Topology density correlator on dynamical domain-wall ensembles with nearly frozen topological charge

OSAKA UNIVERSITY
Live Locally, Grow Globally

Hidenori Fukaya (Osaka Univ.)
[for JLQCD collaboration]

1. Introduction

Long auto-correlation of topology

$$\lim_{a \to 0} S_{\mathrm{gauge}} \to \infty$$

at topology boundaries.

HMC (semi-continuous) updates cannot change topology in the continuum limit.

Long auto-correlation of topology

Related talks & posters :

- Mueller-Preussker, plenary (Mon)
- McGlynn, parallel 1F.
- Namekawa, poster
- Gambhir, poster
- Garcia Ramos, parallel 5D
- Cichy, parallel 5D
- Gerber, this session
- Dromard, this session

Topology fixing with overlap quarks

JLQCD (+TWQCD) collaboration
2006-2012

We have simulated QCD with overlap quarks fixing topology to Q=0 (and to Q=1 for one parameter set) to avoid discontinuity of the quark determinant.

New project launched.

Simulations on bigger & finer lattices started.

Computers @KEK: SR11000 (2 TFLOPS) + BG/L (57 TFLOPS)
→ SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)

Lattice cut-off : 1.8 GeV → 2.4, 3.6, 4.2 GeV

Lattice size : 16^3 x48 → 32^3 x64, 48^3x96, 64^3x128

(Physical size : 1.8 fm → 2.6 fm ~ 4 fm)

Fermion action : overlap fermion→ (improved) DomainWall

Hitachi SR16000

IBM Blue Gene/Q

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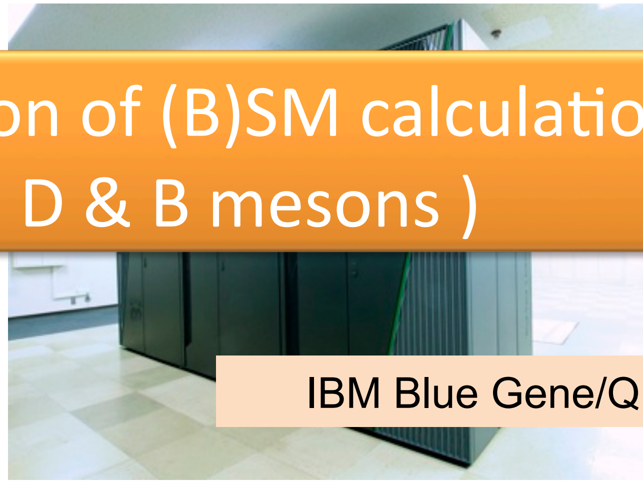
Our goal = 1% precision of (B)SM calculations (in particular, D & B mesons)

Hitachi SR16000



IBM Blue Gene/Q

New project launched.

Domain-wall fermion for sea quarks

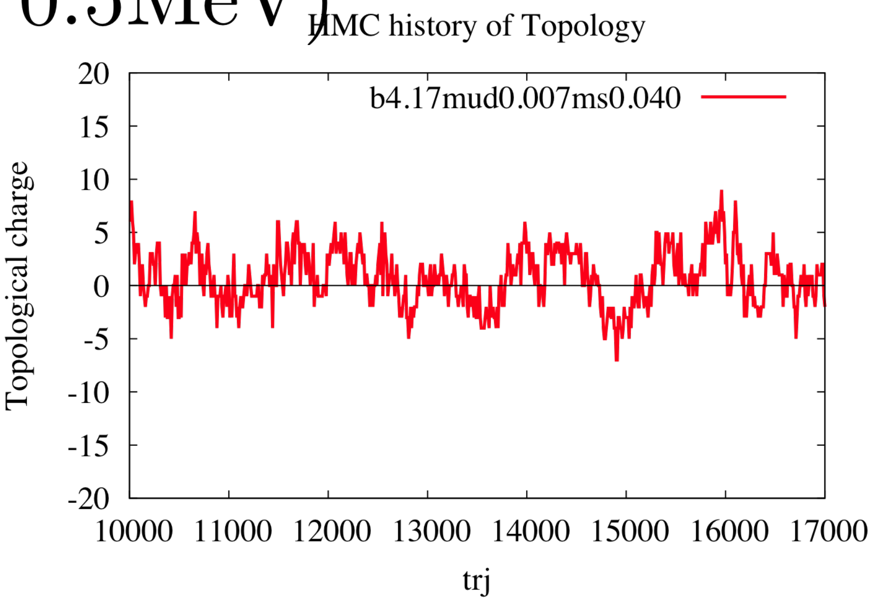
Kaplan 1992; Shamir 1994; Borici 1997; Chiu 1998; Brower et al. 2001

Improved (using scaled Shamir Kernel)

domain-wall + 3 steps of stout smearing

-> $m_{\mathrm{res}}a \lesssim 0.001$ ($\lesssim 0.5\mathrm{MeV}$)

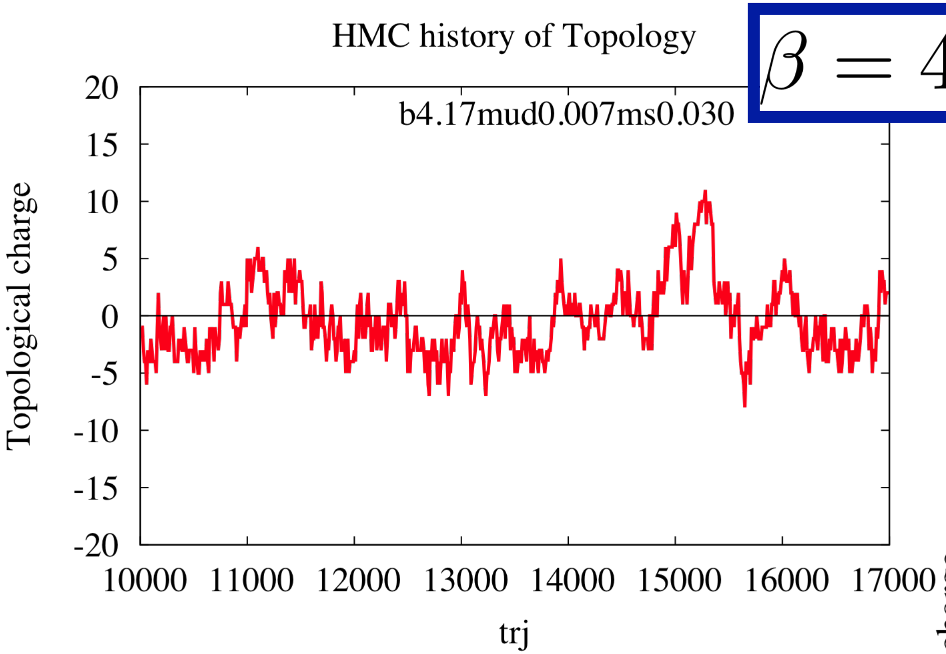
at Ls=8-12.



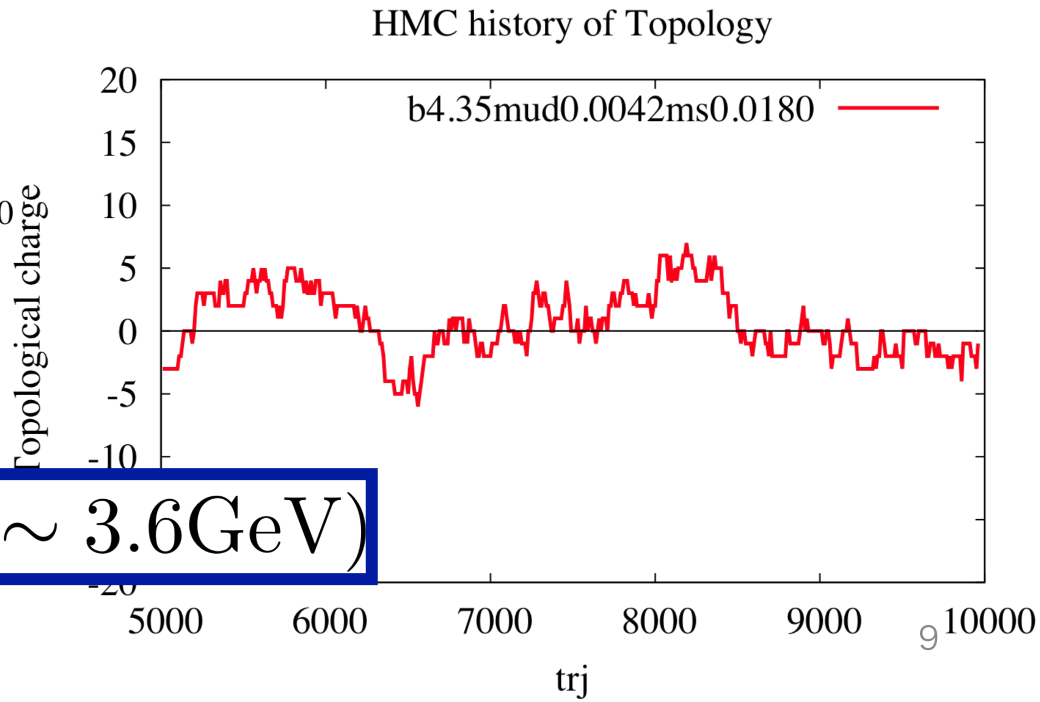
Bonus :

Topology tunnelings are active.

Topology is frozen (again).



$$\beta = 4.17 \ (a^{-1} \sim 2.4\mathrm{GeV})$$

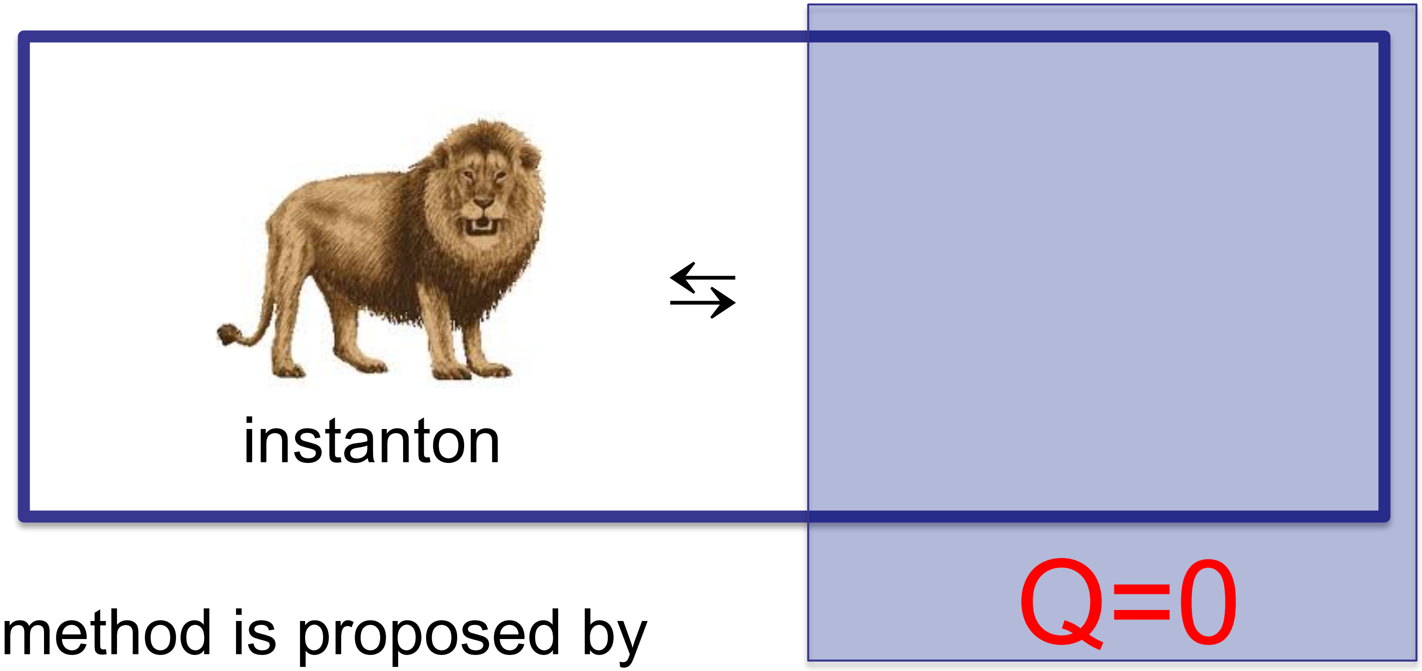


$$\beta = 4.35 \ (a^{-1} \sim 3.6\mathrm{GeV})$$

2. Two ideas

How about sub-volumes ?

