Phase structure study of SU(2) lattice gauge theory with 8 flavors

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* The affiliation was valid until November, 2013.

Outline

- **The Motivations**
- **Simulation details & Study scheme**
- **Results of the phase structure study**

  - Plaquette (General study / Cold-start and hot-start runs)

  - Polyakov loop (General study / The constraint Effective potential)

  - The Dirac operator eigenvalue spectrum (Preliminary)

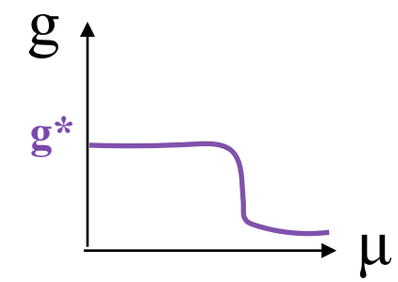
- **Summary & Future work**

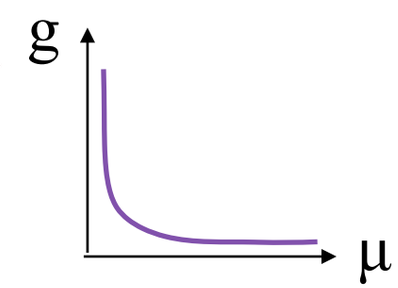
Motivations

- Conformal or not in technicolor :

  (a) Conformal (scale invariance): **No** chiral symmetry breaking.

  (b) **Non-**conformal (ex: QCD-like theories): Chiral symmetry breaking.





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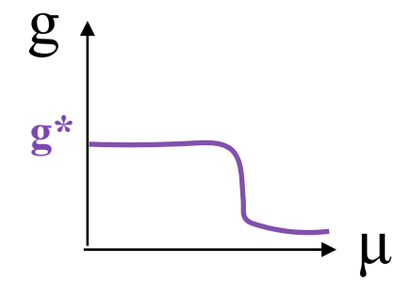
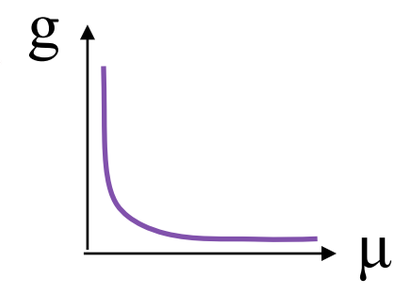


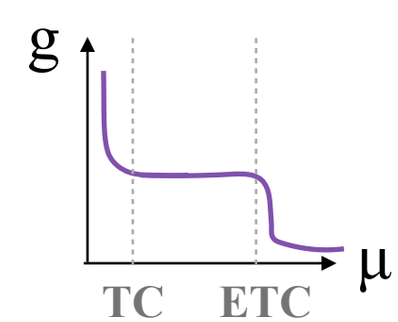


- *Almost* conformal in technicolor :

  (a) The mass anomalous dimension in RG evolution is chosen to be approximately 1.

  (b) With (a) and g ~ constant between the cutoff scale of TC & ETC, we have condensate enhancement in RG evolution.

  (The theory is conformal near the IR fixed point)



Motivations

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  (a) Like the group SU(3), the group SU(2) is a simple Lie group and non-Abelian.

  (b) A simple choice to study chiral symmetry breaking comparing with the group SU(3).

  (c) No well understanding about the theory so far.

  (d) Question for the N_f of the theory developing IR conformal behavior (i.e. **The IR behavior of the theory is determined by an IR fixed point**).

Simulation Details & Study Scheme

- Unimproved staggered fermions & plaquette gauge action.
- Lattice sizes: 6×6^3, 8×8^3, 12×12^3, 16×16^3, and 24×24^3 (Preliminary).
- Periodic boundary conditions for four directions.
- Save one configuration for every ten HMC-trajectories.
- Cold-start & Hot-start simulations.

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- Periodic boundary conditions for four directions.
- Save one configuration for every ten HMC-trajectories.
- Cold-start & Hot-start simulations.
- Go through weak and strong coupling regimes by tuning $\beta_{fund.}$ for different bare fermion mass (m) input values ($\beta_{fund.} = \frac{2N}{g^2}$, where $N=N_{TC}$ & g is the bare coupling).

  - Smallest bare mass value: 0.003.

  - Largest bare mass value: 0.025.

Results of the Phase Structure Study

- Plaquette
  - General study
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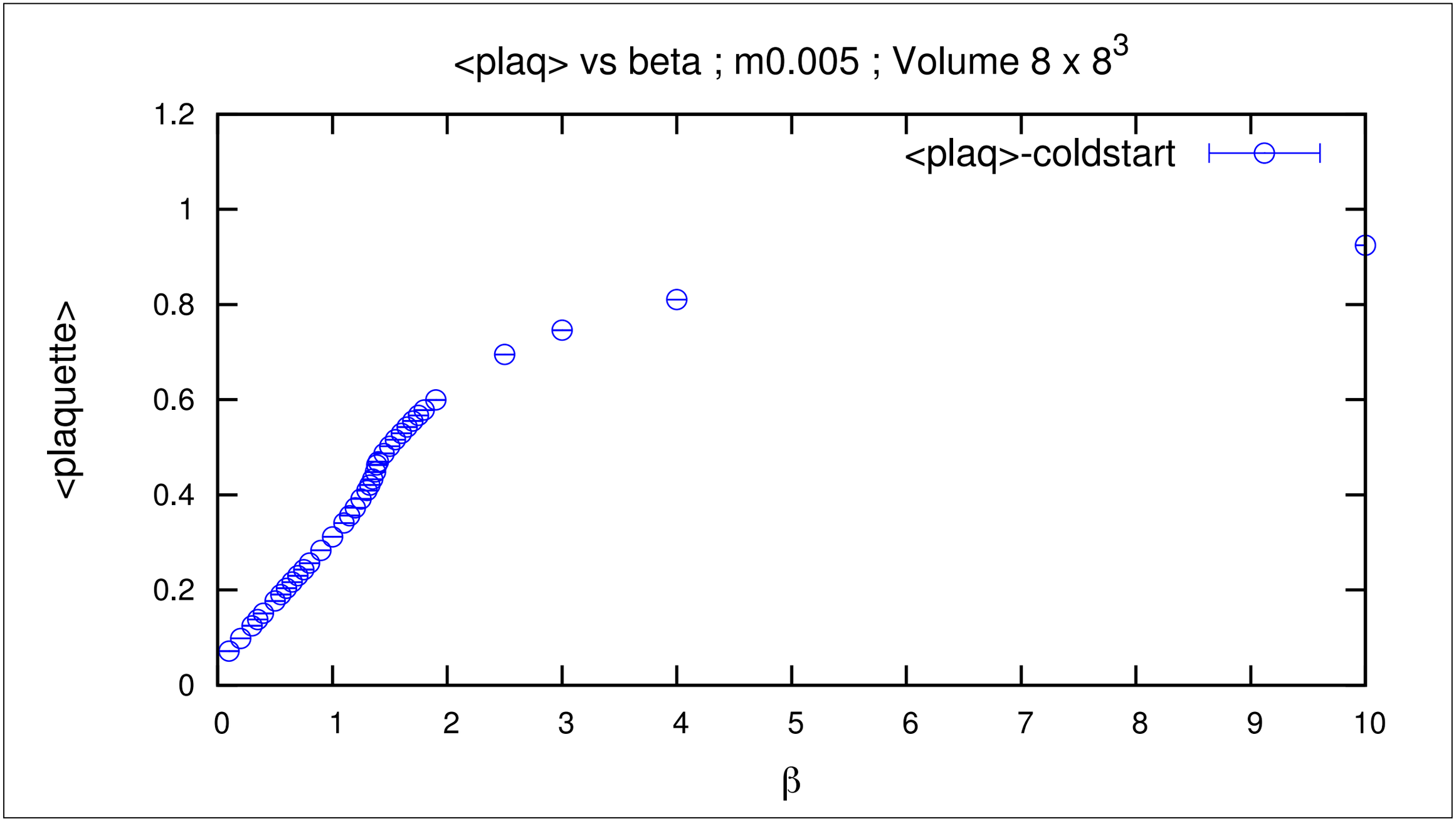
Plaquette—General Study

- <Plaq> vs β plot : (**One phase transition is observed.**)



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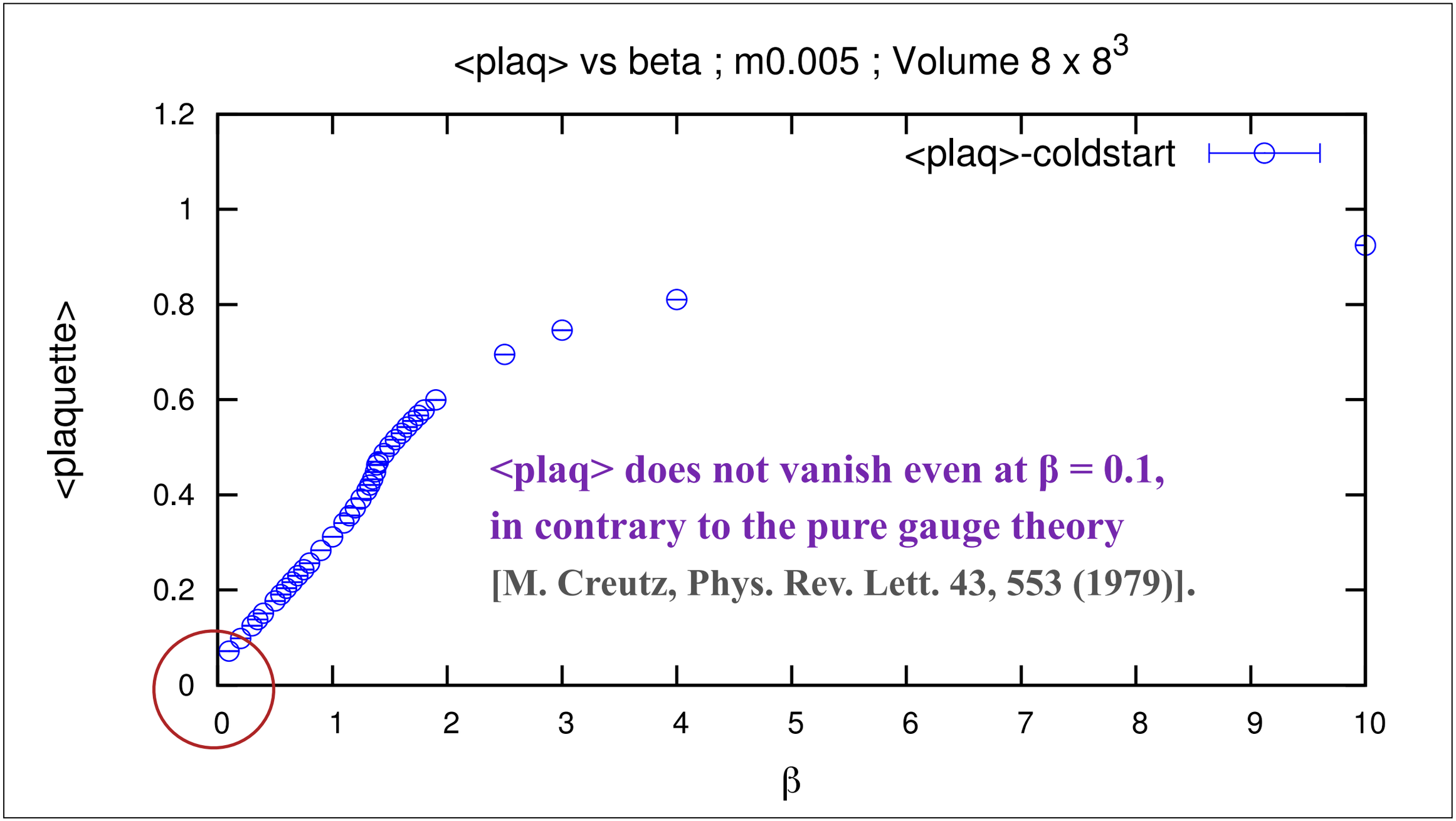

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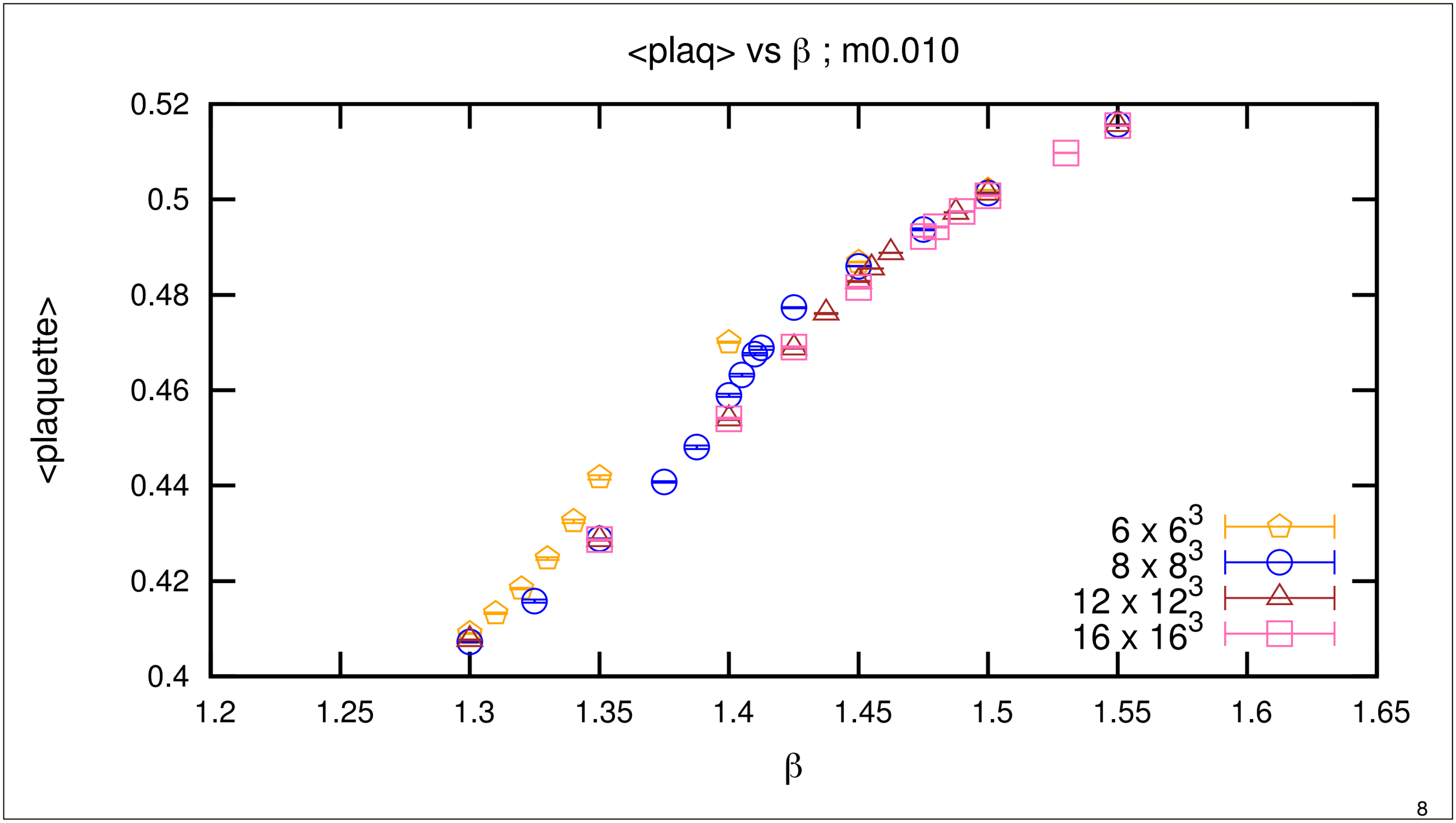

As the volume increases, the shifting of the phase boundary becomes smaller. → **A bulk phase transition.**

