Scalar correlators near the 3-flavor thermal critical point

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Introduction

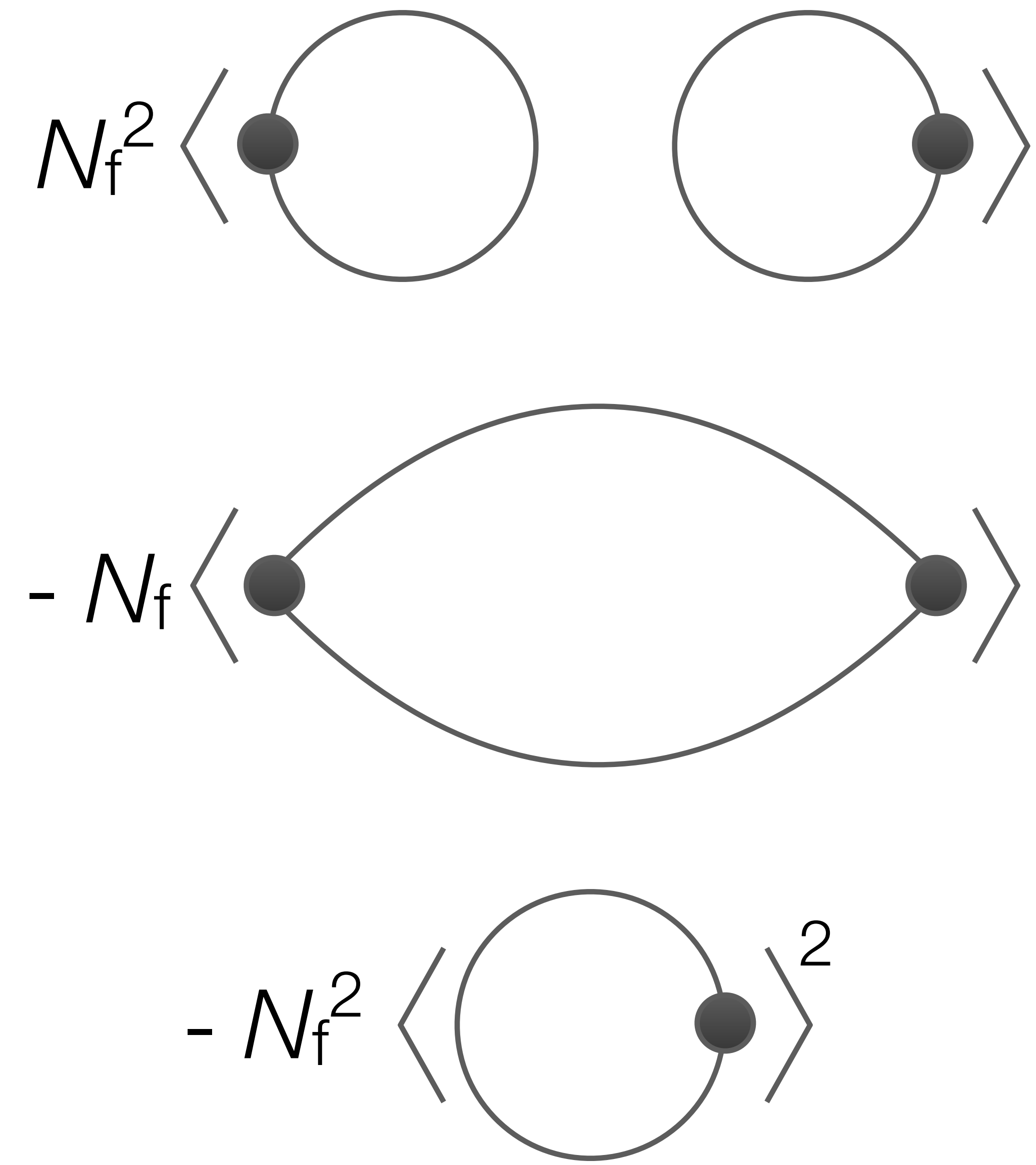
- 3-flavor finite temperature simulations using the clover fermions and the Iwasaki gauge with $N_t = 8$
- Evolution uses BQCD; measurement uses my modified code (developed in Helsinki, optimized for K by Jarno)
- The scalar singlet is the only massless mode at the chiral critical point (Pisarski & Wilczek 1984, Gavin et al 1994)
- Naive staggered simulations ($N_t = 4$) provided evidence (JLQCD 1999, Liao 2002); no improvement since

Contents

- Measurement techniques
  - Hierarchical truncation with stochastic probing
  - Truncated solver method + Probing + Random sources
  - 40× speedup in measuring Tr[D^{-1}] on one configuration
- Physics results
  - First order transition at two different parameter sets
  - Two stable states on both sides of the transition
  - Screening masses with the singlet scalar on the transition line

