The electroweak transition and the equation of state in the $SU(2)$-Higgs-model

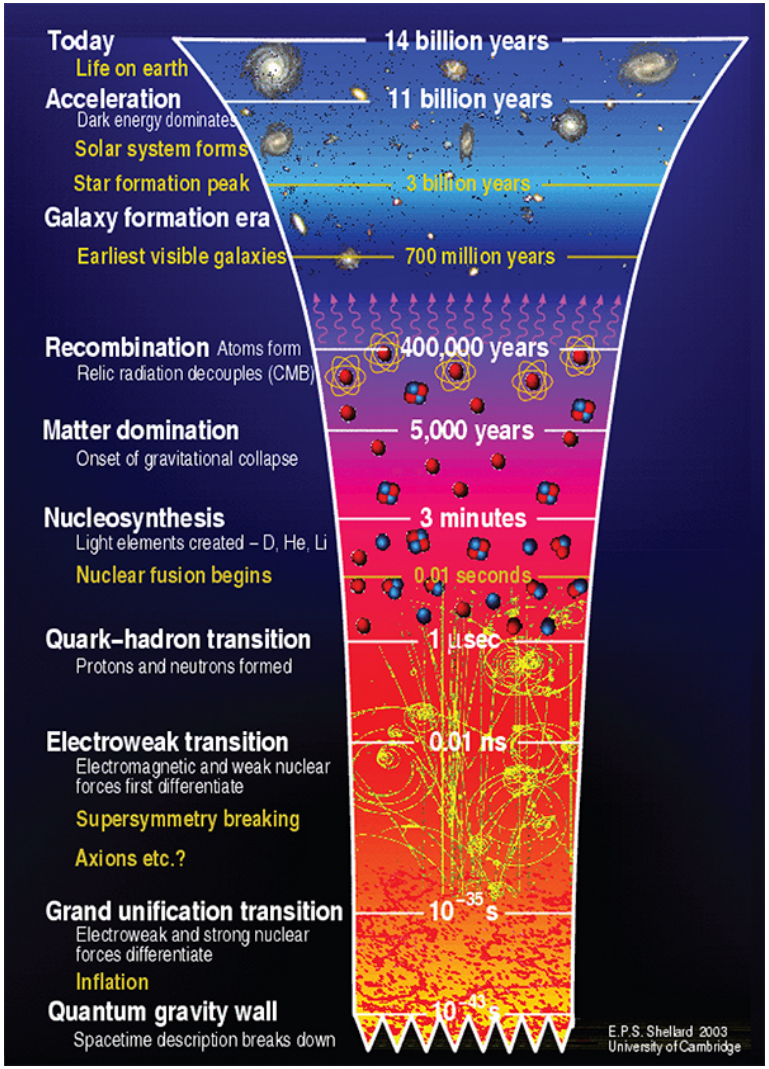
Jana Günther

Bergische Universität Wuppertal

June 27th 2014

Introduction

- with EoS the temporal development of the Universe can be described
- until 2012 calculation impossible, due to unknown m_H
- here first attempts to calculate the EoS in $SU(2)$-Higgs-model



The transition

- in 1996 by Kajantie, Laine, Rummukainen and Shaposhnikov:
- for $m_H \lesssim m_W$ first order phase transition
- for $m_H \gtrsim m_W$ cross over
- in 1998 by Aoki, Fodor, Csikor ...:
- critical endpoint (66.5 ± 1.4) GeV
- physical:
  - $m_W = 80.385$ GeV
  - $m_H = 125.9$ GeV
  - $m_H > m_W \longrightarrow$ cross over

The $SU(2)$-Higgs-model

- electroweak interaction: $SU(2) \times U(1)$
- $SU(2)$-Higgs-model $U(1)$ degrees of freedom are integrated out
- does not describe: strong interaction, fermions, $U(1)$-gauge-fields

The action

The continuum action:

$$S_K = \int \mathrm{d}^4x \, \frac{1}{4} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi D^\mu \phi + m_\mathrm{K} \phi_\mu \phi^\mu + \lambda_\mathrm{K} \left(\phi_\mu \phi^\mu\right)^2$$

discretisation on the lattice:

$$\int \mathrm{d}^4x \, D_\mu \phi D^\mu \phi \longrightarrow \sum_{x \in \Lambda} \sum_{\mu=1}^{4} \left(U_{x\mu} \phi_{x+\hat{\mu}} - \phi_x\right)^\dagger \left(U_{x\mu} \phi_{x+\hat{\mu}} - \phi_x\right)$$

$$\int \mathrm{d}^4x \, m_\mathrm{K}^2 \phi_\mu \phi^\mu + \lambda_\mathrm{K} \left(\phi_\mu \phi^\mu\right)^2 \longrightarrow \sum_{x \in \Lambda} m_\mathrm{K}^2 \phi_x^\dagger \phi_x + \lambda_\mathrm{K} \left(\phi_x^\dagger \phi_x\right)^2$$

Reparametrisation I

$$S_{\mathrm{S}} = \sum_{x\in\Lambda}\sum_{\mu=1}^{4}\left(U_{x\mu}\phi_{x+\hat{\mu}} - \phi_x\right)^{\dagger}\left(U_{x\mu}\phi_{x+\hat{\mu}} - \phi_x\right) + m_{\mathrm{K}}^2\phi_x^{\dagger}\phi_x + \lambda_{\mathrm{K}}\left(\phi_x^{\dagger}\phi_x\right)^2 .$$

$$\begin{aligned}
\lambda_{\mathrm{K}} &= \frac{\lambda}{\kappa} \\
m_{\mathrm{K}}^2 &= \frac{1-2\lambda}{\kappa} - 2 \\
\phi_x &= \sqrt{\kappa}\Phi_x .
\end{aligned}$$

$$S_{\mathrm{S}} = \sum_{x\in\Lambda}\Phi_x^{\dagger}\Phi_x + \lambda\left(\Phi_x^{\dagger}\Phi_x - 1\right)^2 - 2\kappa\sum_{\mu=1}^{4}\Re\left(\Phi_x^{\dagger}U_{x\mu}\Phi_{x+\hat{\mu}}\right) .$$