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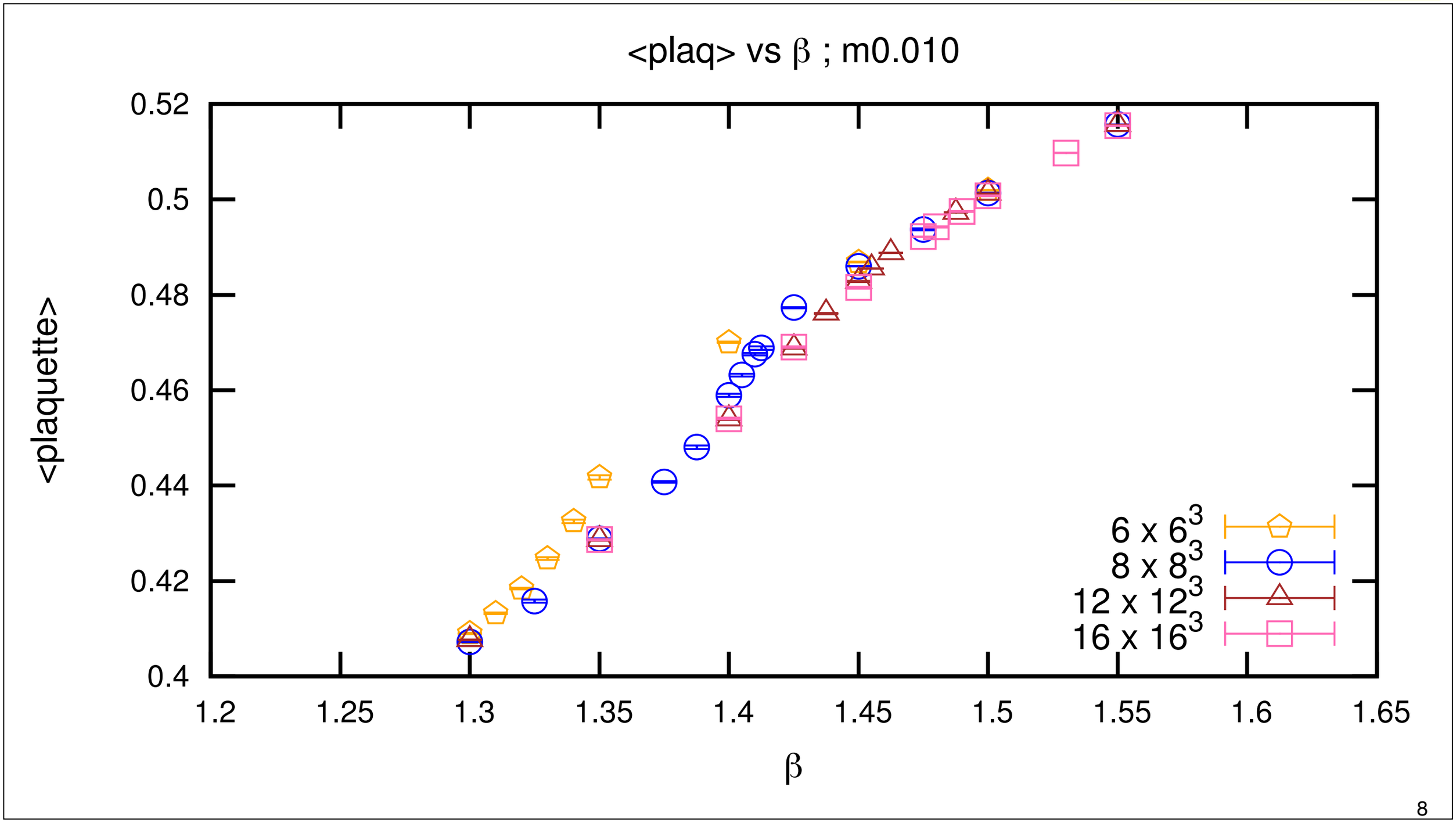

As the volume increases, the shifting of the phase boundary becomes smaller. → **A bulk phase transition.**

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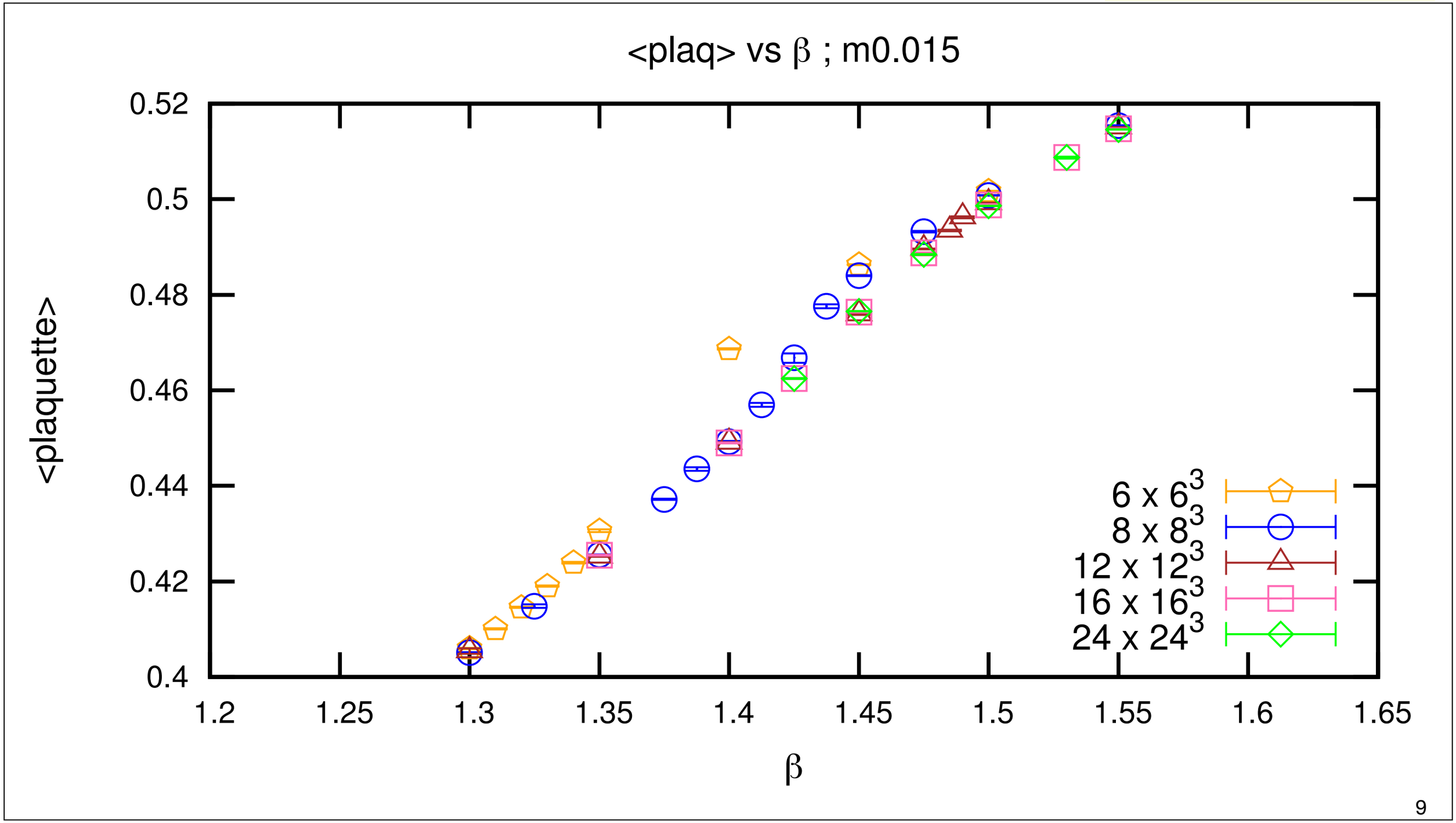

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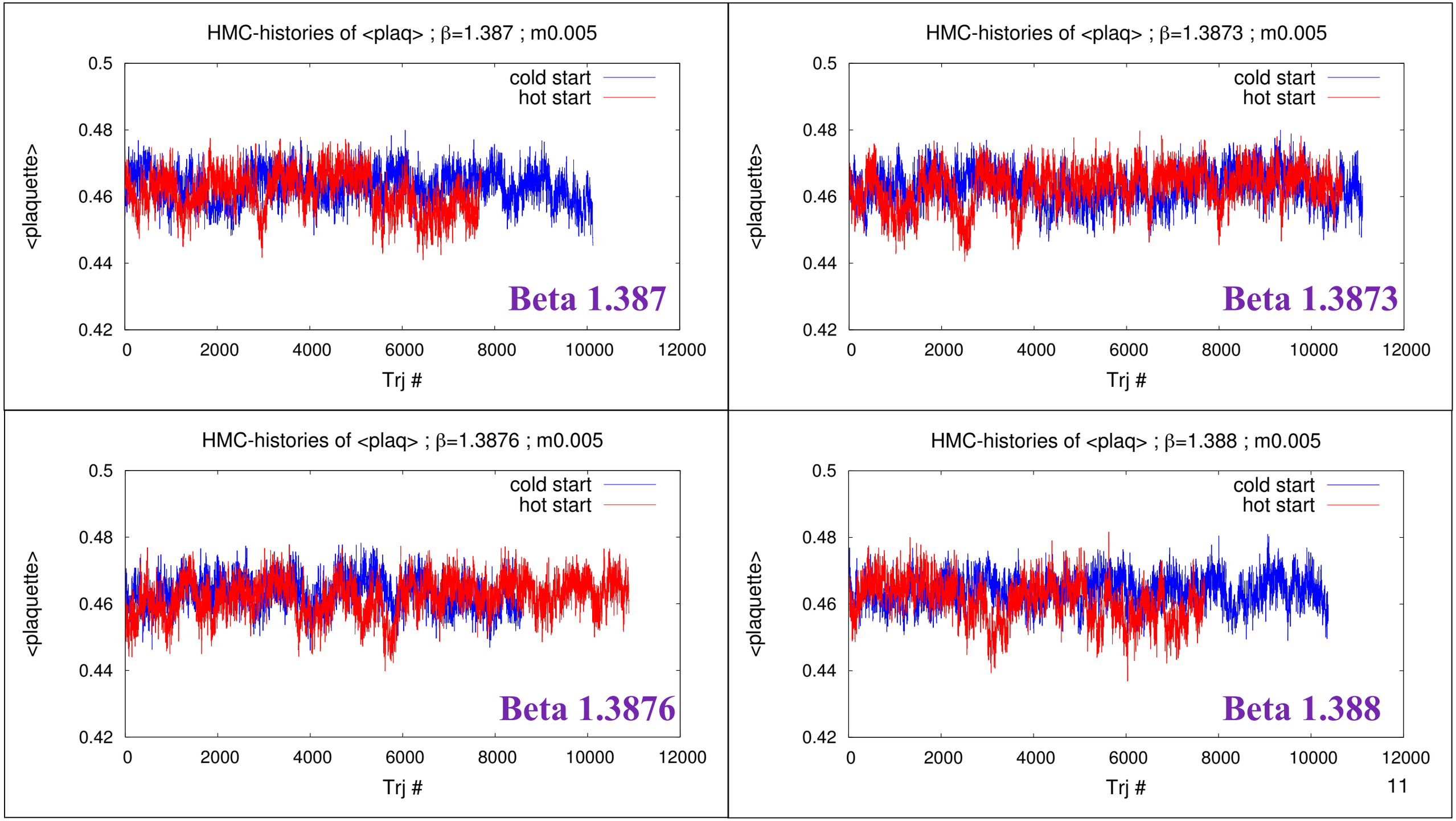
Plaquette—Cold-start & Hot-start Runs

- Study the detail around the phase boundary (β=1.387~1.388) for m=0.005 with HMC-histories in volume 8×8^3 :



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Polyakov loop—General Study

- APE smearing (To average out local fluctuations without changing IR physics).
- One cannot apply smearing too many times because of finite lattice sizes with boundaries.
- In our studies, where isometric lattices are applied, the largest smearing time for $N_t \times N_x N_y N_z = N \times N^3$ lattice is (N - 2).

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  (The confining phase)
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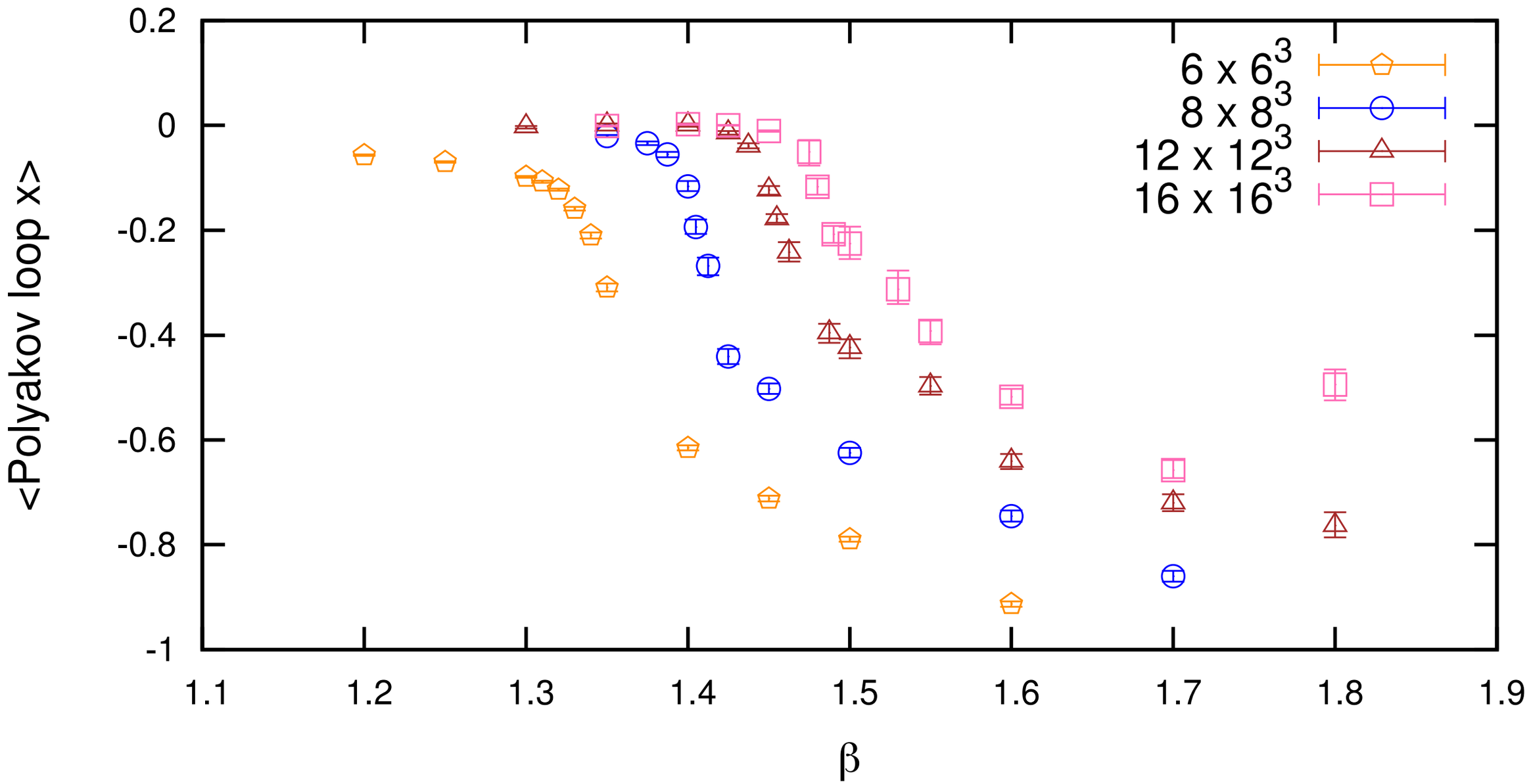