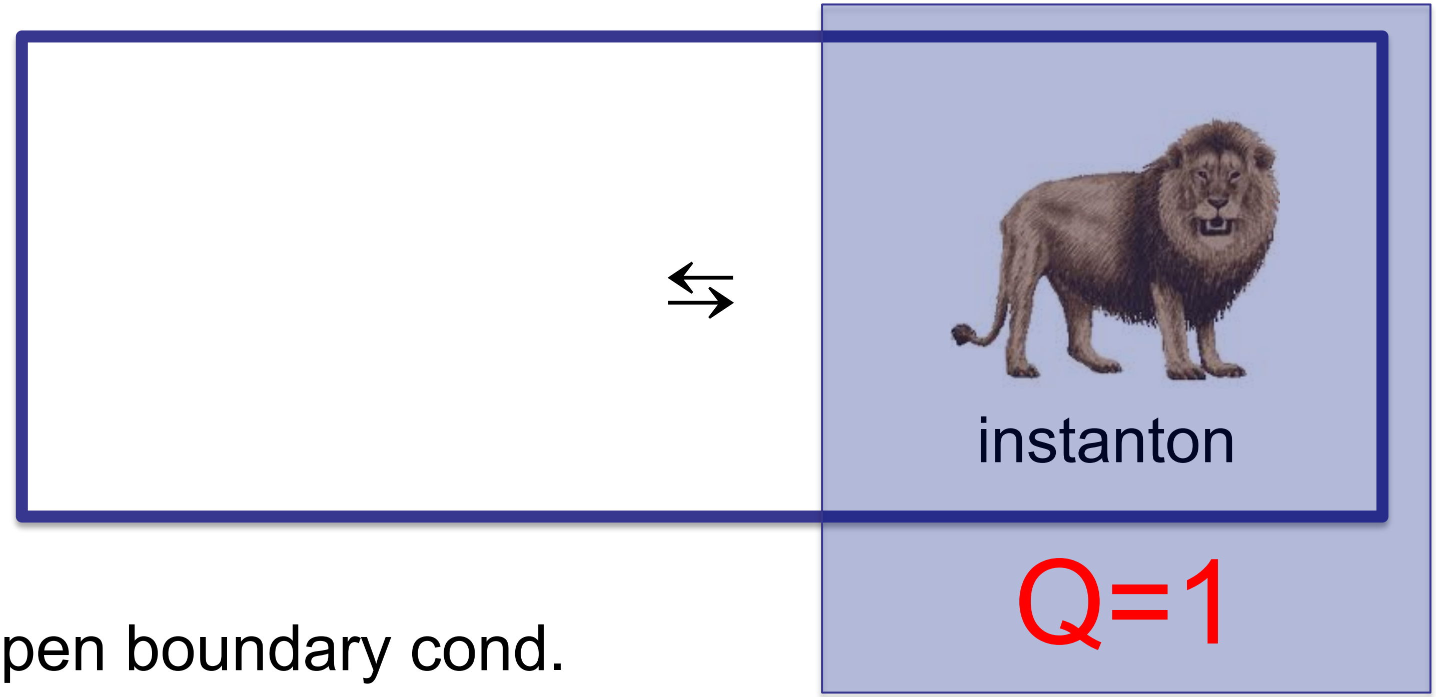
Topology fluctuation in sub-volume may be more frequent.



Similar method is proposed by
[LSD collaboration 2014]

How about sub-volumes ?

Topology fluctuation in sub-volume may be more frequent.



Cf. open boundary cond.
[Luscher & Schaefer 2012]

Can we subtract slow-mode ?

$$\langle O \rangle = \langle O_{Q\text{-indep}} \rangle + \langle O_Q \rangle$$

fast slow

$$\langle O \rangle - \langle O_Q \rangle = \langle O_{Q\text{-indep}} \rangle$$

may have shorter auto-correlation length.

Q dependence = Fourier transform w.r.t. vacuum angle θ

$$Z_Q = \int d\theta e^{i\theta Q} Z(\theta)$$

$$\langle O \rangle_Q = \langle O(\theta=0) \rangle + \left\langle \frac{\partial^2}{\partial\theta^2} O(\theta=0) \right\rangle \frac{1}{2\chi_t V}\left[1 - \frac{Q^2}{\chi_t V}\right] + \cdots$$

$$\chi_t = \frac{1}{V}\frac{\partial^2}{\partial\theta^2} \ln Z(\theta) = \frac{\langle Q^2 \rangle}{V}$$

[Brower et al. 2003, Aoki, F, Hashimoto, Onogi 2007]

General Formula

A remark [Chiu 2009, Aoki, F, 2009]

Topology is global physics ~ pion zero-momentum mode → ChPT helps us :

$$\chi_t = \frac{1}{V}\frac{\partial^2}{\partial\theta^2}\ln Z(\theta) = \bar{m}\Sigma. \qquad \Sigma : \text{chiral condensate.}$$

$$c_4 = \frac{1}{V}\frac{\partial^4}{\partial\theta^4}\ln Z(\theta) = -\bar{m}\Sigma\left(\sum_f^{N_f}\frac{\bar{m}^3}{m_f^3}\right).$$

…

where $\bar{m} = \sum_f^{N_f}\frac{1}{m_f}$

But we don't use ChPT in this talk.

Our work

We try these two ideas in HMC updates.

1. Topology fluctuation in sub-V.
2. Subtract Q-dependent part (= slow moving part) :

$$\langle O\rangle - \underbrace{\langle O_Q\rangle}_{\text{slow}} = \underbrace{\langle O_{Q\text{-indep}}\rangle}_{\text{fast}}$$

Our work

Our target = topology density operator with wilson flow cooling [Luscher 2010] :

$$q(x) = c\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$$

Smeared region = $\sqrt{8t} \sim 0.5\mathrm{fm}.$

1. fluctuation at sub-V is still active,
2. smooth enough to give a well-determined topological charge: $\mathrm{plaq} < 0.067$

3. Lattice set-up

Lattice set-up

Improved domain-wall fermions

Borici 1997; Chiu 1998; Brower et al. 2001

Kernel = scaled Shamir Kernel:

$$2H_T = \gamma_5 \frac{2D_W}{2 + D_W}$$

Sgn function = Tanh:

$$\mathrm{sgn}_{\tanh}(2H_T) = \frac{(1+2H_T)^{L_s} - (1-2H_T)^{L_s}}{(1+2H_T)^{L_s} + (1-2H_T)^{L_s}} = \tanh(L_s \tanh^{-1}(2H_T))$$

Ls = O(10)

m_{res}<0.5MeV, Chiral symmetry ~ 10^{-3}

Lattice set-up

Simulation parameters

Lattice size : 32^3x64(x12), 48^3x96(x8)

Symanzik gauge action with β=4.17, 4.35

3 steps of stout smearing

m_{ud}= 1/6 m_s, m_s ~ physical point

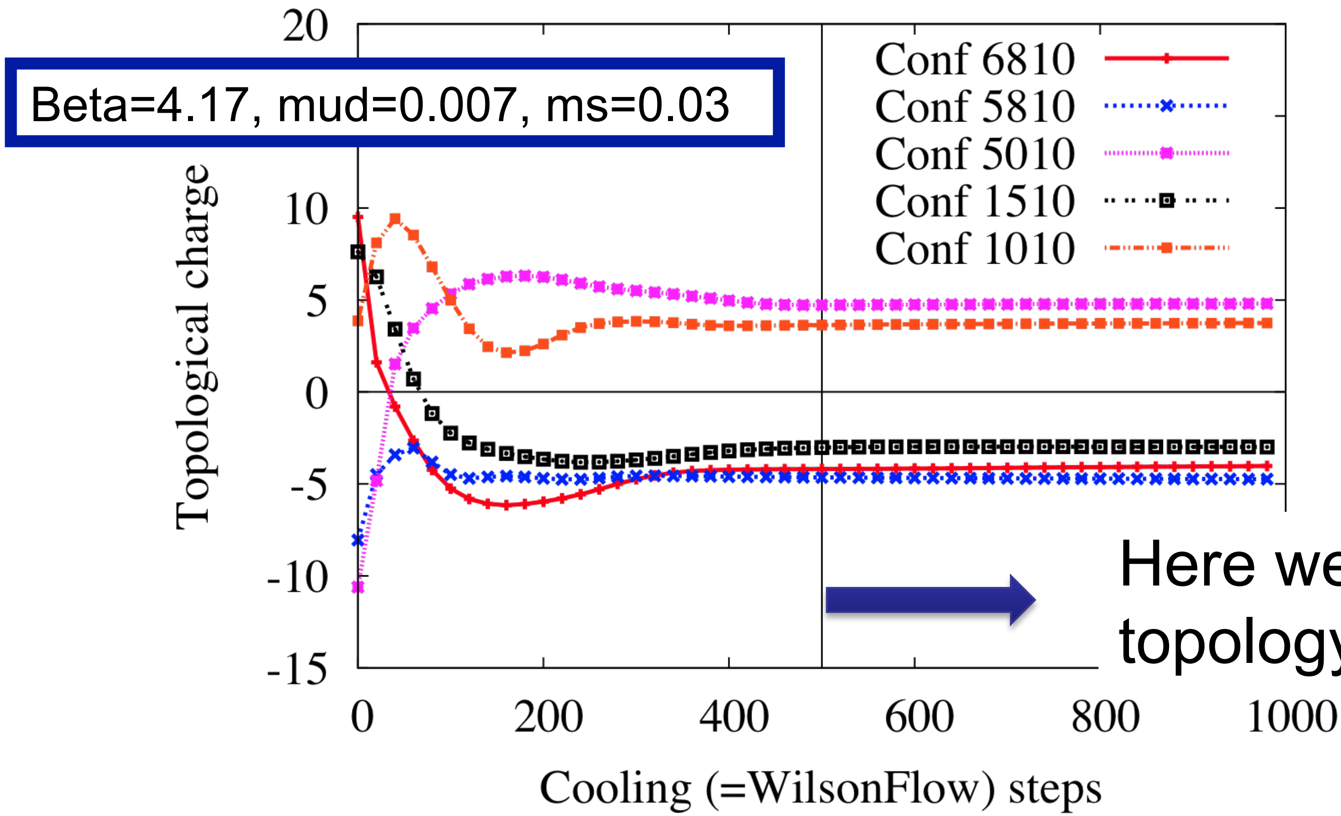
1/a ~ 2.4 GeV, 3.6 GeV

[Simulated w/ Iroiro++ code G. Cossu et al.]