Reparametrisation II

$$S_S = \sum_{x\in\Lambda} \Phi_x^\dagger \Phi_x + \lambda \left(\Phi_x^\dagger \Phi_x - 1\right)^2 - 2\kappa \sum_{\mu=1}^{4} \Re\left(\Phi_x^\dagger U_{x\mu} \Phi_{x+\hat{\mu}}\right).$$

writing the scalar field as matrix:

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \Phi_2^* & \Phi_1 \\ -\Phi_1^* & \Phi_2 \end{pmatrix} = \varphi$$

the full lattice action:

$$S[U,\varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2}\,\mathrm{tr}\, U_{pl}\right)$$

$$+ \sum_x \left(\frac{1}{2}\,\mathrm{tr}(\varphi_x^\dagger \varphi_x) + \lambda \left(\frac{1}{2}\,\mathrm{tr}(\varphi_x^\dagger \varphi_x) - 1\right)^2 - \kappa \sum_{\mu=1}^{4} \mathrm{tr}(\varphi_x^\dagger U_{x\mu} \varphi_{x+\mu})\right).$$

Used Algorithms

For simulations on GPUs:

- Heatbath-Algorithm
- Overrelaxation-Algorithm

For comparison on CPUs:

- HMC

Successfull comparison with

- Simulating the electroweak phase transition in the SU(2) higgs model
- by Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay
- published 1995 in Nucl. Phys. B439

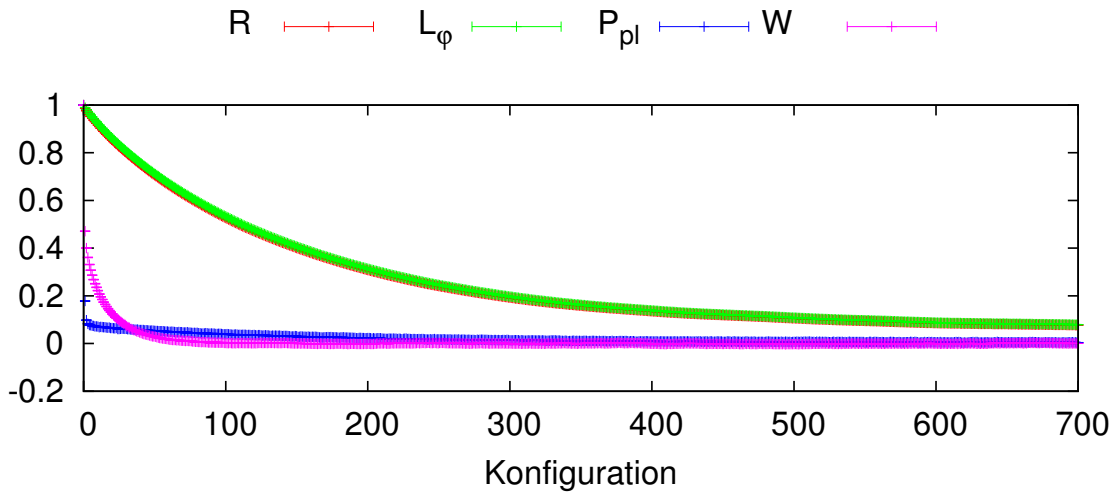
Simple observables

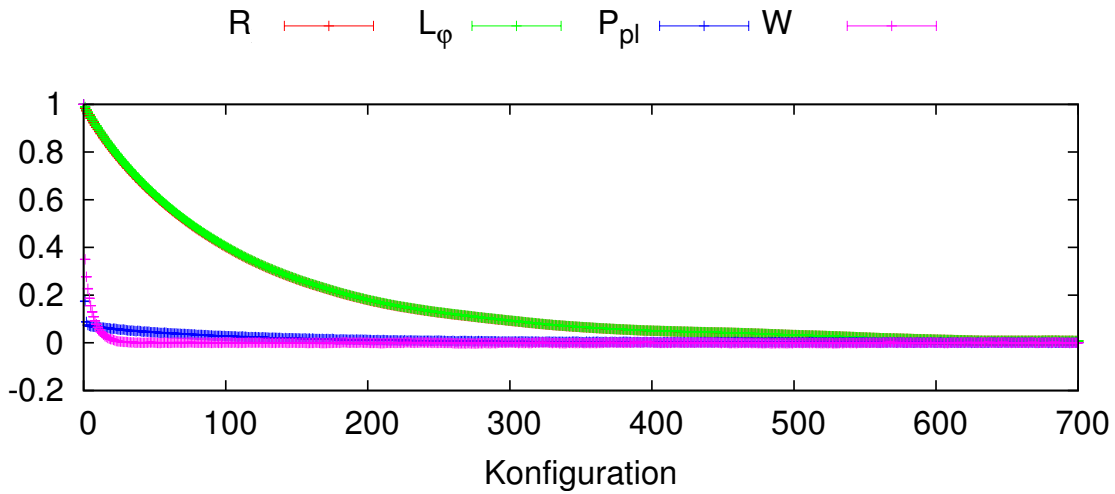
$$S[U,\varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2}\,\mathrm{tr}\, U_{pl}\right)$$
$$+ \sum_{x} \left(\frac{1}{2}\,\mathrm{tr}(\varphi_x^\dagger \varphi_x) + \lambda \left(\frac{1}{2}\,\mathrm{tr}(\varphi_x^\dagger \varphi_x) - 1\right)^2 - \kappa \sum_{\mu=1}^{4} \mathrm{tr}(\varphi_x^\dagger U_{x\mu} \varphi_{x+\mu})\right)$$

The observables:

$$R_x = \det \varphi_x = \frac{1}{2}\,\mathrm{tr}\left(\varphi_x^\dagger \varphi_x\right) = \rho_x^2$$
$$L_{\varphi,x\mu} = \frac{1}{2}\,\mathrm{tr}\left(\varphi_x^\dagger U_{x\mu} \varphi_{x+\hat{\mu}}\right)$$
$$P_{Pl} = 1 - \frac{1}{2}\,\mathrm{tr}\, U_{pl}$$
$$Q_x = \left(\rho_x^2 - 1\right)^2$$
$$S_x = 6\beta P_{Pl} + R_x + \lambda Q_x - 8\kappa L_{\varphi,x\mu}$$

Autocorrelation time without and with overrelaxation





Correlators

Used correlators for the higgs boson mass:

$$R_x = \det \varphi_x$$
$$L_{\varphi,x\mu} = \frac{1}{2} \operatorname{tr} \left(\varphi_x^\dagger U_{x\mu} \varphi_{x+\hat{\mu}} \right)$$