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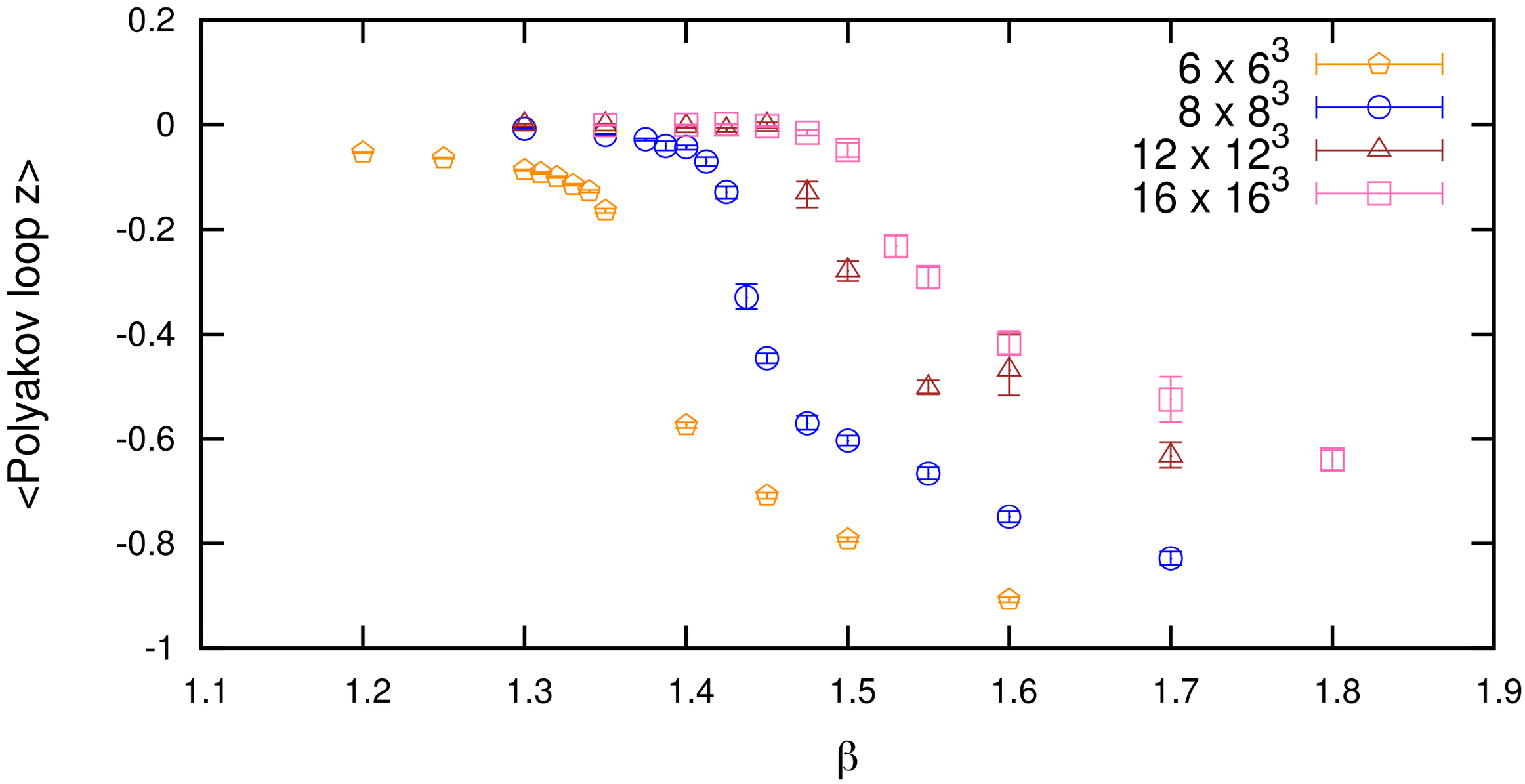
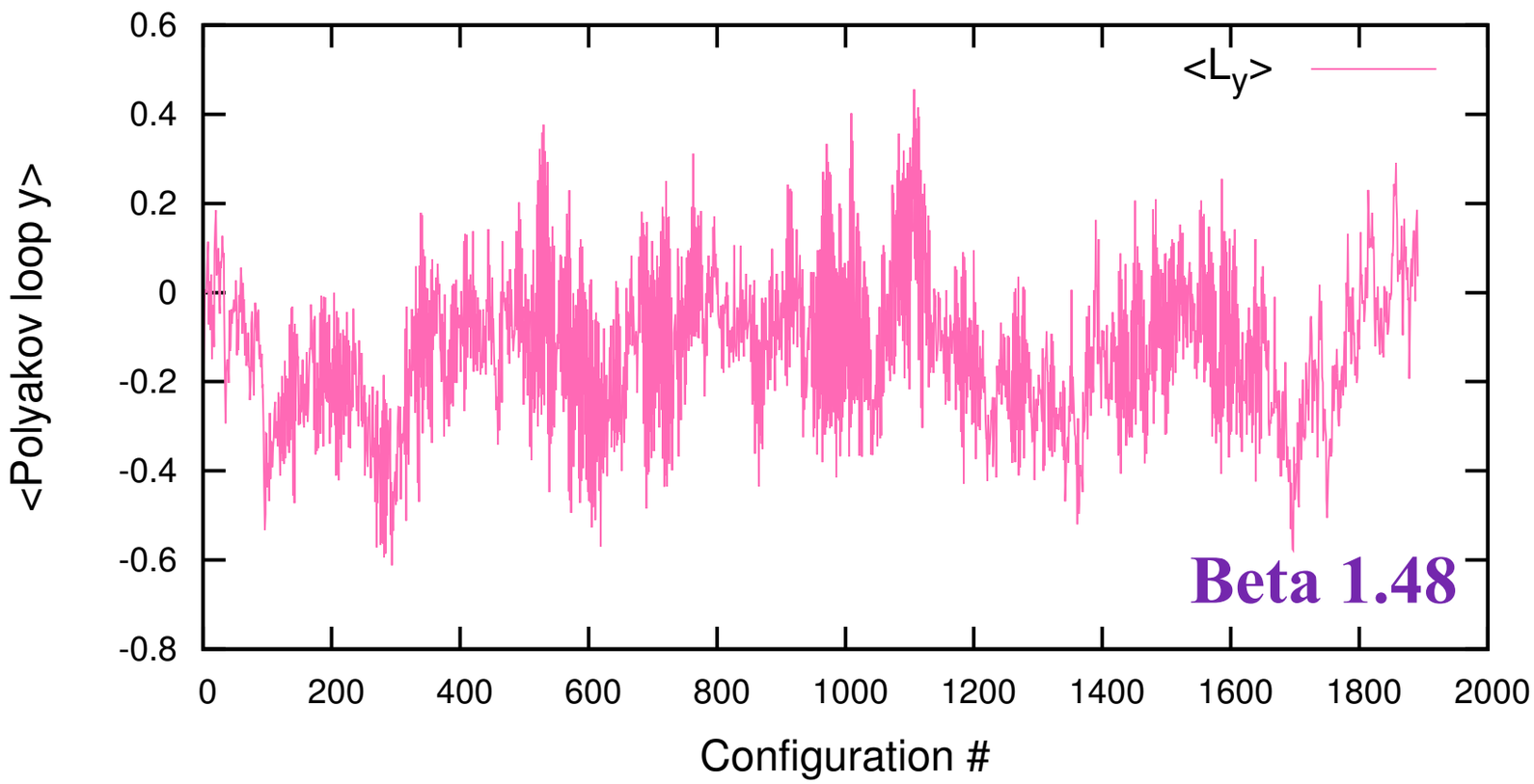
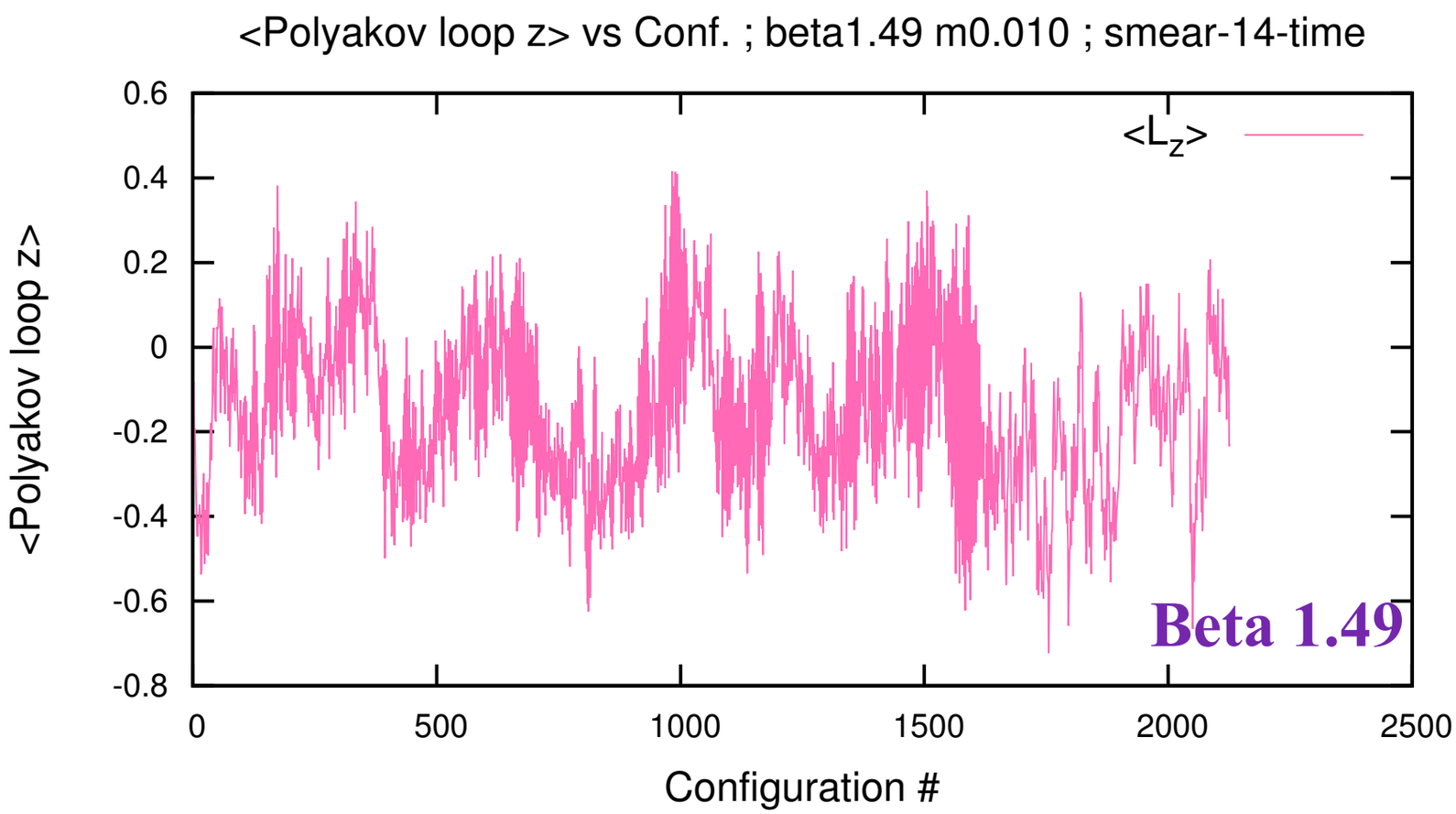
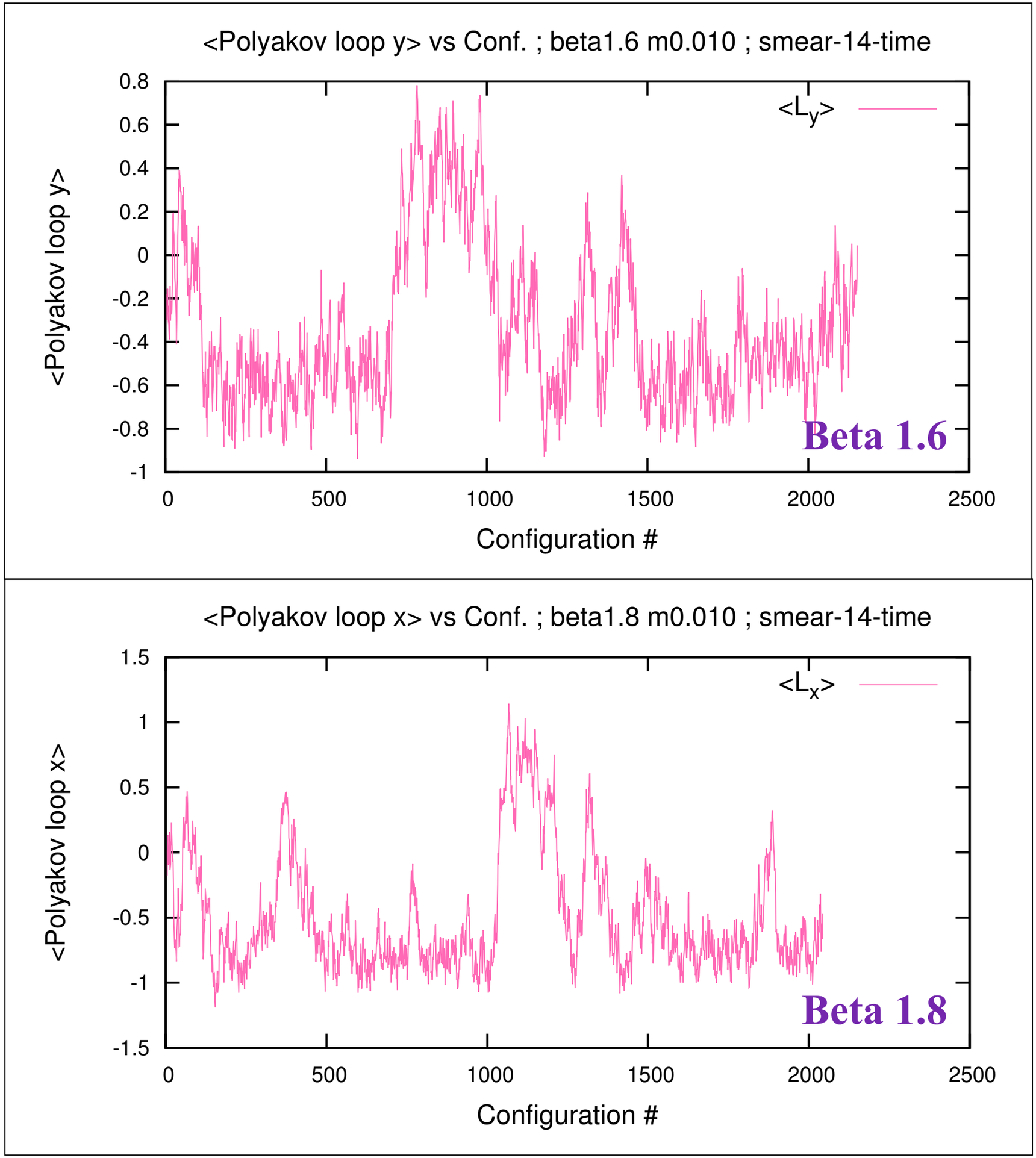






Mass 0.010
Volume(16×16^3)

Indications of tunneling around the phase transition point.



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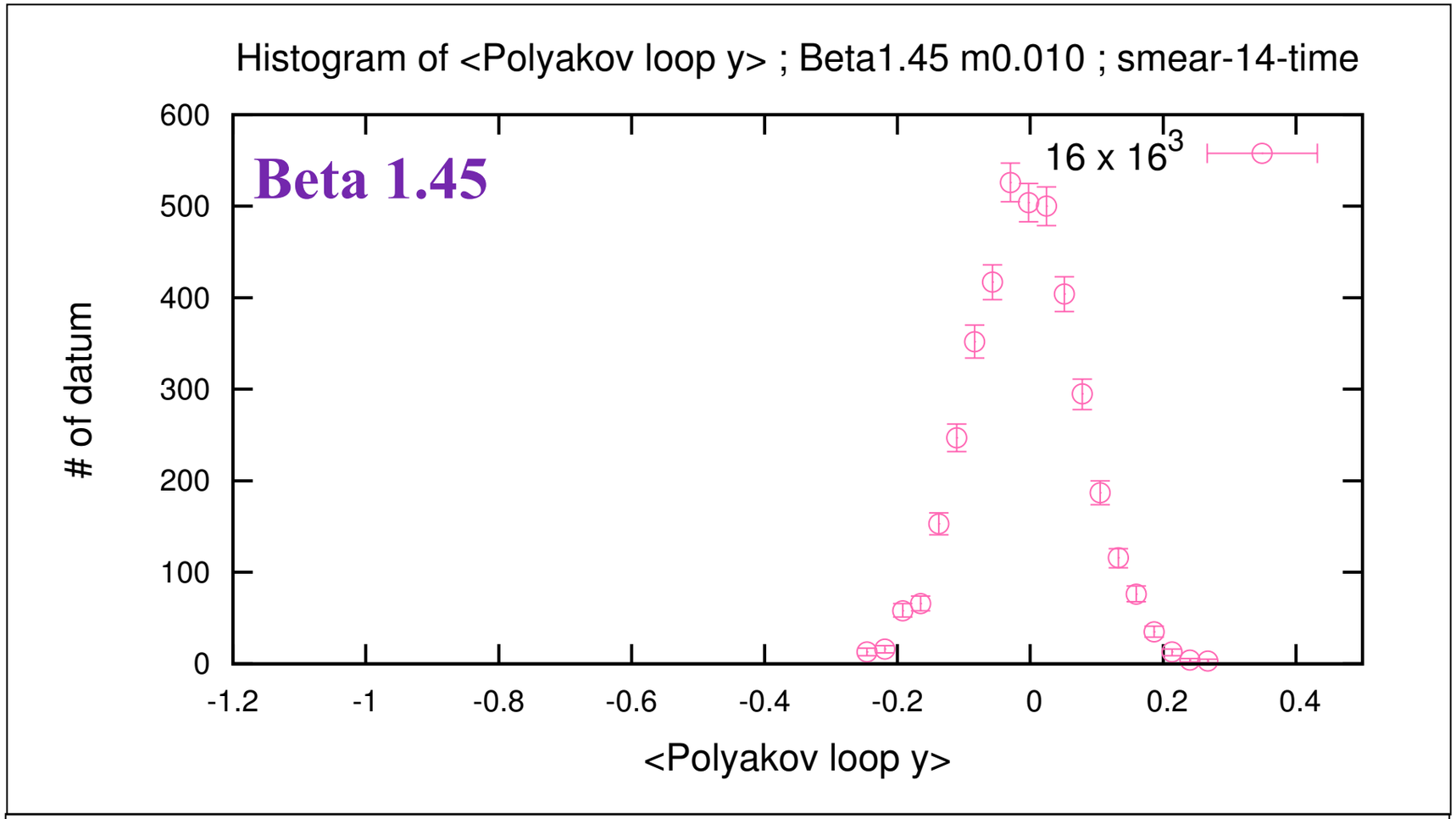
Tunnelings are ubiquitous in the deconfining phase.