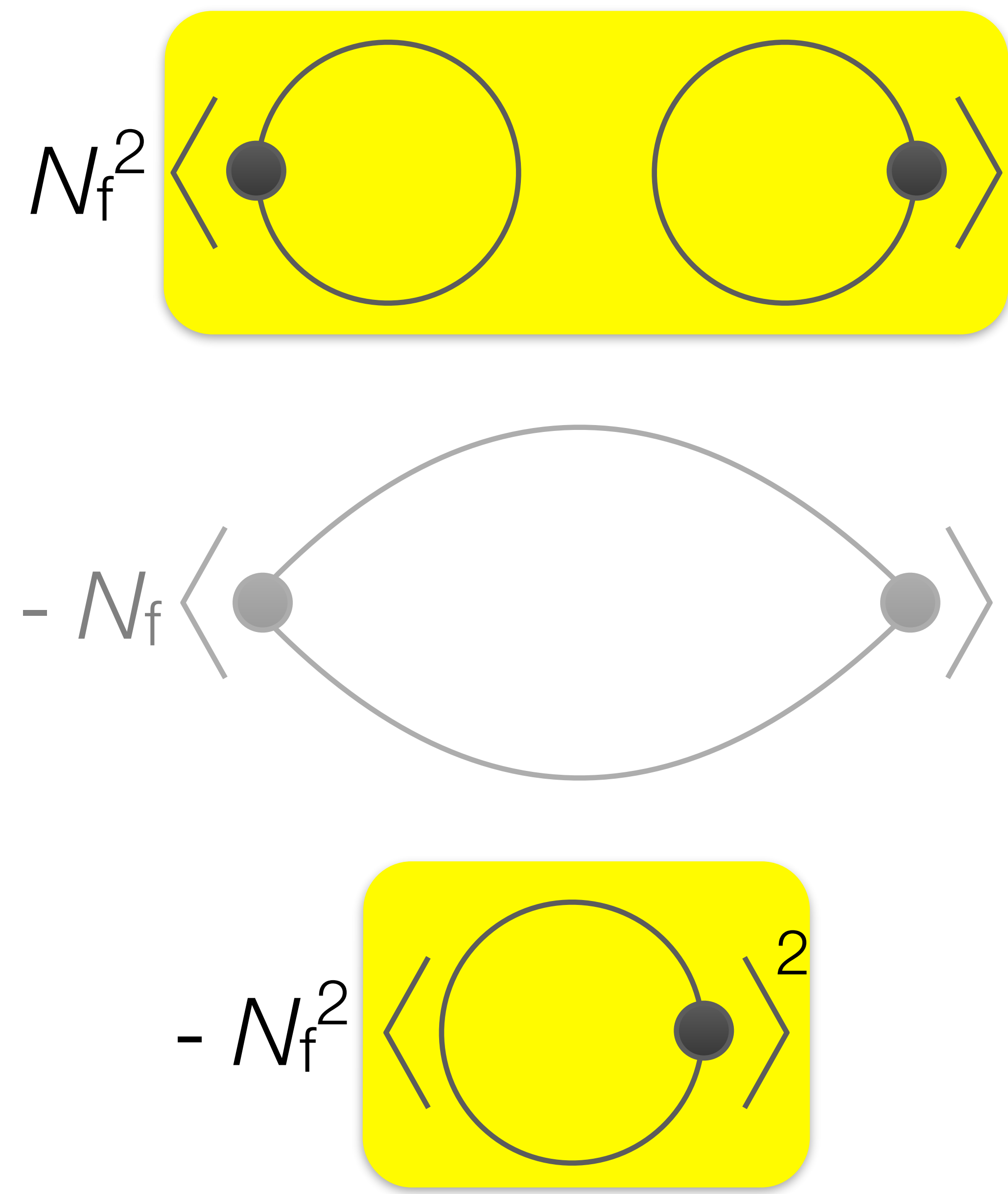
Singlet propagators

It is hard



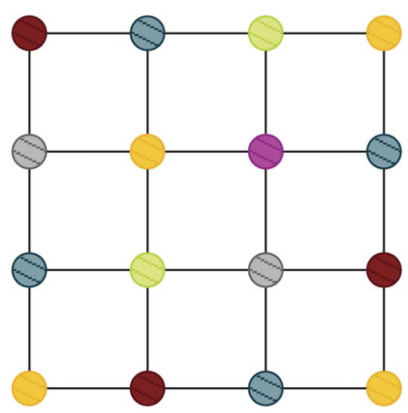
Singlet propagators

It is hard



Nearest neighbor action

Minimum links between same colored site, $d_{\min} = 3$



$$
\begin{array}{cccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
$$

$$
V_6^{\mathrm{T}} = \left[\begin{array}{cccc|cccc|cccc|cccc}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

$$
(B_\varepsilon V_6)^{\mathrm{T}} = \left[\begin{array}{cccc|cccc|cccc|cccc}
b_{11} & * & * & b_{44} & * & * & * & * & * & b_{10,10} & * & * & * & * & * & b_{16,16} \\
* & b_{22} & * & * & * & * & * & b_{88} & * & * & * & * & b_{13,13} & * & * & * \\
* & * & b_{33} & * & b_{55} & * & * & * & * & * & * & b_{12,12} & * & b_{14,14} & * & * \\
* & * & * & * & * & b_{66} & * & * & * & * & * & * & * & * & b_{15,15} & * \\
* & * & * & * & * & * & b_{77} & * & b_{99} & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & b_{11,11} & * & * & * & * & *
\end{array}\right]
$$

Ignore the off-diagonals
if they fall off quickly

Probing

Tang & Saad 2012

Probing as a form of space-time dilution

- Number of diluted vectors for $32^3 \times 8$ using the greedy multi-coloring algorithm (Saad 2003)

d_{min}	2	3	4	5	6	7	8
No.	2	23	16	120	210	411	256

- Not only the upfront cost is impractical
  - Uniform sources generate bias from off-diagonal terms
  - It is hard to pick d_{min} beforehand (one solution offered by Stathopoulos et al 2013)

"Use no force,

but the random source.

Never let the odds stop you."

–A Lattice Field Theorist

$d_{\min}$	2	3	4	5	6	7	8
No.	2	23	16	120	210	411	256
$\sigma_{\mathrm{imp}}/\sigma$ (25 CG iter) @ same cost	0.82		0.58				0.28