Used correlators for the W boson mass:

$$W_{xrk}^{(n)} = \frac{1}{2} \operatorname{tr} \left(\sigma_r \varphi_x^\dagger U_{xk} U_{x+\hat{k},k} U_{x+2\hat{k},k} \cdots U_{x+(n-1)\hat{k},k} \varphi_{x+n\hat{k}} \right)$$

(Bunk, Ilgenfritz, Kripfganz, Schiller: The finite-temprature phase transition in lattice SU(2) Higgs theory at weak couplings, Nucl. Phys. B403 (1993))

Renormalized coupling

The renormalized coupling

$$g_R^2 = \frac{16}{3}\pi A$$

can be calculated from the static potential

$$V(R) = C - \frac{A}{R}e^{-MR} + DG(M,R)$$

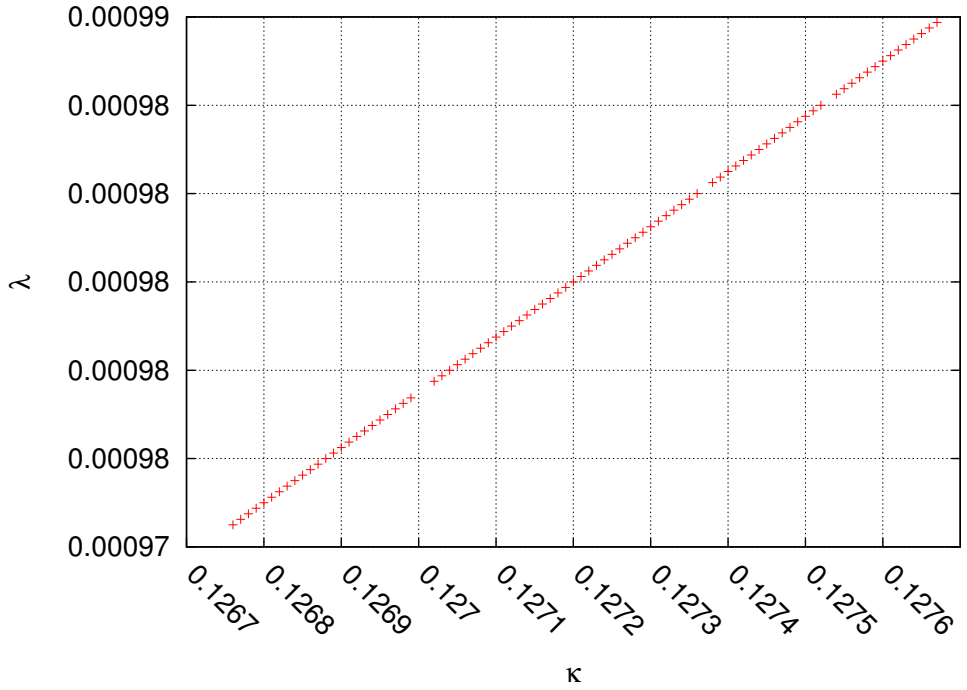
- A, C, D, M are fit parameter (M is screening mass)
- $G(M,R)$ are lattice corrections (1986 by Langguth,Montvay, Weisz)

The static potential can be calculated from the Wilson-Loops

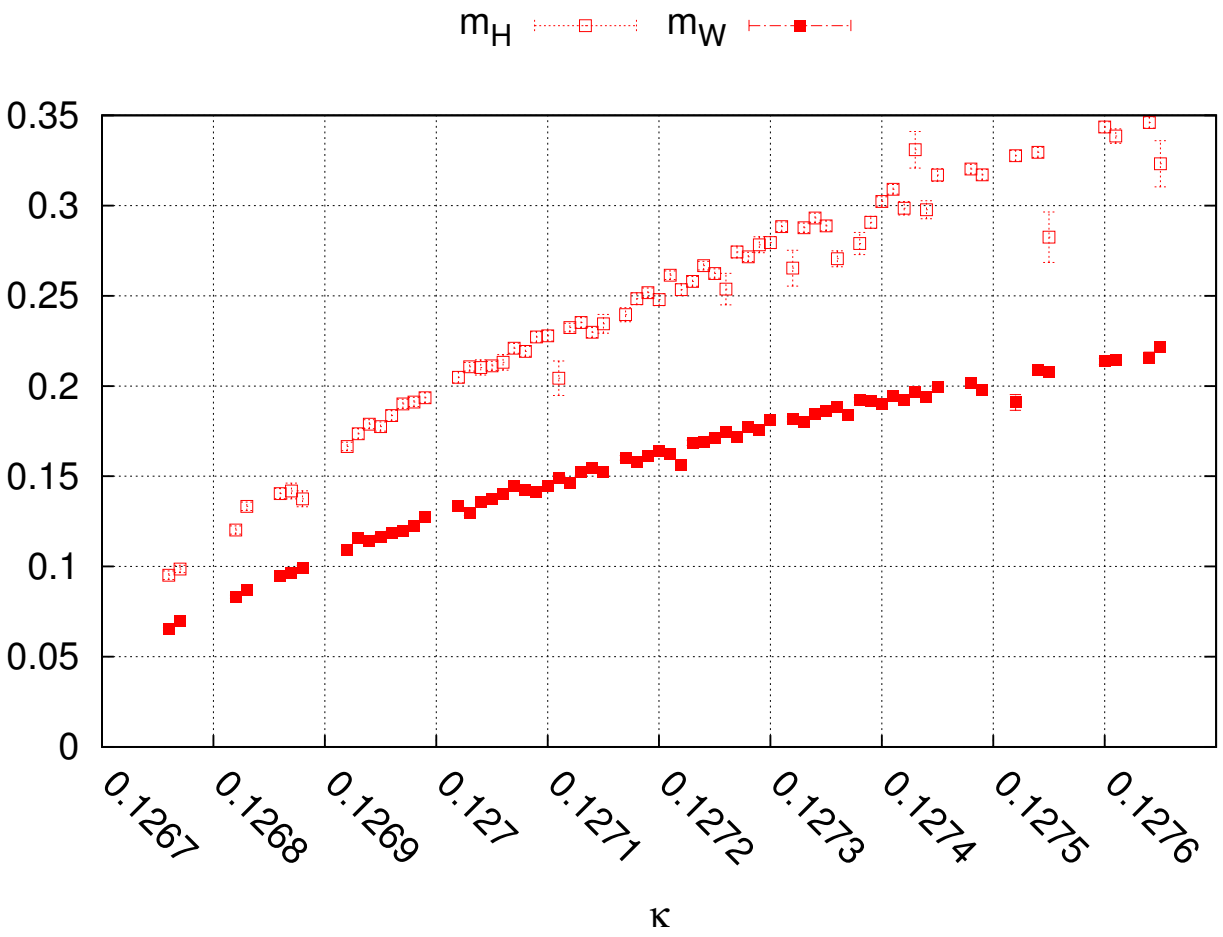
$$V(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\mathcal{W}(R,T)$$

