Charmonium spectra and dispersion relation with improved Bayesian analysis in lattice QCD

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Motivation

Relativistic Heavy Ion Collisions

- Dynamical property of QGP medium
- Charmonium: Observable of Relativistic Heavy Ion Collisions
  - J/ψ suppression [Matsui & Satz 1986]
  - Color Debye screening
    Bound state melts at upper T_c

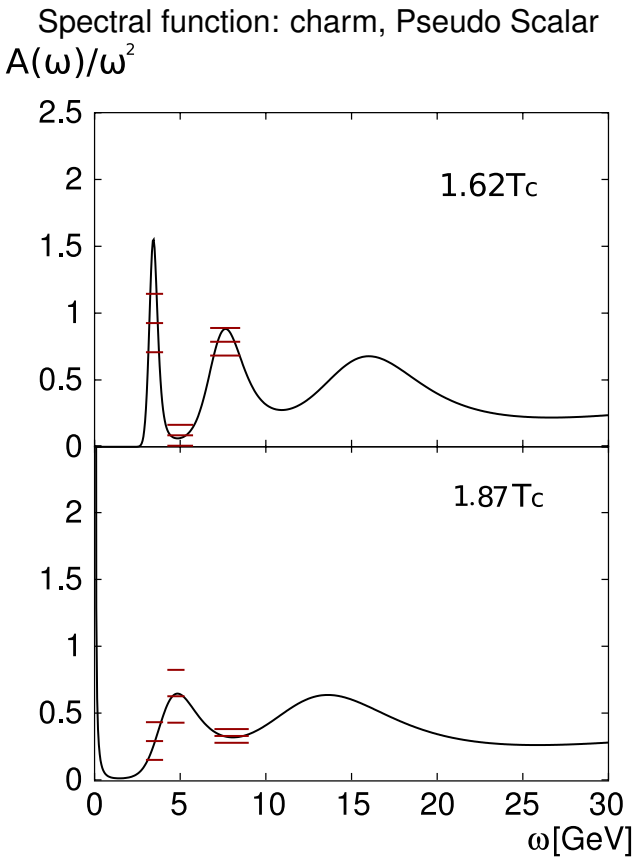
Lattice QCD: First principle calculation

- Imaginary time correlation function
- To study the dynamical property, real time information is needed.
- Analytic continuation: Imaginary time $\Longrightarrow$ Real time

ill-posed Problem

Motivation: Maximum Entropy Method

- Reconstruct the most probable spectral function.
  Lattice & default model
- MEM enables us to estimate the statistical error of the reconstructed image.
- However, this error is large. (transport coefficient, dispersion relation, etc. . .)



M.Asakawa and T.Hatsuda, PRL. 92, (2004).

Purpose

1. Extend MEM and reduce the error of MEM
2. Analyze the dispersion relation of charmonium at finite temperature

Spectral function and Lattice

Correlator and Spectral function

$$D(\tau, \vec{p}) = \int d^3x e^{i\vec{p}\cdot\vec{x}} \left\langle J_i(\tau, \vec{x}) J_i^\dagger(0, \vec{0}) \right\rangle$$
$$= \int_0^\infty K(\tau, \omega) A(\omega, \vec{k}) d\omega$$
$$K(\tau, \omega) = \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}}$$

$D(\tau, \vec{p})$: Imaginary time Correlator
Lattice QCD

$J_i(\tau, \vec{x})$: $\bar{c} i \gamma_i c$ $(i = 1, 2, 3)$
Vector current

ill-posed problem

- Imaginary time correlator → $\mathcal{O}(10)$ data points
- Spectral function → continuous

Inverse Laplace transform

Maximum Entropy Method

1. Correlator from Lattice QCD $\Longrightarrow$ Likelihood function
   - χ^2 with Covariance matrix
2. Prior knowledge $\Longrightarrow$ Prior probability
   - Shannon-Jaynes entropy
   - pQCD at high energy
   - Bad default model with Big error

Reconstructed Image A_{out}

$$P(A,\alpha) = [\text{Likelihood function}](A) \times \alpha[\text{Prior probability}](A)/Z$$

$$A_{\text{out}} = \int d\alpha \int [dA]\, A(\omega) P(A,\alpha)$$

α controls the weight

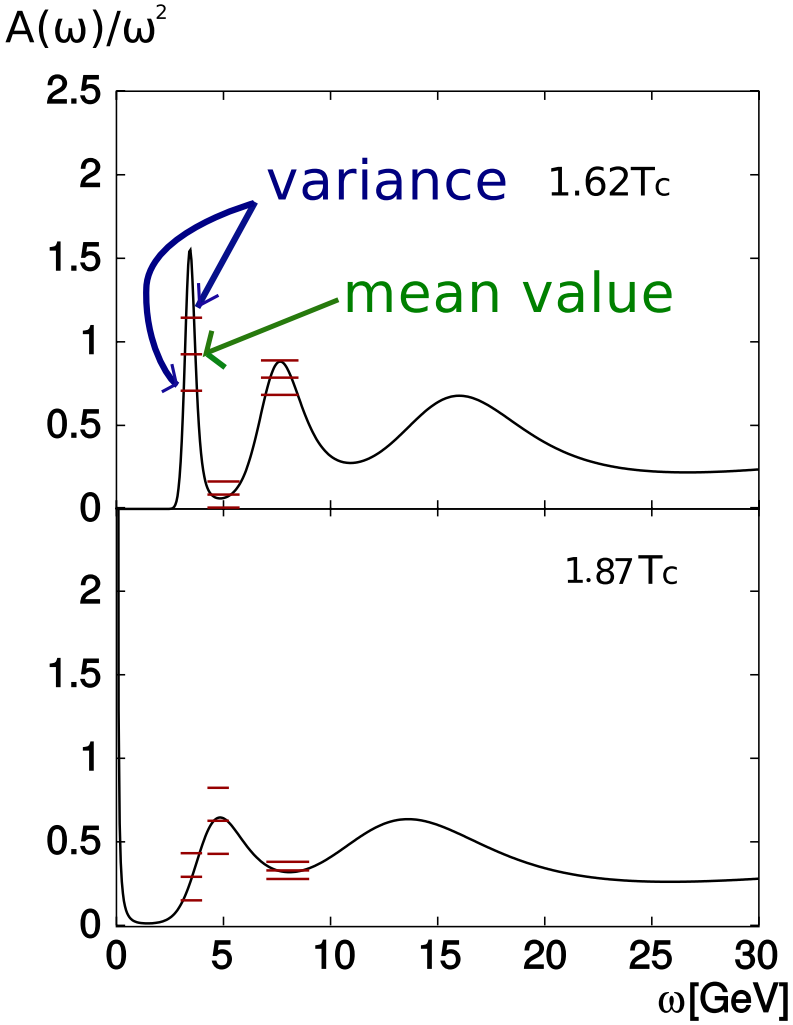
Error estimate

Error: $I = [\omega_1, \omega_2]$

$$\langle (\delta A_{\rm out})^2 \rangle_I$$
$$= \int d\alpha \int [dA] \int_{I\times I} d\omega d\omega'$$
$$\delta A(\omega)\delta A(\omega') P(A,\alpha)$$
$$/ \int_{I\times I} d\omega d\omega'$$