Require spin-color separation to get an improvement: $\times 12$

“Use no force,

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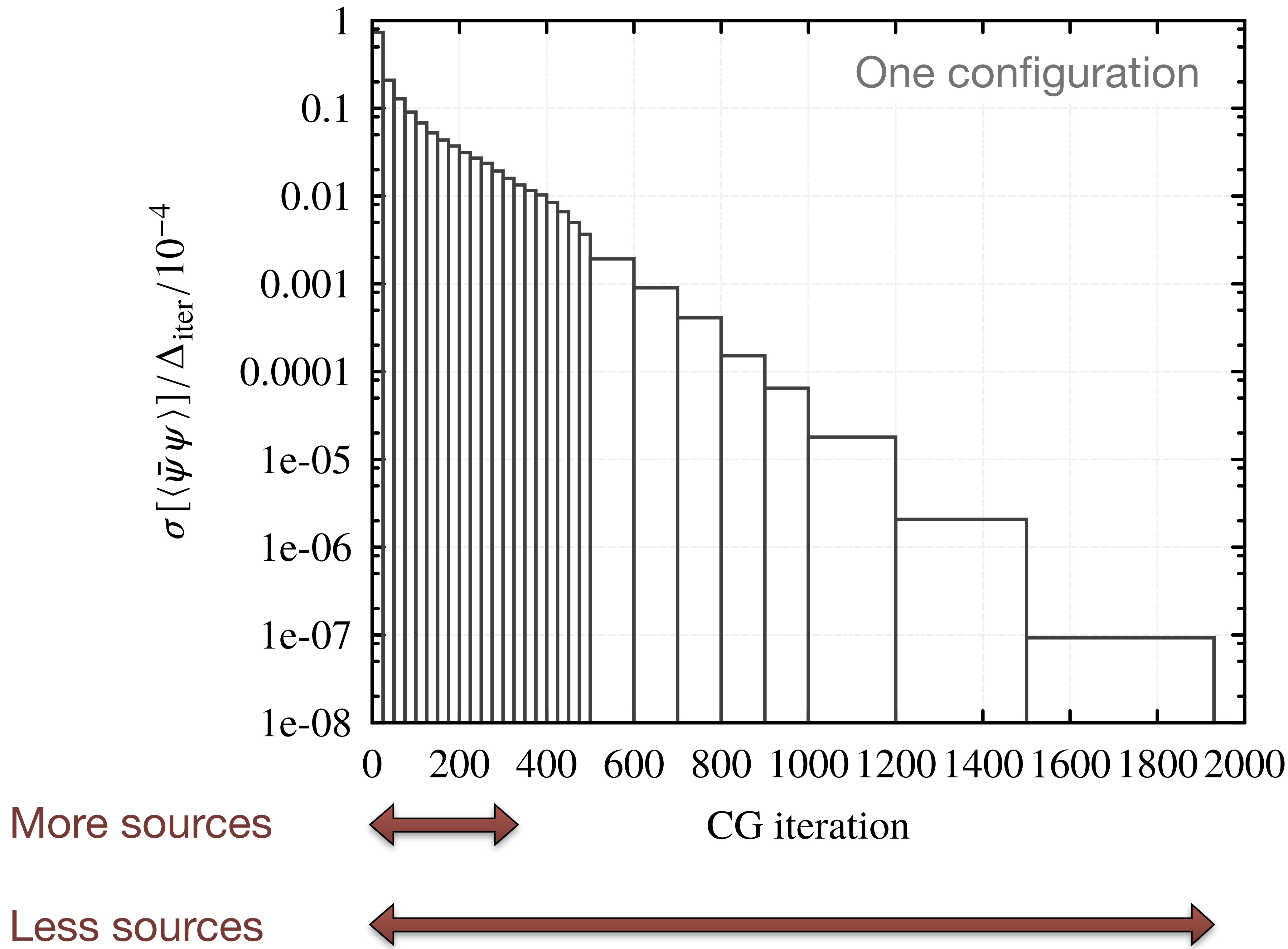
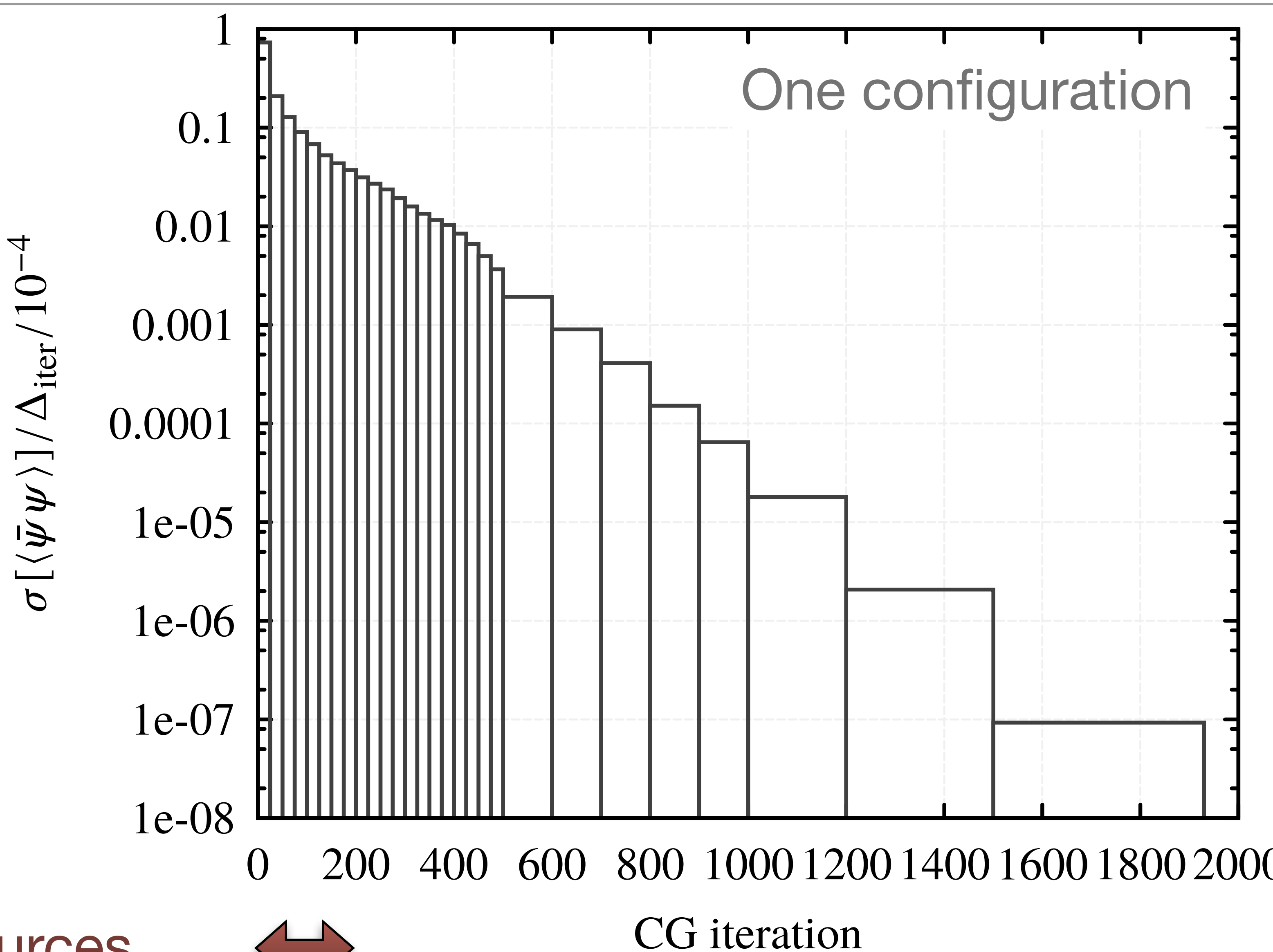
“BUT I CAN’T PAY THE COST!”



Truncated solver method (TSM)

