A Perturbative Study of the Chirally Rotated Schrödinger Functional in QCD

Pol Vilaseca Mainar
Stefan Sint

Overview:

•Introduction: Schrödinger Functional schemes
and automatic O(a) improvement.

•The Chirally Rotated Schrödinger Functional (χSF)

- Definition
- Renormalization & Improvement
- Correlation Functions

•A perturvatibe study

- Determination of coefficients
- Checks of automatic O(a) improvement.

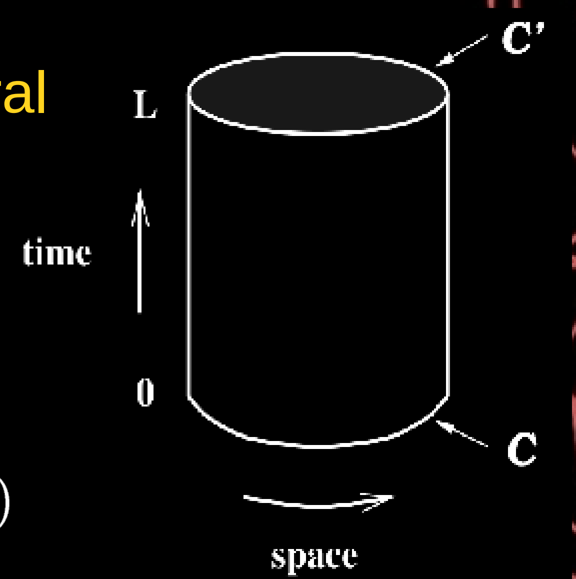
- Applications
- Concluding Remarks

Introduction:

- Finite Volume (FV) schemes based on the Schrödinger Functional widely used in non perturbative renormalization.

- Hypercylindrical Euclidean manifold with temporal boundaries. [Lüscher et al. '92]

$$\mathcal{Z}[C, C'] = \int \mathcal{D}[A, \psi, \overline{\psi}] e^{-S[A,\psi,\overline{\psi}]}$$



- Boundary conditions:
  - -Gauge fields $\quad C_k = \dfrac{i}{L}\mathrm{diag}(\phi_{1k}, \ldots, \phi_{Nk})$
  - -Fermion fields

$$P_+\psi \mid_{x_0=0} = P_-\psi \mid_{x_0=T} = 0$$

$$\overline{\psi}P_- \mid_{x_0=0} = \overline{\psi}P_+ \mid_{x_0=T} = 0$$

- Successfully applied in severel renormalization problems: (coupling in QCD, running quark masses, BSM, composite operators, ...)

Introduction:

- In any SF formulation, there are extra sources of cutoff effects generated at the boundaries. (Extra dim 4 operators localized at the boundaries).

- These can be removed through Symanzik improvement.

- For Wilson Fermions and gauge action, O(a) effects are due to dim 5 operators in the bulk and dim 4 at the boundaries.

- O(a) improvement is achieved by adding

  -Bulk: (dim 5) $\overline{\psi} i \sigma_{\mu\nu} F_{\mu\nu} \psi \longrightarrow c_{\mathrm{SW}}$

  -Boundaries: (dim 4)

    -Gauge: $\mathrm{tr}\,\{F_{kl} F_{kl}\} \longrightarrow c_{\mathrm{t}}$

    -Fermion: $\overline{\psi} P_{\pm} D_0 \psi \longrightarrow \widetilde{c}_{\mathrm{t}}$

Automatic O(a) improvement

If $\mathcal{R}_5$ is a symmetry of the massless continuum theory,

$$\mathcal{R}_5 : \psi \to i\gamma_5\psi$$
$$\overline{\psi} \to \overline{\psi} i\gamma_5$$

$$\mathcal{O}_{\mathrm{even}} \longrightarrow O(1),\ O(a^2),\ \ldots$$
$$\mathcal{O}_{\mathrm{odd}} \longrightarrow O(a),\ O(a^3),\ \ldots$$

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Yes !!!! $\boxed{\widetilde{\gamma}_5 = \gamma_5 \tau_1 \qquad \widetilde{Q}_\pm = \frac{1}{2}\left(1 \pm i\gamma_0\gamma_5\tau_3\right)}$