Wilson flow cooling

After Wilson flow at $\sqrt{8t} \sim 0.5\mathrm{fm}$

topological charge does not change.



Here we measure topology density.

Good agreement with Dirac index

Index theorem : $Q = n_+ - n_-$

$n_\pm$: # of L/R zero-modes of overlap Dirac op.

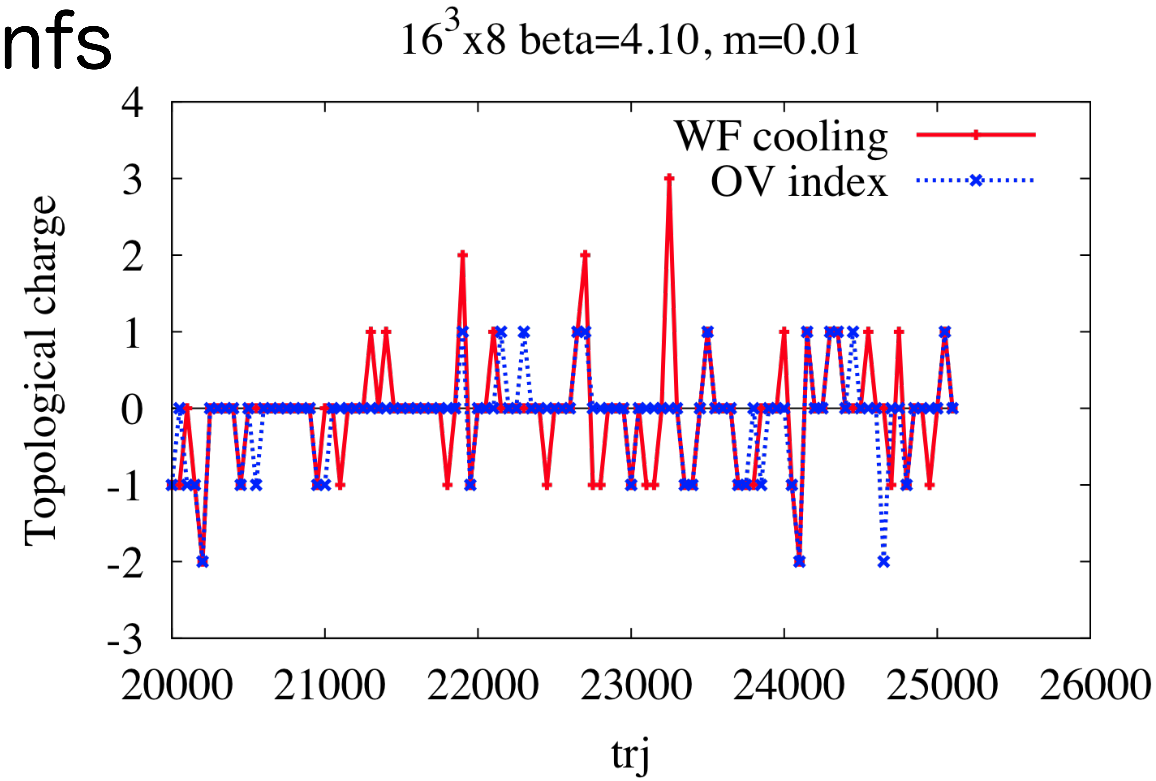
on non-cooled confs

Agreement

~ 80-90%.

*Check at finite T runs.

Tomiya's talk.

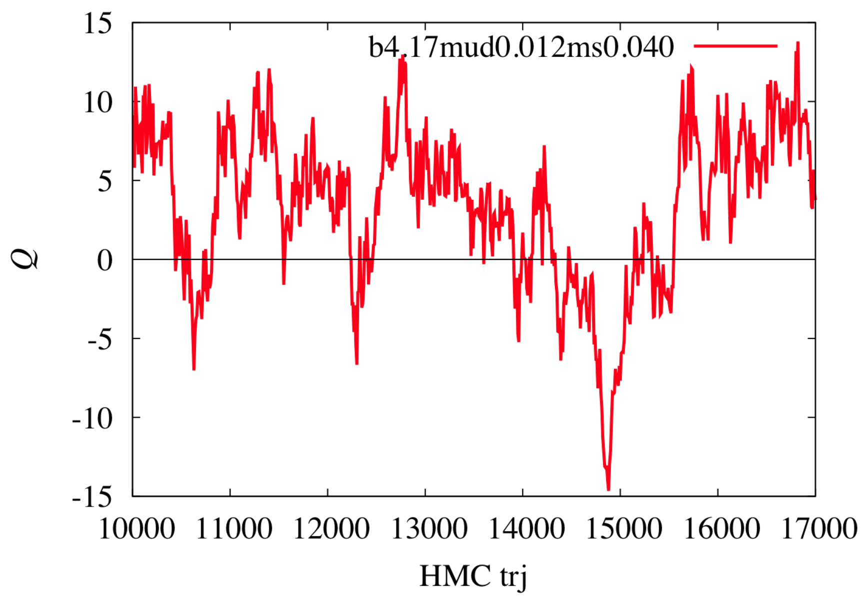


3. (Preliminary)Results

Global topology

$$Q = \sum_x q(x)$$

$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$



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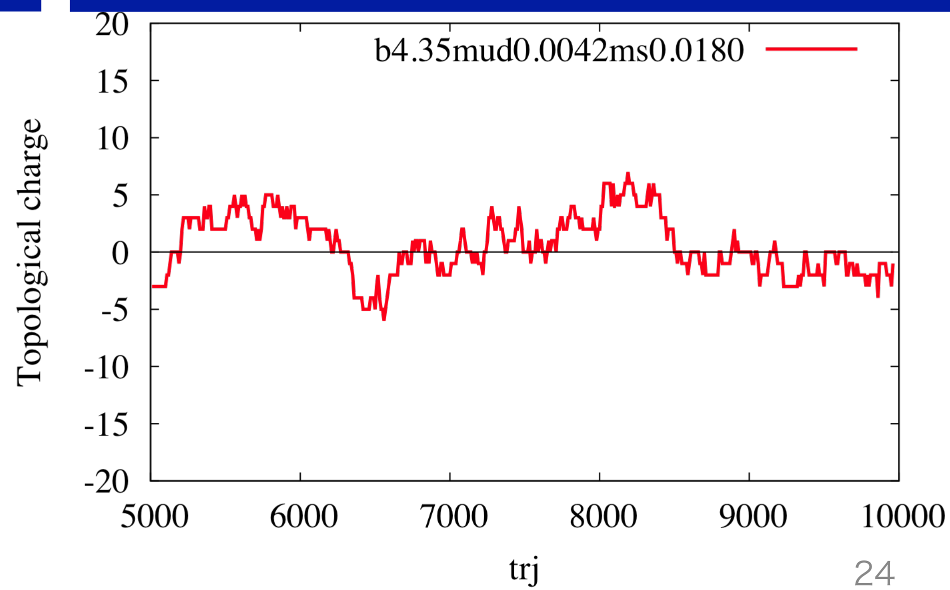
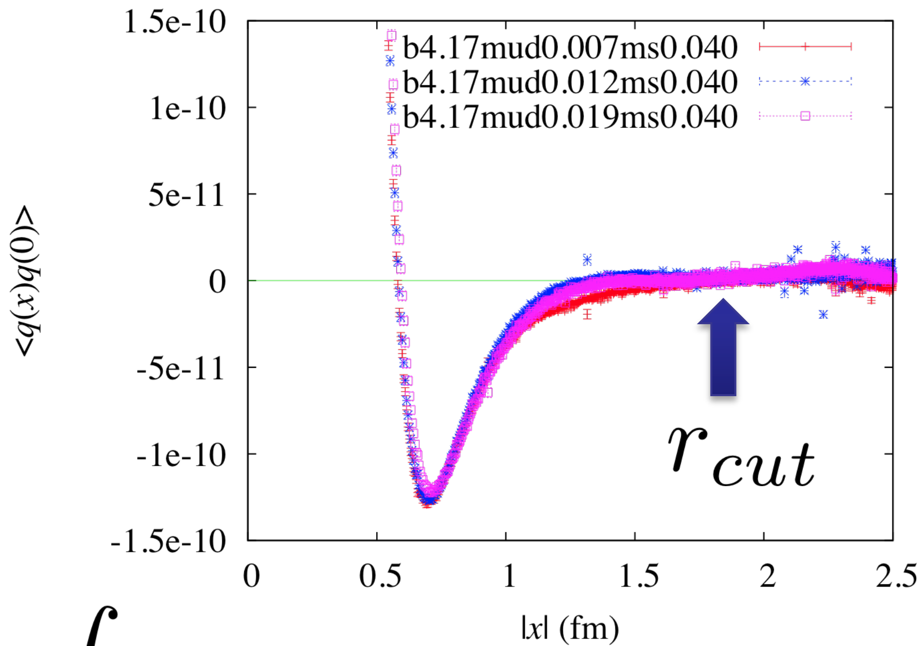


Sub-volume topology



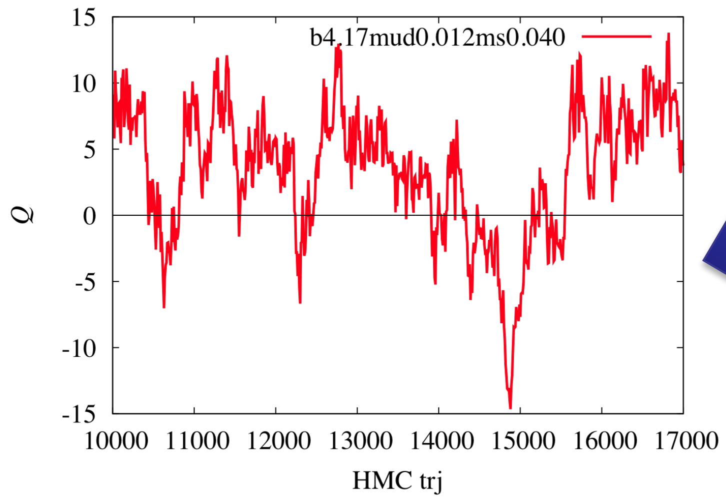
$$\chi_t^{\text{sub}} \equiv \int_{|x|<r_{cut}} d^4x \langle q(x)q(0) \rangle$$

with $r_{cut} = 1.5\text{–}2\text{fm}$ is calculated using FFT.

