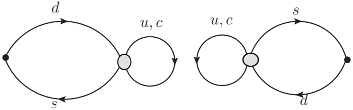
Type 1



Type 2



Type 3



Type 4

- In our first exploratory study on 16^3 ensembles with $m_\pi = 420$ MeV, $(1/a = 1.73$ GeV) we only evaluated Type 1 and Type 2 graphs.

  N.Christ, T.Izubuchi, CTS, A.Soni & J.Yu, arXiv:1212.5931

- In our more recent study, we evaluated all the diagrams.

  Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu, arXiv:1406.0916

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu, arXiv:1406.0916

- We have performed a full calculation of Δm_K, using 800 gauge configurations (separated by 10 time units) on a $24^3\times 64\times 16$ lattice, with DWF and the Iwasaki gauge action, $m_\pi=330$ MeV, $m_K=575$ MeV, $m_c^{\overline{\rm MS}}(2\,\text{GeV})=949$ MeV, $1/a=1.729(28)$ GeV and $am_{\rm res}=0.00308(4)$.

For details of the ensembles see arXiv:0804.0473 and 1011.0892

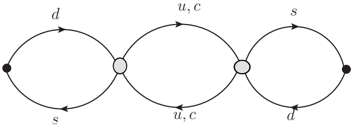
- At these unphysical parameters we find

$$\Delta m_K = 3.19(41)(96)\times 10^{-12}\,\text{MeV}\,,$$

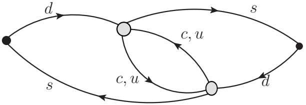
to be compared to the physical value 3.483(6)$\times 10^{-12}$ MeV.

  - Agreement with physical value may well be fortuitous, but it is nevertheless reassuring to obtain results of the correct order.
  - Systematic error dominated by discretization effects related to the charm quark mass, which we estimate at 30%.
  - Here $m_K<2m_\pi$ and so we do not have exponentially growing two-pion terms.

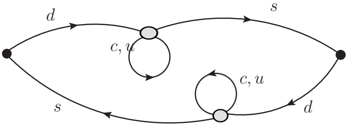
Complete calculation of Δm_K (cont.)



Type 1



Type 2



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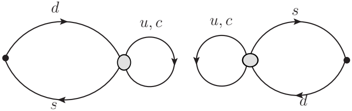
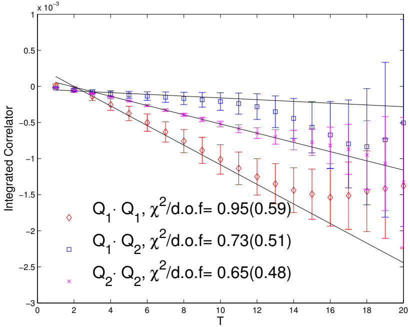
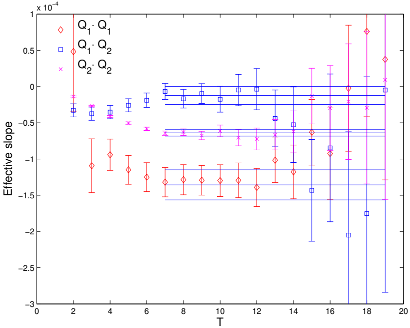


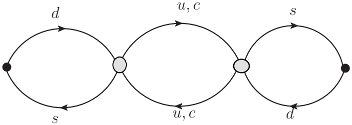
Type 4

- Coulomb-gauge fixed wall sources used for the kaons.
- Point source propagators calculated for each of the 64 time slices (Types 1&2).
- Random-source propagators on each time slice (Types 3&4).

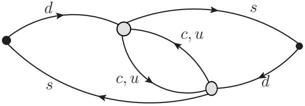

x 10^−3
Integrated Correlator
T
Q1· Q1, χ2/d.o.f= 0.95(0.59)
Q1· Q2, χ2/d.o.f= 0.73(0.51)
Q2· Q2, χ2/d.o.f= 0.65(0.48)


x 10^−4
Effective slope
T
Q1· Q1
Q1· Q2
Q2· Q2

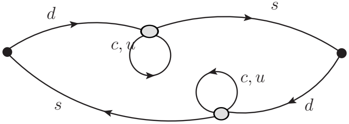
Violation of the OZI rule



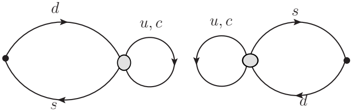
Type 1



Type 2



Type 3



Type 4

- One possible surprise(?) from this calculation is the large size of the disconnected diagrams of type 4.