Collins et al 2007

Multi-level truncation



More sources

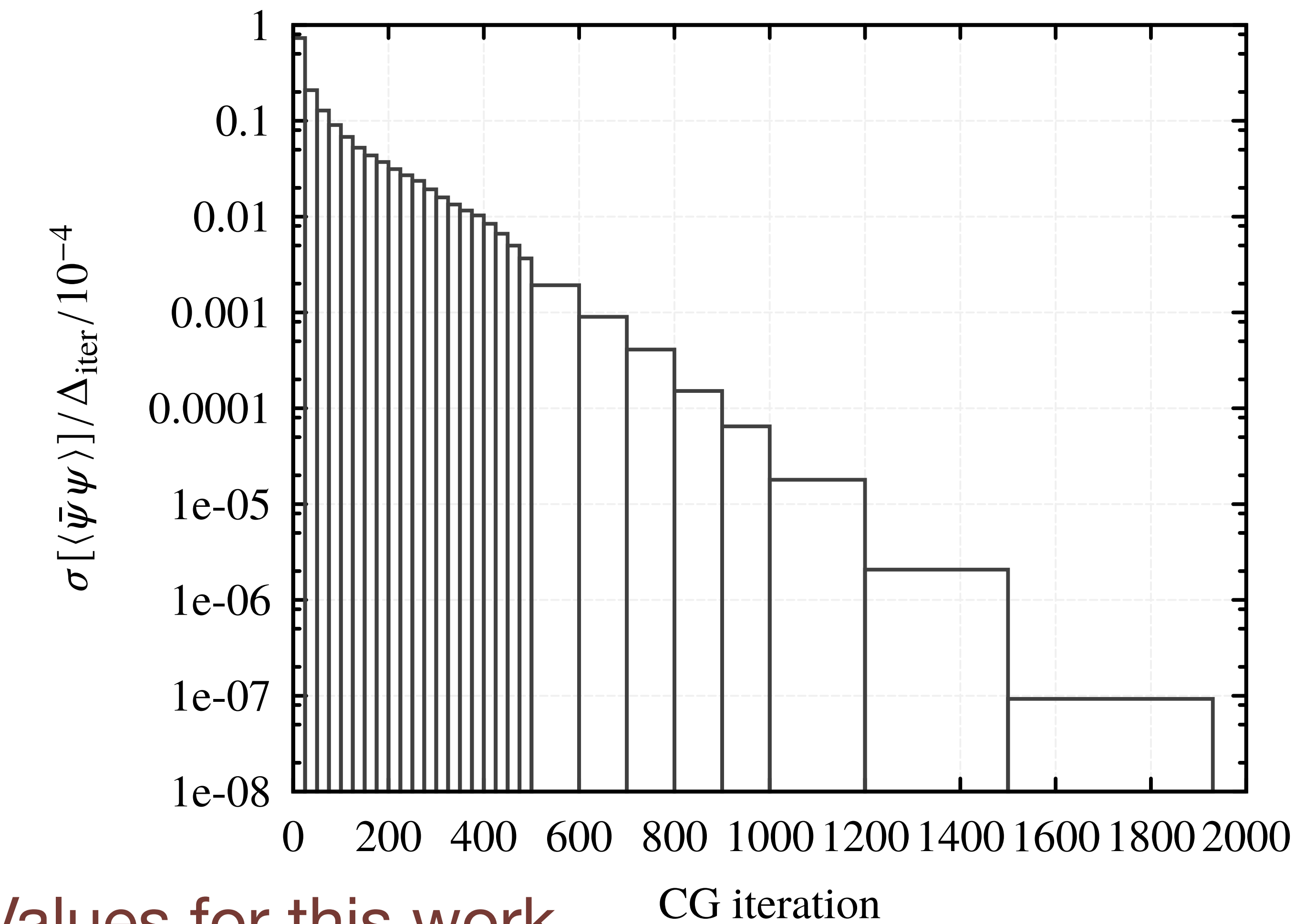
Less sources

Much less

Hierarchical truncation with stochastic probing

- Spin-color separated, space-time diluted random sources with minimum distance between non-zero entries, d_{min}, as large as possible

- Apply different d_{min} hierarchically on CG truncations, respecting cost constraints