Chirally Rotated SF

χSF [Sint '05-'10]

$$\widetilde{Q}_{+}\psi\mid_{x_0=0}=\widetilde{Q}_{-}\psi\mid_{x_0=T}=0$$

$$\overline{\psi}\widetilde{Q}_{+}\mid_{x_0=0}=\overline{\psi}\widetilde{Q}_{-}\mid_{x_0=T}=0$$

Implements in the SF the mechanism of automatic O(a) improvement

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SF and χSF related through a chiral rotation
(Identical in the continuum and chiral limits).

$$\psi \longrightarrow R(\alpha)\psi$$

$$\overline{\psi} \longrightarrow \overline{\psi}R(\alpha)$$

$$R(\alpha) = e^{i\gamma_5\tau^3\alpha/2} \quad \alpha = \pi/2$$

Automatic O(a) improvement is expected after renormalization and O(a) imp (of boundaries):

$m_{\rm c}$ $z_{\rm f}$

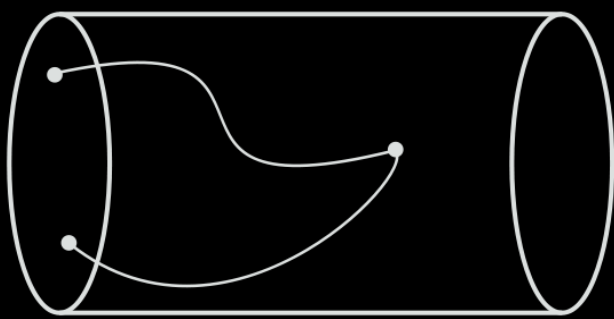
$d_{\rm s}$ $c_{\rm t}$

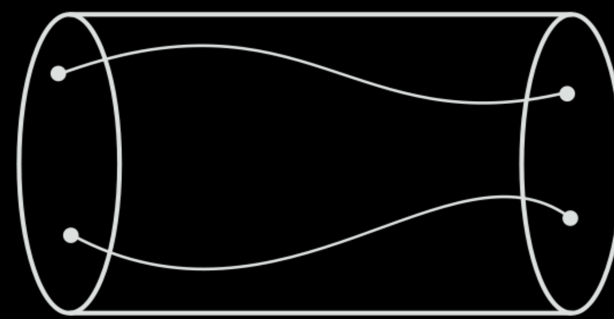
χSF correlation functions

- Boundary to bulk and boundary to boundary

g_X

l_Y





g_1

l_1

$$g_X^{f_1 f_2}(x_0) = -\frac{1}{2}\langle X^{f_1 f_2}(x_0)\mathcal{Q}_5^{f_2 f_1}\rangle$$

$$X = A_0, V_0, S, P$$

$$g_1^{f_1 f_2} = -\frac{1}{2}\langle \mathcal{Q}_5^{f_1 f_2}\mathcal{Q}_5'^{f_2 f_1}\rangle$$

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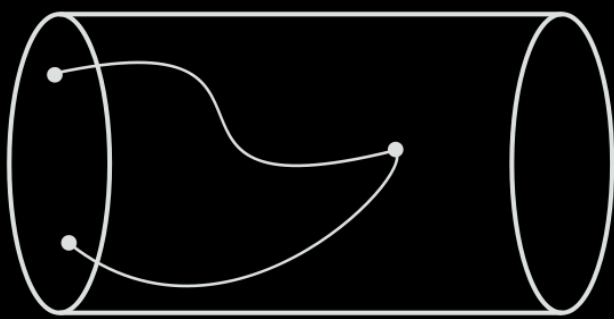