$$\delta A(\omega) = A(\omega) - A_\alpha(\omega)$$

Reconstruct continuum function from $\mathcal{O}(10)$

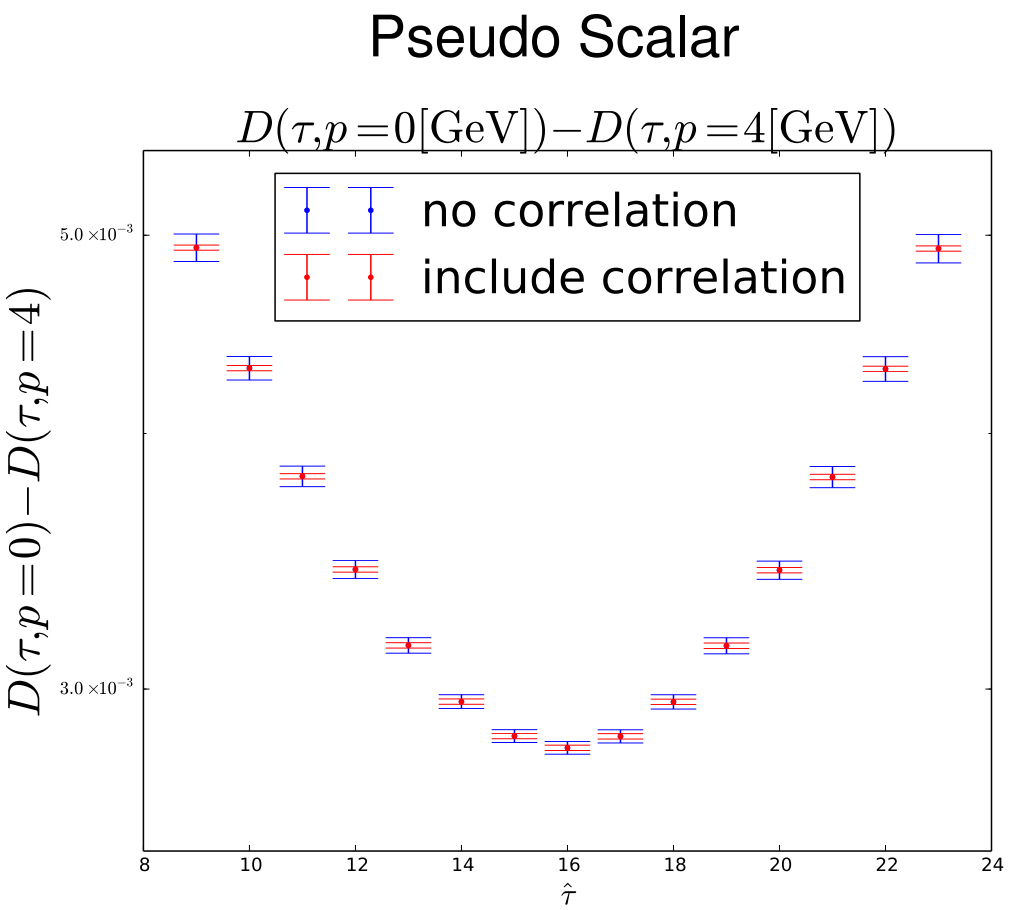


M.Asakawa and T.Hatsuda, PRL. 92, (2004).

Charmonium melt between $1.62T_c$ and $1.87T_c$

Extend Bayesian Analysis

Strong correlation between correlators measured on same gauge configurations



$$\sigma = \sigma[D(p=0)] + \sigma[D(p=4)]$$

$$\sigma = \sigma[D(p=0) - D(p=4)]$$

Correlators measured on same configuration have strong correlation

Improvement of MEM

Conventional MEM

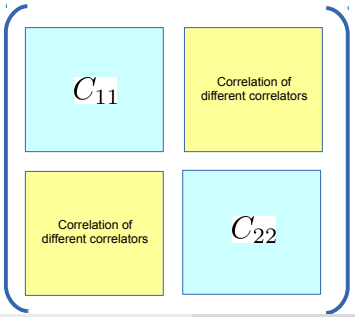
$$\vec{D}(p_0) \xrightarrow{\text{MEM}} A_1(\omega) \qquad \vec{D}(p_n) \xrightarrow{\text{MEM}} A_n(\omega)$$

Extended MEM

$$\left(\vec{D}(p_0), \cdots, \vec{D}(p_n)\right) \xrightarrow{\text{MEM}} (A_1(\omega), \cdots, A_n(\omega))$$