Diagrams	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
Type 1,2	1.479(79)	1.567(36)	3.677(52)	6.723(90)
All	0.68(10)	-0.18(18)	2.69(19)	3.19(41)

- Type 3 contributions are small.

Work in progress

- At this conference Ziyuan Bai presented preliminary results from the RBC-UKQCD collaboration study on the $32^3 \times 64$ DWF&DSDR coarse lattice which had been used in the first computation of $K \to (\pi\pi)_{I=2}$ decay amplitudes.

m_π	m_K	m_c	a^{-1}	L	no. of configs.
171 MeV	492 MeV	592/750 MeV	1.37 GeV	4.6 fm	212

  - $m_K > 2m_\pi \Rightarrow$ allows us to study the effect of the two-pion intermediate state.

- We use the freedom to perform chiral rotations, to transform

$$H_W \to H'_W = H_W + c_S\,(\bar{s}d) + c_P\,(\bar{s}\gamma^5 d)$$

with c_S and c_P chosen so that

$$\langle 0\,|\,H'_W\,|\,K\rangle = 0 \qquad \text{and} \qquad \langle \eta\,|\,H'_W\,|\,K\rangle = 0\,.$$

  - Even though $m_\eta > m_K$, we find that the large errors associated with the $\eta \Rightarrow$ it is difficult to control the exponential suppression. We therefore find that it is more effective to eliminate the η (rather than the pion).

Preliminary results

Z.Bai - This conference

m_c	Δm_K
750 MeV	$(4.6 \pm 1.3) \times 10^{-12}$ MeV
592 MeV	$(3.8 \pm 1.7) \times 10^{-12}$ MeV

- Only statistical errors are shown.
- The contributions from $\pi\pi$ intermediate states is small ($\Delta m_K(\pi\pi)_{I=0} = -0.133(99) \times 10^{-12}$ MeV, $\Delta m_K(\pi\pi)_{I=2} = -6.54(25) \times 10^{-16}$ MeV).
- For $I = 0$ the FV effects are $O(20\%)$ of the 4% contribution (i.e. $\leq$ 1%).
- Very promising indeed and in near-future calculations we will perform computations at physical kinematics and also on ensembles with unquenched charm quarks.
- For prospects for the calculation of ε_K see:

N.H.Christ, T.Izubuchi, CTS, A.Soni and J.Yu, arXiv:1402.2577,

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni and J.Yu, arXiv:1406.0916.

See parallel talk by Xu Feng

Some comments from F.Mescia, C.Smith, S.Trine hep-ph/0606081:

- Rare kaon decays which are dominated by short-distance FCNC processes, $K \to \pi \nu \bar{\nu}$ in particular, provide a potentially valuable window on new physics at high-energy scales.
- The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are also considered promising because the long-distance effects are reasonably under control using ChPT.
  - They are sensitive to different combinations of short-distance FCNC effects and hence in principle provide additional discrimination to the neutrino modes.
  - A challenge for the lattice community is therefore either to calculate the long-distance effects reliably or at least to determine the Low Energy Constants of ChPT.
- We, N.Christ, X.Feng, A.Portelli, CTS and RBC-UKQCD, are attempting to meet this challenge.

There are three main contributions to the amplitude:

1. Short distance contributions: F.Mescia, C,Smith, S.Trine hep-ph/0606081

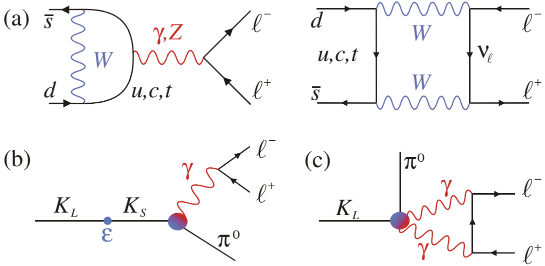
$$H_{\rm eff} = -\frac{G_F \alpha}{\sqrt{2}} V_{ts}^* V_{td} \{ y_{7V} (\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu \ell) + y_{7A} (\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\} + \text{h.c.}$$

   - Direct CP-violating contribution.
   - In BSM theories other effective interactions are possible.