Values for this work

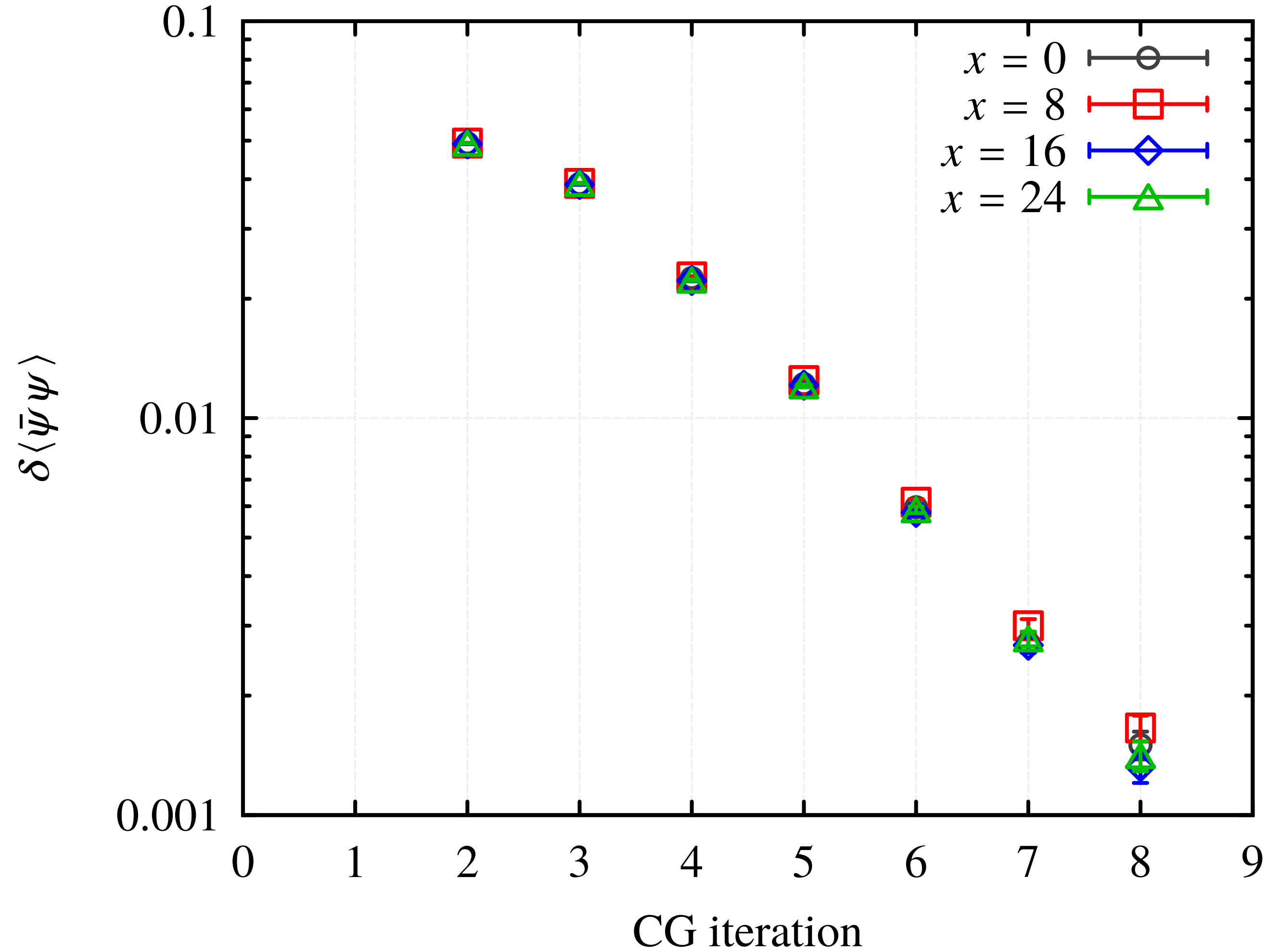
$d_{\mathrm{min}} = 8$

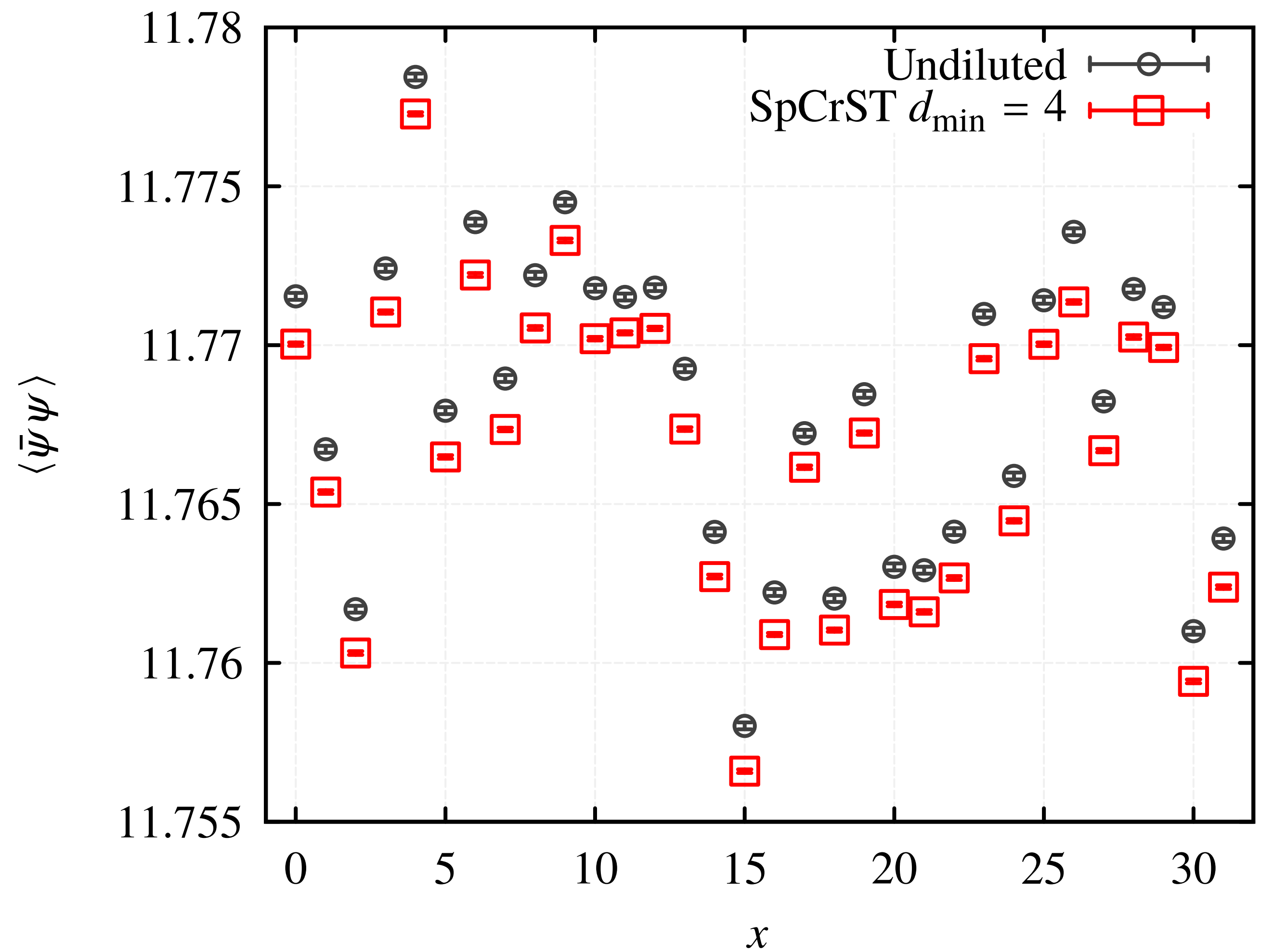
$d_{\mathrm{min}} = 4$

$d_{\mathrm{min}} = 2$

Truncation with & without dilution

- Compare 'undiluted' random sources and 'spin-color separated, space-time diluted' random sources
- Measured on one configuration up to the 8th CG iteration
- Exponential decay (no proof yet)





Computed improvement on one configuration

- Computed from variance/covariance distribution of CG iterations
- R_{imp}, reduction in variance with equal cost
- Constraint of cost ~500 full CG inv.

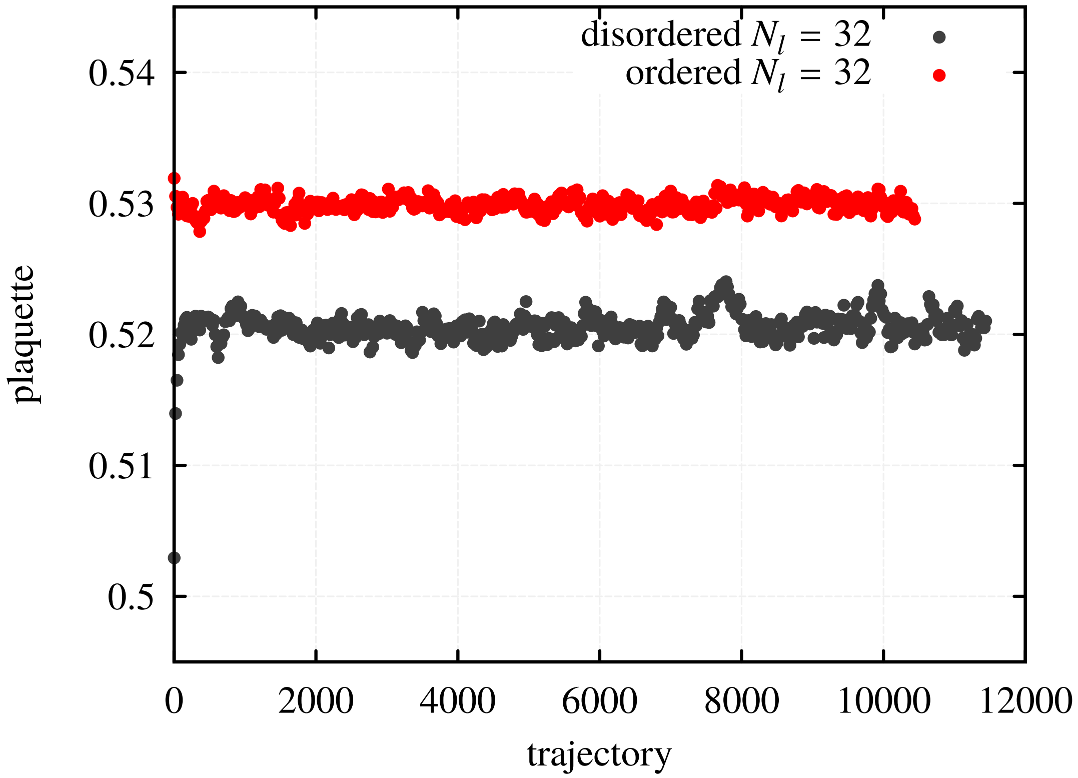
Scenario	TSM	2L-TSM	3L-TSM	2L-HTwSP
Cost	500.3	493.2	486.2	501.9
R_{imp}	0.203(4)	0.119(4)	0.102(3)	0.0231(6)
N_{h}	99	21	5	1
$\mathrm{Iter}_{\mathrm{h}}$	2049	2049	2049	2049
C_{h}	1	1	1	24
N_{l}	2990	1312	442	4
$\mathrm{Iter}_{\mathrm{l}}$	275	425	475	475
C_{l}	1	1	1	192
N_{l2}		8200	2964	1
$\mathrm{Iter}_{\mathrm{l2}}$		50	150	200
C_{l2}		1	1	3072
N_{l3}			13255	
$\mathrm{Iter}_{\mathrm{l3}}$			25	
C_{l3}			1	