More information $\Longrightarrow$ The error of MEM will be reduced

(Covariance Matrix)=



Lattice setup

- Quenched QCD
- Wilson Fermion
- Anisotropic lattice: $a_\sigma/a_\tau = 4.0$

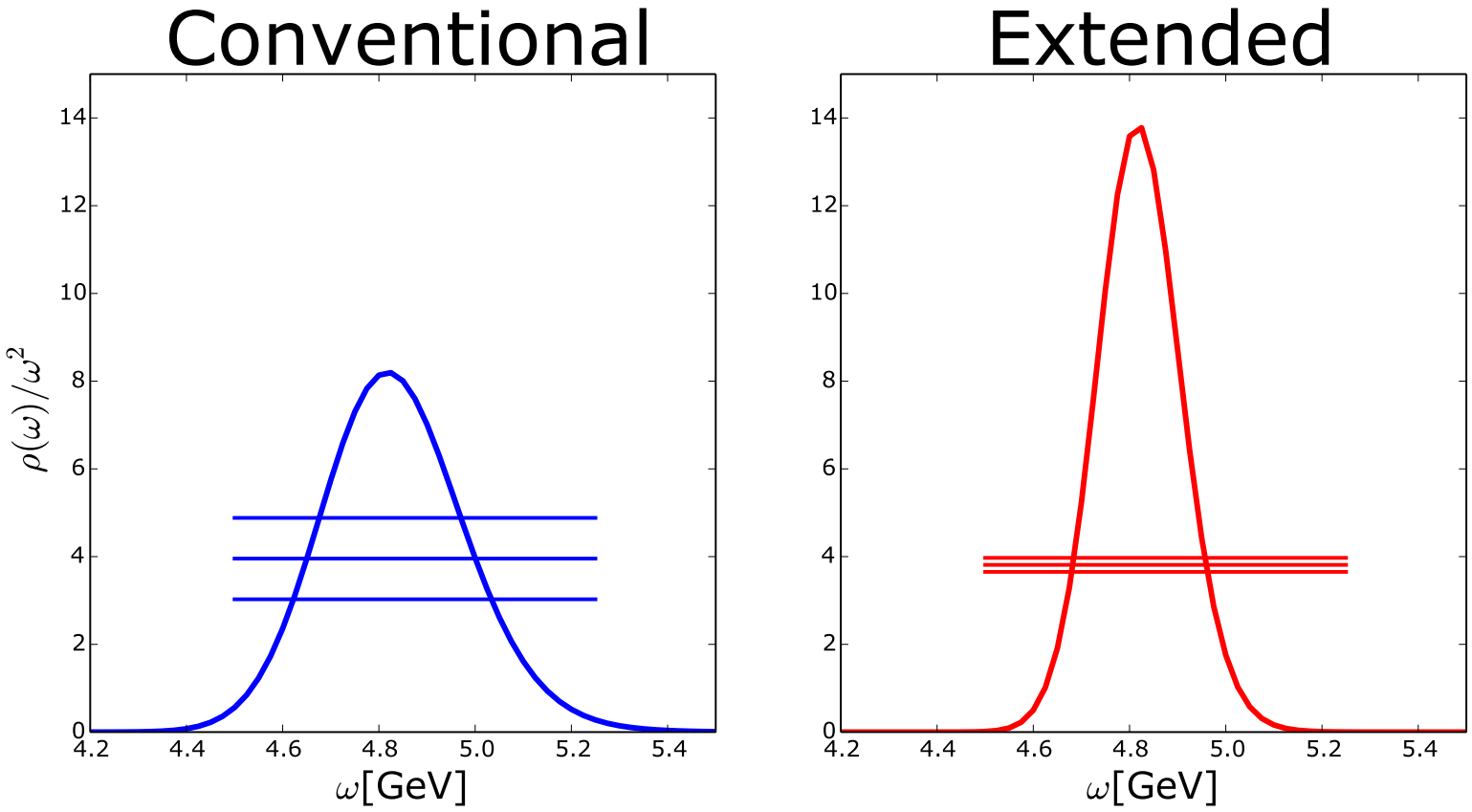
setup

N_τ	T/T_c	N_σ	L_σ[fm]	a_τ[fm]	a_σ/a_τ	β	N_{conf}
32	2.33	64	2.496	0.00975	4	7.0	396
40	1.87	64	2.496	0.00975	4	7.0	400
42	1.78	64	2.496	0.00975	4	7.0	427
44	1.70	64	2.496	0.00975	4	7.0	407
46	1.62	64	2.496	0.00975	4	7.0	401
96	0.78	64	2.496	0.00975	4	7.0	207

[Nonaka et al, 2011]

Reduction of Error

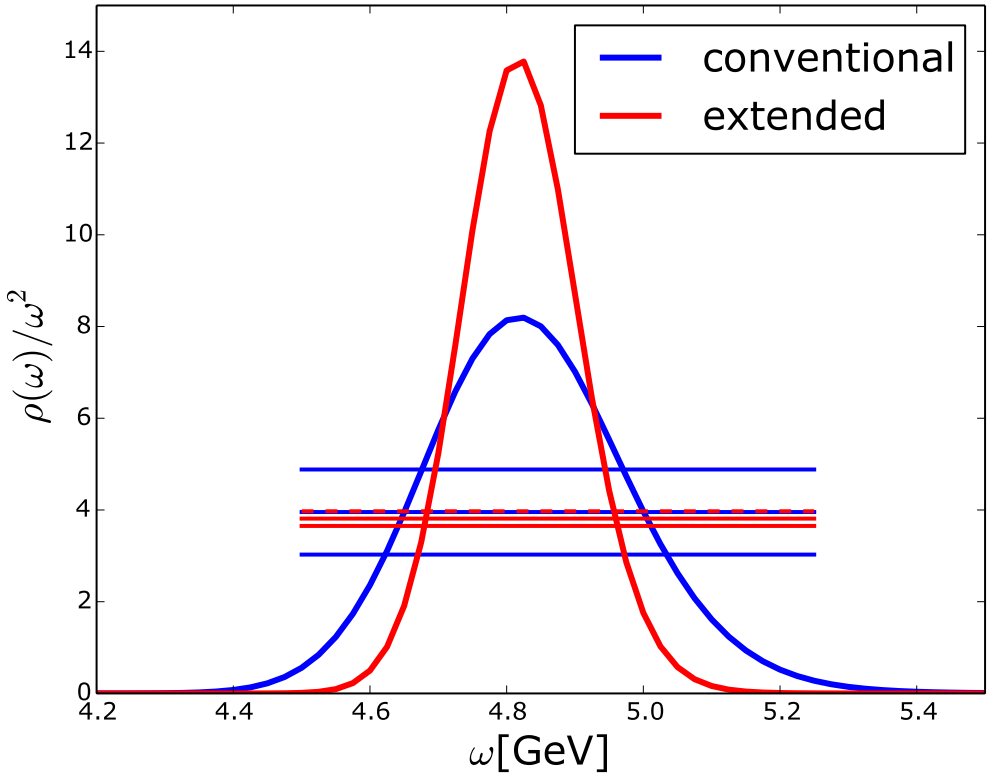
Drastic Reduction of Error



Pseudo scalar, 1.70T_c, p=0

- Analyze two correlators (0, 4 GeV) together
- The width of the peak becomes narrow

Drastic Reduction of Error

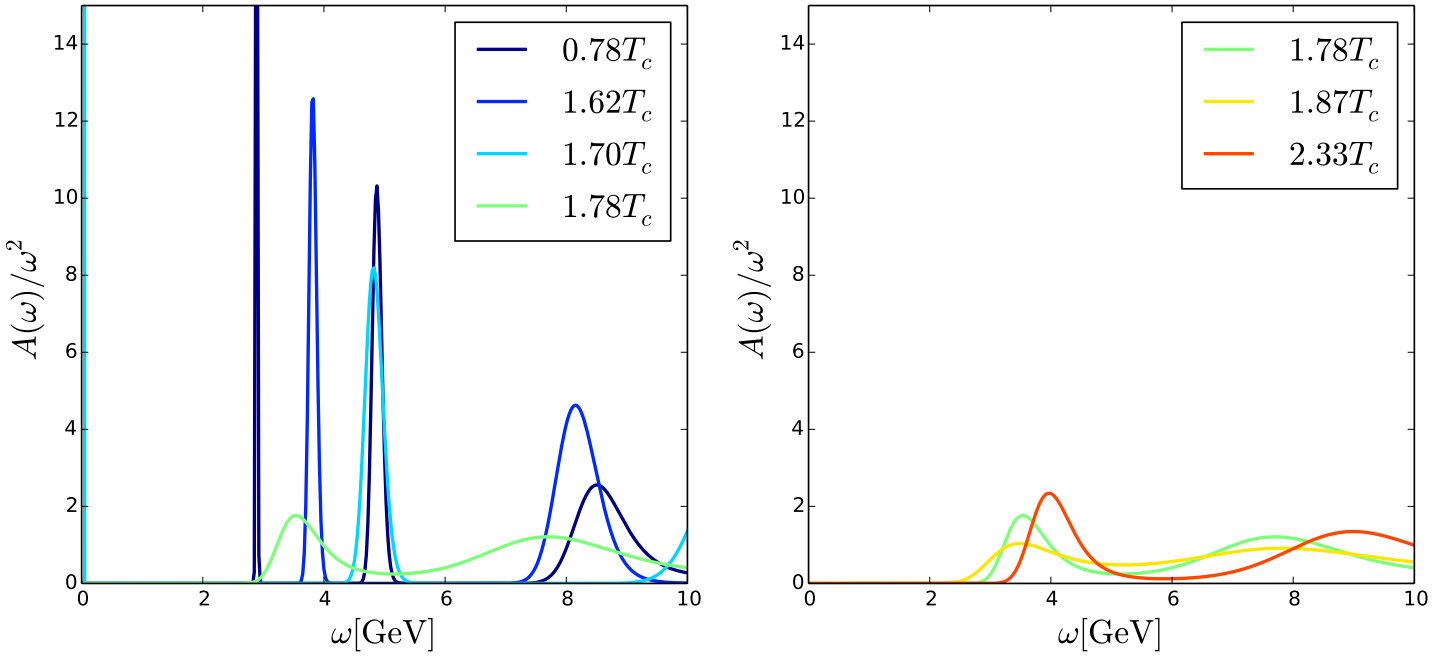


Pseudo scalar, 1.70T_c, p=0

- Analyze two correlators (0, 4 GeV) together
- The width of the peak becomes narrow