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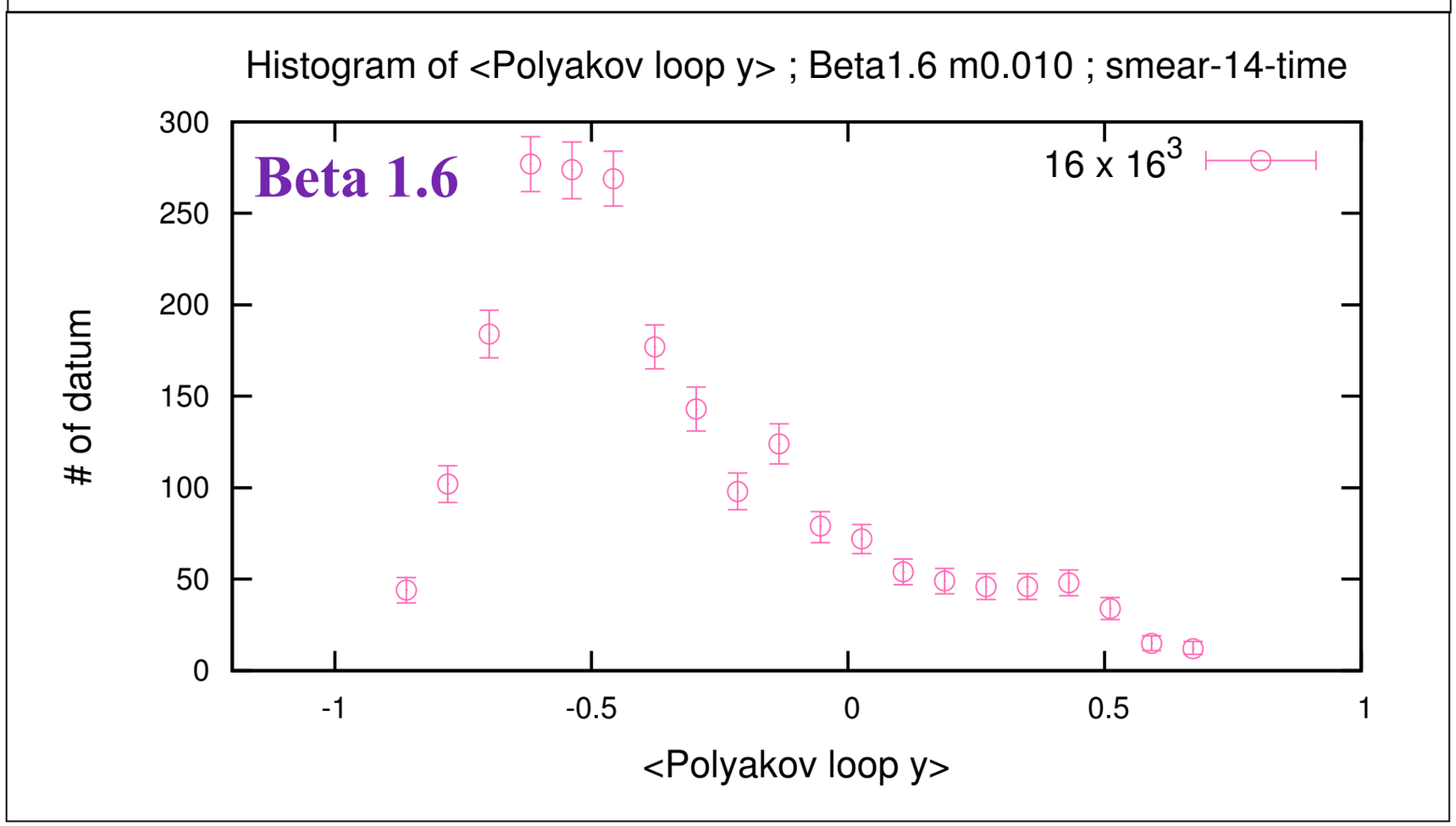
Relatively *strong* coupling regime



Relatively *weak* coupling regime

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[R. Fukuda and E. Kyriakopoulos, Nucl. Phys. B85 (1975) 354-364]

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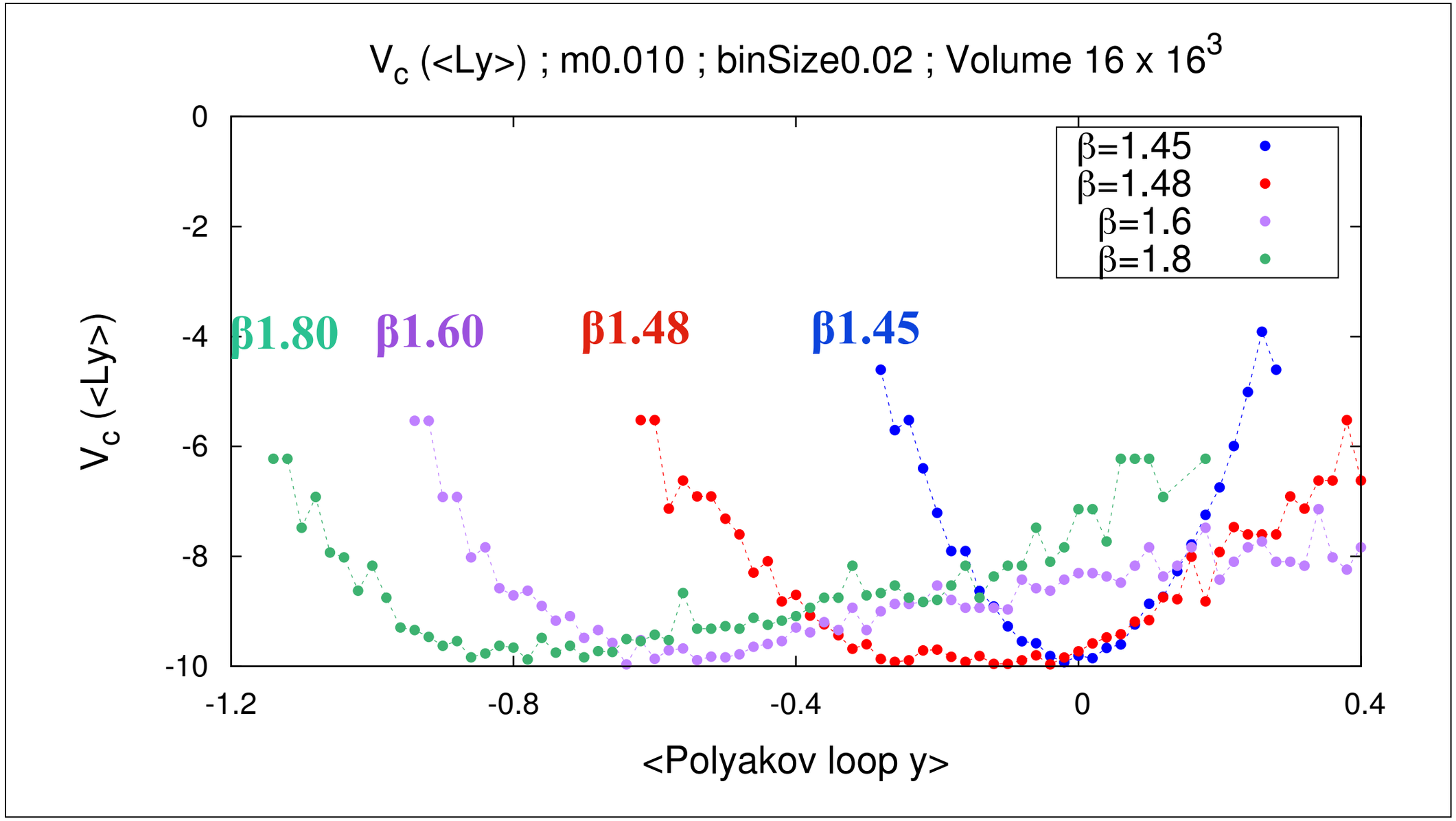
→ $V_c(<\mathbf{L}>) = -ln(\mathbf{N}),$ where **N** is the number of data points falling into specific <**L**>.

~ probability distribution (P) for <L>

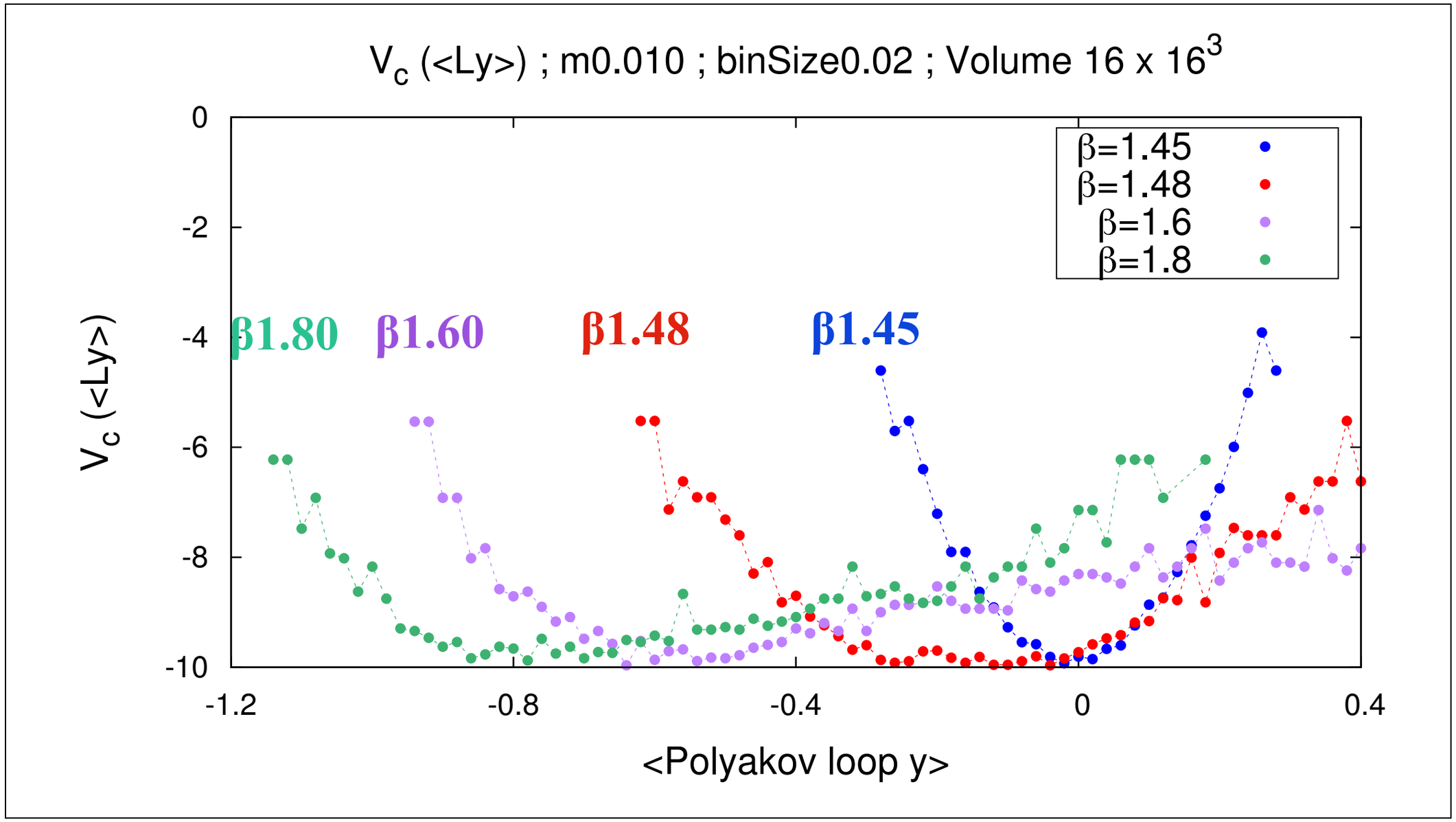


Relatively *strong* coupling regime → **Symmetric potential**

Around the phase transition point → **Double-well shape potential (1st order)**

Broadened shape potential (2nd order)

In relative *weak* coupling regime → **Tilted, asymmetric potential**



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In relative *weak* coupling regime → **Tilted, asymmetric potential**

(1) The simulation was very close to the phase boundary.

(2) Combining this study with histories of <Ly>, we may have a weakly 1st order phase transition

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The Dirac Operator Eigenvalue Spectrum

- The chiral condensate (an order parameter for the spontaneous chiral symmetry breaking) can be determined from

$$\boxed{< \bar{\psi}\psi >= \ -\pi\rho(0).}$$

→ $\rho(\lambda)$ is the density with eigenvalue λ [T.Banks and A.Casher, Nucl. Phys. B169, 103(1980); E. Marinari, G. Parisi, and C. Rebbi, Rhys. Rev. Lett.47, 1795 (1981) & H. Leutwyler and A. Smilga, Phys. Rev. D46 (1992) 5607].

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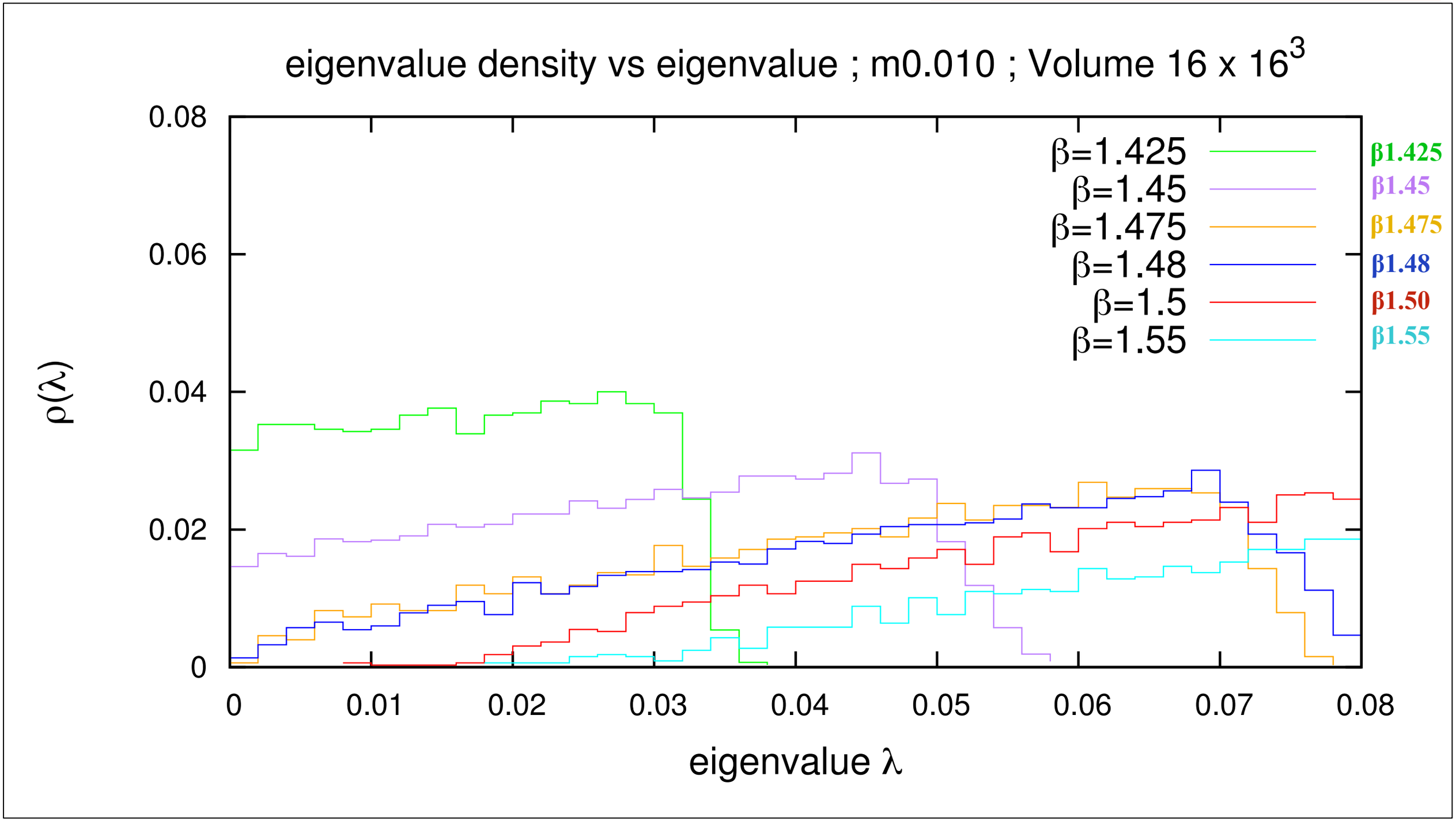
→ We focus on studies of **(1)** $\rho(\lambda)$ with fixed **V** & **M** at different **β**.