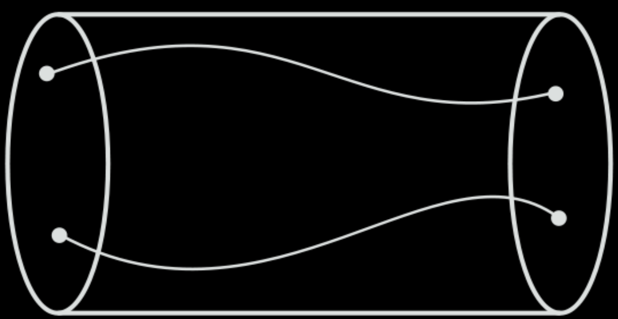

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$$\mathrm{g}_X^{f_1 f_2}(x_0) = -\frac{1}{2}\langle X^{f_1 f_2}(x_0)\mathcal{Q}_5^{f_2 f_1}\rangle \qquad \mathrm{g}_1^{f_1 f_2} = -\frac{1}{2}\langle \mathcal{Q}_5^{f_1 f_2}\mathcal{Q}_5'^{f_2 f_1}\rangle$$

$$X = A_0, V_0, S, P$$

- These are related to standard SF correlationfunctions through the chiral twist

$$\langle O[R(\pi/2)\psi, \overline{\psi}R(\pi/2)]\mathcal{Q}_5^{f_1 f_2}\rangle_{\chi SF} = \langle O[\psi, \overline{\psi}]\mathcal{O}_5^{f_1 f_2}\rangle_{SF}$$

$$R(\alpha) = e^{i\gamma_5 \tau^3 \alpha/2}$$

χSF correlation functions

Dictionary between SF and cSF correlation functions

$$\left.\begin{aligned} f_A &= \mathrm{g}_A^{uu} = \mathrm{g}_A^{dd} = -i\mathrm{g}_V^{ud} = i\mathrm{g}_V^{du} \\ f_P &= i\mathrm{g}_S^{uu} = -i\mathrm{g}_S^{dd} = \mathrm{g}_P^{ud} = \mathrm{g}_P^{du} \end{aligned}\right| \quad \text{even}$$

$$\text{odd} \quad \left|\begin{aligned} f_V &= \mathrm{g}_V^{uu} = \mathrm{g}_V^{dd} = -i\mathrm{g}_A^{ud} = i\mathrm{g}_A^{du} \\ f_S &= i\mathrm{g}_P^{uu} = -i\mathrm{g}_P^{dd} = \mathrm{g}_S^{ud} = \mathrm{g}_S^{du} \end{aligned}\right.$$

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$$R(\alpha) = e^{i\gamma_5\tau^3\alpha/2}$$

Renormalization and improvement

Renormalization and improvement conditions evaluated order by order in perturbation theory.

- $m_{\rm c}$: $m_{\rm PCAC} = 0$
- $z_{\rm f}$: require a $\mathcal{P}_5$ odd observable to vanish.

  $$\mathrm{g}_A^{ud} = 0; \quad \mathrm{g}_V^{uu} = 0; \quad \mathrm{g}_P^{uu} = 0; \quad \mathrm{g}_S^{ud} = 0$$

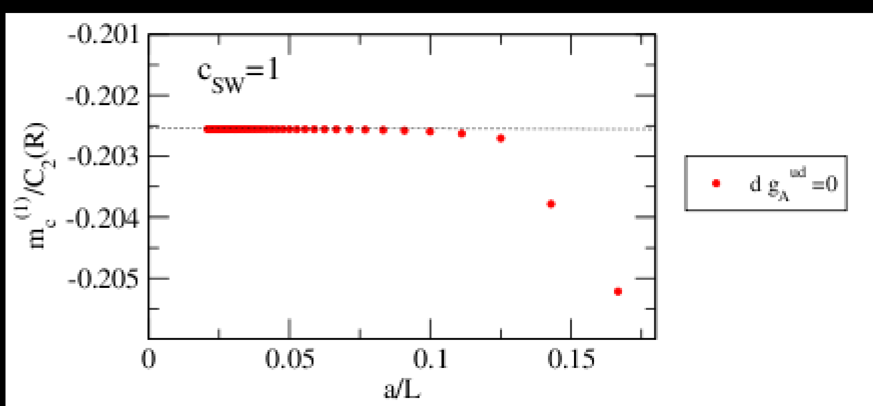
- $d_{\rm s}$: require absence of O(a) effects in $\mathcal{P}_5$ even observable
- $c_{\rm t}$: require absence of O(a) terms in the 1-loop coupling