Dispersion relation

Pseudo scalar spectral function at finite temperature



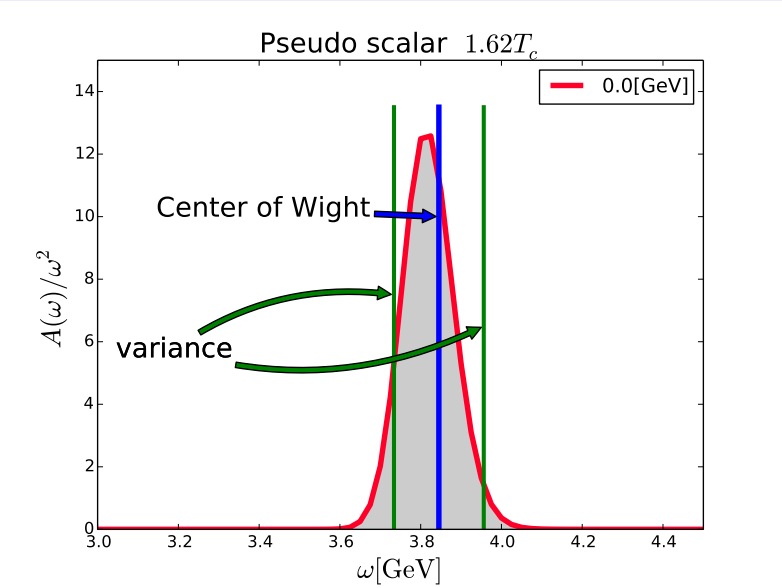
Bound states of charmonium suvives up to $\mathbf{1.70T_c}$

$\Longrightarrow$ Analyze the dispersion relation of charmonium below 1.70Tc

Dispersion relation

- Momentum dependence of spectral peaks

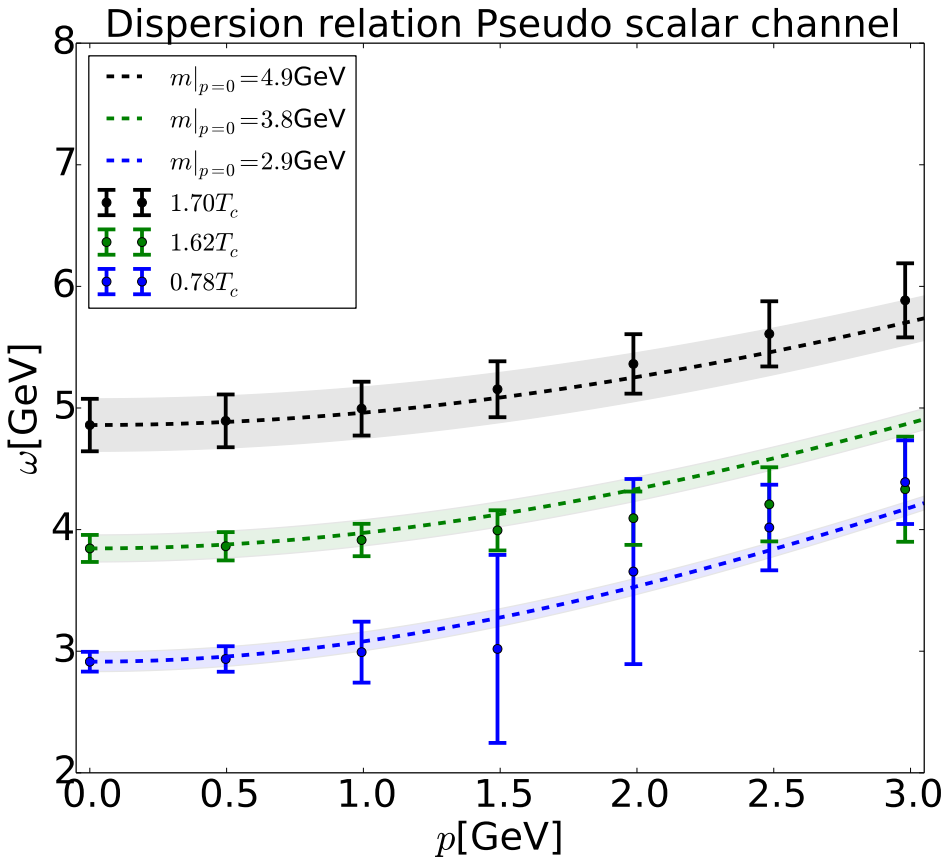
Error estimate



$$\text{(Center of weight)} = \frac{\langle \omega \frac{A(\omega)}{\omega^2} \rangle_I}{\langle \frac{A(\omega)}{\omega^2} \rangle_I}$$

$$\textbf{(variance)} = \frac{\sqrt{\langle \{\omega \delta(\frac{A(\omega)}{\omega^2})\}^2 \rangle_I}}{\langle \frac{A(\omega)}{\omega^2} \rangle_I}$$

Dispersion relation: Conventional Method

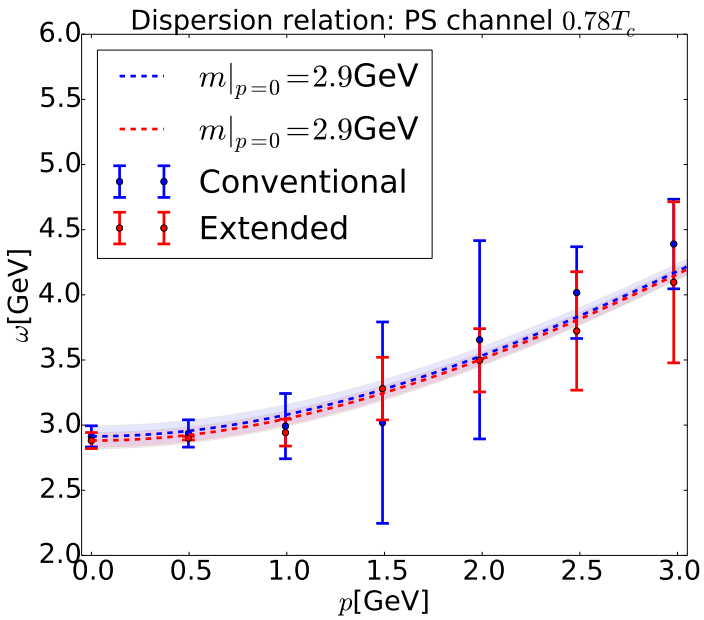


- Mass Shift: Temperature ↑ $\Longrightarrow$ Mass ↑
- Dispersion relation from Lorentz invariance