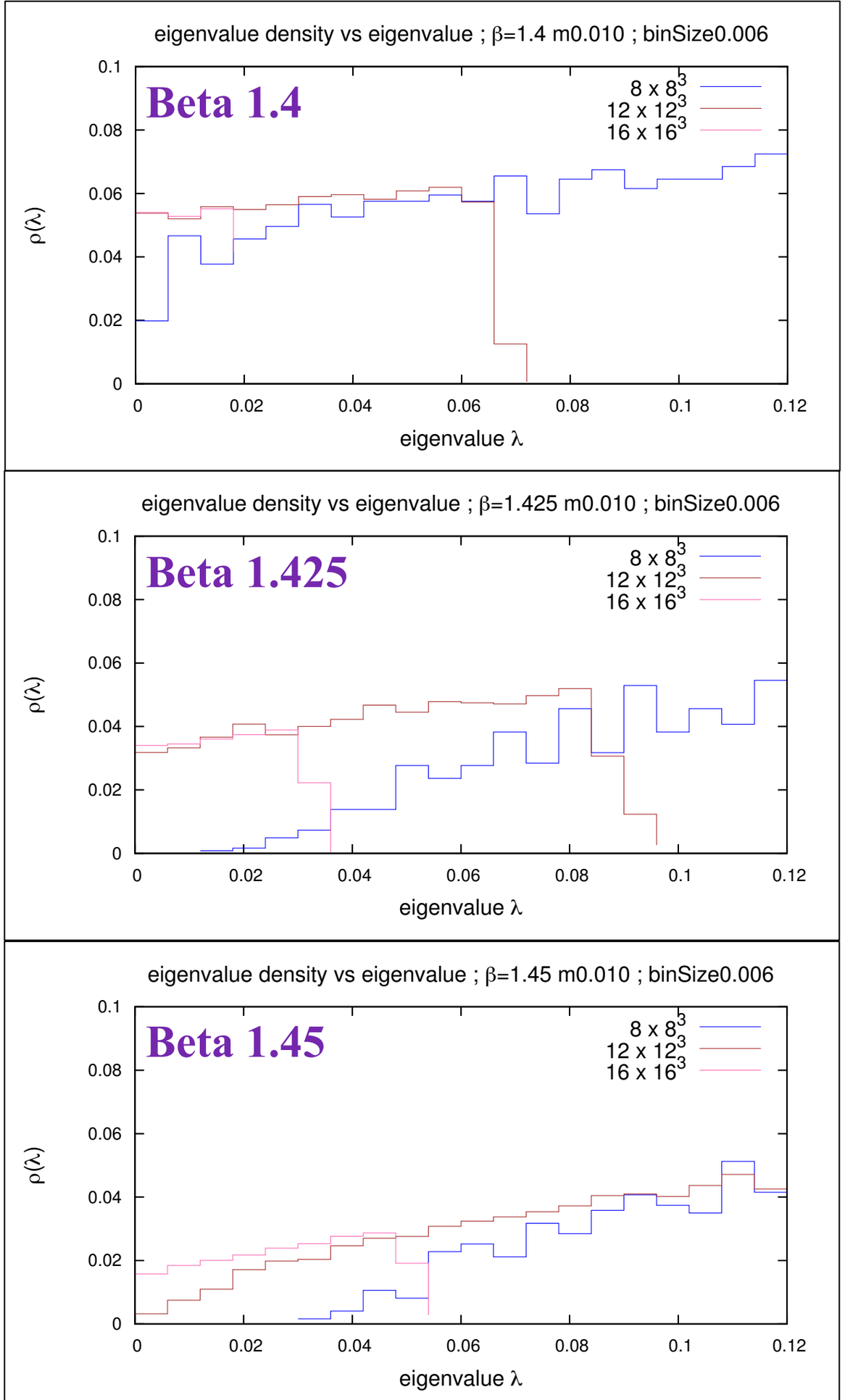
(2) $\rho(\lambda)$ with fixed **β** & **M** in different **V**.

(1) ρ with fixed V & M at different β

- ρ(0) ~ 0
  (**Chirally-symmetric phase**)
- ρ(0) > 0
  (**Chirally-broken phase**)



(2) ρ with fixed β & M in different V



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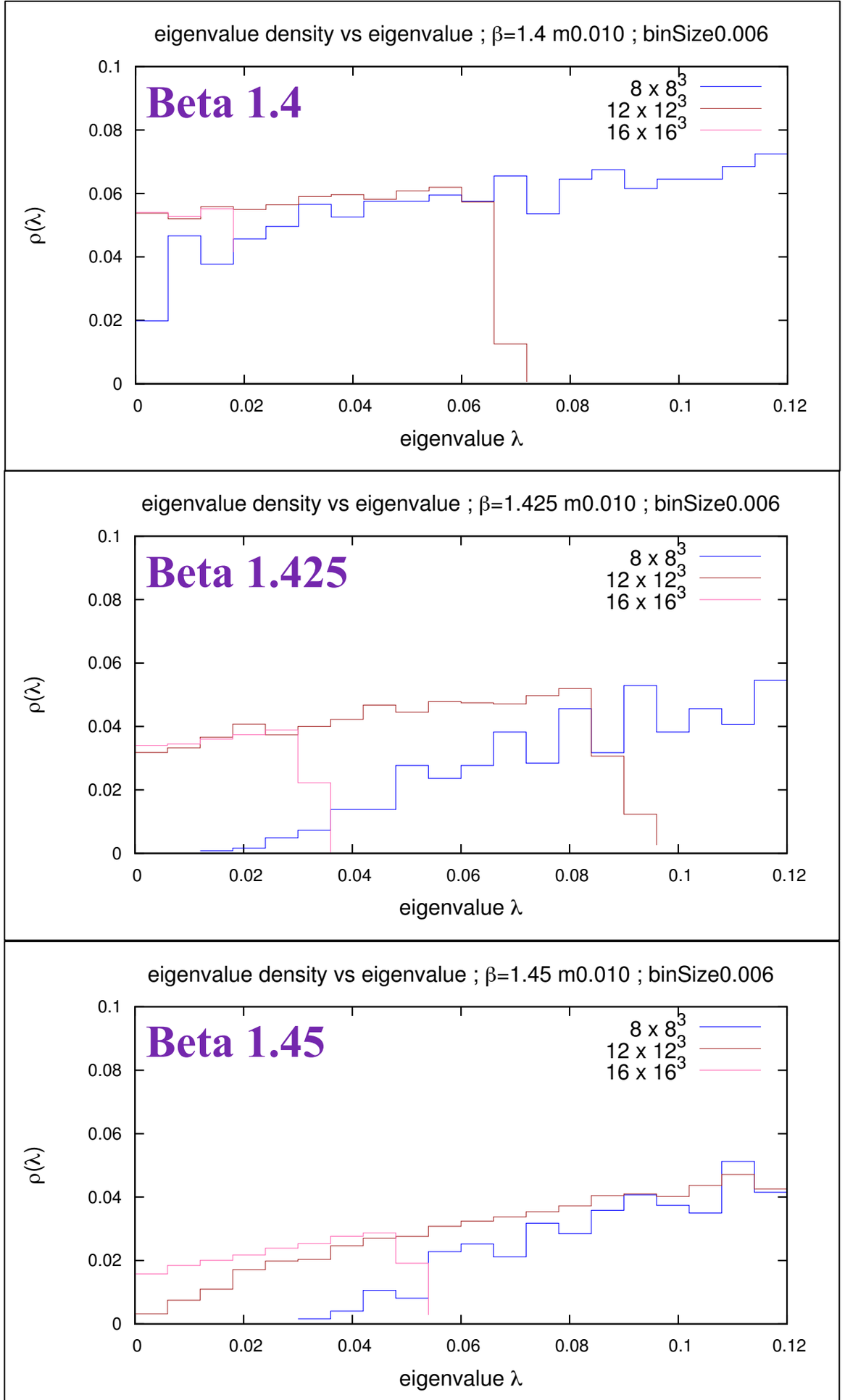

smaller β

larger β

As β increases, the volume dependence becomes more and more apparent.

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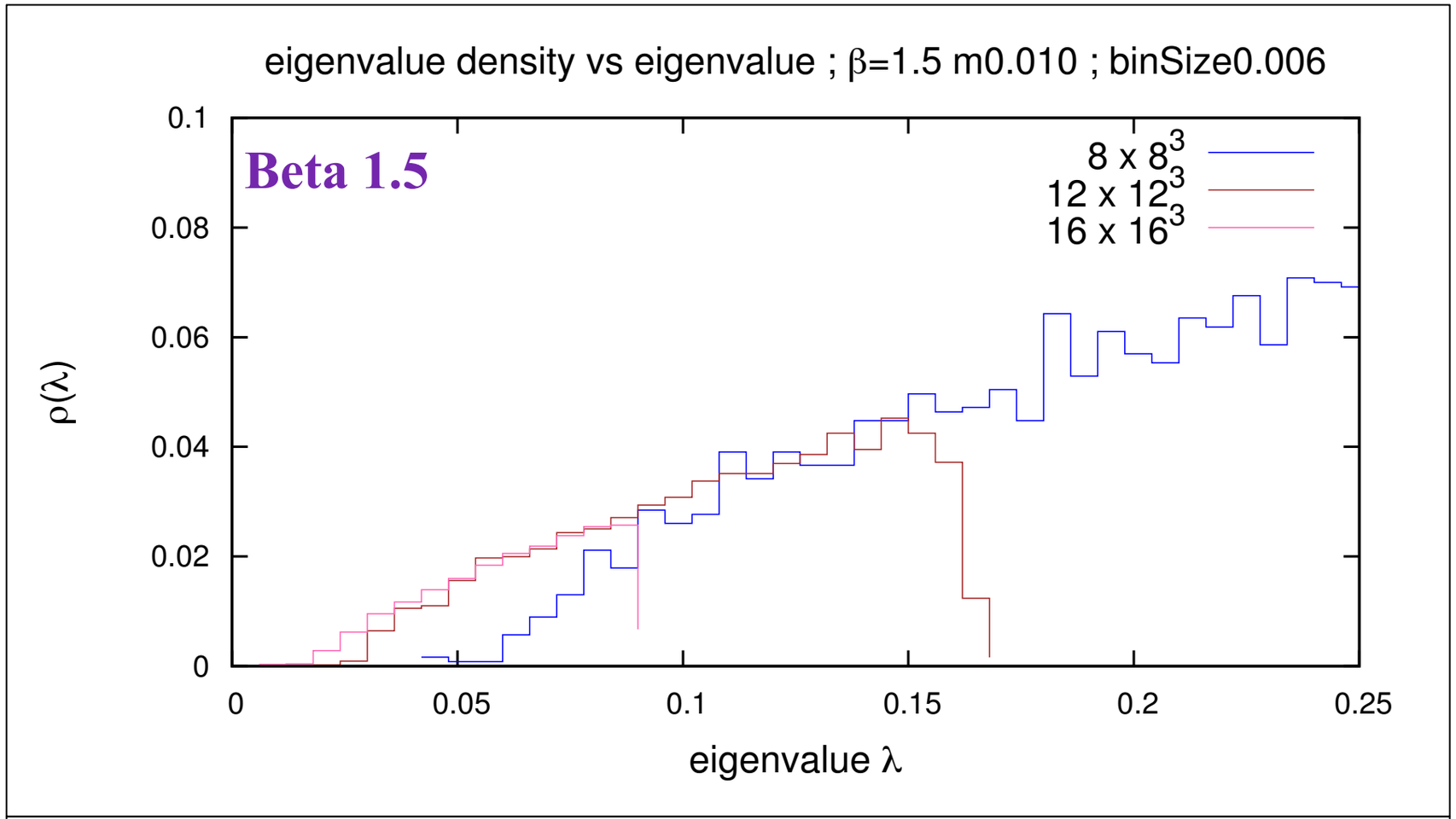