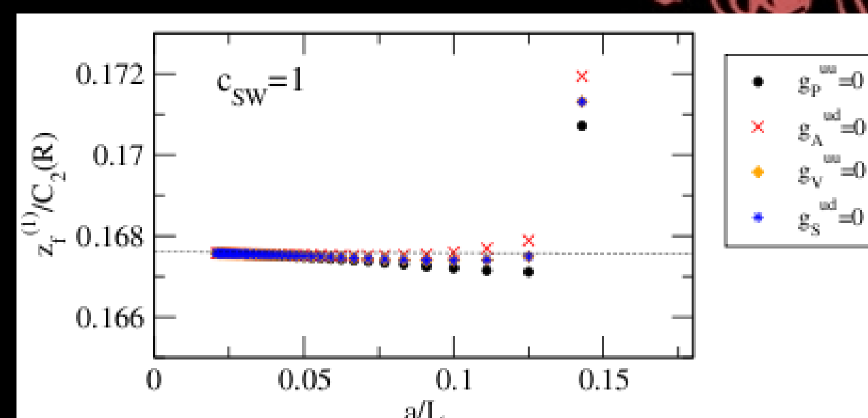
Renormalization and improvement

For m_c and z_f:

$$m_c^{(1)} : \begin{cases} -0.2025565(1) \times C_2(\mathcal{R}), & c_\mathrm{SW} = 1 \\ -0.325721(7) \times C_2(\mathcal{R}), & c_\mathrm{SW} = 0, \end{cases}$$

$$z_f^{(1)} : \begin{cases} 0.167572(2) \times C_2(\mathcal{R}), & c_\mathrm{SW} = 1 \\ 0.33023(6) \times C_2(\mathcal{R}), & c_\mathrm{SW} = 0, \end{cases}$$





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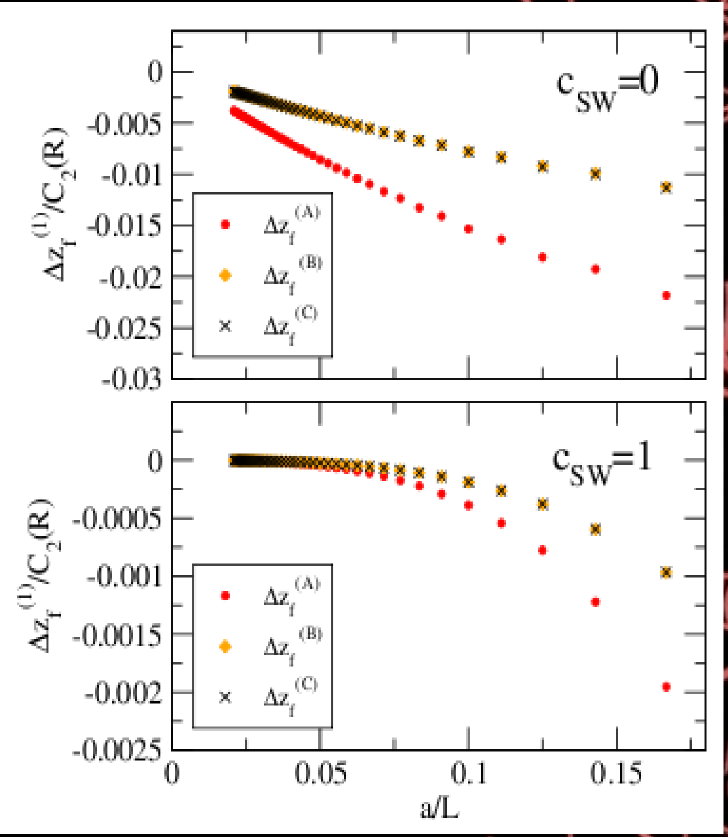
Different renormalization conditions