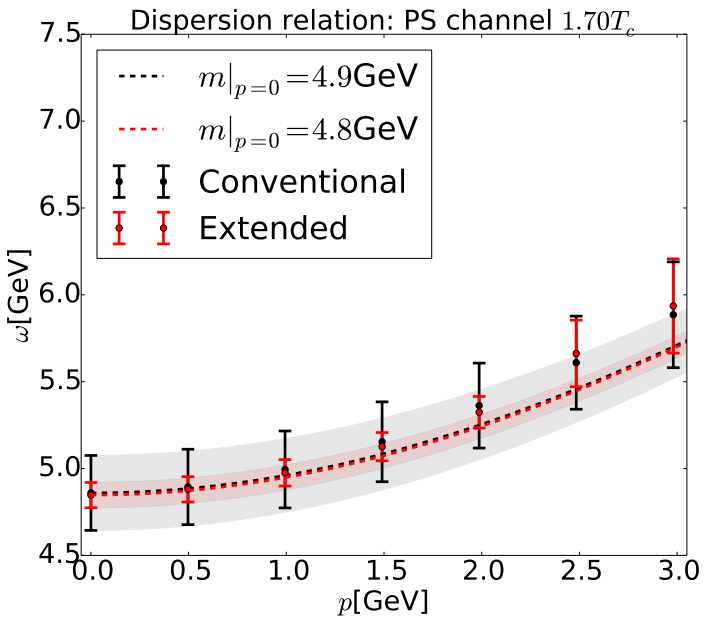
  $$\omega = \sqrt{m|^2_{p=0} + p^2}$$
- Large error at middle and high momentum

Dispersion relation: Extended MEM

$0.78T_c$



$1.70T_c$



- Big improvement at low and middle momentum
- Dispersion relation follows Lorentz invariance in the finite temperature medium

Conclusion and future work

Conclusion

1. We extend the MEM analysis to the product space of the correlators to take advantage of more data and the strong correlation among Euclidean correlators with different momenta.
   - Improved MEM reduce the error of reconstructed image.
2. We analyze the dispersion relation of charmonium at finite temperature
   - Mass shift is observed.
   - Above Tc the dispersion relation of charmonium follows Lorentz invariance.

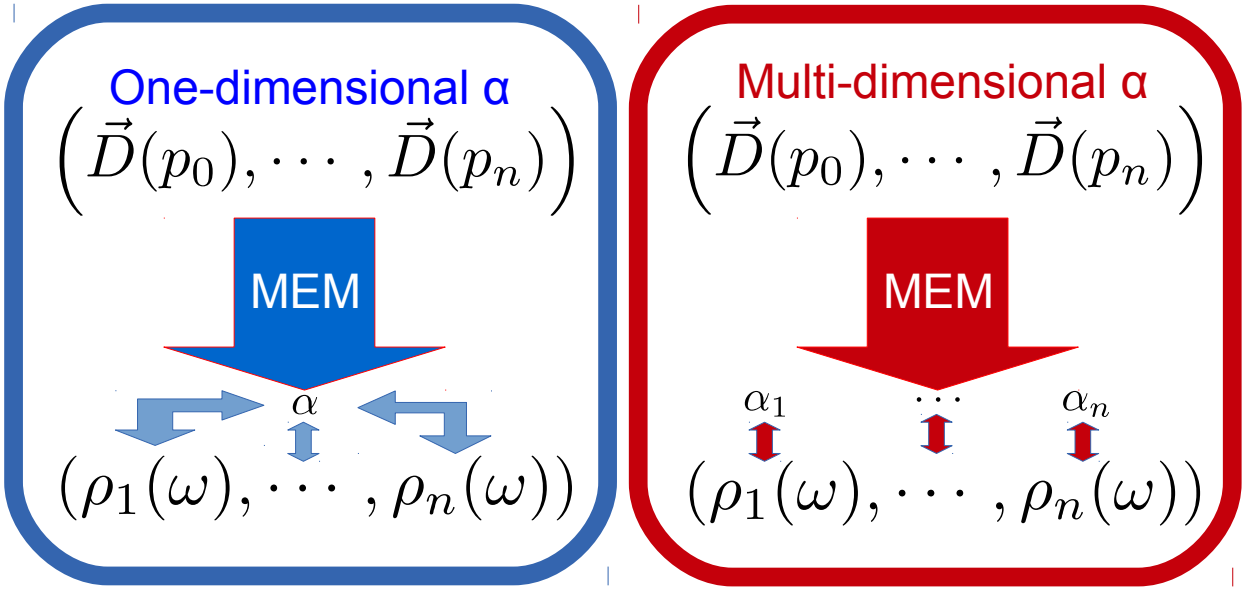
Future work

- Multi-dimensional α
- Other temperature
- Analysis for more than 3 correlators
- Analysis for between different channel, . . .

Multi-dimensional α

- α which maximize $\boldsymbol{P(A,\alpha)}$ is different for each correlator.
- Reconstructed image with worse default model compared with others is influenced by default model more than necessary.

Make α multi-dimensional



Likelihood function

Likelihood function: χ^2

$$\exp(-L) = \exp\left[-\frac{1}{2}\sum_{i,j}\left(D(\tau_i) - D_A(\tau_i)\right) C^{-1}_{ij}\left(D(\tau_j) - D_A(\tau_j)\right)\right]$$

$$D(\tau_i) = \frac{1}{N_{\mathrm{conf}}}\sum_{m=1}^{N_{\mathrm{conf}}} D^m(\tau_i)$$

Covariance matrix

$$C_{ij} = \frac{1}{N_{\mathrm{conf}}(N_{\mathrm{conf}}-1)}\sum_{m=1}^{N_{\mathrm{conf}}}(D^m(\tau_i) - D(\tau_i))(D^m(\tau_j) - D(\tau_j))$$