Line of constant physics

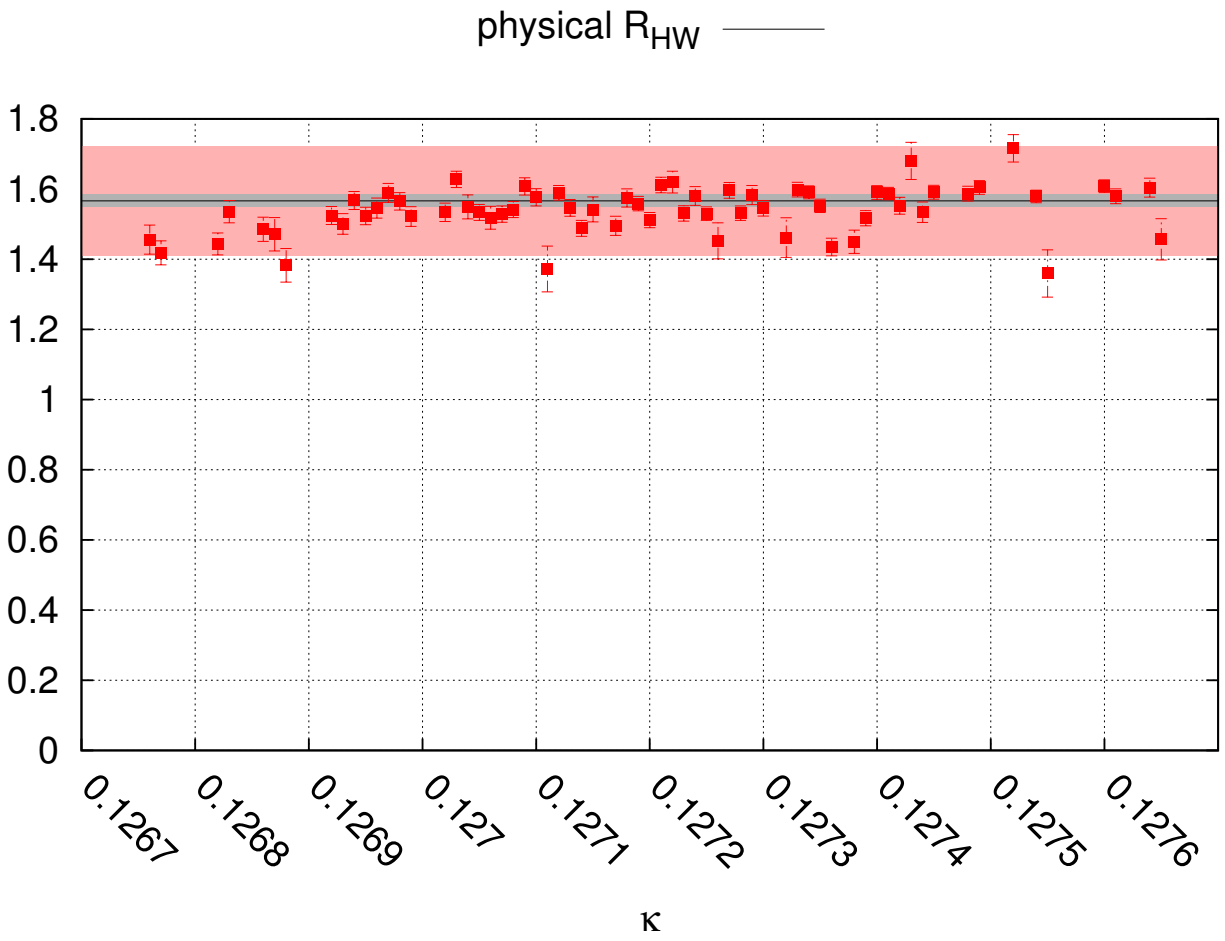
- Latttice: 32^4
- β is constant
- λ depends linear on κ