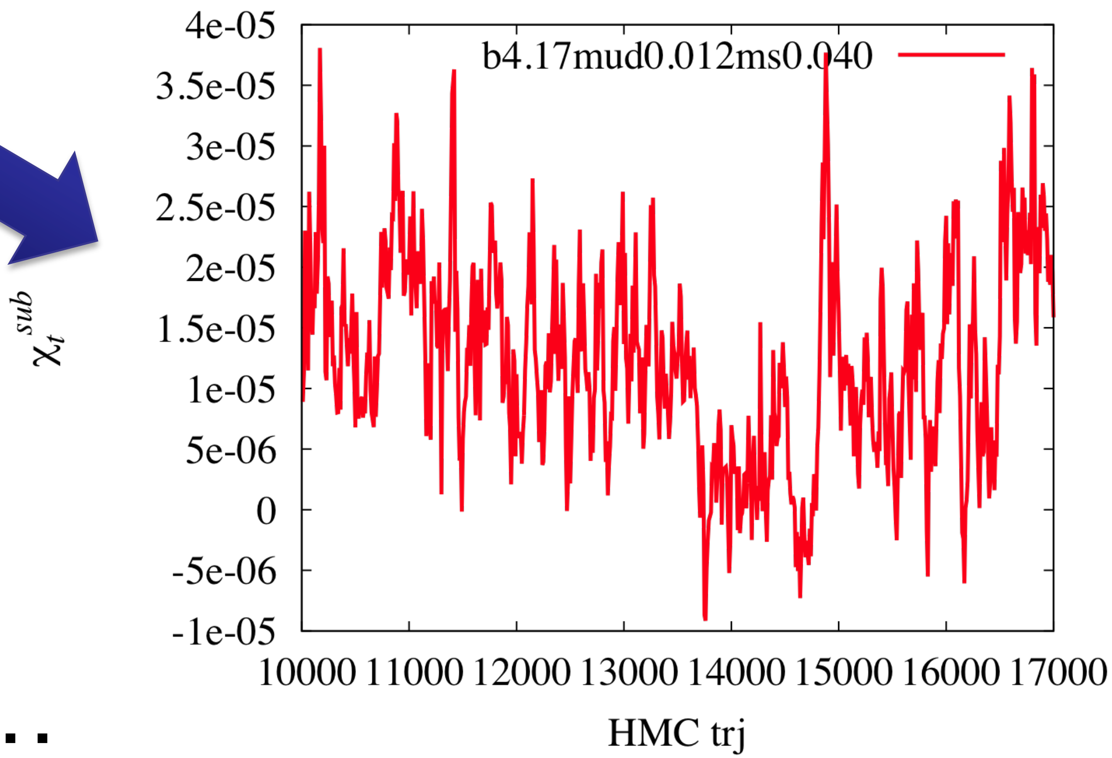
Sub-volume topology

Local Q fluctuation is more frequent.

Global Q



$\chi_t^{\rm sub}$

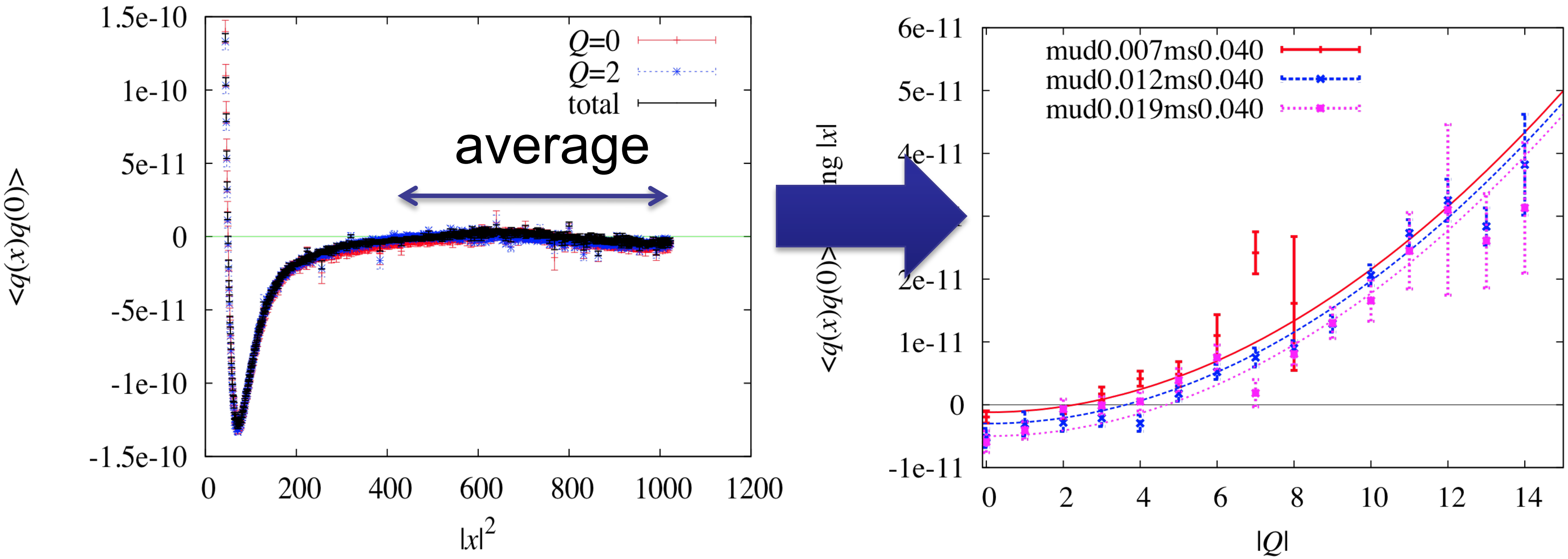


But bias due to global Q looks remaining…

Removing global topology bias on local topology

$$\langle q(x)q(0)\rangle_Q \to_{|x|\to\text{large}} \frac{1}{V}\left[\frac{Q^2}{V} - \chi_t\right] + O(V^{-2}),$$

[Aoki, F, Hashimoto, Onogi 2007]



Note : coefficent of Q^2 has no parameter to tune.

Removing global topology bias on local topology

$$\langle q(x)q(0)\rangle_Q \to_{|x|\to \mathrm{large}} \frac{1}{V}\left[\frac{Q^2}{V} - \chi_t\right] + O(V^{-2}),$$

Let us remove bias from global topology,

$$\langle \chi_t^{Q\mathrm{-indep.}}\rangle = \frac{V}{V - V_{sub}}\left(\chi_t^{sub} - \frac{V_{sub}}{V^2}Q^2\right)$$

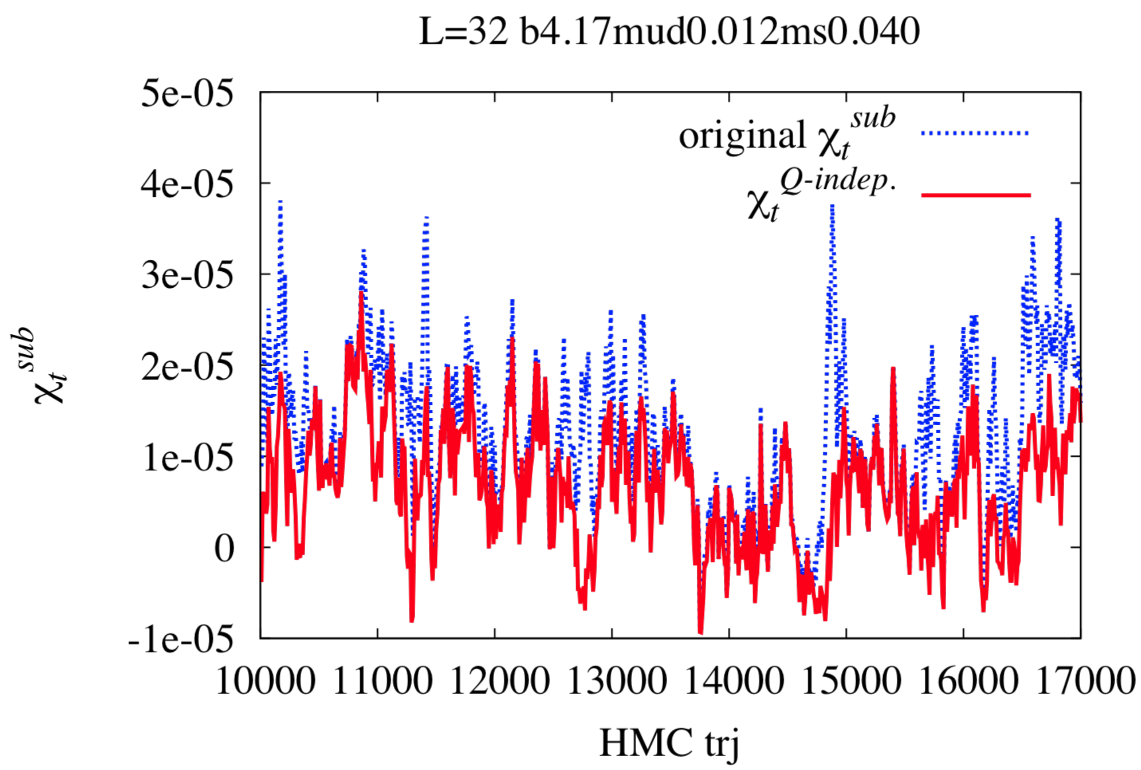
Note : if there is no bias in Q,

$$\langle \chi_t^{Q\mathrm{-indep.}}\rangle = \langle \chi_t^{\mathrm{sub}}\rangle = \langle \chi_t\rangle.$$

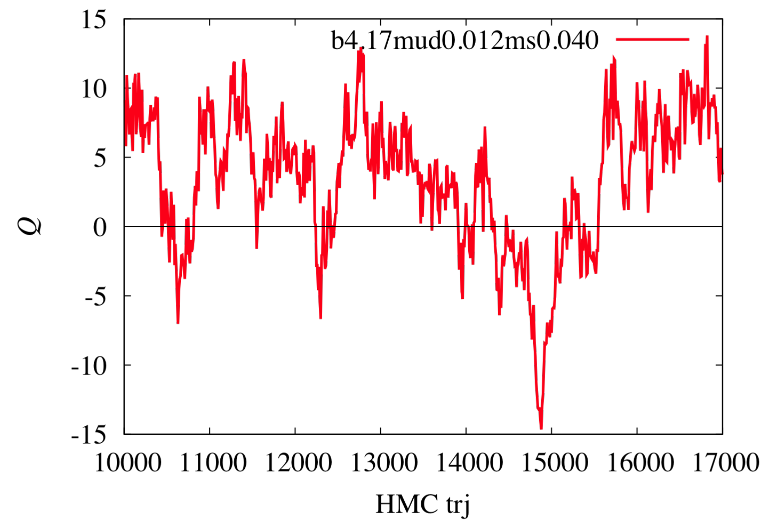
Removing global topology bias on local topology

$$\langle \chi_t^{Q\text{–indep.}} \rangle = \frac{V}{V - V_{sub}} \left(\chi_t^{sub} - \frac{V_{sub}}{V^2} Q^2 \right)$$