Prior probability

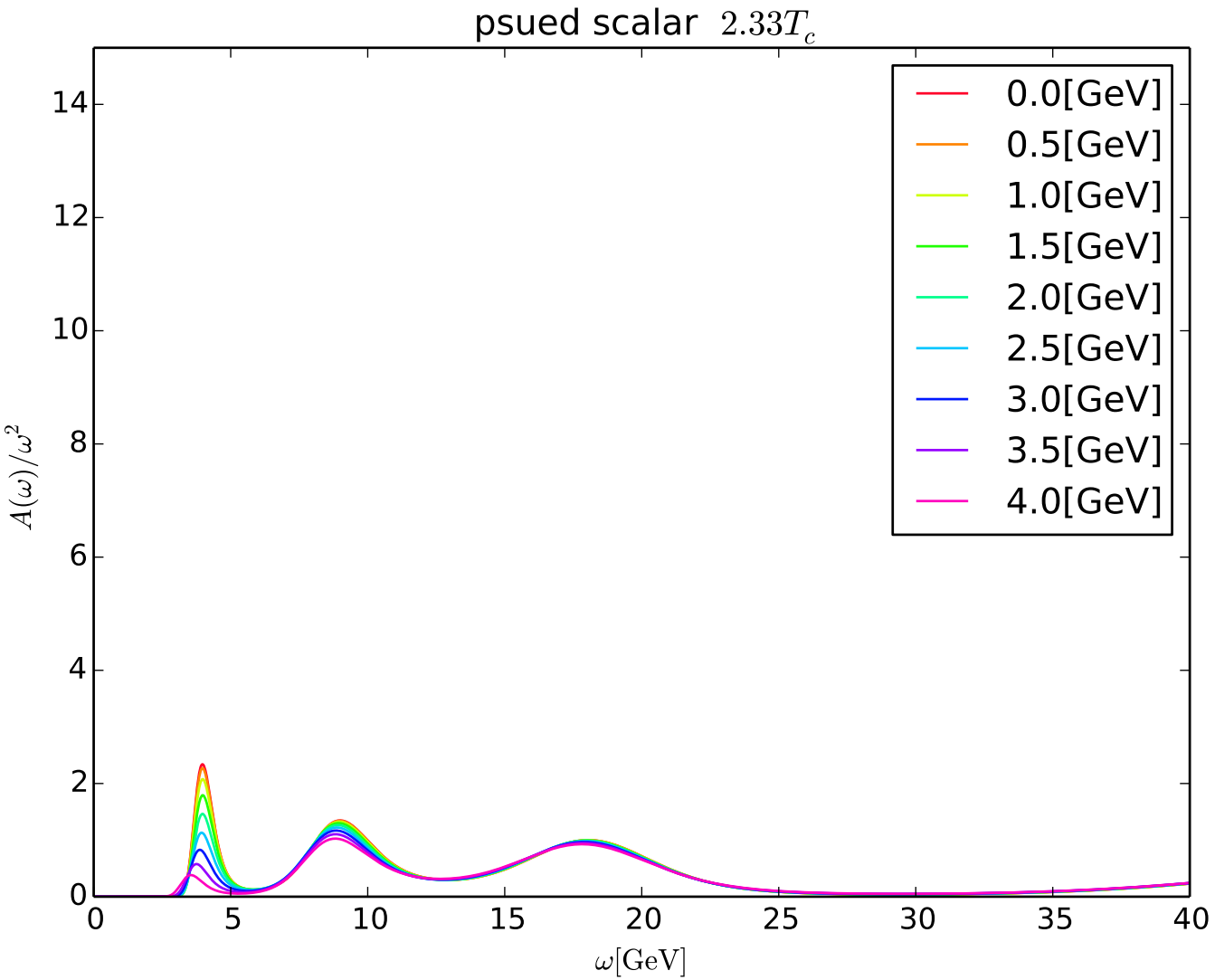
Prior probability: Shannon-Jaynes entropy

$$\exp(\alpha S) = \exp\left(\alpha \int_0^\infty \left[A(\omega) - m(\omega) - A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right] d\omega\right)$$

1. Consistency with Prior knowledge
2. default model $m(\omega)$
   - pQCD: $m(\omega) = m_0\omega^2$
   - $m_0 = 0.40$(vector),1.15(pseudo scalar)
   - Bad default model $\Longrightarrow$ Big error