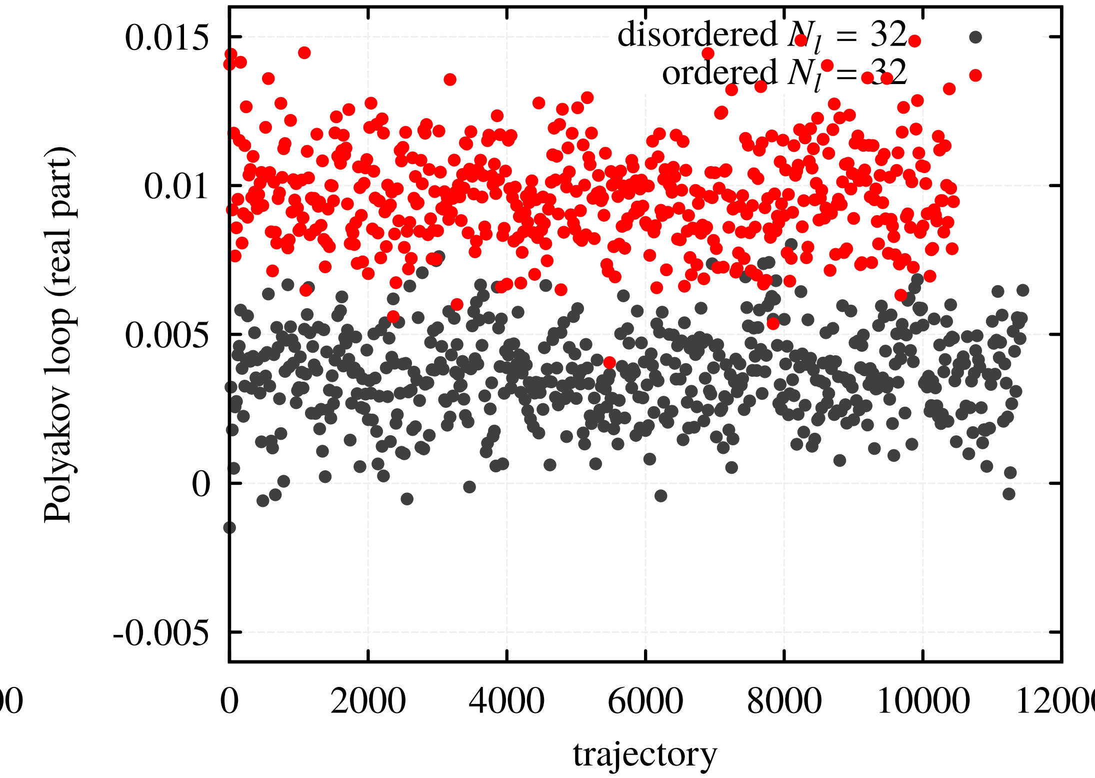
Physics results

Preliminary

$\beta = 1.74$

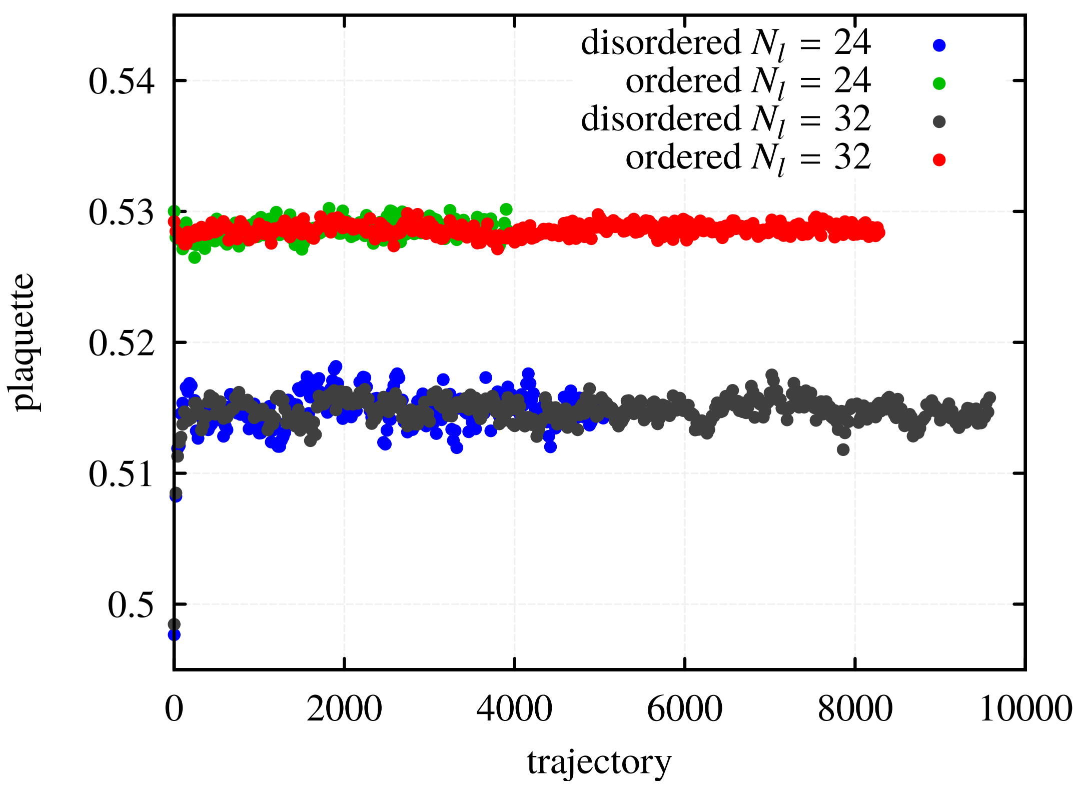
$\kappa = 0.14054$

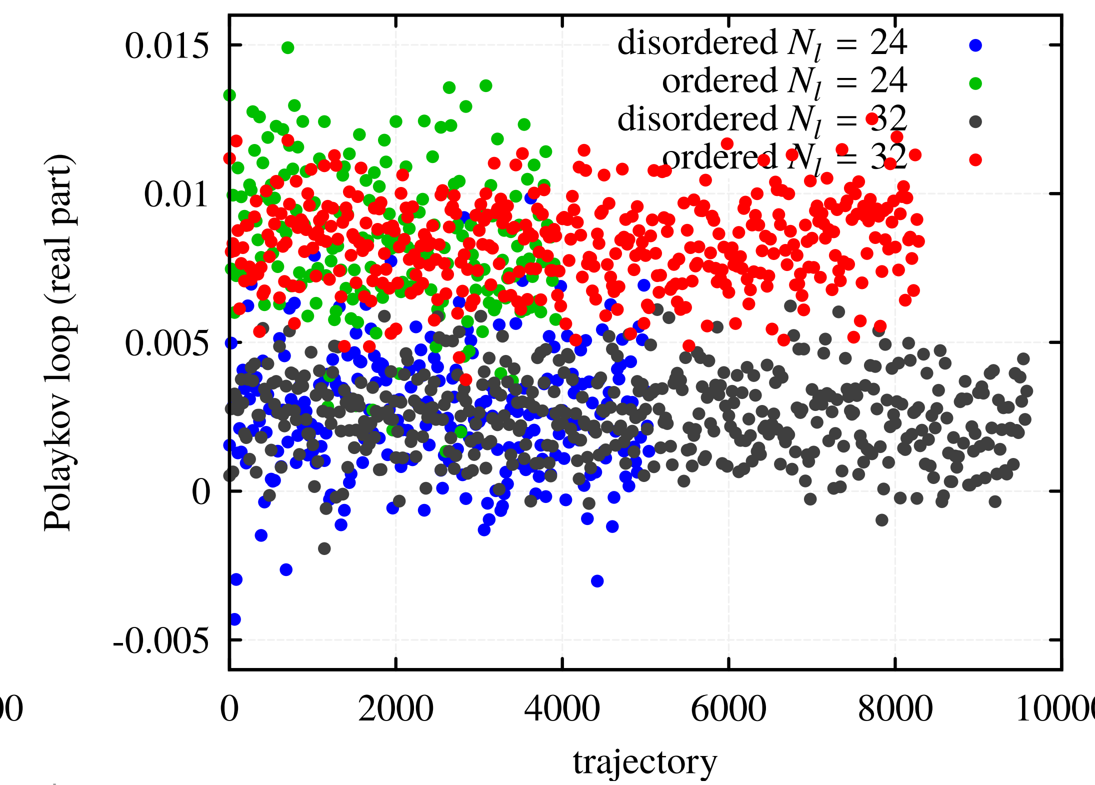




$\beta = 1.73$

$\kappa = 0.14085$



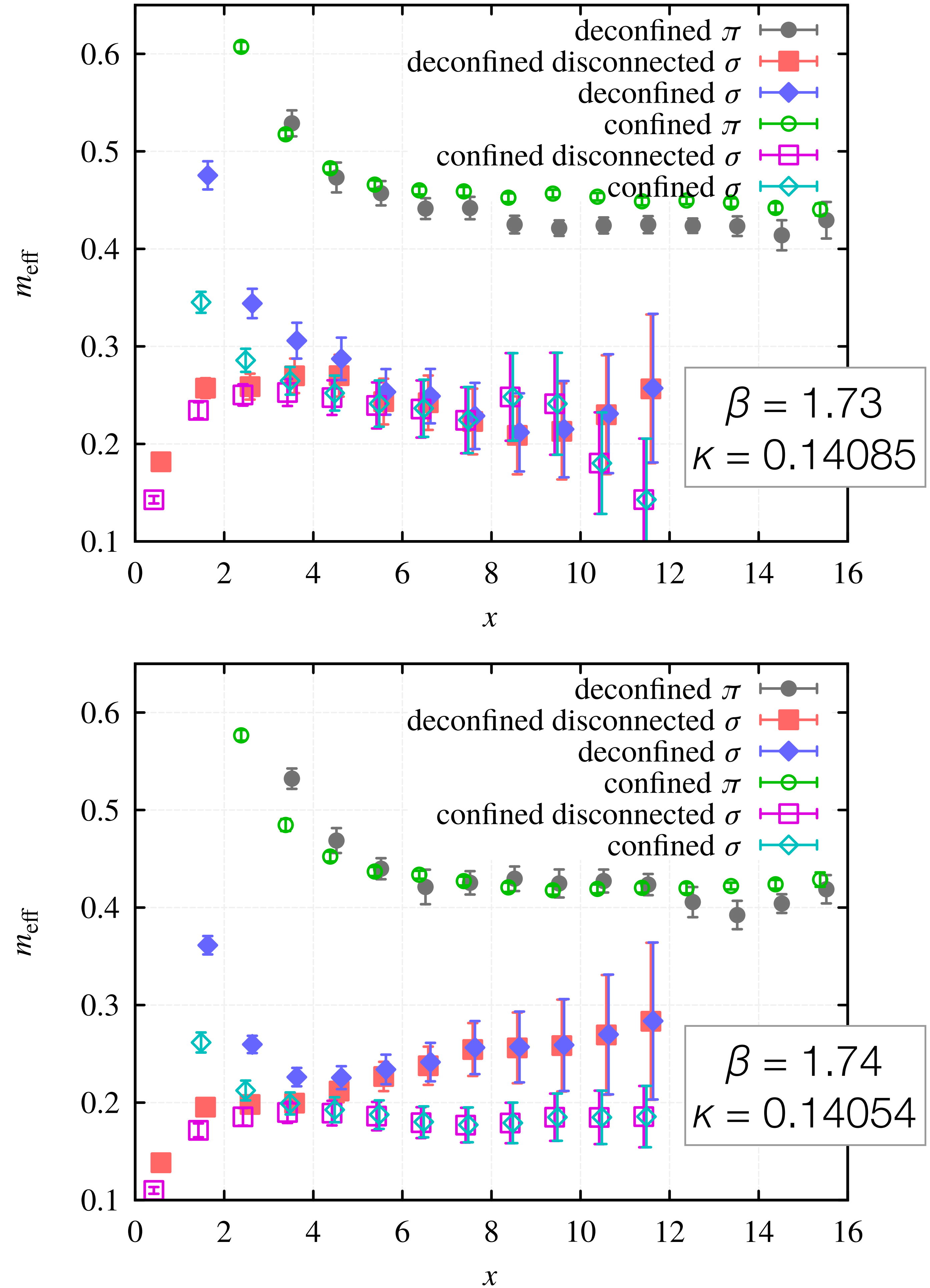


Evolution of trajectories

Stable 2 states at 2 parameter sets

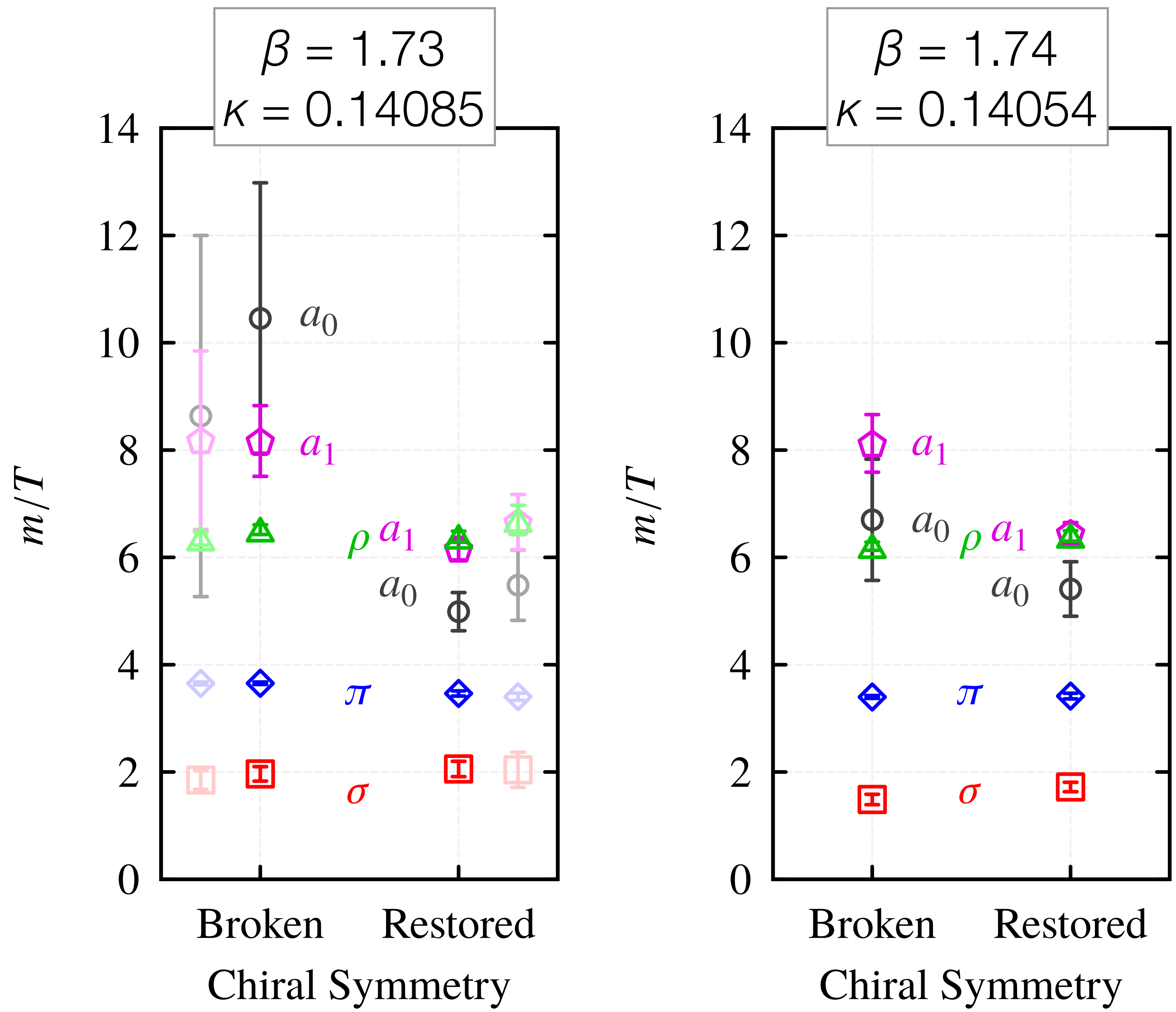
Effective masses

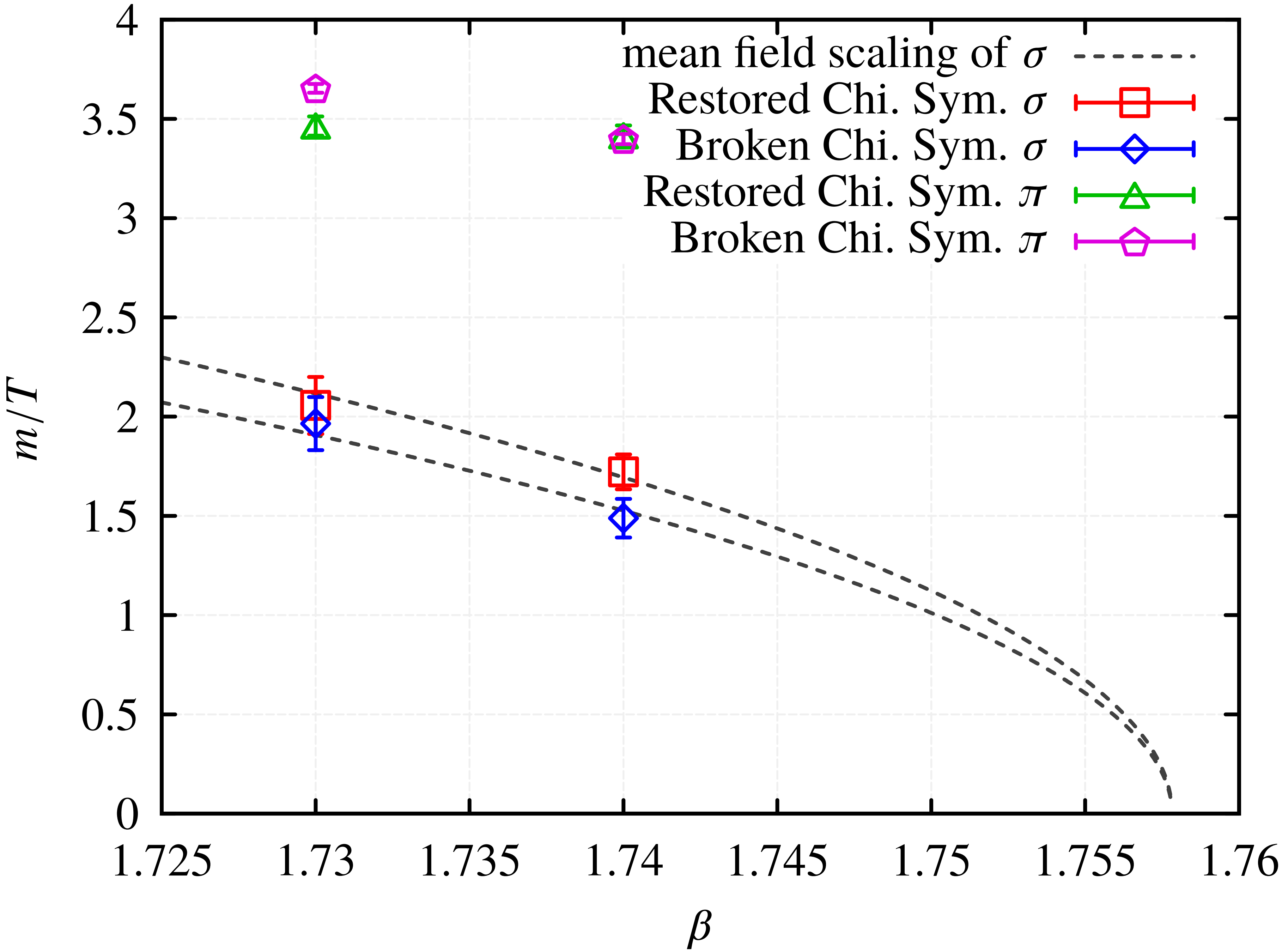
- Disconnected part dominates the singlet scalar propagator
- Singlet scalar states extracted from $x = 3 \sim 8$, while other non-singlet mesons are extracted from $x = 6 \sim 12$
- At $\beta = 1.73$, deconfined pion is clearly lighter than confined pion
- Need to understand statistics and autocorrelations



Screening masses at the transition

- ρ, π, and σ are almost constant across the transition
- a_1 becomes degenerate with ρ in the chiral symmetric phase
- a_0 drops and becomes closer to π





Extrapolating to the critical point

Clear trend

Summary

- We developed the method of hierarchical truncation with stochastic probing, which is easy to implement and gives 40x speedup in measuring the quark condensate.
- We observed σ screening mass twice as light as π on both sides of the first order transition close to the endpoint.
- We will increase the statistics and expand the parameter space.