2. Long-distance indirect CP-violating contribution

$$A_{ICPV}(K_L \to \pi^0 \ell^+ \ell^-) = \varepsilon A(K_1 \to \pi^0 \ell^+ \ell^-)\,.$$

3. The two-photon CP-conserving contribution $K_L \to \pi^0 (\gamma^* \gamma^* \to \ell^+ \ell^-)$.



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- The current phenomenological status for the SM predictions is nicely summarised by: V.Cirigliano et al., arXiv1107.6001

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 \right\}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} = 10^{-12} \times \left\{ 3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}} \right)^2 \right\}$$

  - $\lambda_t = V_{td} V_{ts}^*$ and Im $\lambda_t \simeq 1.35 \times 10^{-4}$.
  - $|a_S|$, the amplitude for $K_S \to \pi^0 \ell^+ \ell^-$ at $q^2 = 0$ as defined below, is expected to be $O(1)$ but the sign of a_S is unknown. $|a_S| = 1.06^{+0.26}_{-0.21}$.
  - For $\ell = e$ the two-photon contribution is negligible.
  - Taking the positive sign (?) the prediction is

$$\begin{aligned} \mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} &= (3.1 \pm 0.9) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} &= (1.4 \pm 0.5) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPC}} &= (5.2 \pm 1.6) \times 10^{-12}. \end{aligned}$$

  - The current experimental limits (KTeV) are:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \quad \text{and} \quad \mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}.$$

G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026

- We now turn to the CPC decays $K_S \to \pi^0\ell^+\ell^-$ and $K^+ \to \pi^+\ell^+\ell^-$ and consider

$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\,\mathrm{T}\{J^\mu_{\mathrm{em}}(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,,$$

where Q_i is an operator from the effective Hamiltonian.

- EM gauge invariance implies that

$$T_i^\mu = \frac{\omega_i(q^2)}{(4\pi)^2}\,\Big\{q^2(p+k)^\mu - (m_K^2 - m_\pi^2)\,q^\mu\Big\}\,.$$

- Within ChPT the Low energy constants a_+ and a_S are defined by

$$a = \frac{1}{\sqrt{2}}\,V_{us}^*V_{ud}\left\{C_1\,\omega_1(0) + C_2\,\omega_2(0) + \frac{2N}{\sin^2\theta_W}f_+(0)C_{7V}\right\}$$

where $Q_{1,2}$ are the two current-current GIM subtracted operators and the C_i are the Wilson coefficients. (C_{7V} is proportional to y_{7V} above).

G.D'Ambrosio, G.Ecker, G.Isidori and J.Portoles, hep-ph/9808289

- Phenomenological values: $a_+ = -0.578 \pm 0.016$ and $|a_S| = 1.06^{+0.26}_{-0.21}$.

Can we do better in lattice simulations?

- The generic non-local matrix elements which we wish to evaluate is

$$
\begin{aligned}
X &\equiv \int_{-\infty}^{\infty} dt_x\, d^3x\, \langle \pi(p)\,|\,\mathrm{T}\,[J(0)\,H(x)]\,|K\rangle \\
&= i\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{m_K - E_n + i\varepsilon} - i\sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi + i\varepsilon}\,,
\end{aligned}
$$

  - $\{|n\rangle\}$ and $\{|n_s\rangle\}$ represent complete sets of non-strange and strange sets.

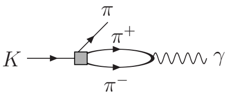
- In Euclidean space we envisage calculating correlation functions of the form

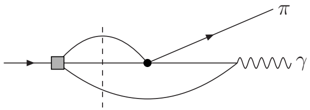
$$
C \equiv \int_{-T_a}^{T_b} dt_x\, \langle \phi_\pi(\vec{p},t_\pi)\,\mathrm{T}\,[J(0)\,H(t_x)]\,\phi_K^\dagger(t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-E_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,,
$$

where

$$
\begin{aligned}
X_{E_-} &= -\sum_n \frac{\langle \pi(p)\,|\,J(0)\,|n\rangle\,\langle n\,|H(0)\,|\,K\rangle}{E_K - E_n}\left(1 - e^{(E_K - E_n)T_a}\right) \quad \text{and} \\
X_{E_+} &= \sum_{n_s} \frac{\langle \pi(p)\,|\,H(0)\,|n_s\rangle\,\langle n_s\,|J(0)\,|\,K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s} - E_\pi)T_b}\right).
\end{aligned}
$$

- We can remove the single pion intermediate state.
- Which intermediate states contribute?
  - Are there any states below M_K?
  - We can control q^2 and stay below the two-pion threshold.