$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$

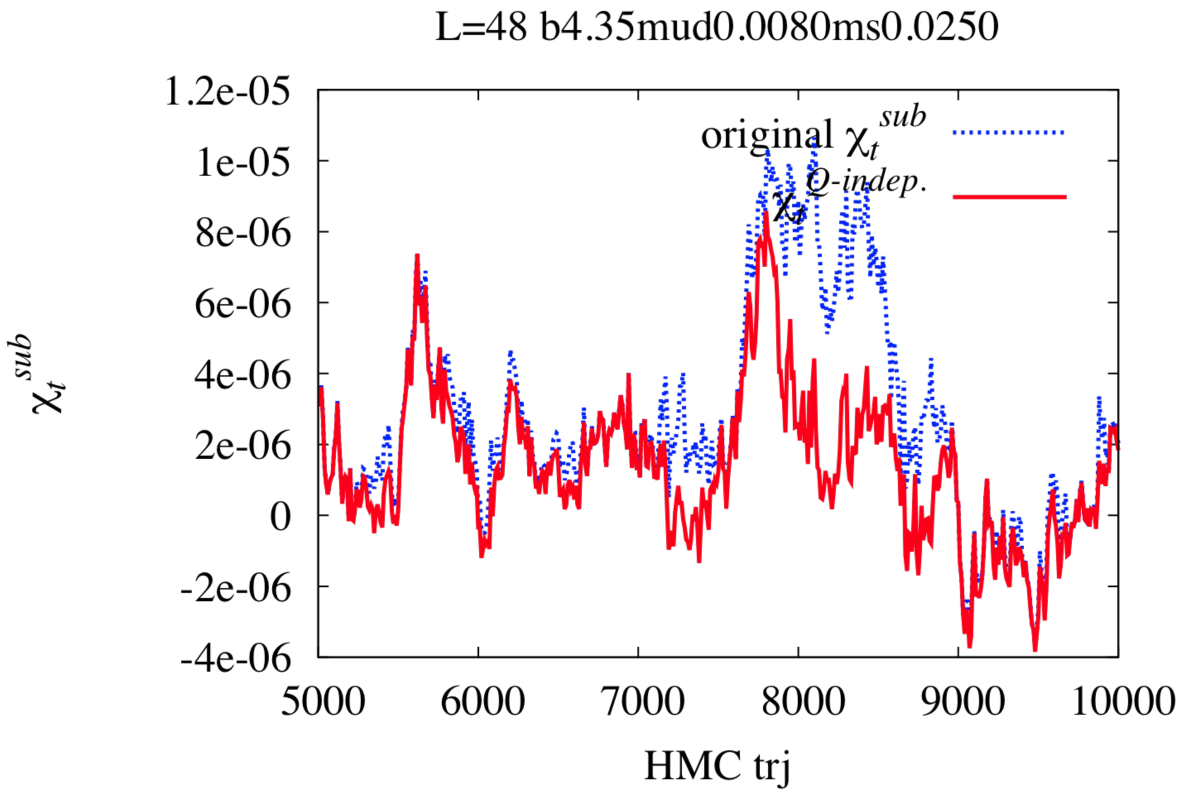


Global Q

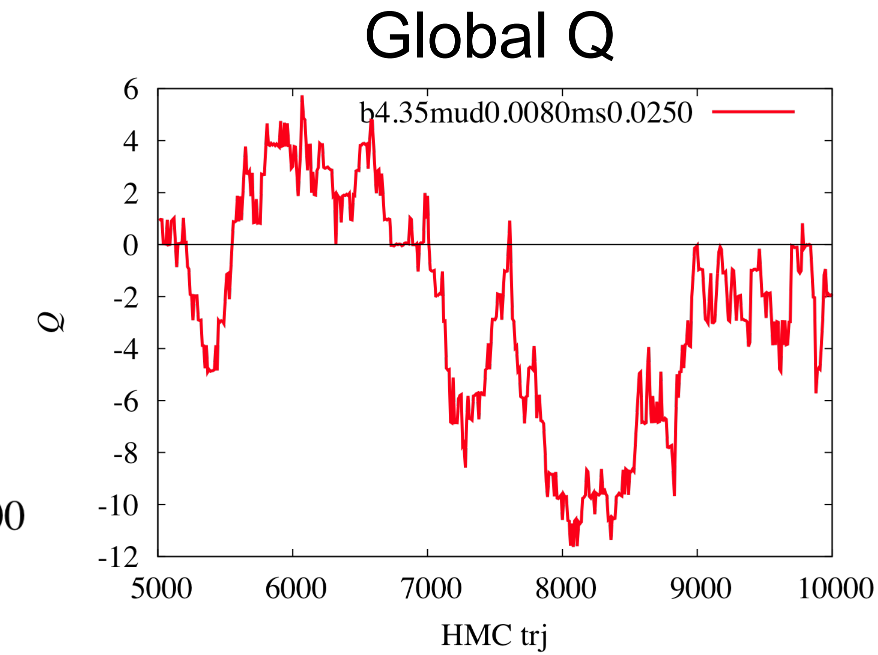


Removing global topology bias on local topology

$$\langle \chi_t^{Q\text{--indep.}} \rangle = \frac{V}{V - V_{sub}} \left(\chi_t^{sub} - \frac{V_{sub}}{V^2} Q^2 \right)$$



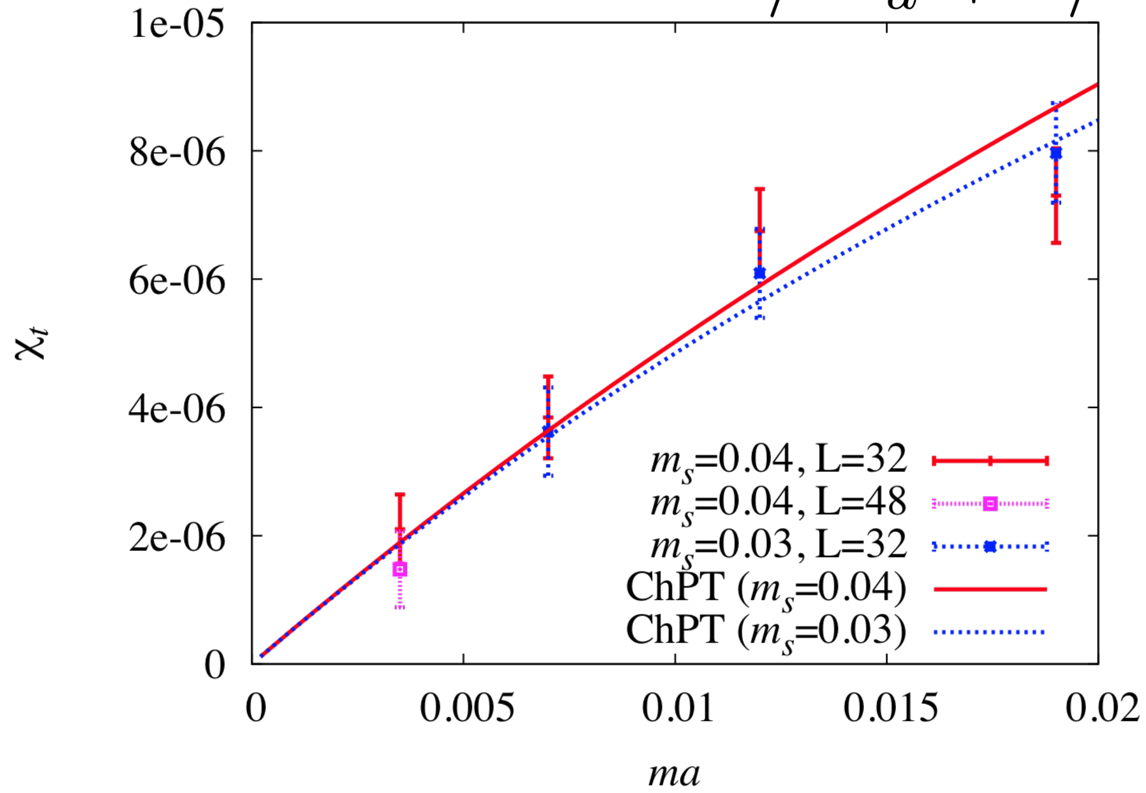
$\boxed{\beta = 4.35\ (a^{-1} \sim 3.6\mathrm{GeV})}$



Topological susceptibility

$\chi_t^{Q-\text{indep.}}$ agrees with ChPT prediction :

$$\chi_t^{\text{ChPT}} = \frac{\Sigma}{1/m_u + 1/m_d + 1/m_s}$$



Fit with the chiral condensate

$$\Sigma^{1/3} = 250\text{MeV}.$$

$$\boxed{\beta = 4.17\ (a^{-1} \sim 2.4\text{GeV})}$$

Topological susceptibility

$\chi_t^{Q-\text{indep.}}$ agrees with ChPT prediction :

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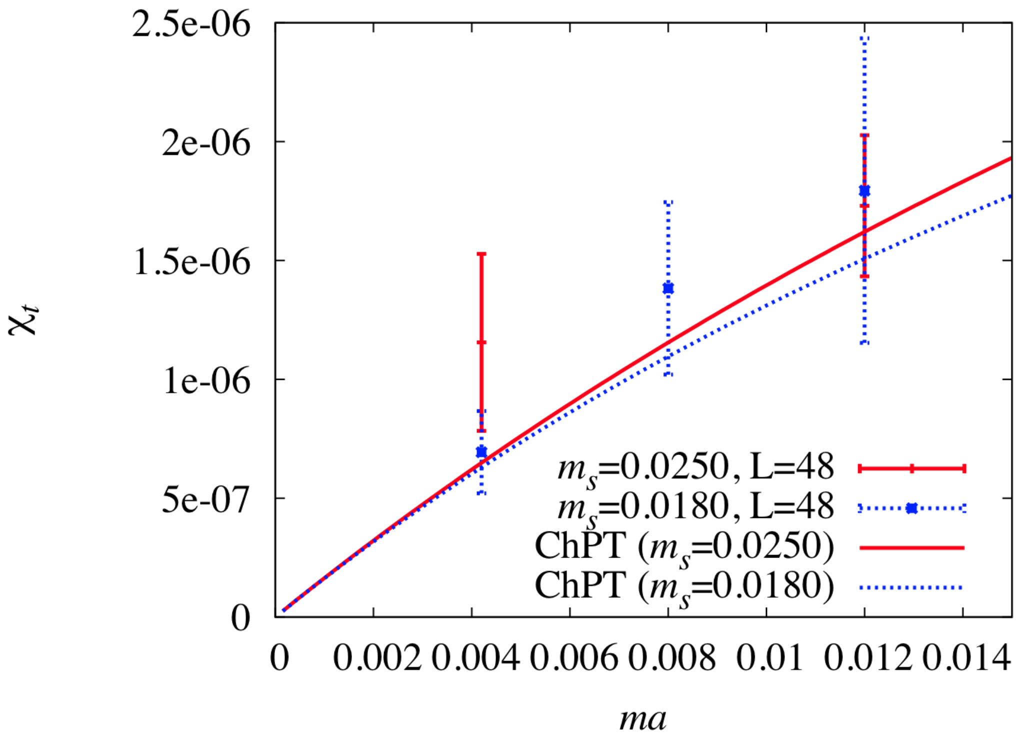