→ O(a) differences in z_f

$$\begin{cases} \Delta z_f^{(A)} = z_f^{(1)}\Big|_{g_A^{ud}} - z_f^{(1)}\Big|_{g_P^{uu'}} \\ \Delta z_f^{(B)} = z_f^{(1)}\Big|_{g_V^{uu'}} - z_f^{(1)}\Big|_{g_P^{uu'}} \\ \Delta z_f^{(C)} = z_f^{(1)}\Big|_{g_S^{ud}} - z_f^{(1)}\Big|_{g_P^{uu'}} \end{cases}$$



Renormalization and improvement

For d_s we demand O(a) effects to be absent from the ratio (several θ)

$$\left.\frac{\left[g_P^{ud}(x_0,\theta,a/L)\right]_R}{\left[g_P^{ud}(x_0,0,a/L)\right]_R}\right|_{x_0=T/2} \longrightarrow \quad d_\mathrm{s}^{(1)} = -0.0009(3)\times C_2(\mathcal{R})$$

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For c_{t} we demand O(a) effects to be absent from the SF coupling (standard definition through a background field).

$$\frac{1}{\overline{g}^2} = \frac{\partial\Gamma/\partial\eta|_{\eta=0}}{\partial\Gamma_0/\partial\eta|_{\eta=0}}$$

$$\longrightarrow c_t^{(1,1)} : \begin{cases} 0.006888(3), & c_{\mathrm{SW}} = 1 \\ -0.00661445(5), & c_{\mathrm{SW}} = 0, \end{cases}$$

Check automatic O(a) improvement

i) Correct realization of the boundary conditions

Boundary bilinears defined with oposite projectors

$$\mathcal{Q}_5 \sim \overline{\zeta}\Gamma\widetilde{Q}_{\pm}\zeta \qquad S(x,y)\overline{Q}_{+}\big|_{y_0=0} = 0$$

$$\widetilde{Q}_{\mp}$$

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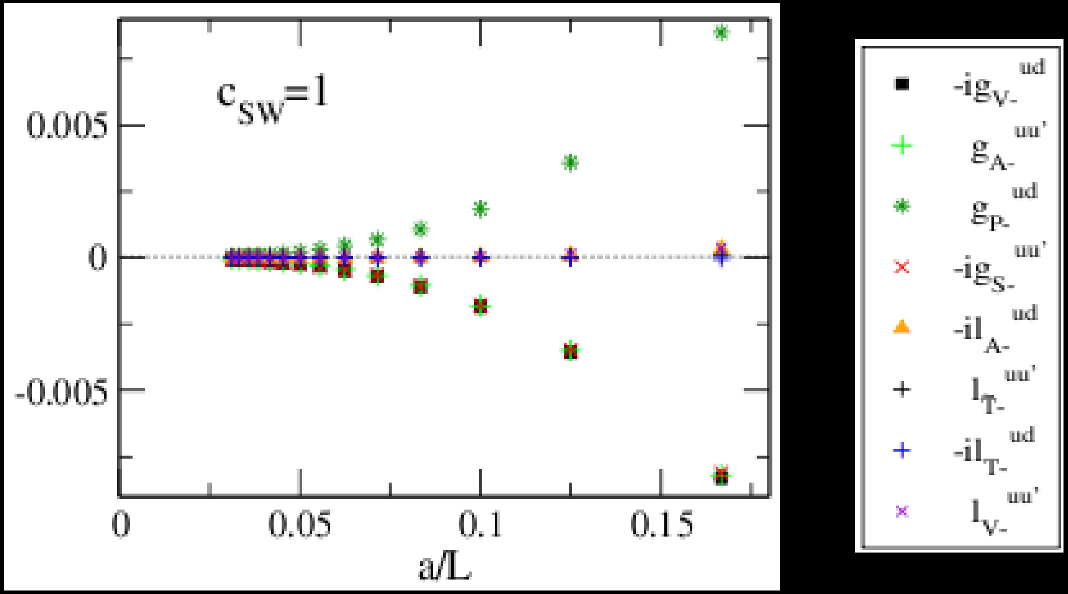