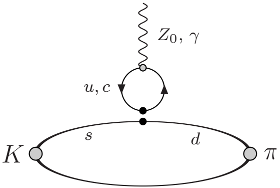
  - Do the symmetries protect us completely from two-pion intermediate states at low q^2?
  - Are the contributions from three-pion intermediate states negligible?
  - Answers to the above questions will affect what the finite-volume corrections are?
  - The ChPT-based phemomenology community neglect such possibilities as they are higher order in the chiral expansion.

All to be investigated further!

- It looks as though the FV corrections are much simpler than for ΔM_K and may be exponentially small?

$$T_i^{\mu} = \int d^4x\, e^{-iq\cdot x}\, \langle \pi(p)\,|\, \mathrm{T}\{J^{\mu}(x)\, Q_i(0)\,\}\,|\, K(k)\rangle\,,$$

- Each of the two local Q_i operators can be normalized in the standard way and for J we imagine taking the conserved vector current.
- We must treat additional divergences as $x \to 0$.

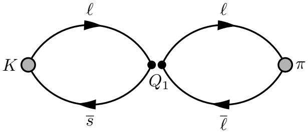


- Quadratic divergence is absent by gauge invariance $\Rightarrow$ Logarithmic divergence.
  - Checked explicitly for Wilson and Clover at one-loop order.

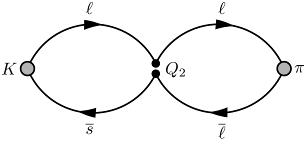
    G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - Absence of power divergences does not require GIM.
  - Logarithmic divergence cancelled by GIM.
  - For DWF the same applies for the axial current.
    - To be investigated further!

Many diagrams to evaluate!

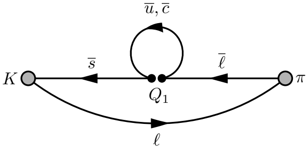
- For example for K^+ decays we need to evaluate the diagrams obtained by inserting the current at all possible locations in the three point function (and adding the disconnected diagrams):



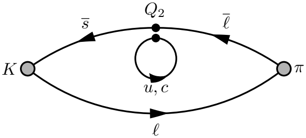
W



C



S



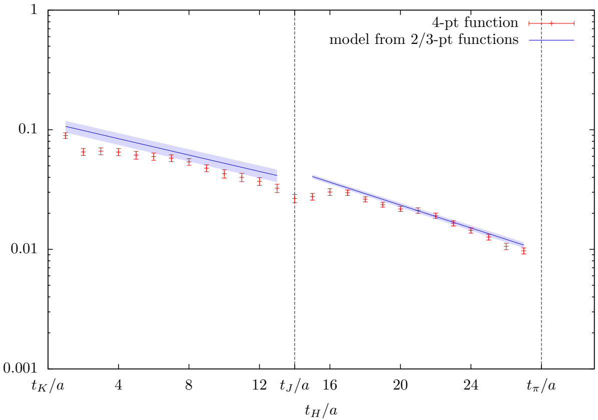
E

- W=Wing, C=Connected, S=Saucer, E=Eye.
- For K_S decays there is an additional topology with a gluonic intermediate state.
- For the first exploratory study, we have only considered the W and C diagrams.

Exploratory numerical study

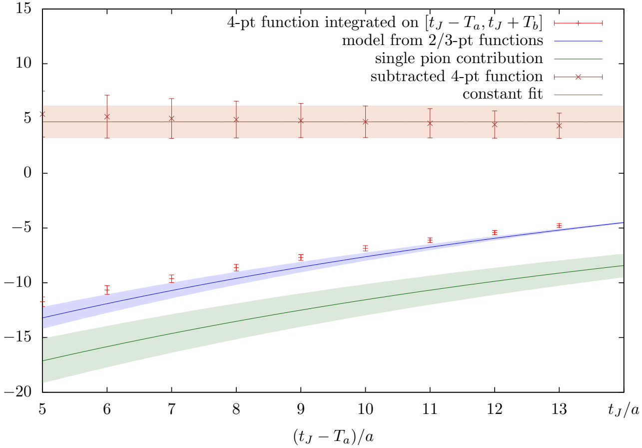
- The numerical study is performed on the $24^3 \times 64$ DWF+Iwasaki RBC-UKQCD ensembles with $am_l = 0.01$ ($m_\pi \simeq 420$ MeV), $am_s = 0.04$, $a^{-1} \simeq 1.73$ fm.
- 127 configurations were used with $\vec{k} = (1,0,0)\,\frac{2\pi}{L}$ and $\vec{p} = 0$.
- The calculation is performed using the conserved vector current (5-dimensional), J^0.