Fit with

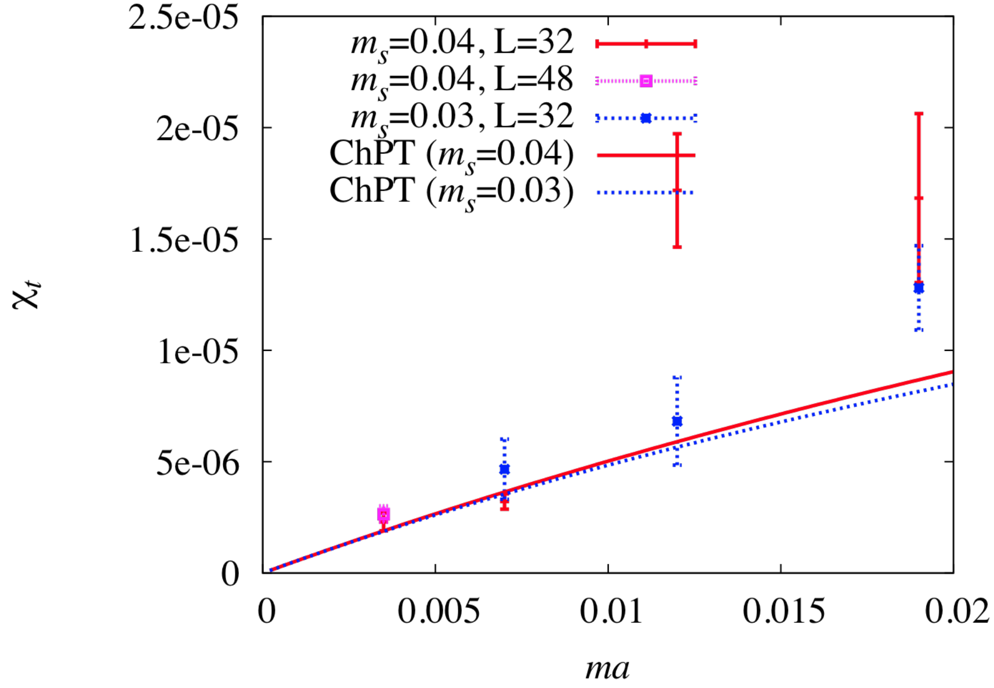
$$\Sigma^{1/3} = 250\text{MeV}.$$

$$\boxed{\beta = 4.35 \; (a^{-1} \sim 3.6\text{GeV})}$$

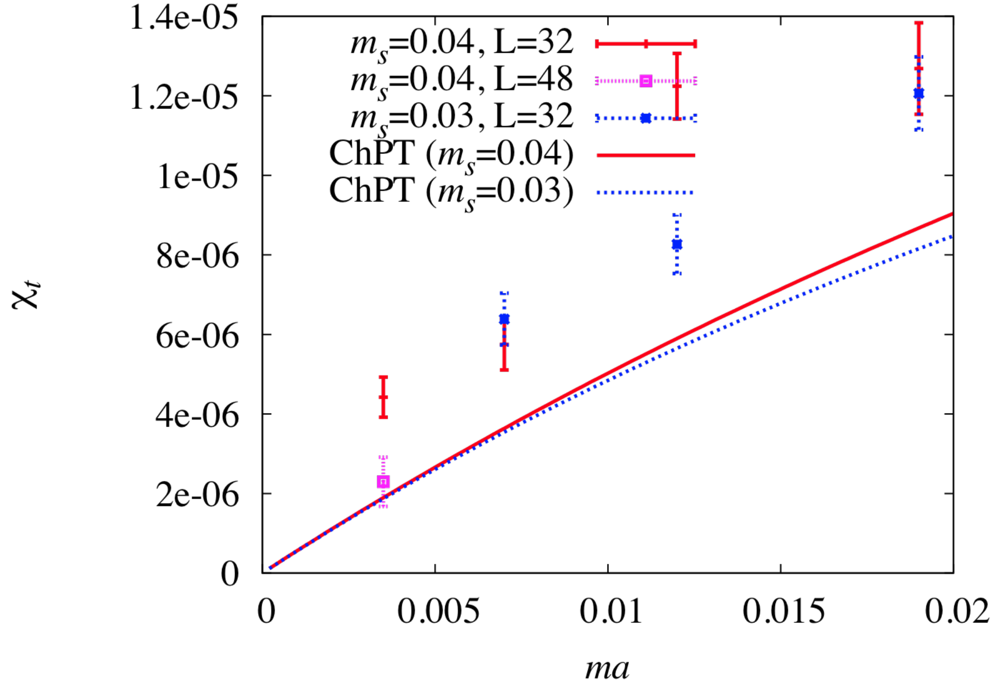
Topological susceptibility

while others do not
(due to biased global Q).

χ_t from Q



χ_t^{sub}



4. Summary

Think globally, act locally.

1. Local (sub-volume) topology fluctuation is more frequent than global Q.
2. Removing global Q dependent part, we can correct the global topological bias.

$$\langle O \rangle - \langle O_Q \rangle = \langle O_{Q\text{-indep}} \rangle$$

slow fast

Similar method is proposed by LSD collaboration 2014

Difference from conventional method

In the literature, people often use

$$\langle Q^2 \rangle - \langle Q \rangle^2$$

But this is theoretically incorrect :

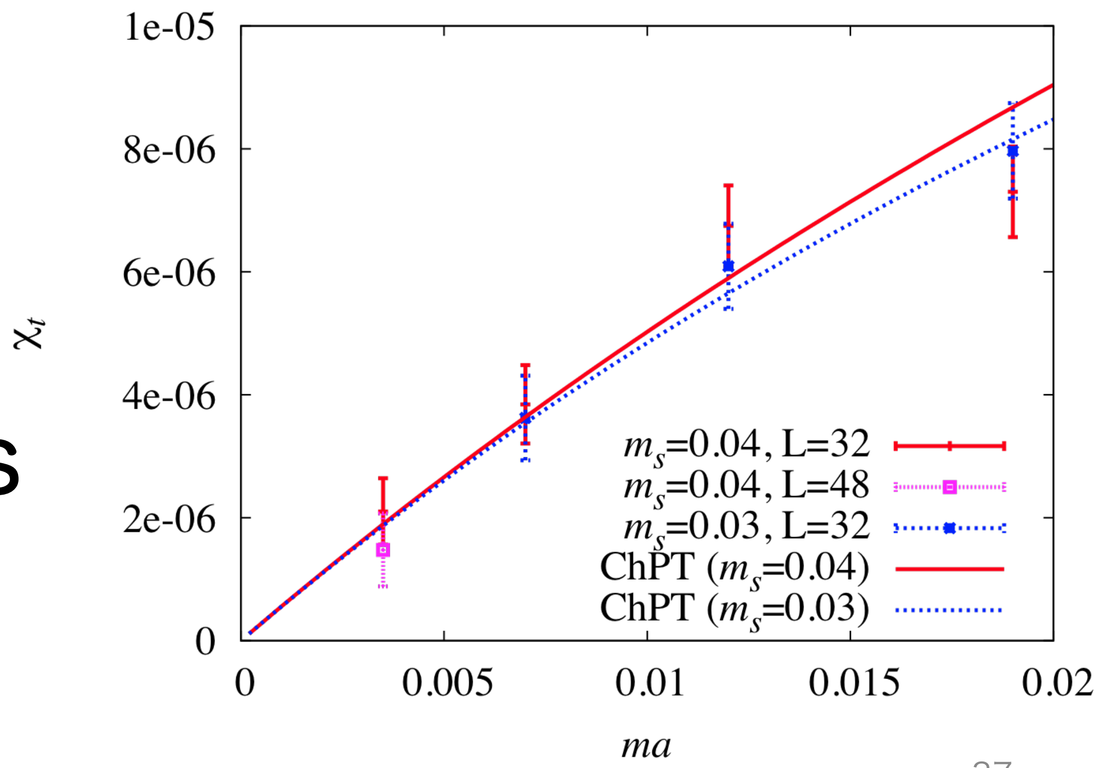
$$\langle Q \rangle = 0.$$

and the global bias and long auto-correlation stay the same.

Finally we'd like to stress

$$q(x) = c\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$$

is pure gluonic quantity, which shows a clear sea quark mass dependence !

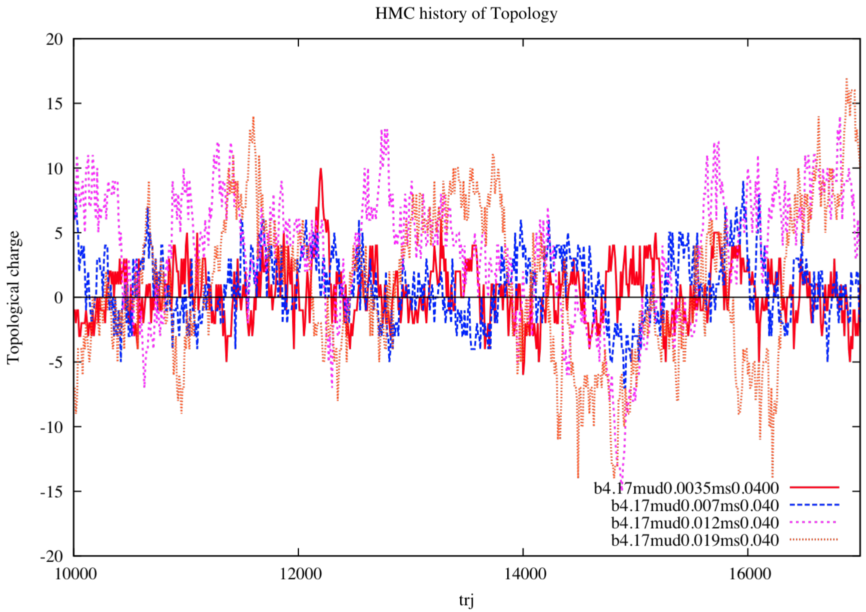


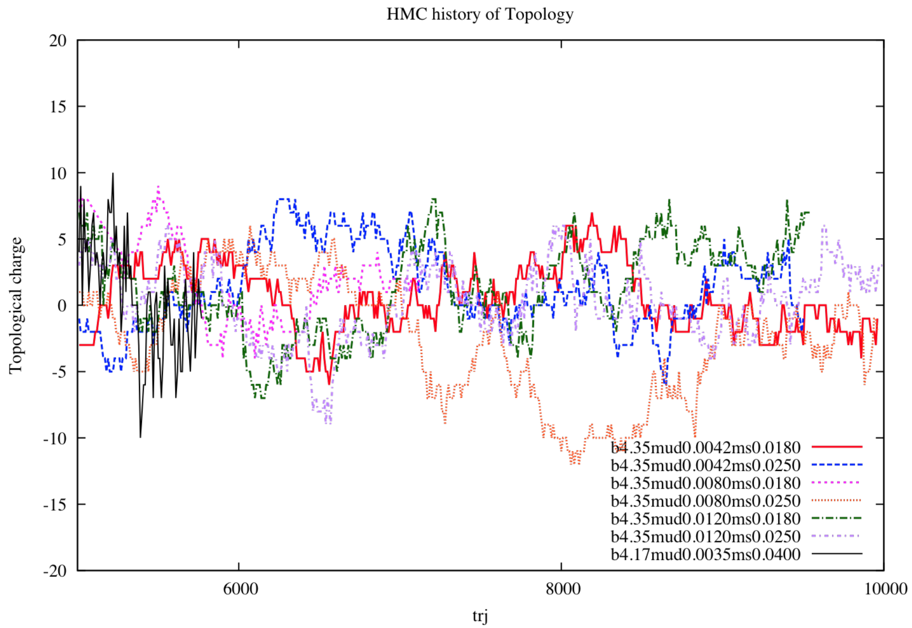
Back-up slides

History of global topology

$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$

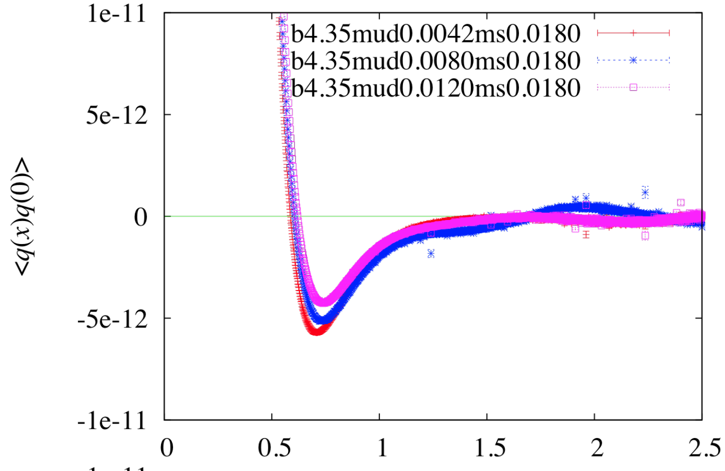
$\beta = 4.35\ (a^{-1} \sim 3.6\mathrm{GeV})$