Probability of image A

$$P(A, \alpha) = \exp(\alpha S - L)$$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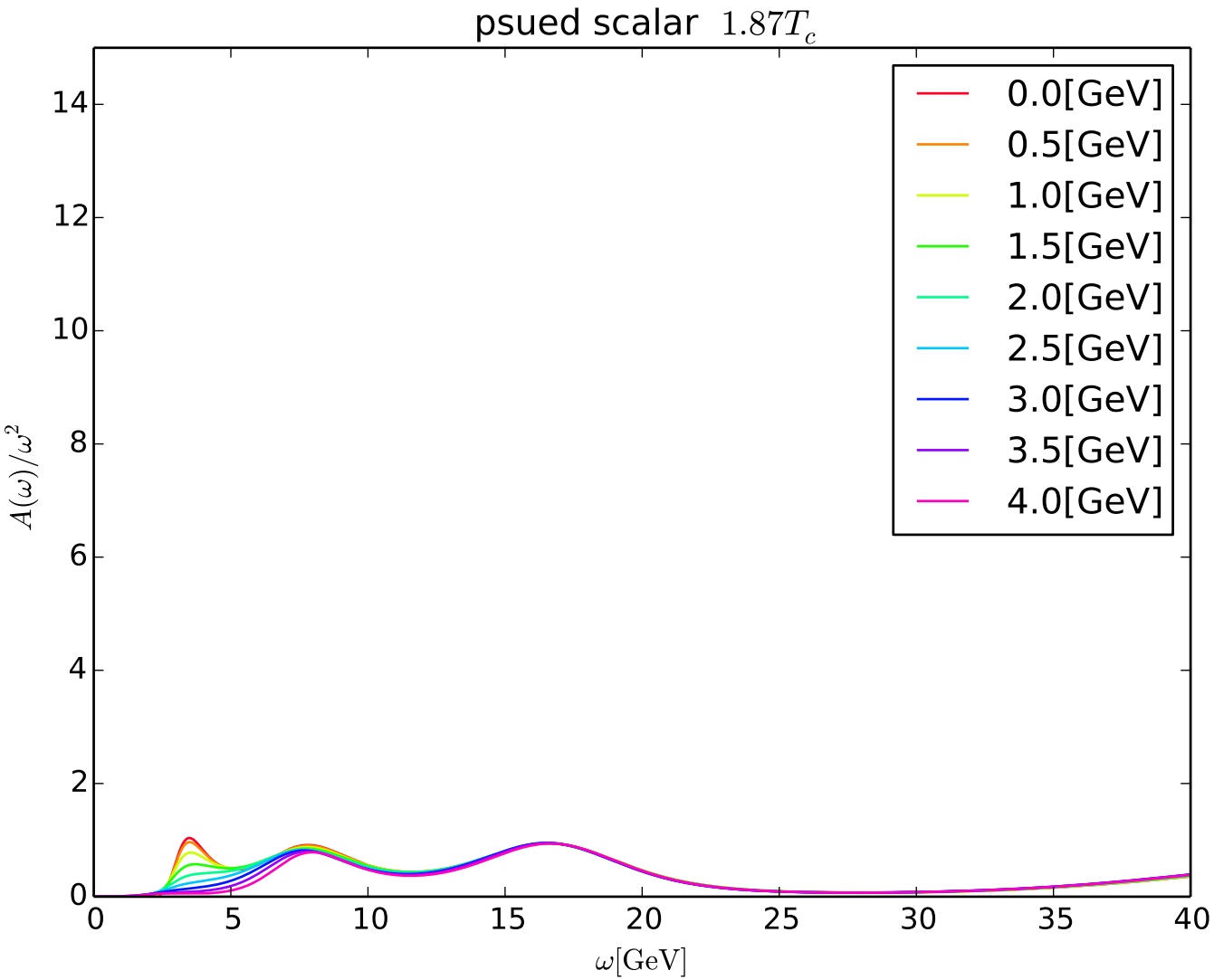
psued scalar $1.87T_c$
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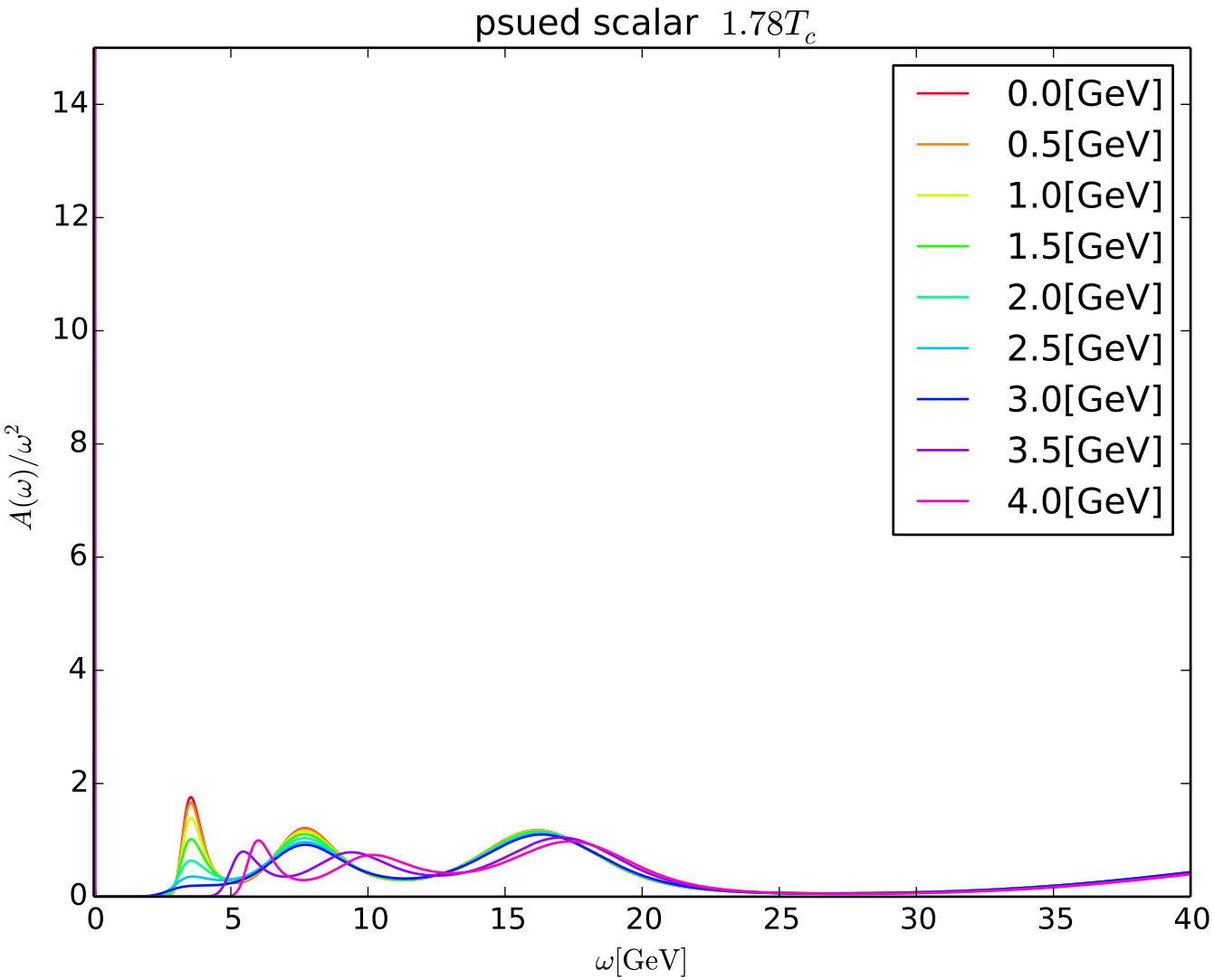