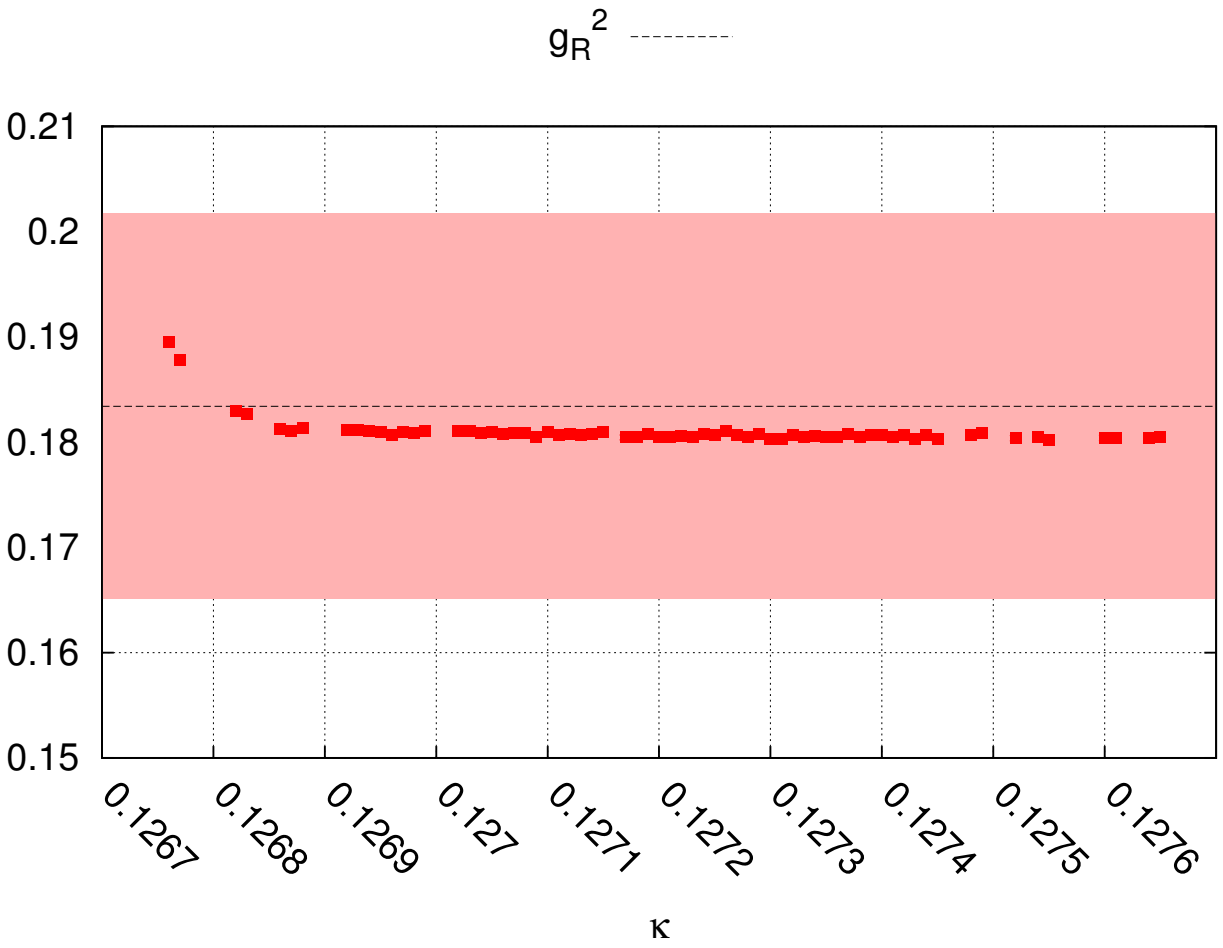
Masses


m_H
m_W
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1267
0.1268
0.1269
0.127
0.1271
0.1272
0.1273
0.1274
0.1275
0.1276
κ

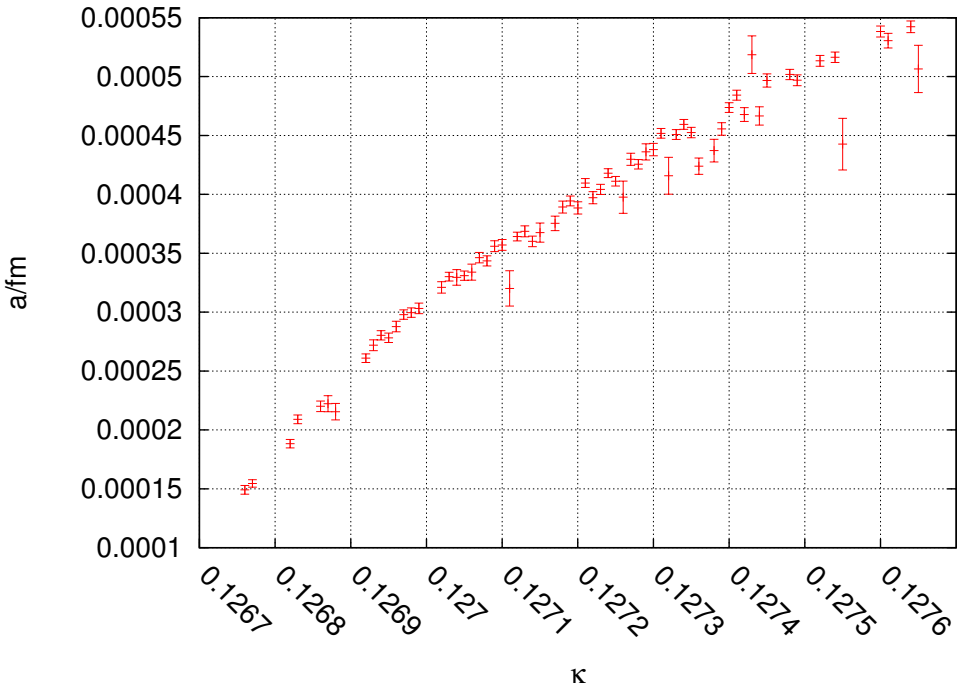
$$R_{HW} = \frac{m_H}{m_W}$$



g_R^2

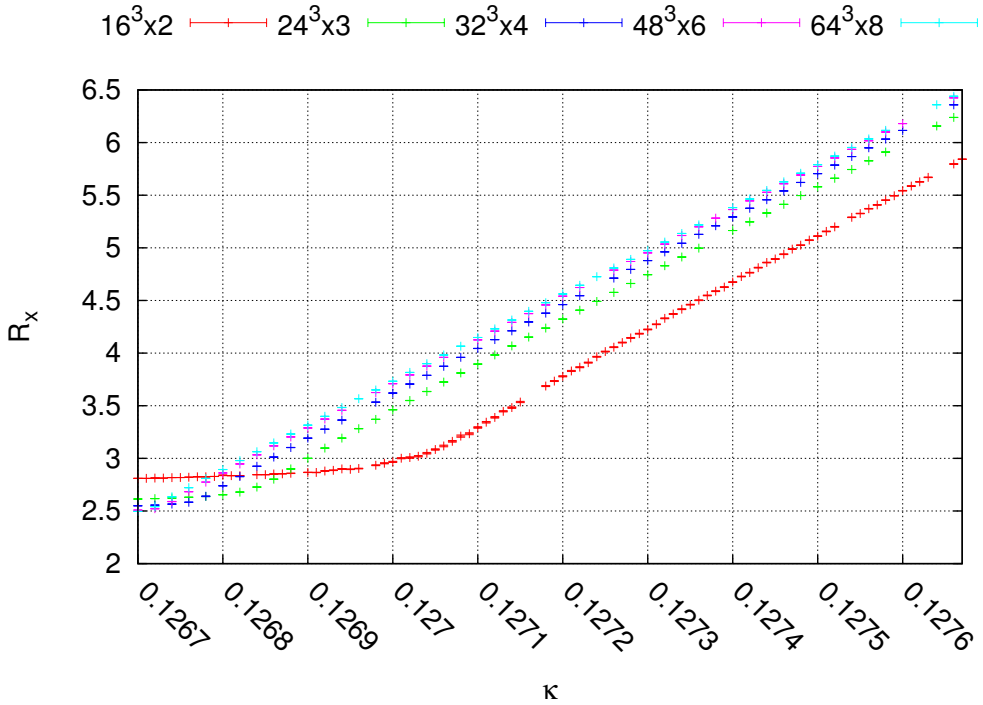

g_R^2
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.1267
0.1268
0.1269
0.127
0.1271
0.1272
0.1273
0.1274
0.1275
0.1276
κ