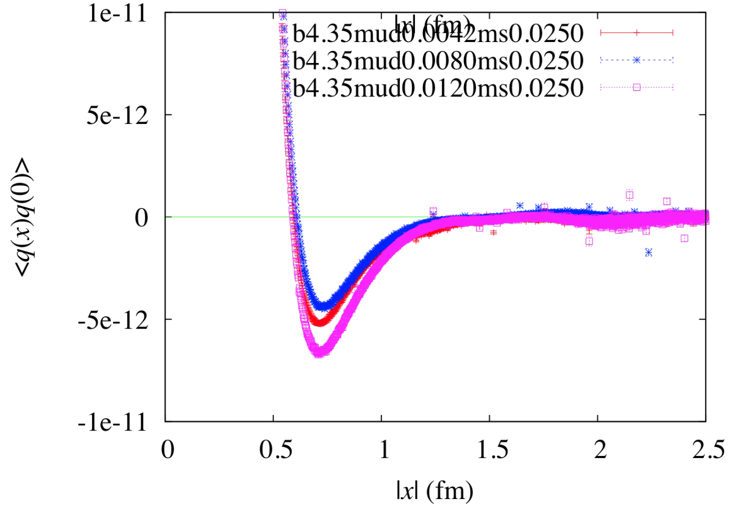




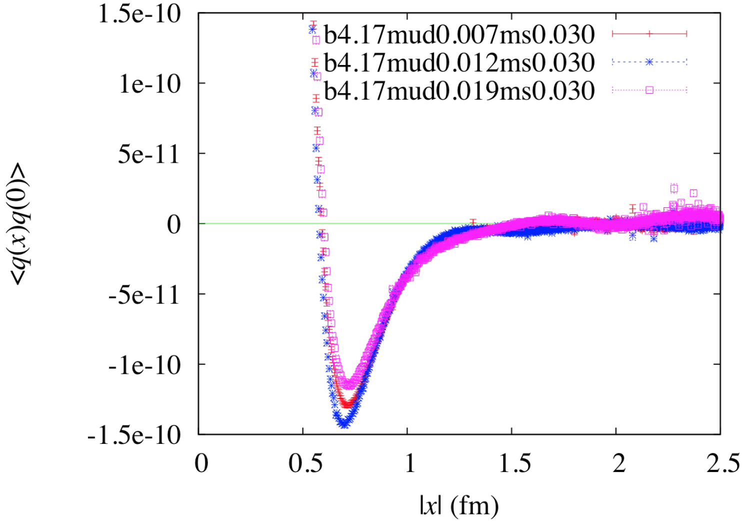
Topology density correlators

$\beta = 4.35\ (a^{-1} \sim 3.6\mathrm{GeV})$

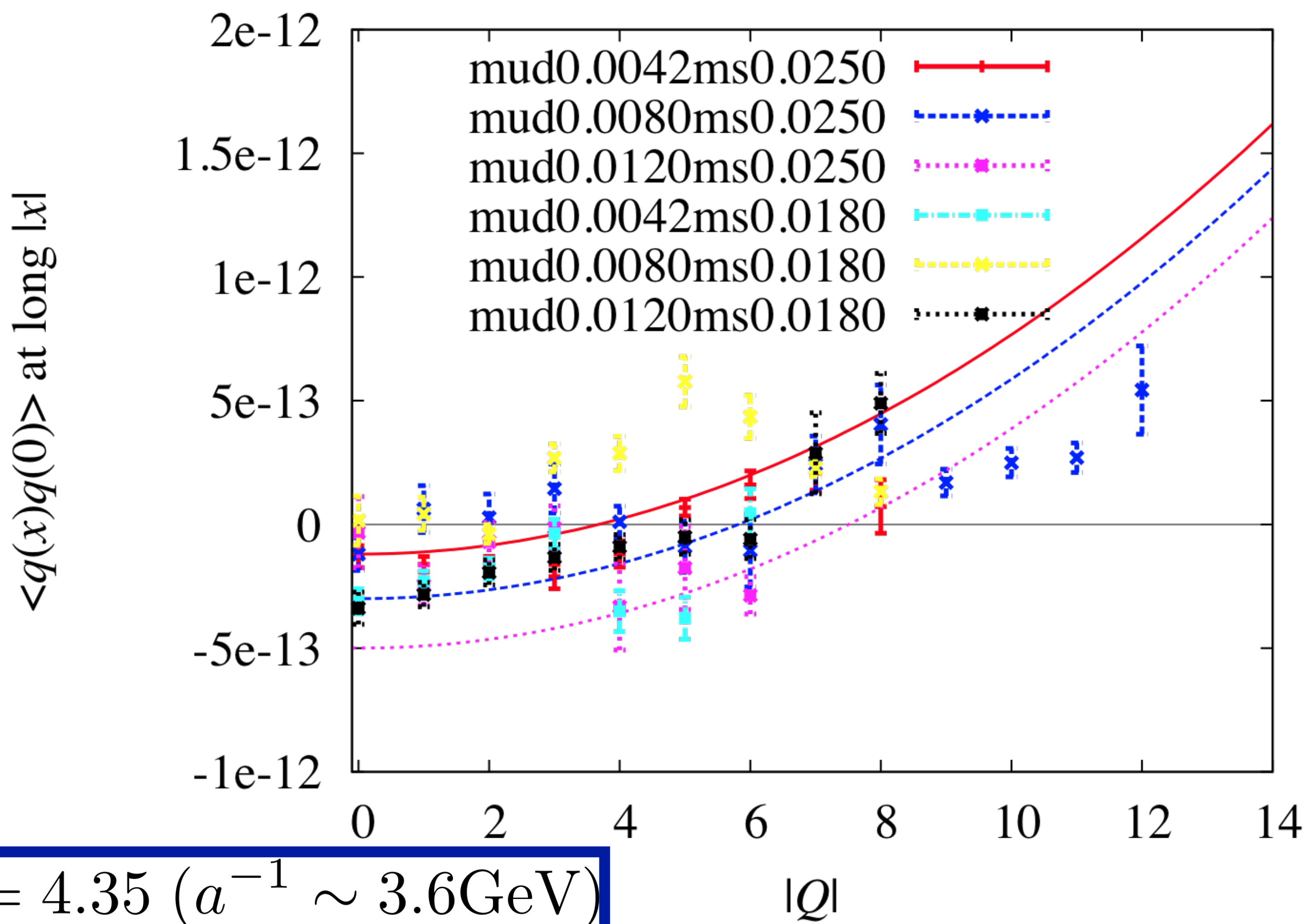




$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$

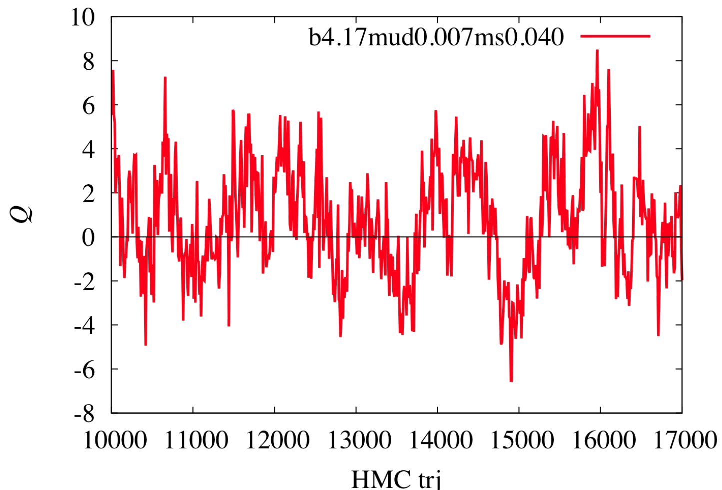


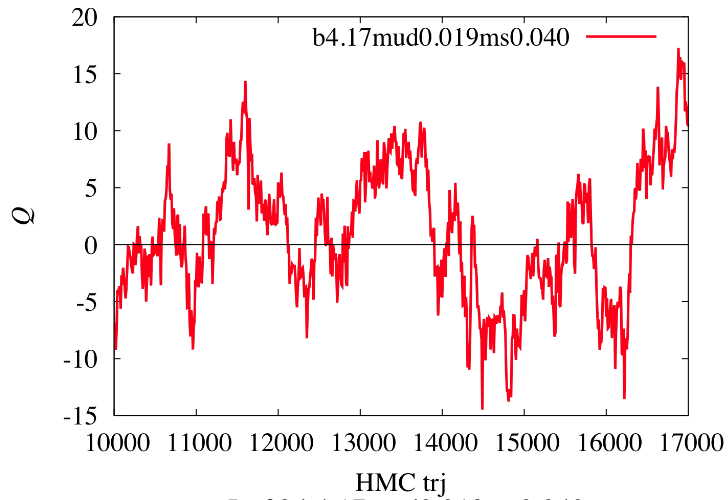
Topology dependence of long distance qq correlator



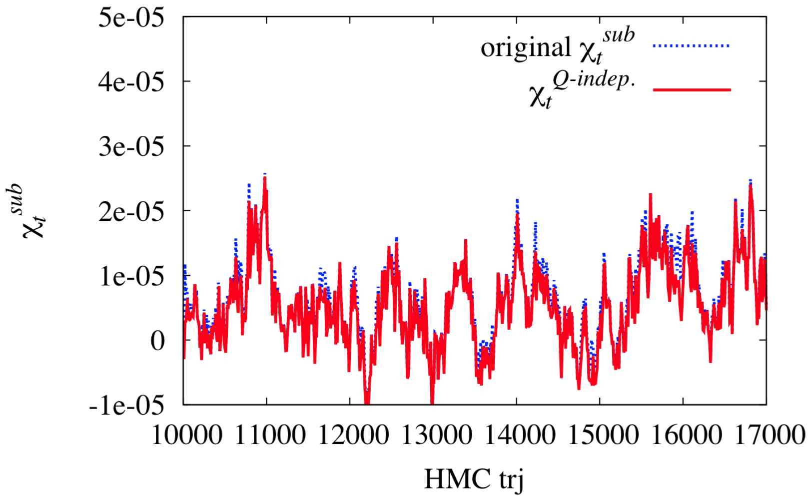
$\beta = 4.35\ (a^{-1} \sim 3.6\text{GeV})$

Global vs. sub-V topology (1)