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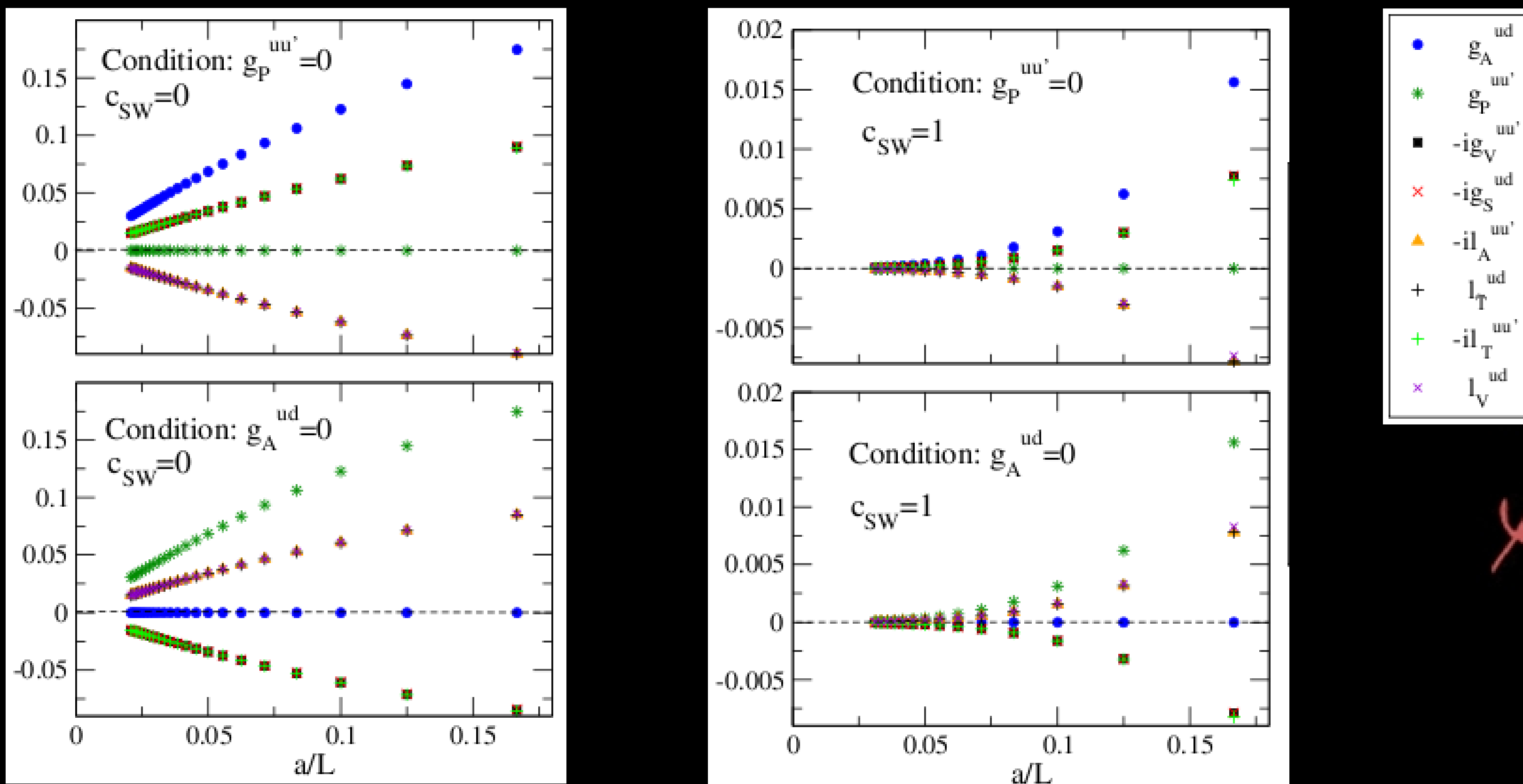

Check automatic O(a) improvement

iii) Universality between SF and χSF: Ratios between renormalized correlation functions converge.

$$\frac{g_{\mathrm{A}}^{uu'}(T/2)/\sqrt{g_1}}{f_{\mathrm{A}}(T/2)/\sqrt{f_1}} \longrightarrow 1 \qquad \frac{g_{\mathrm{P}}^{ud}(T/2)/\sqrt{g_1}}{f_{\mathrm{P}}(T/2)/\sqrt{f_1}} \longrightarrow 1$$