Unintegrated 4-point Correlation Function



- $t_K = 0$, $t_\pi = 28$ and $t_J = 14$. x-coordinate is t_H.
- Blue band - Result from 2&3 point-functions assuming ground state contributions between t_J and t_H. (No fit here.)

Integrated 4-point Correlation Function



- In this plot $T_b = 9$, so that the integral is from the x-coordinate to 23.
- It appears that the subtraction of the exponentially growing term can be performed and a constant result obtained.
- These are just the beginnings - much work still to be done.

3. Electromagnetic corrections to weak matrix elements

N.Carrasco, V.Lubicz, G.Martinelli, CTS, F.Sanfillipo, N.Tantalo, C.Tarantino, M.Testa (in preparation)

- For a review of electromagnetic mass-splittings see the talk by A.Portelli at this conference.
- The evaluation of (some) weak matrix elements are now being quoted with $O(1\%)$ precision e.g. FLAG Collaboration, arXiv:1310.8555

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}
130.2(1.4)	156.3(0.8)	209.2(3.3)	248.6(2.7)	190.5(4.2)	227.7(4.5)

(results given in MeV)

- We therefore need to start considering electromagnetic (and other isospin breaking) effects if we are to use these results to extract CKM matrix elements at a similar precision.
- For illustration, I consider f_π but the discussion is general. I do not use ChPT. For a ChPT based discussion of f_π, see J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479
- At $O(\alpha^0)$

$$\Gamma(\pi^+\to\ell^+\nu_\ell)=\frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\,m_\pi\,m_\ell^2\left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2.$$

Infrared Divergences

- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+ \nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+ \nu_\ell) + \Gamma(\pi^+ \to \ell^+ \nu_\ell \gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

  where the suffix denotes the number of photons in the final state.

- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

  F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for our community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.

- At this stage we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - A cut-off Δ of $O(10\,\mathrm{MeV})$ appears to be appropriate both experimentally and theoretically.
  - (In the future, as techniques and resources improve, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.)
- We now write

$$\boxed{\Gamma_0 + \Gamma_1(\Delta) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\mathrm{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta))\,.}$$

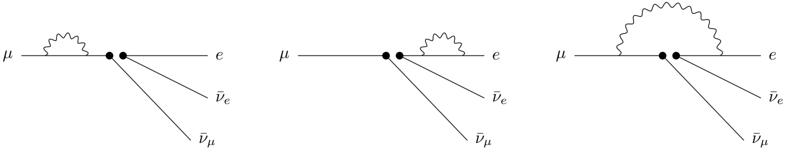
  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated nonperturbatively and Γ_0^{pt} in perturbation theory. The subtraction in the first term is performed for each momentum and then the sum over momenta is performed (see below).

1. The result for the width is expressed in terms of G_F, the Fermi constant $(G_F = 1.16632(2) \times 10^{-5}\,\mathrm{GeV}^{-2})$. This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right].$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

- This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:



together with the diagrams with a real photon.

- The diagrams are evaluated in the W-regularisation in which the photon propagator is modified by: A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}. \qquad \left(\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} + \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}\right)$$

2. Most (but not all) of the EW corrections which are absorbed in G_F are common to other processes (including pion decay) $\Rightarrow$ factor in the amplitude of $(1+3\alpha/4\pi(1+2\bar{Q})\log M_Z/M_W)$, where $\bar{Q}=\frac{1}{2}(Q_u+Q_d)=1/6$.

   A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

3. We therefore need to calculate the pion-decay diagrams in the effective theory (with $H_{\text{eff}}\propto(\bar{d}_L\gamma^\mu u_L)(\bar{\nu}_{\ell,L}\gamma_\mu \ell_L)$) in the W-regularization. These can be related to the lattice theory by perturbation theory, e.g. for Wilson fermions:

$$O_{LL}^{W-\text{reg}}=\left(1+\frac{\alpha}{4\pi}\left(2\log a^2M_W^2-15.539\right)+O(\alpha\alpha_s)\right)O_{LL}^{\text{bare}}\,.$$

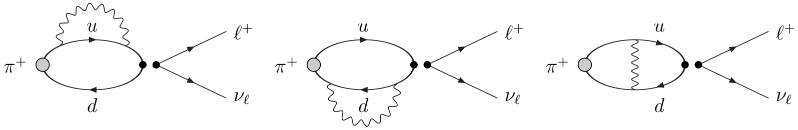
4. We now return to the master formula:

$$\boxed{\Gamma_0+\Gamma_1(\Delta)=\lim_{V\to\infty}(\Gamma_0-\Gamma_0^{\text{pt}})+\lim_{V\to\infty}(\Gamma_0^{\text{pt}}+\Gamma_1(\Delta))\,.}$$

   - The term which is added and subtracted is not unique, but we require that both terms are free of ir divergences and independent of the ir regulator.
   - Kinoshita performed the calculation for a pointlike pion, (i) integrating over all phase space and (ii) imposing a cut-off on the charged-lepton energy.