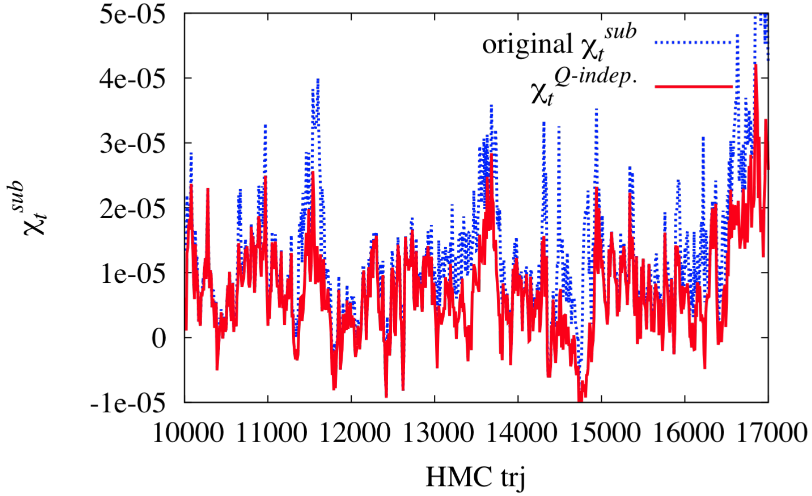




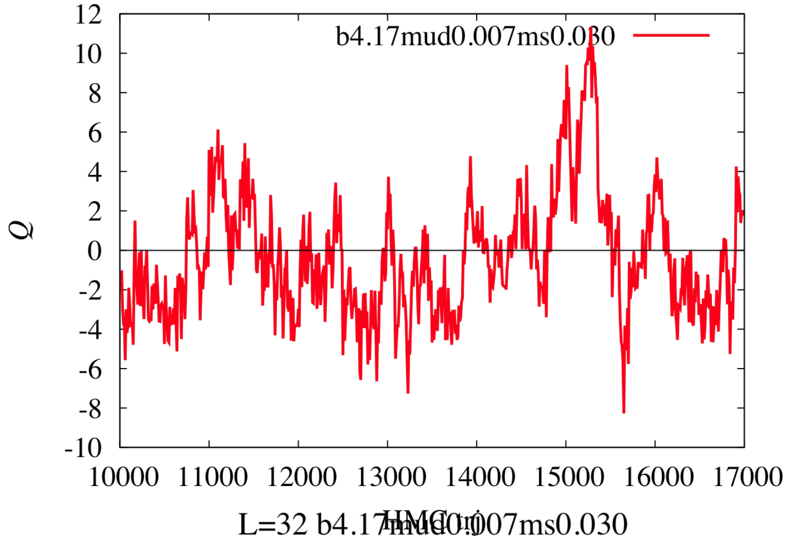
L=32 b4.17mud0.007ms0.040

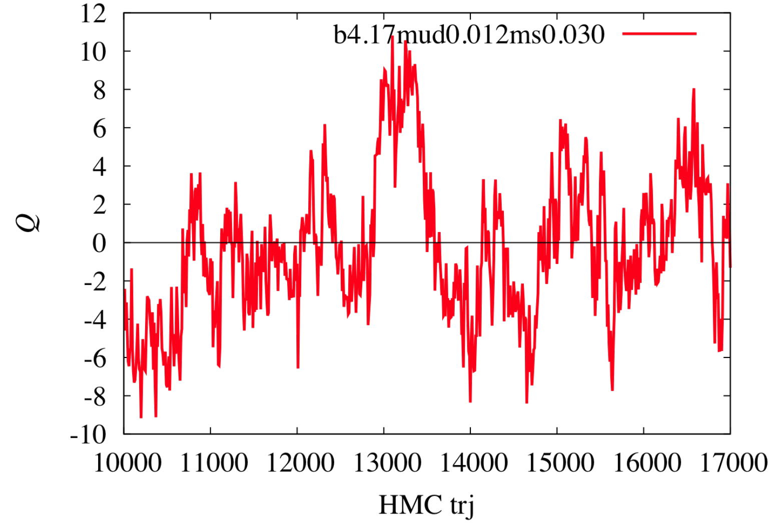


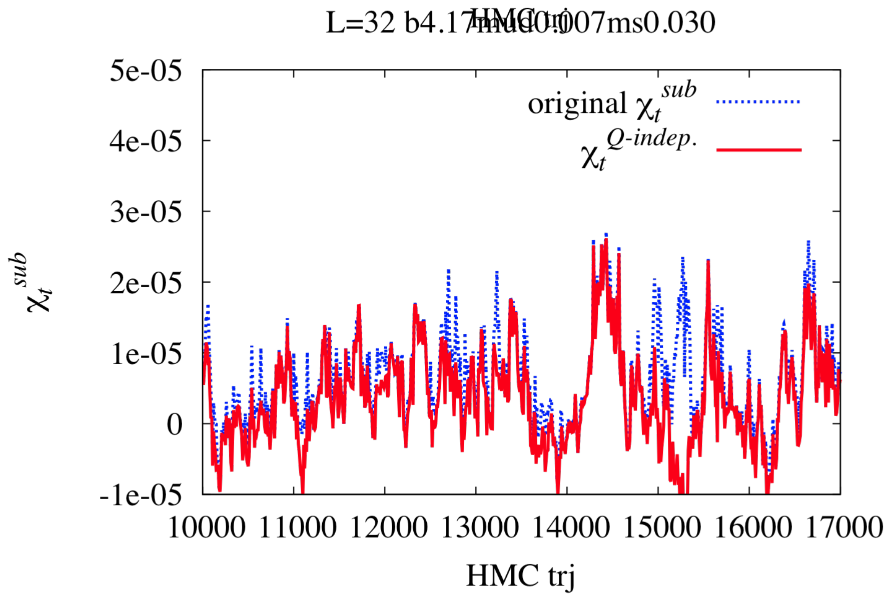
L=32 b4.17mud0.019ms0.040

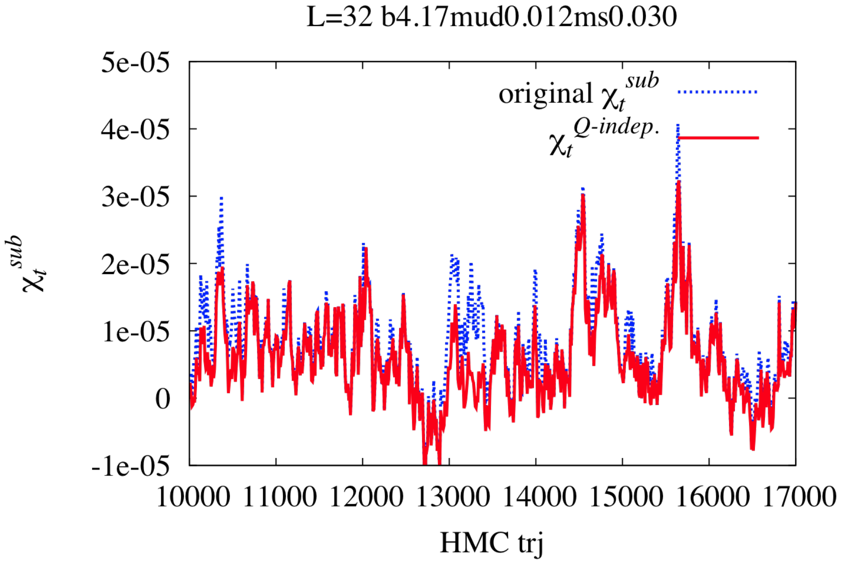


Global vs. sub-V topology (2)

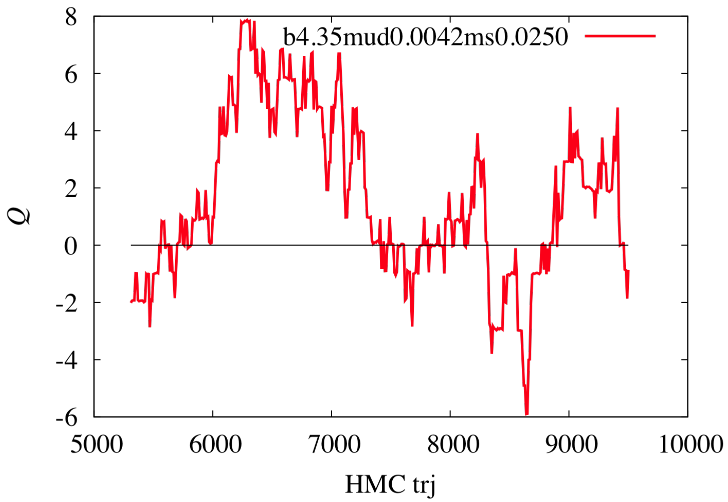


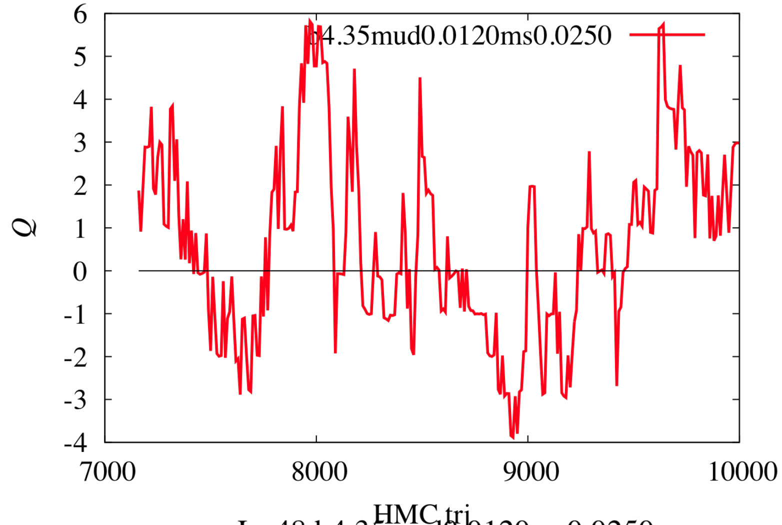


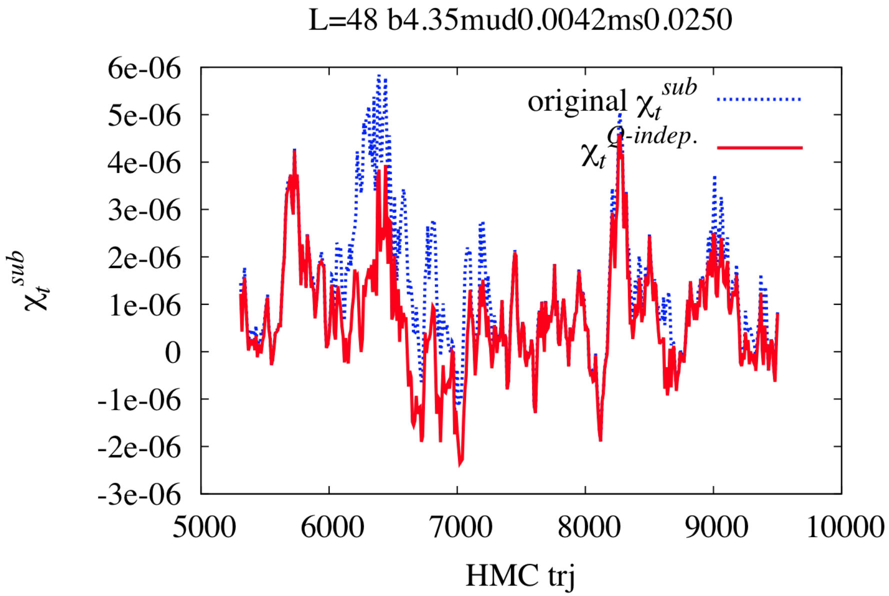


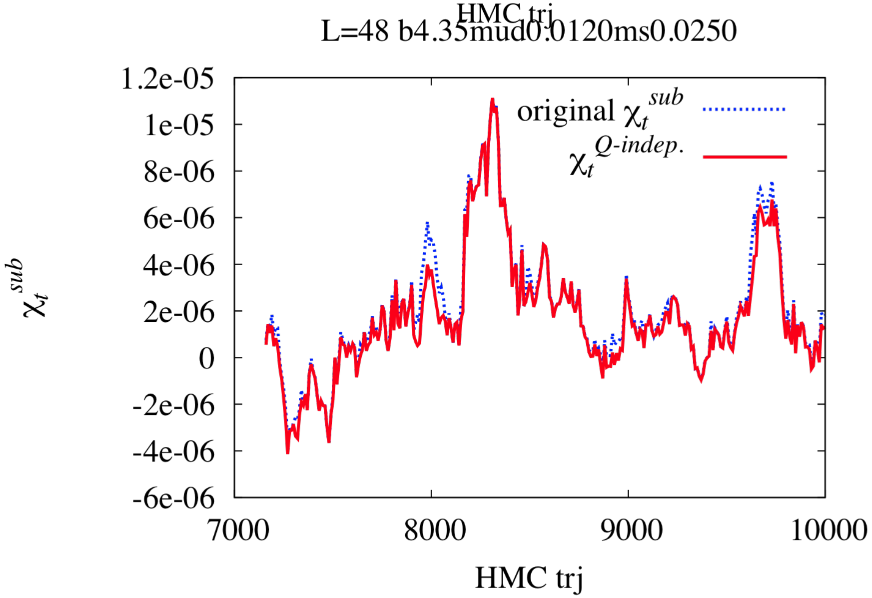


Global vs. sub-V topology (3)









Global vs. sub-V topology (4)