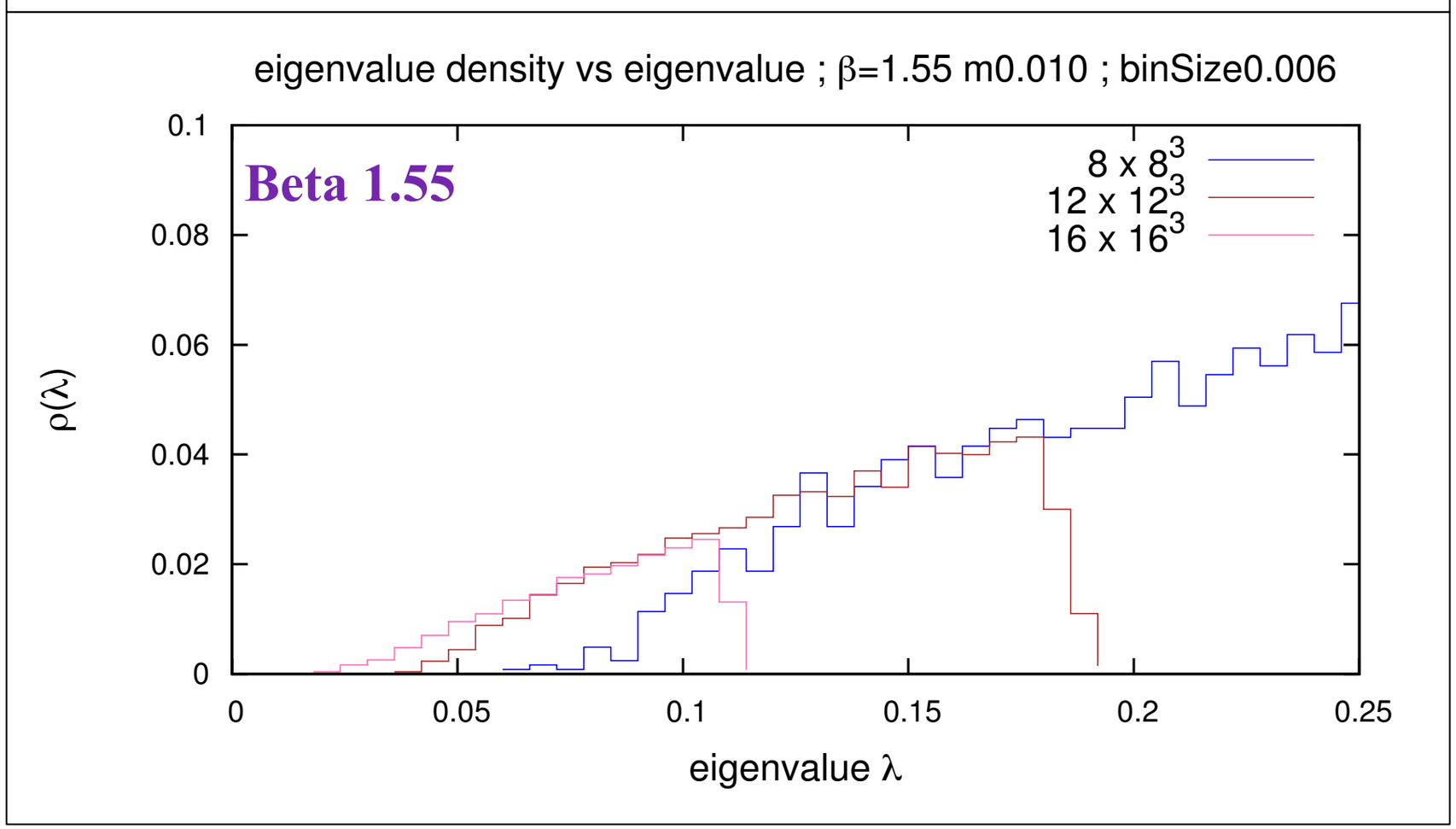

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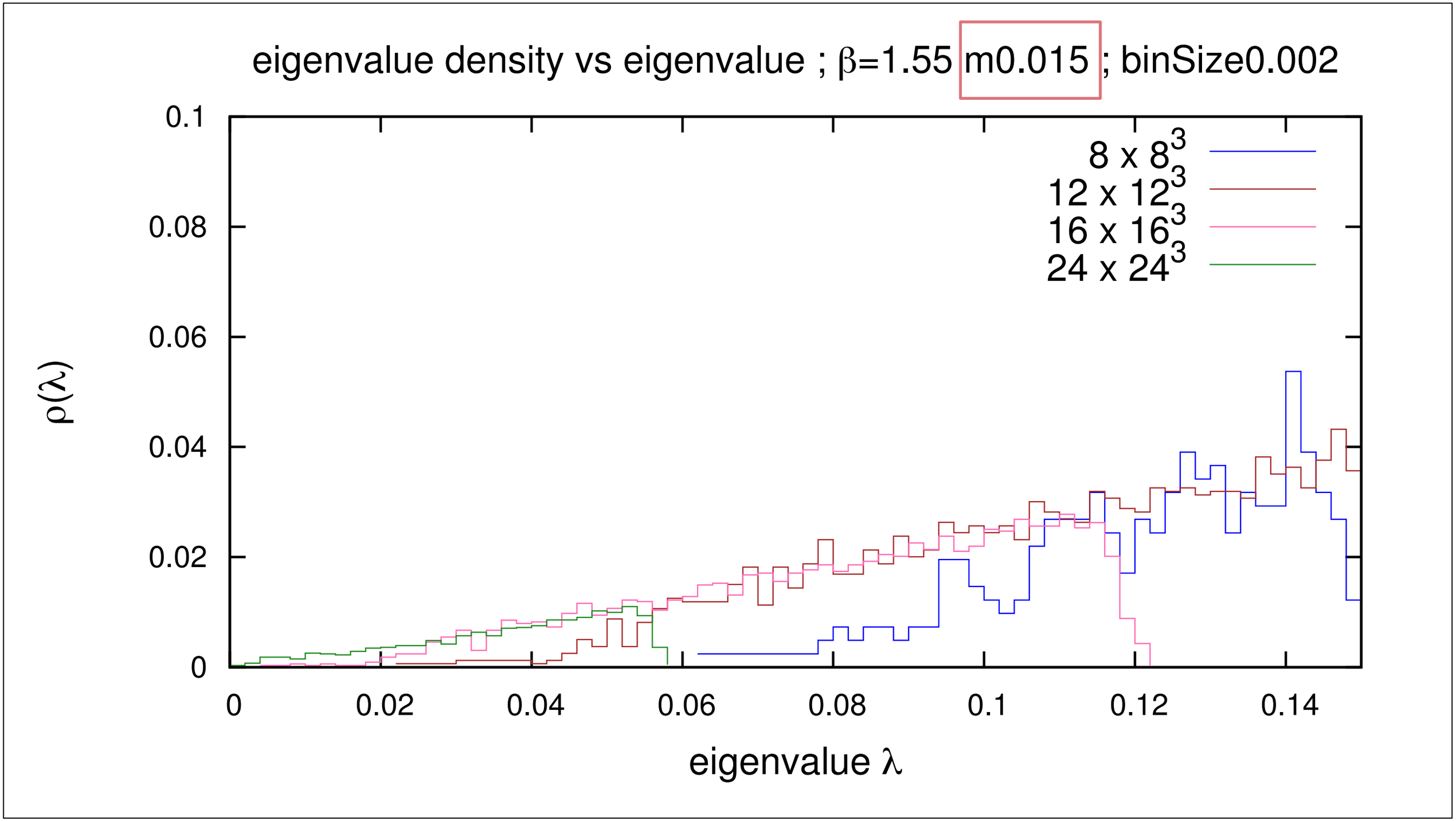

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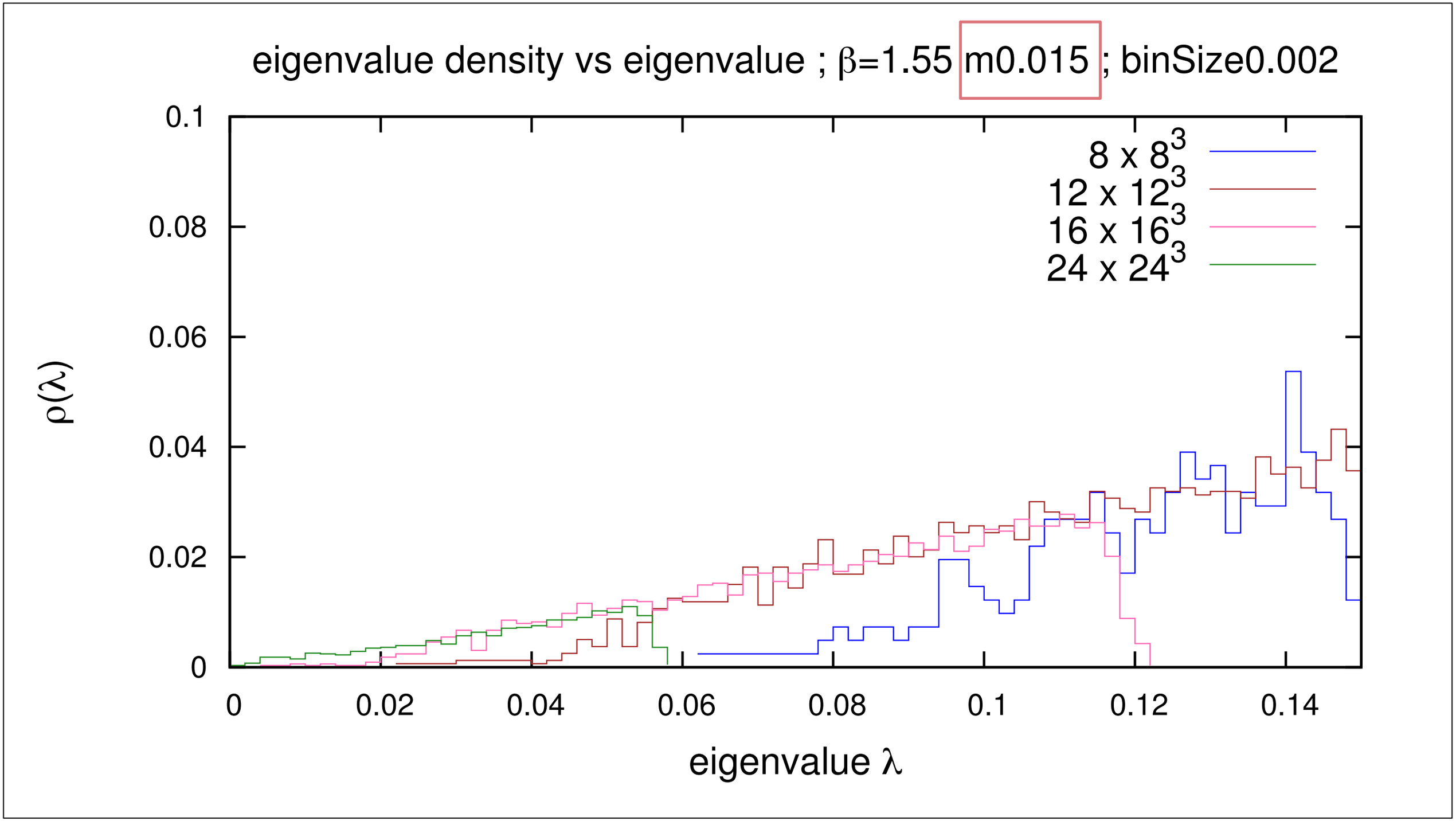
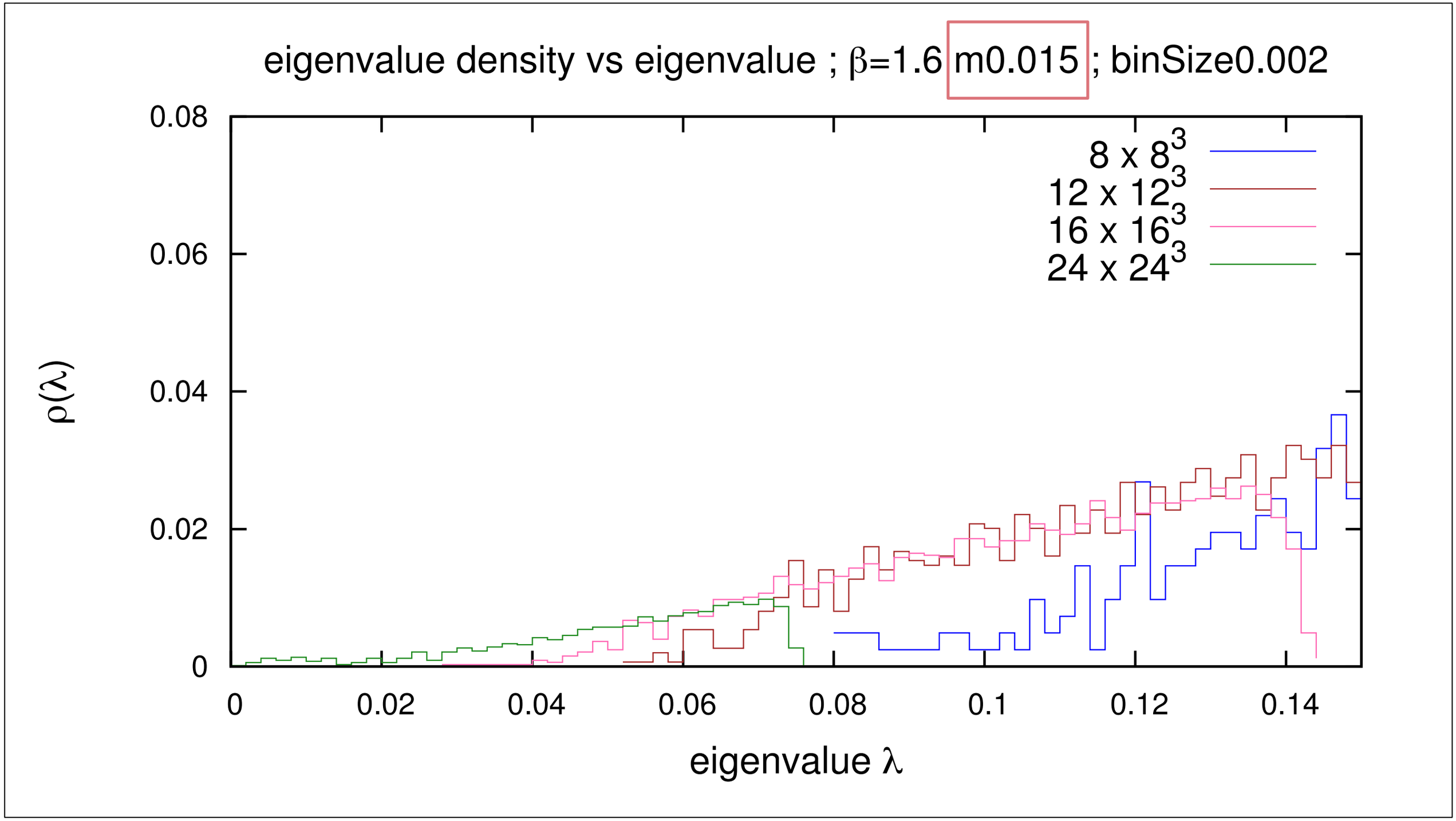


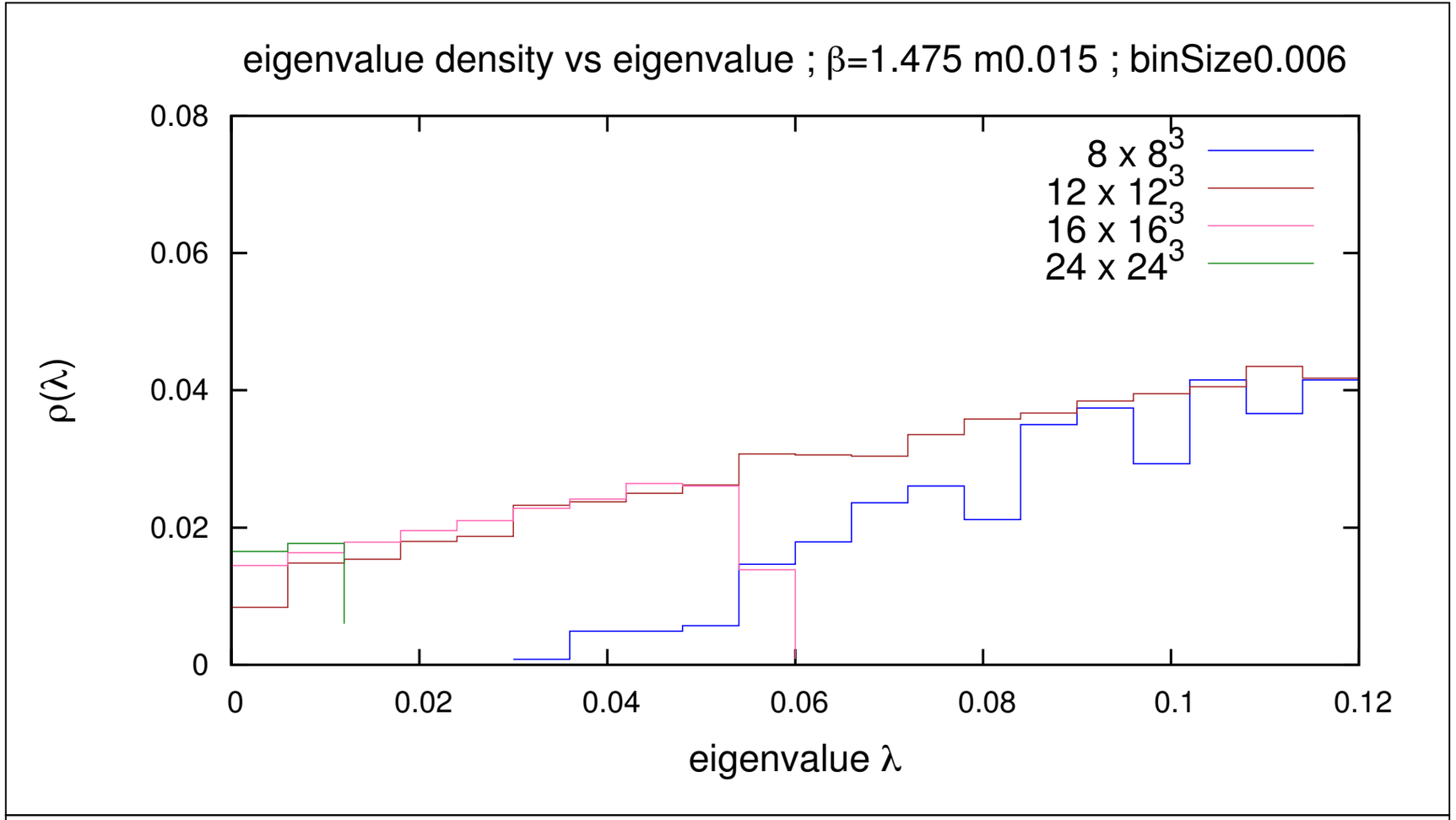
→ **The volume effect gradually decreases in larger volumes, which means that if we have the chiral phase transition, it is possibly to be bulk.**