The scale



Electroweak scale is 0.0008 fm (246 GeV)

The transition



$$R_x = \det \varphi_x = \frac{1}{2} \operatorname{tr}\left(\varphi_x^\dagger \varphi_x\right) = \rho_x^2$$

Equation of state

- Interaction measure:

$$I = -\frac{N_t^3}{N_s^3} m_H \left(\frac{\partial \ln Z}{\partial \beta} \frac{\partial \beta}{\partial \kappa} \frac{\mathrm{d}\kappa}{\mathrm{d}m_H} + \frac{\partial \ln Z}{\partial \lambda} \frac{\partial \lambda}{\partial \kappa} \frac{\mathrm{d}\kappa}{\mathrm{d}m_H} + \frac{\partial \ln Z}{\partial \kappa} \frac{\mathrm{d}\kappa}{\mathrm{d}m_H} \right)$$
$$= -N_t^4 m_H \left(-6\langle P_{\mathrm{Pl}} \rangle \frac{\partial \beta}{\partial \kappa} \frac{\partial \kappa}{\partial m_H} - \langle Q \rangle \frac{\partial \lambda}{\partial \kappa} \frac{\partial \kappa}{\partial m_H} + 8\langle L_\varphi \rangle \frac{\partial \kappa}{\partial m_H} \right)$$

- Pressure:

$$\frac{p}{T^4} = N_t^4 \int_{(\kappa_0,\beta_0,\lambda_0)}^{(\kappa,\beta,\lambda)} \mathrm{d}(\kappa,\beta,\lambda) \left(\frac{1}{N_t N_s^3} \begin{pmatrix} \frac{\partial \ln Z}{\partial \kappa} \\ \frac{\partial \ln Z}{\partial \beta} \\ \frac{\partial \ln Z}{\partial \lambda} \end{pmatrix} - \frac{1}{N_{s0}^3 N_{t0}} \begin{pmatrix} \frac{\partial \ln Z_0}{\partial \kappa} \\ \frac{\partial \ln Z_0}{\partial \beta} \\ \frac{\partial \ln Z_0}{\partial \lambda} \end{pmatrix} \right)$$
$$= N_t^4 \sum -6\Delta\beta\langle P_{\mathrm{Pl}} \rangle - \Delta\lambda\langle Q \rangle + 8\Delta\kappa\langle L_\varphi \rangle$$

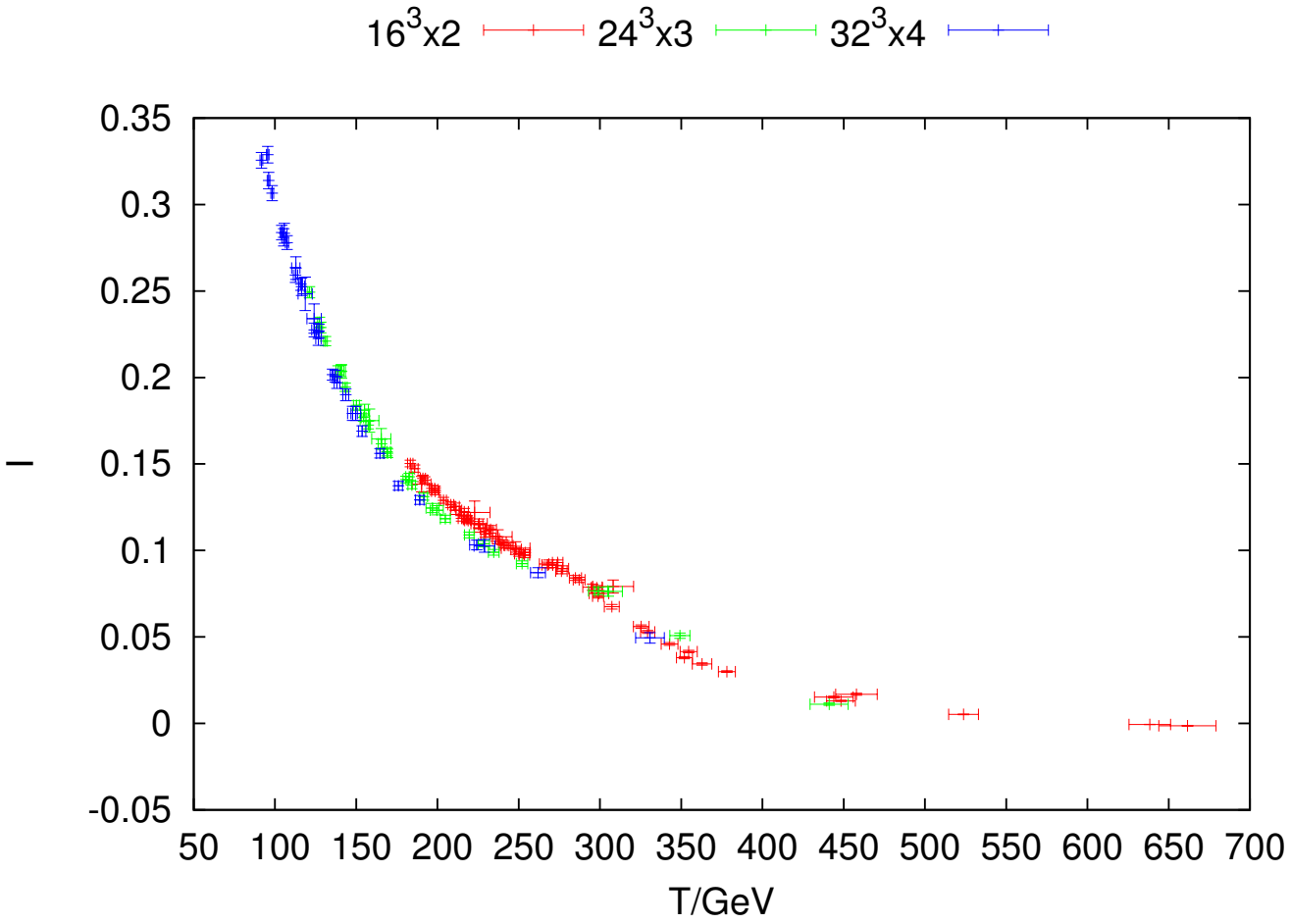
- Energy density:

$$\frac{\epsilon}{T^4} = I + \frac{3p}{T^4}$$

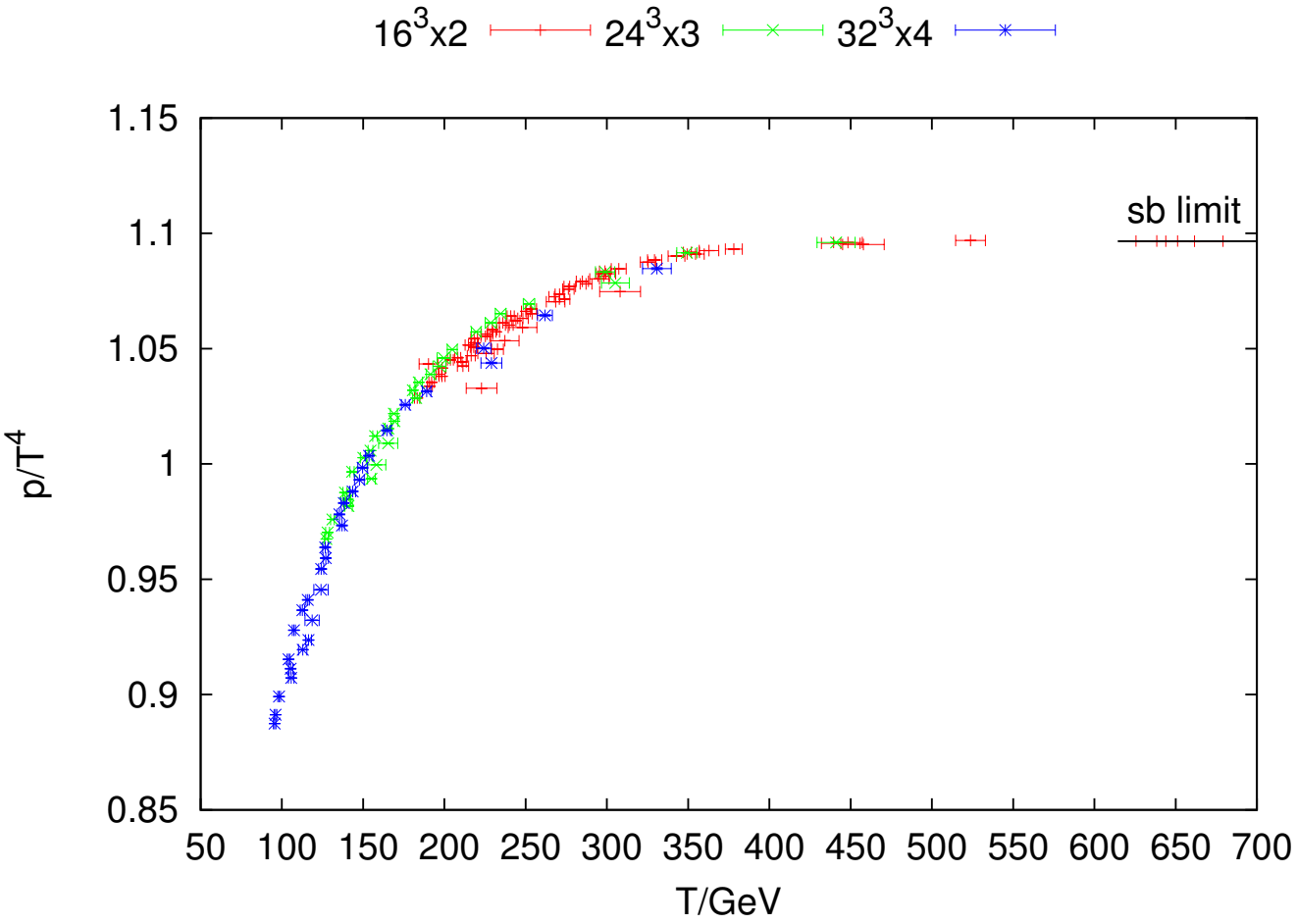
- Entropy density:

$$\frac{s}{T^3} = \frac{\epsilon}{T^4} + \frac{p}{T^4}$$

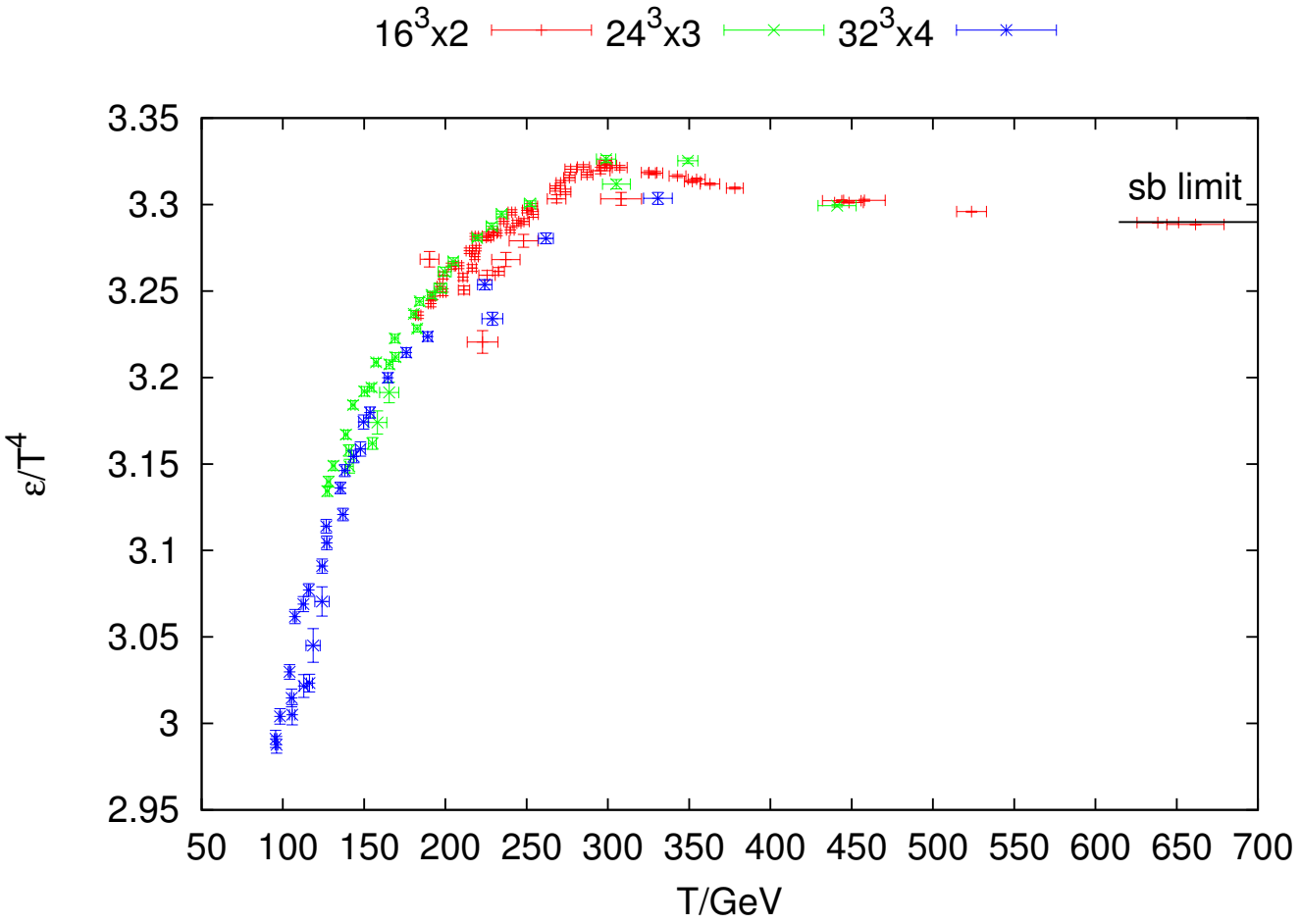
Interaction measure I



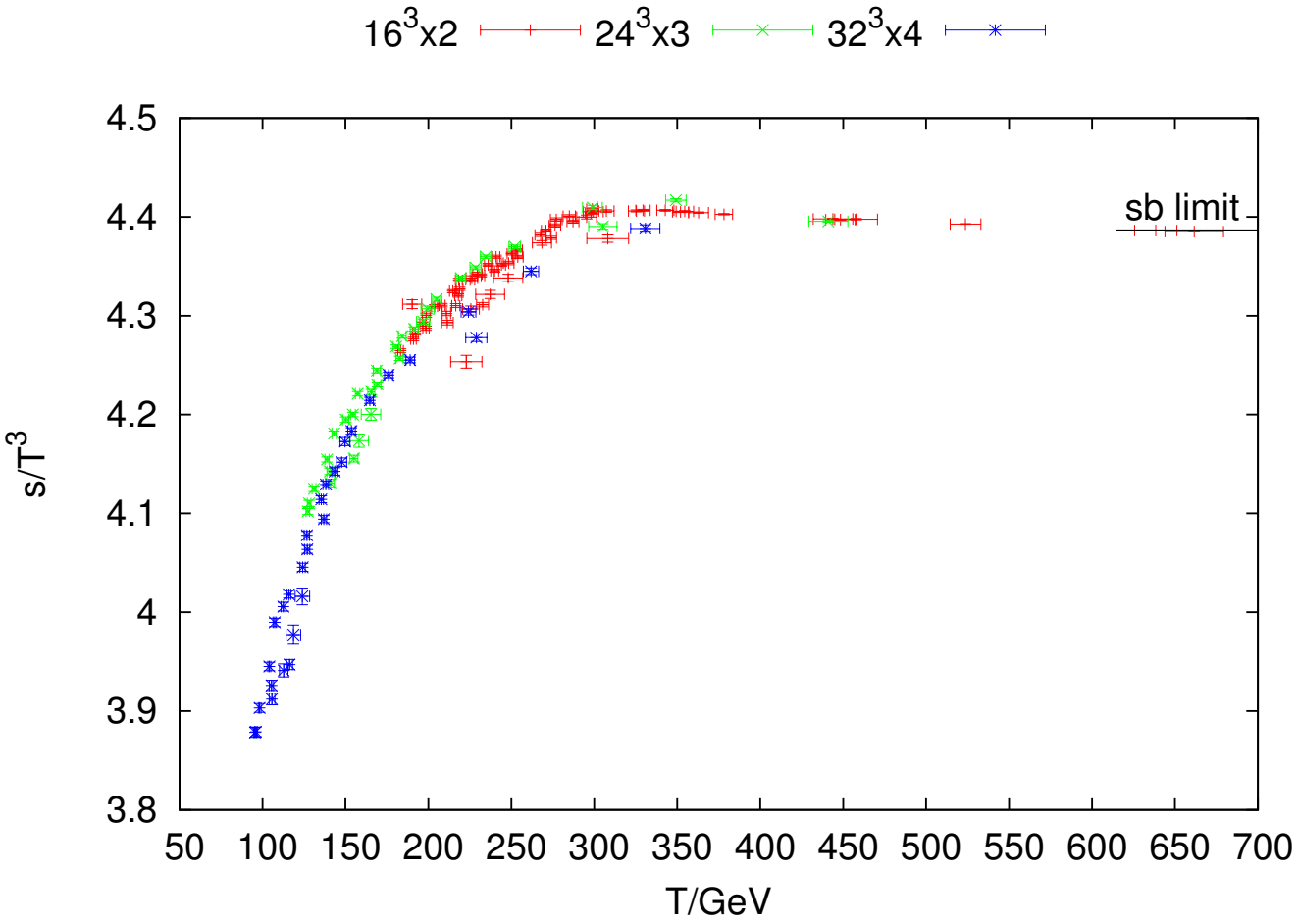
Pressure p


16³x2
24³x3
32³x4
sb limit
p/T⁴
T/GeV
1.15
1.1
1.05
1
0.95
0.9
0.85
50
100
150
200
250
300
350
400
450
500
550
600
650
700

Energy density ϵ



Entropy density s



Work in progress

- Larger lattices will hopefully make it possible to got to smaller masses.
- First results from tuning on 48^4 lattices with GPUs.
- Blue Gene/Q program is ready for larger lattices.