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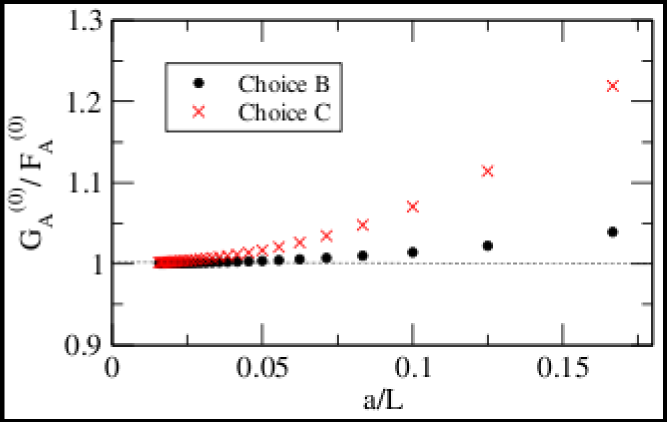
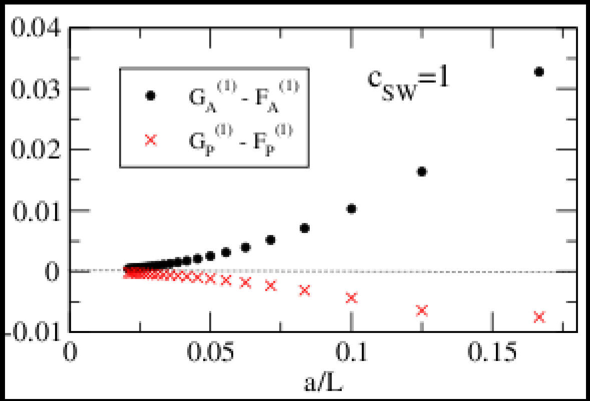
Tree-level;