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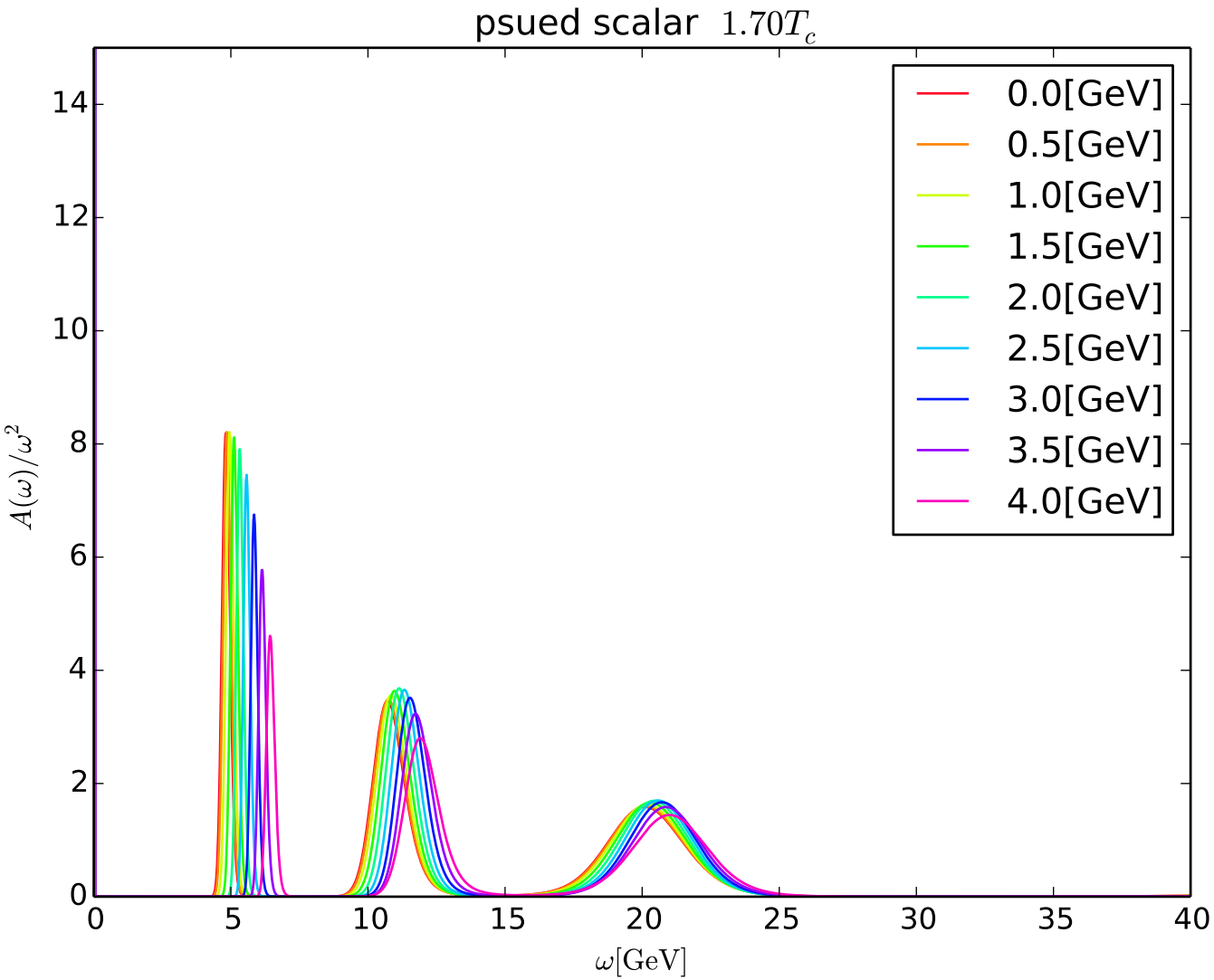
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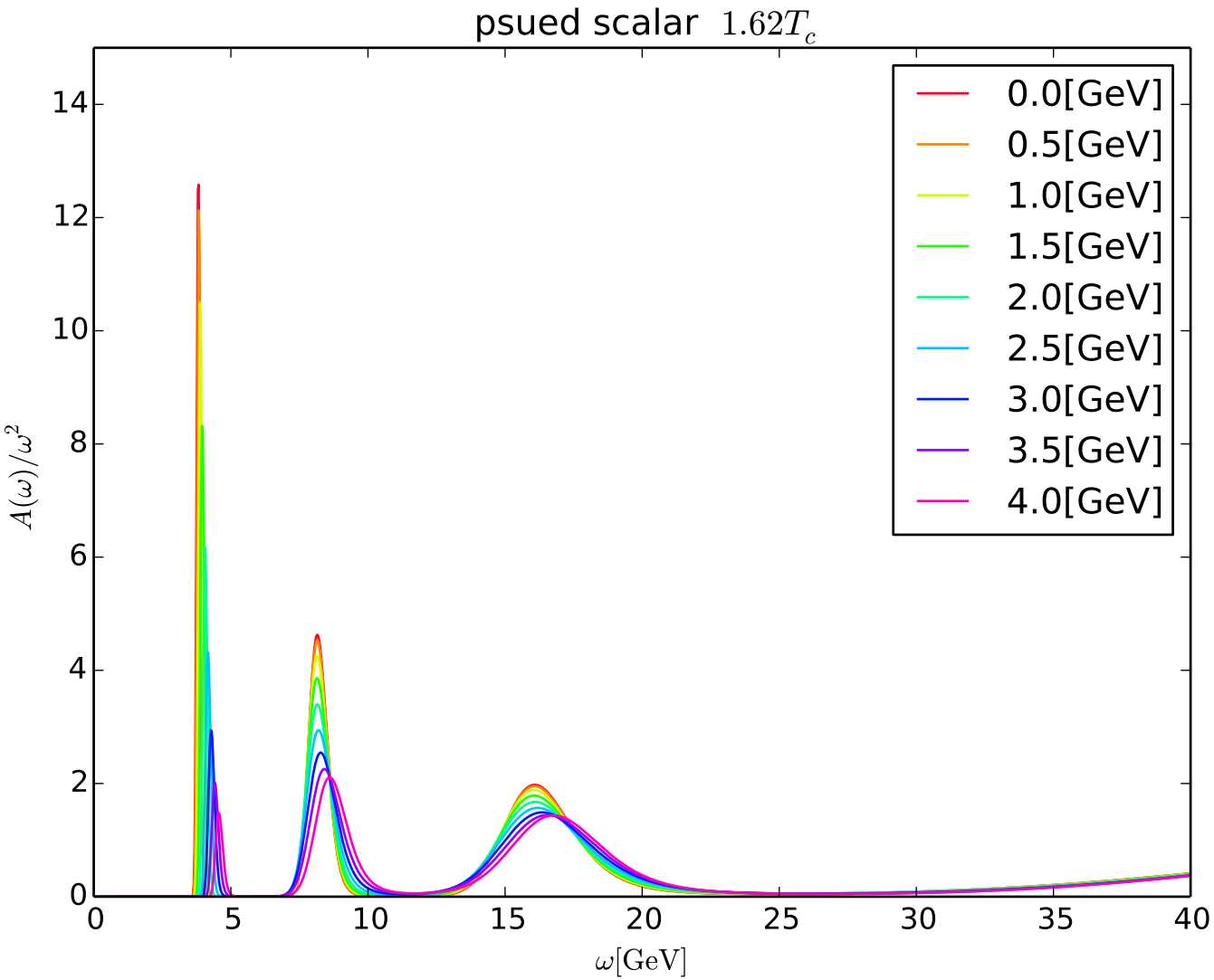
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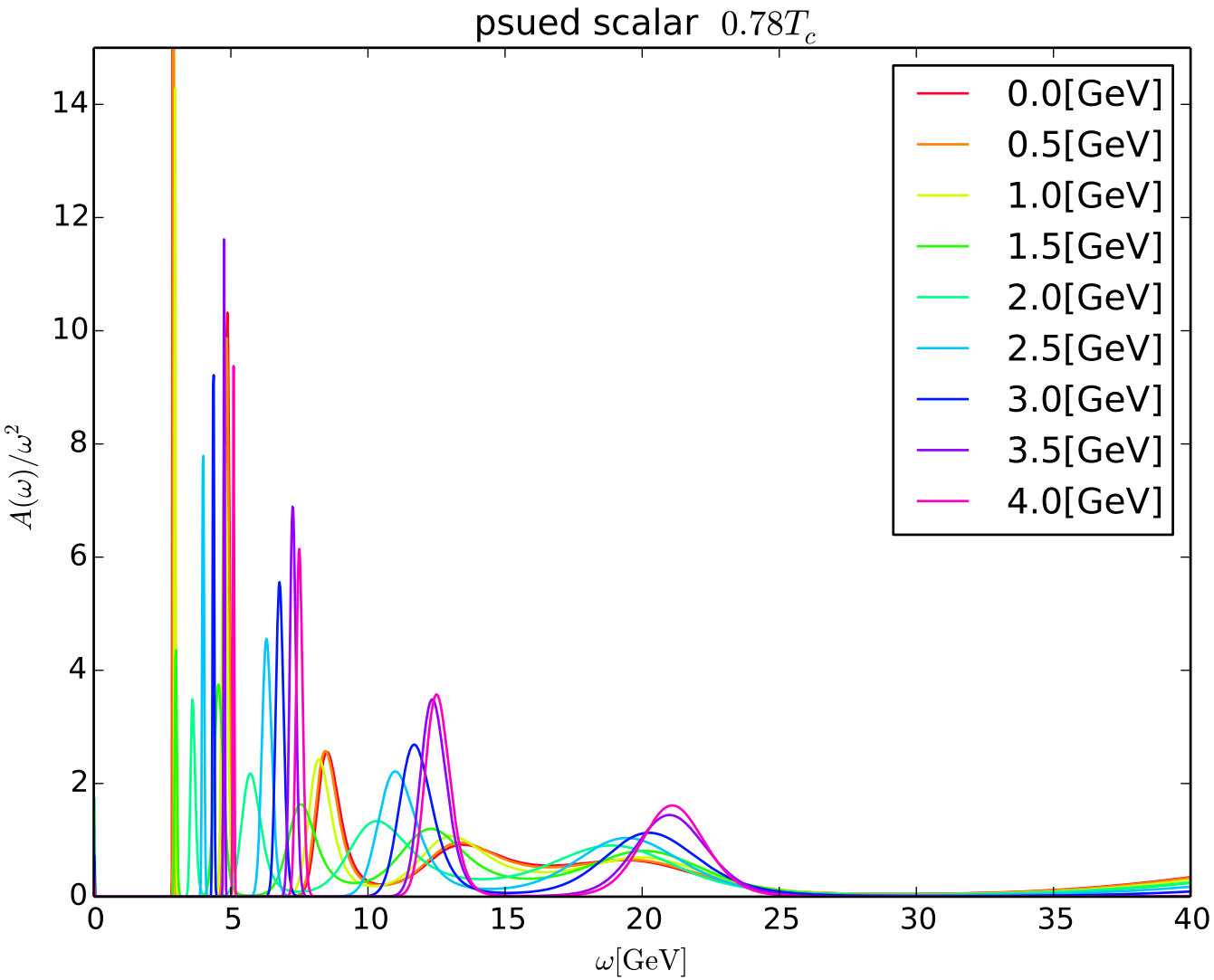
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