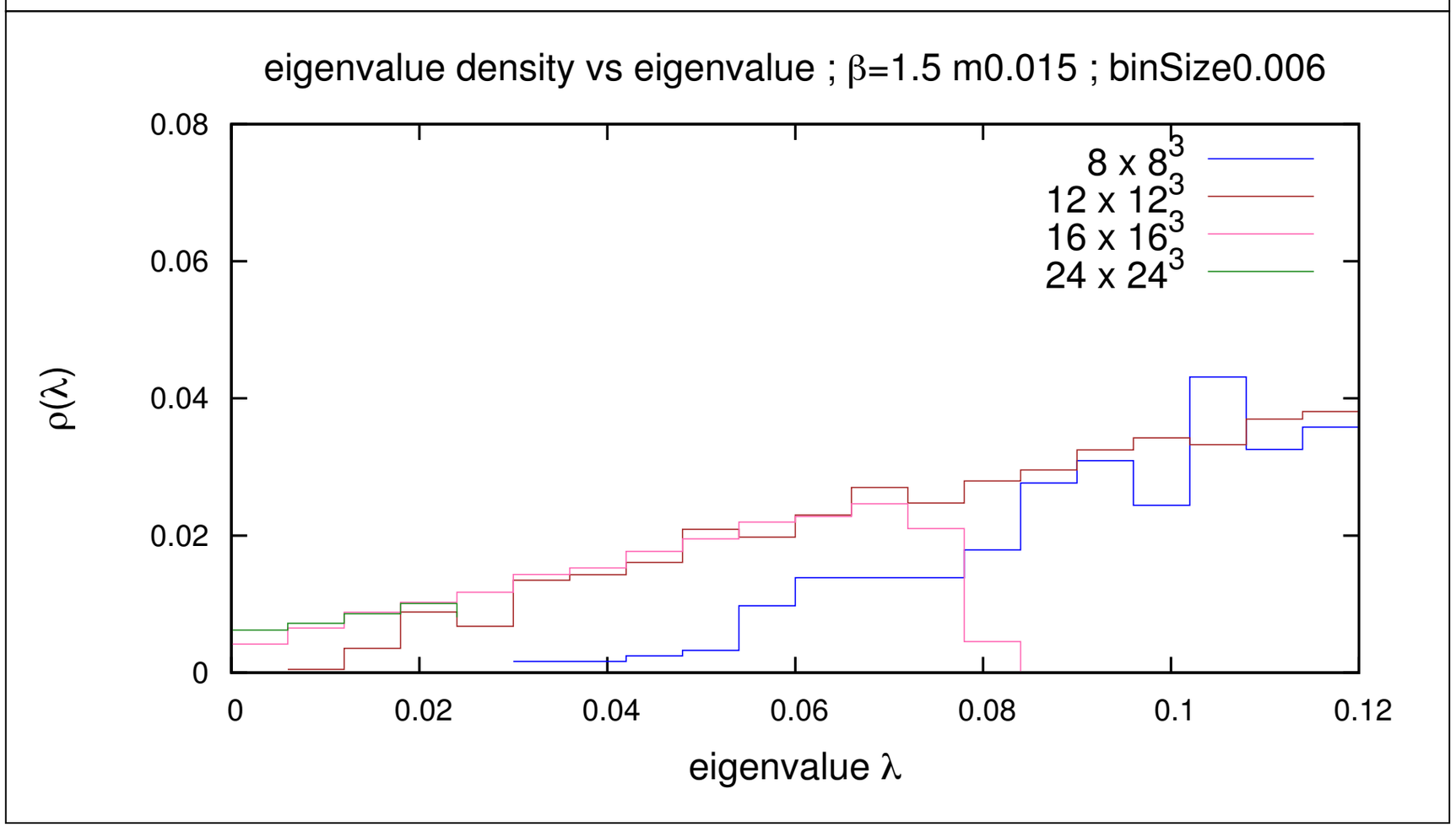
(2) ρ with fixed β & M in different V



→ **If the chiral phase transition is bulk, what can we conclude so far?**

→ **Any other possible realization to explain the observation?**





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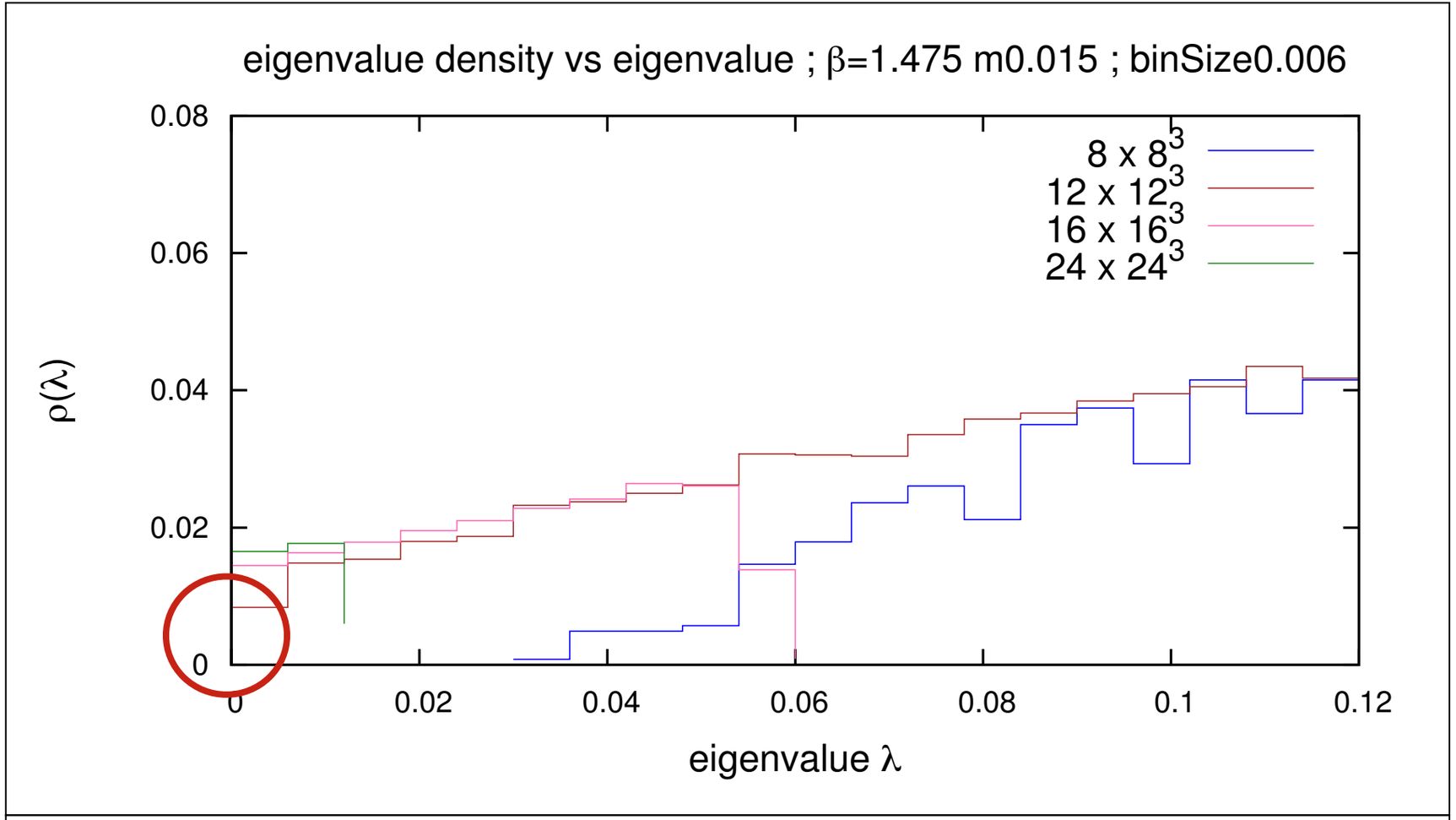
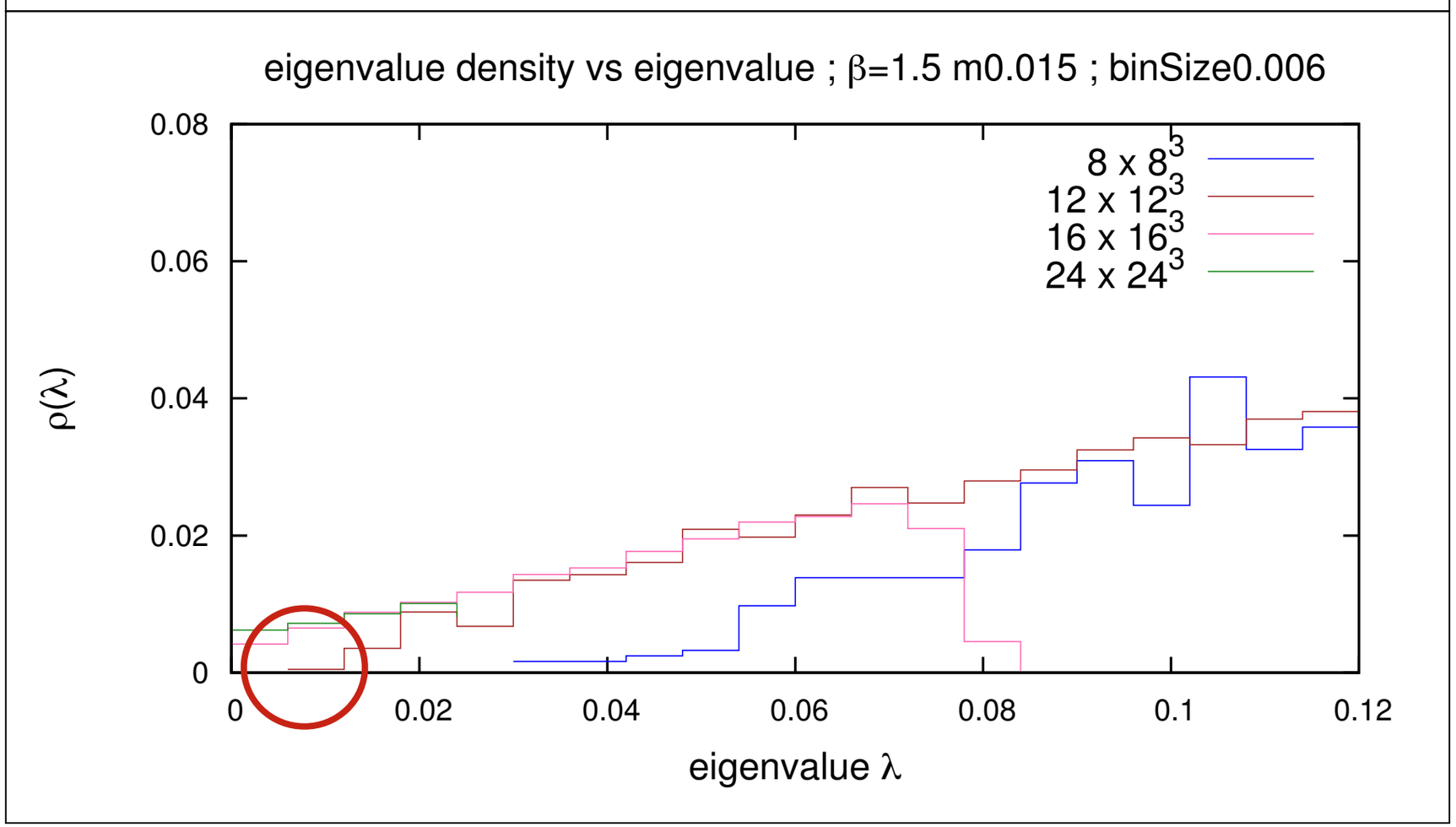
larger β





smaller β

larger β

Summary & Future Work

- From studies of plaquette and Polyakov loops, one weakly first order, bulk phase transition is observed.
- From the Dirac operator eigenvalue spectrum, one phase transition is observed, yet the properties & order of this phase transition take further investigations.

- String tension studies (More precise identification for confinement).
- Spectrum studies (ex: Looking for light scalar bound states).
- Include results of the distributions of $\rho(\lambda)$ for lighter mass in various volumes and β.
- Study the chiral condensate for fixed topology pertaining to the onset of the ϵ-regime.

- The End -
Thanks for Your Attention! :)

Backup: Motivations

- Why technicolor (Beyond Standard Model) ?

(a) Standard Model (SM) with Higgs: Higgs field provides the mechanism for electroweak (EW) symmetry breaking. Yet, it is a scalar and it has problems like triviality & hierarchy.

(b) **Technicolor**:

(i) It does not have fundamental scalar fields.

(ii) It is asymptotically free, giving non-trivial theories *without* triviality & hierarchy problems.

Backup: Motivations

- Technicolor (TC) :

  (a) **Technigauge** (Gauge field) + **Techniquarks** (Massless fermions).

  (b) Techniquarks carry both technicolor & EW charges.

  (c) Chiral symmetry breaking in technicolor $\longrightarrow$ EW symmetry breaking.

  (d) Cutoff scale : $\Lambda_{TC} \approx \Lambda_{EW}$.

- Extended-technicolor (ETC) :

  To explain fermion masses, besides having technicolor, one introduces an interaction coupling SM fermions & techniquarks.

Backup: A Related Work

- Phase structure study of SU(3) lattice gauge theory with 12 flavors **(Anqi Cheng, Anna Hasenfratz, David Schaich, arXiv:1111.2317 [hep-lat]).**

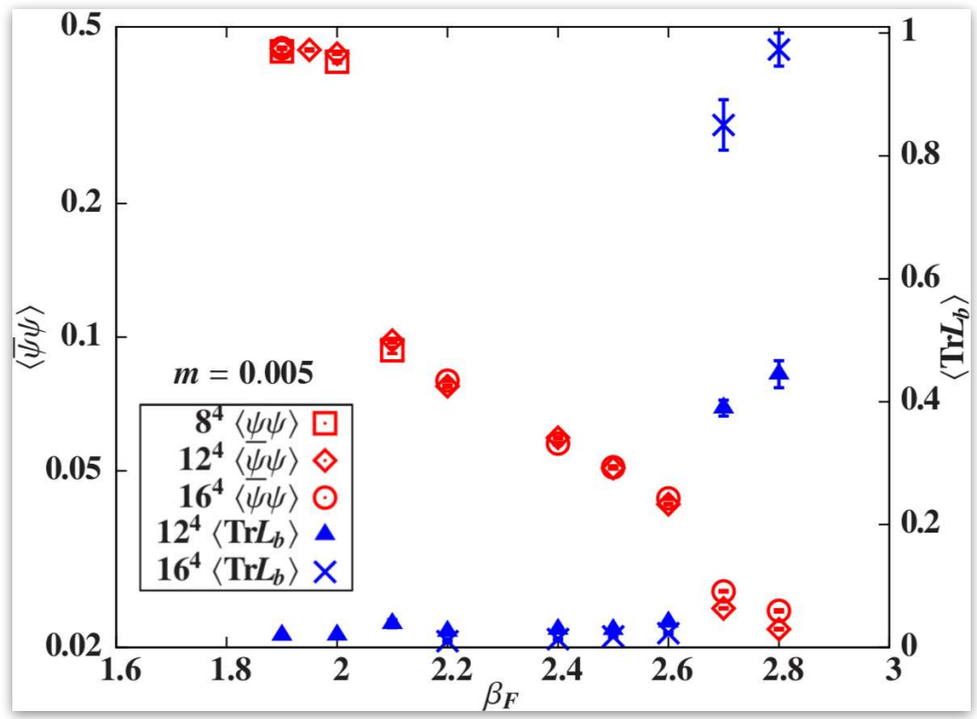


FIG. 1. The chiral condensate $\langle\overline{\psi}\psi\rangle$ (on a log scale) and the blocked Polyakov loop $\langle \mathrm{Tr} L_b \rangle$ indicate two well-separated transitions at $m = 0.005$.

Backup: Plaquette—General Study

- Definition & Picture (A plaquette on ***μν***−plane as an example):

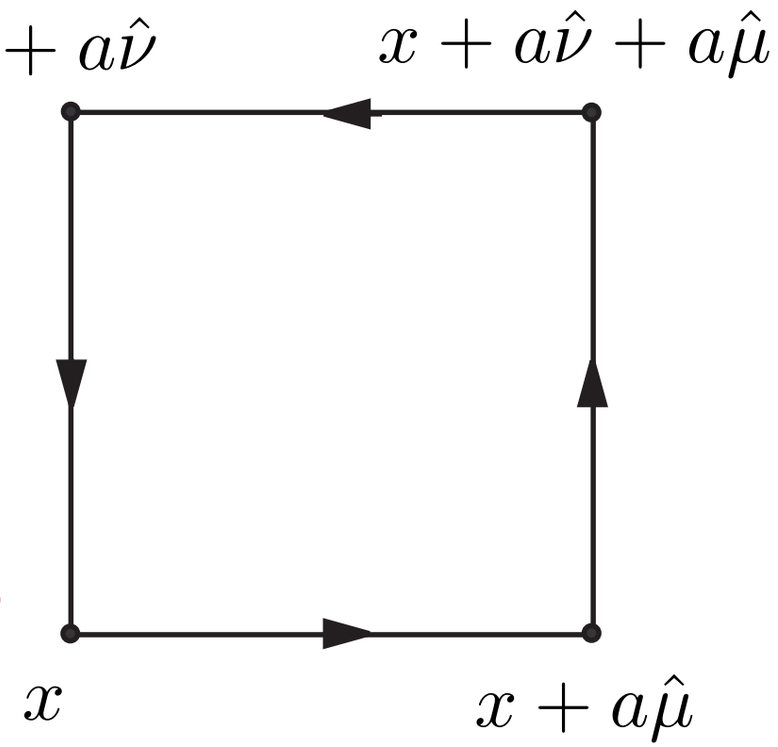
$$\boxed{\square_{\mu\nu}^{\gamma\delta} = U_{\mu}^{\gamma l}(x) U_{\nu}^{lm}(x + a\hat{\mu}) U_{\mu}^{mn\dagger}(x + a\hat{\nu}) U_{\nu}^{n\delta\dagger}(x),}$$

→ x: Lattice site.