     T.Kinoshita, PRL 2 (1959) 477
   - We have reproduced these results and extended them to a cut-off on the photon energy.

5 Consider now the evaluation of the first term in the master formula.



(a) (b) (c)

- The correlation function for this set of diagrams is of the form:

$$C_1(t) = \frac{1}{2}\int d^3\vec{x}\, d^4x_1\, d^4x_2\; \langle 0|T\{J_W^\nu(0)j^\mu(x_1)j_\mu(x_2)\phi^\dagger(\vec{x},t)\}\,|0\rangle\;\Delta(x_1,x_2)\,,$$

where $j_\mu(x) = \sum_f Q_f \bar{f}(x)\gamma_\mu f(x)$, J_W is the weak current, ϕ is an interpolating operator for the pion and Δ is the photon propagator.

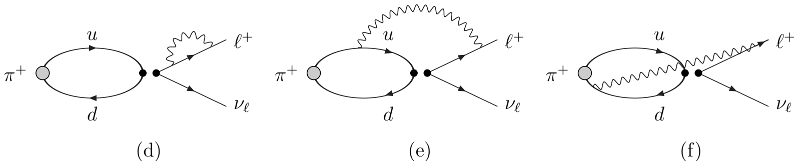
- Combining C_1 with the lowest order correlator:

$$C_0(t) + C_1(t) \simeq \frac{e^{-m_\pi t}}{2m_\pi} Z^\phi\,\langle 0|J_W^\nu(0)\,|\pi^+\rangle\,,$$

where now $O(\alpha)$ terms are included.

- $e^{-m_\pi t} \simeq e^{-m_\pi^0 t}(1-\delta m_\pi t)$ and Z^ϕ is obtained from the two-point function.

The procedure (cont.)



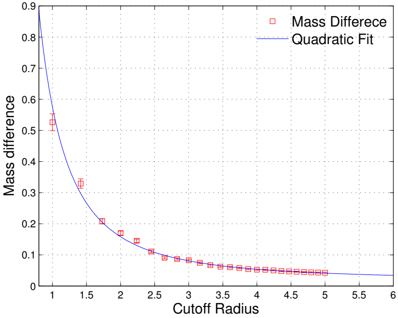
- Diagrams (e) and (f) are not simply generalisations of the evaluation of f_π. The leptonic part is treated using perturbative propagators.
  (There are also disconnected diagrams to be evaluated.)
- We have to be able to isolate the finite-volume ground state (pion).
- The Minkowski $\leftrightarrow$ Euclidean continuation can be performed (the time integrations are convergent).
- Finite volume effects, expected to be $O(1/(L\Lambda_{\rm QCD})^n)$, being investigated.
- The next step will be to start implementing this procedure.
- As we learn how to do such calculations it will be useful to consider simpler quantities such as $\Gamma(\pi\to\mu\nu_\mu(\gamma))/\Gamma(\pi\to e\nu_e(\gamma))$.

- We warmly thank Norman, Bob and Peter, the LOC and all their colleagues at Columbia and Brookhaven who have helped to make Lattice 2014 such a stimulating, enjoyable and beautifully organised conference.

Photos courtesy of H.Wittig.

- In this talk I have described the current status of three projects involving long-distance effects:
  1. $\Delta m_K = m_{K_L} - m_{K_S}$.
  2. Rare kaon decays.
  3. Electromagnetic corrections to leptonic decays.
- The early results and indications are very promising indeed, but much more work needs to be done.

- As an example consider the behaviour of the integrated $Q_1 - Q_1$ correlation function without GIM subtraction but with an artificial cut-off, $R = \sqrt{\{(t_2 - t_1)^2 + (\vec{x}_2 - \vec{x}_1)^2\}}$ on the coordinates of the two Q_1 insertions.



N.Christ, T.Izubuchi, CTS, A.Soni & J.Yu, arXiv:1212.5931

- The plot exhibits the quadratic divergence as the two operators come together.
- The quadratic divergence is cancelled by the GIM mechanism.