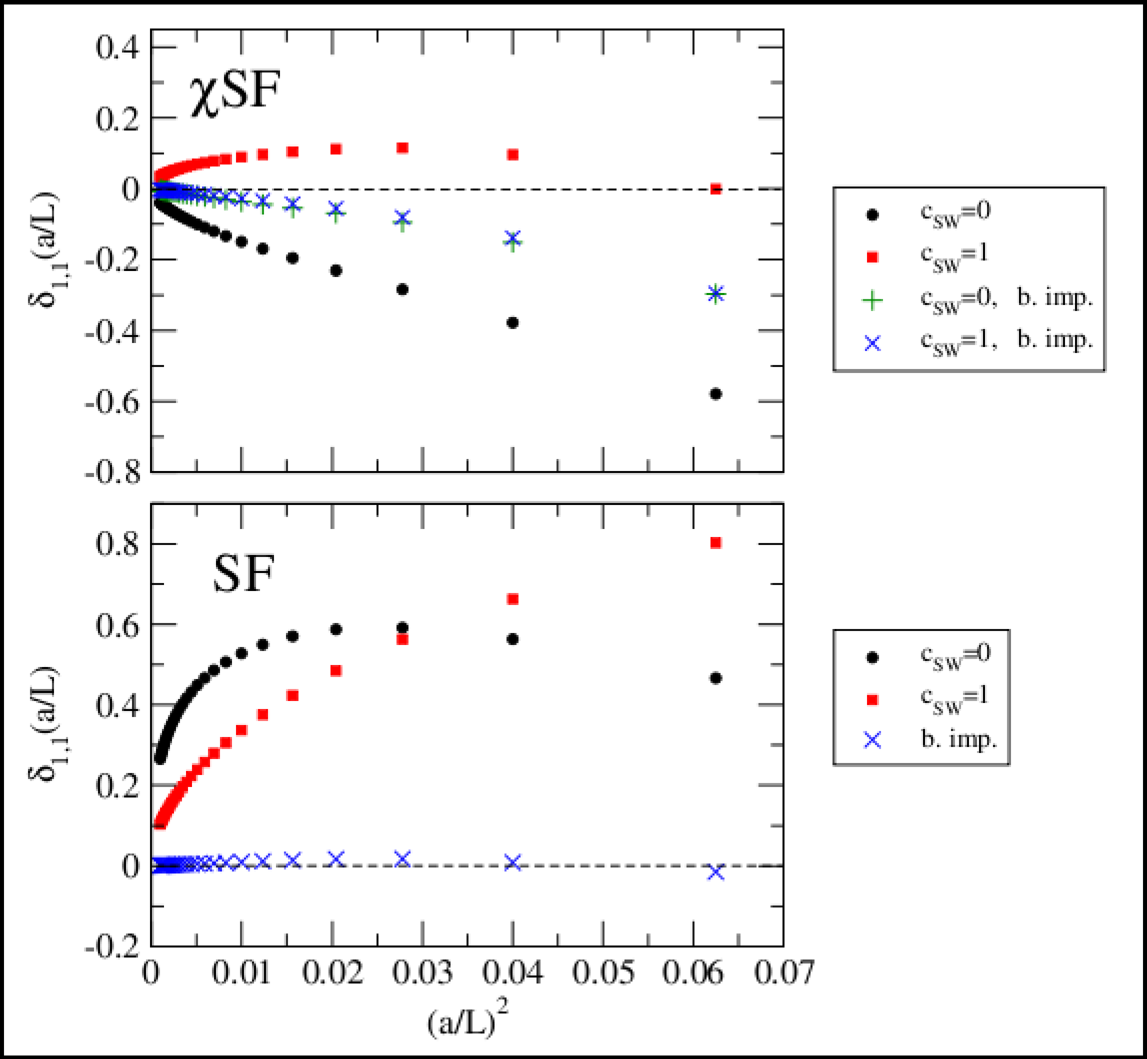


1-loop:



Setup ready!!!!!

Cutoff effects in the SSF



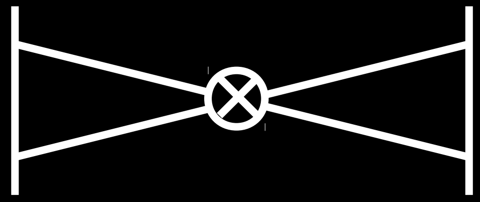
[S. Sint, P.V. '11-'12]

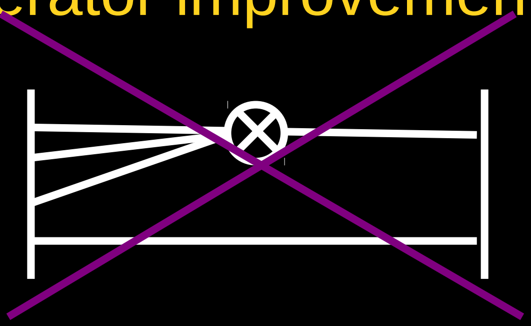
Applications:

- Computation of finite renormalization factors (see Mattia Dalla Brida's talk) [B. Leder, S. Sint, '10]

$$Z_A = i\,\frac{\mathrm{g}_{\widetilde{V}}^{ud}(T/2)}{\mathrm{g}_{A}^{uu}(T/2)} \qquad\qquad Z_V = \frac{\mathrm{g}_{\widetilde{V}}^{ud}(T/2)}{\mathrm{g}_{V}^{ud}(T/2)}$$

- Renormalization of 4-fermion operators:
  Simpler observables & no operator improvement





- Twist-2 operators. [J. Gonzalez Lopez et al, '12]

Conclusions

- The χSF implements the mechanism of automatic O(a) improvement in the SF setup.

- We have determined to 1-loop in PT the necessary coefficients for the renormalization and O(a) improvement of the setup: $m_{\mathrm{c}}^{(1)},\ z_{\mathrm{f}}^{(1)},\ d_{\mathrm{s}}^{(1)},\ c_{\mathrm{t}}^{(1)}$

- We have confirmed that after fixing these parameters, automatic O(a) improvement is at hold (at 1-loop).

- The running coupling has been computed to 1-loop in pt.

- See Mattia's talk for a Nf=2 dynamical calculation.

...... thank you very much !!!