→ a: Lattice spacing.

→ γ, *l*, *m*, *n*, *δ*: Color indices.

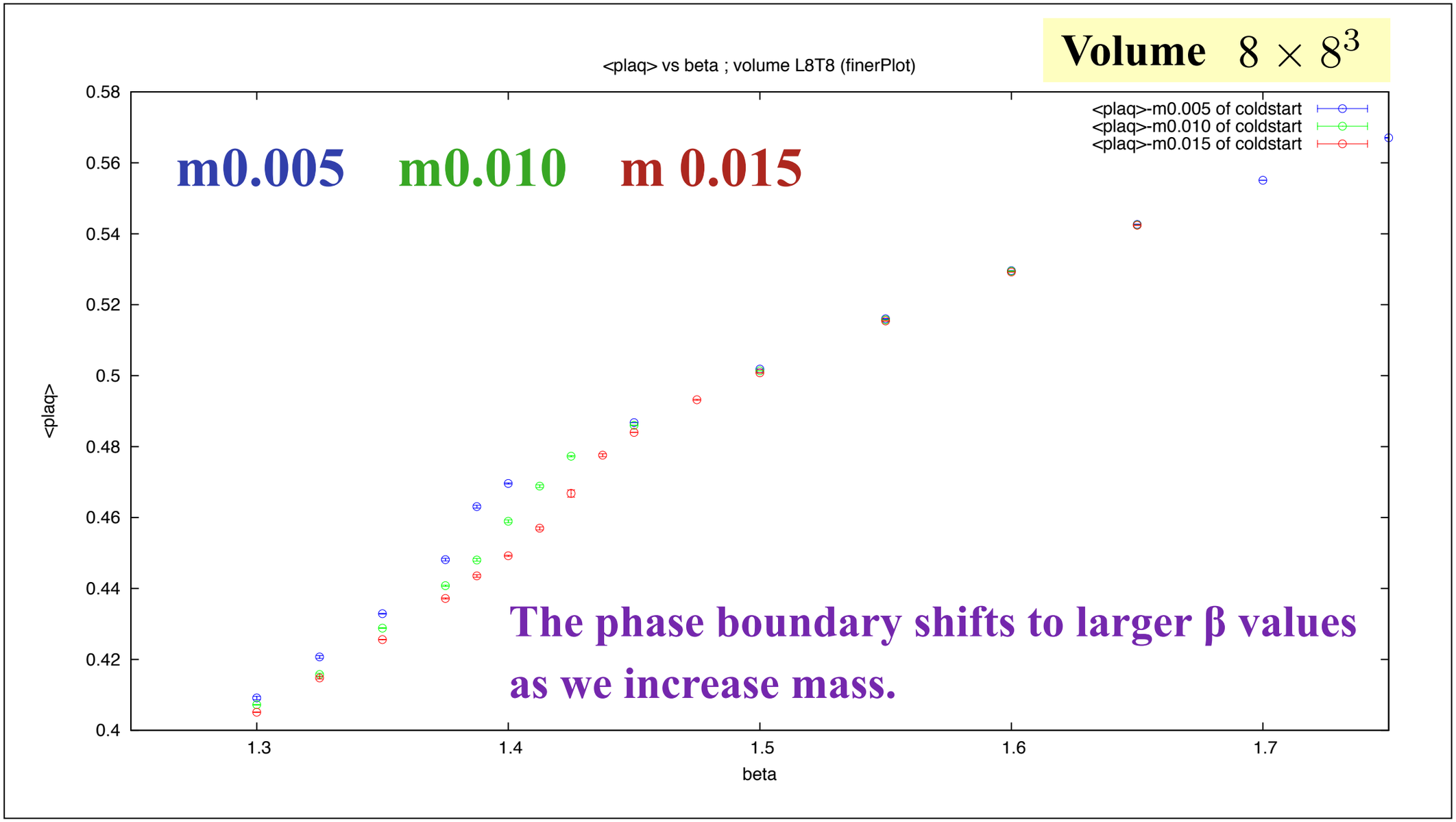
The smallest Wilson loop



- It constructs the lattice Yang-Mills action.

Backup: Plaquette—General Study (Fixed Volume)

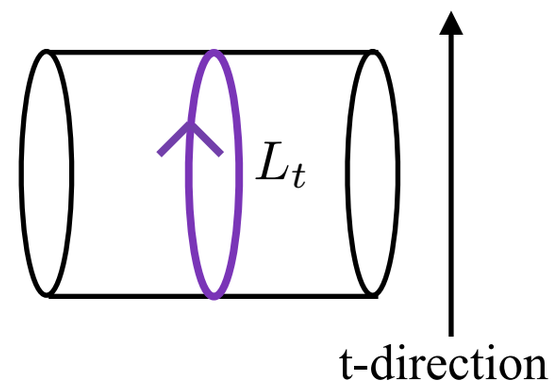
- Zoom-in <Plaq> vs β plot : (**One phase transition is observed.**)



Backup: Polyakov loop—General Study

- Definition & Picture (t-direction as an example):

$$L_t(n) = \frac{1}{N_x N_y N_z} Tr[\prod_{i=0}^{N_t-1} U_t(n+i\hat{t})],$$



where n is the lattice site.

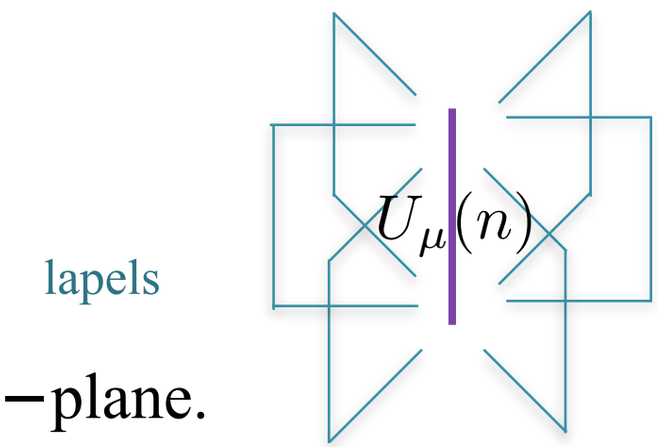
→ **<Polyakov loop (0, 1, 2, 3)>** is **<Polyakov loop (x, y, z, t)>** throughout the slides.

- In the pure gauge theory, Polyakov loop is an order parameter for confining-deconfining phase transition.
- With dynamical fermions, it is no longer a good order parameter, yet giving some information for the phase structure study.

Backup: Polyakov loop—General Study (APE smearing)

- Our smearing procedure : (for one link variable U)

$$U_\mu(n) \to (1 - \alpha) \times U_\mu(n) + \frac{\alpha}{6} \times \sum_{i \neq j} P^{1,2}_{i\,j},$$



→ P is the oriented defect plaquette lying on the $i\,j$ −plane.

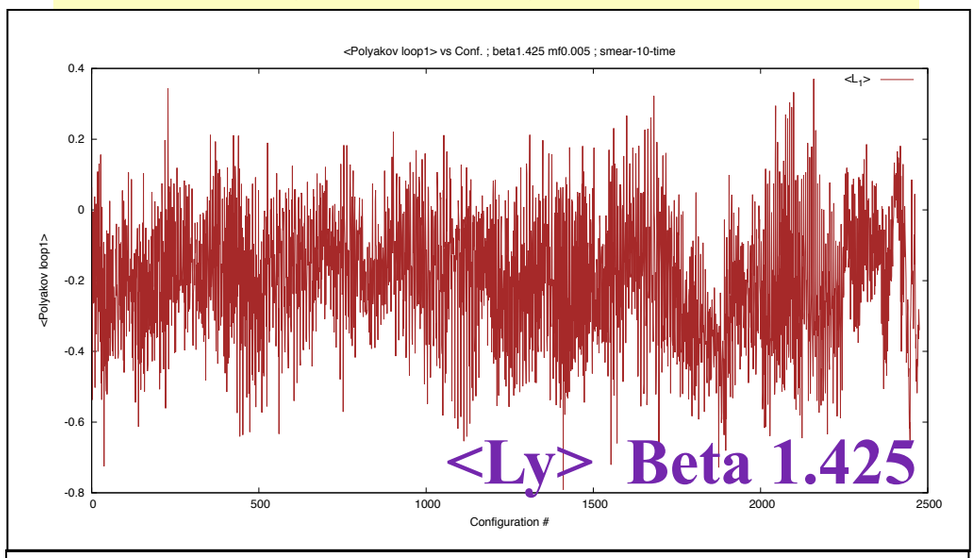
→ Upper label of P: 1, 2 corresponds to the left or right edge of $U_\mu(n)$.

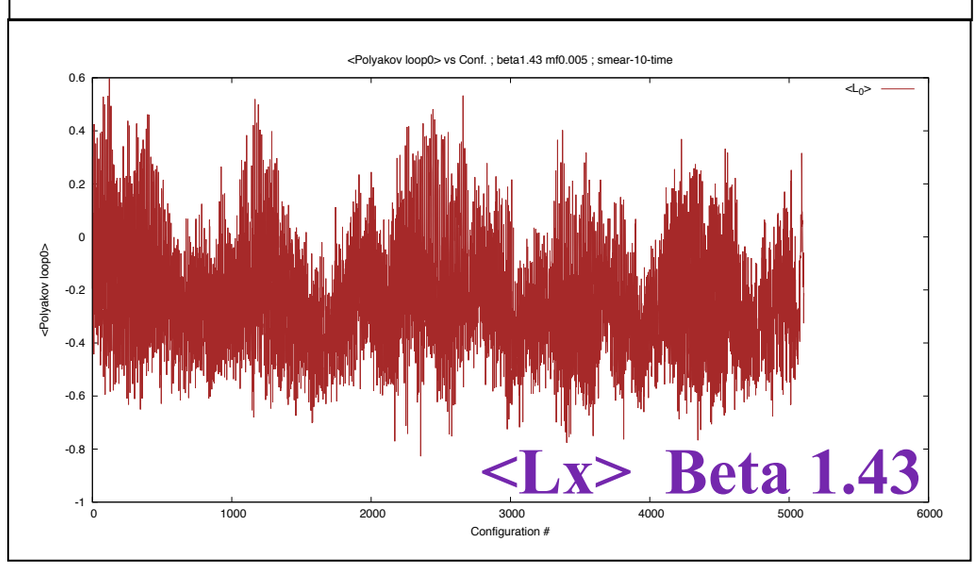
→ Smeared link is projected to SU(2) group and α = 0.5 (fixed).

Backup: Polyakov loop—General Study

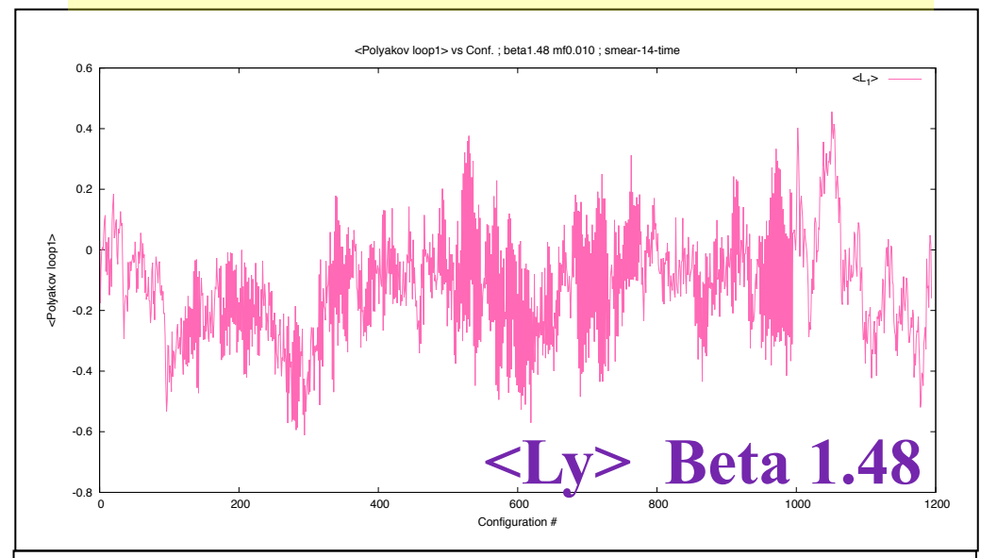
- HMC-histories for smeared <**L**>: Around the phase transition point, there are indications of tunneling.

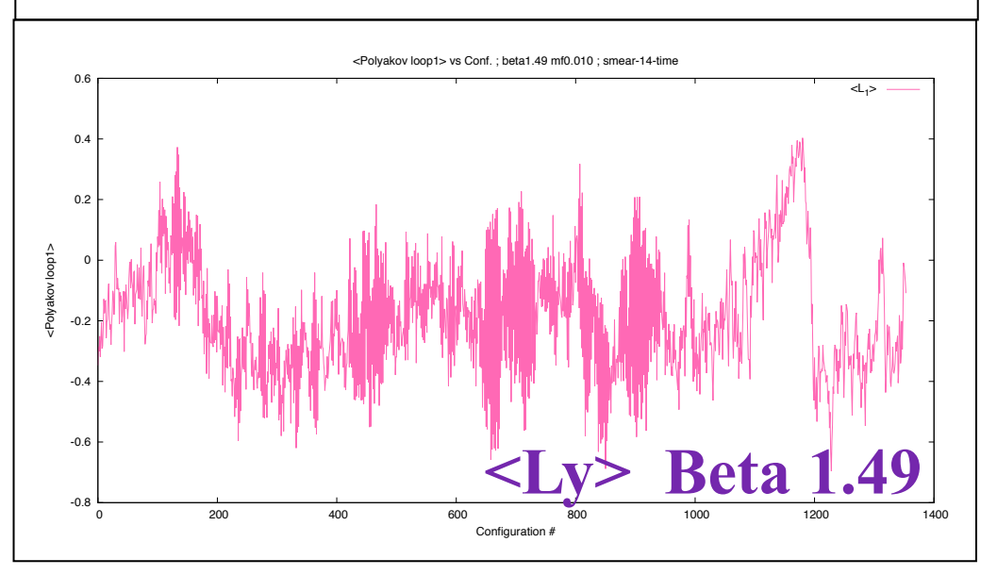
Volume(12×12^3) Mass 0.005





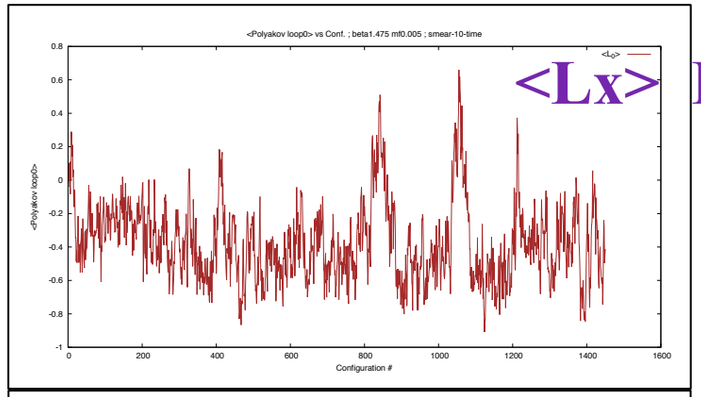
Volume(16×16^3) Mass 0.010



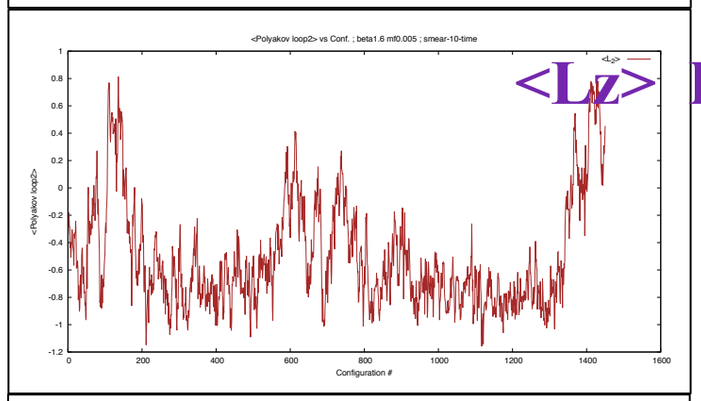


- HMC-histories for smeared <**L**>: In the deconfining phase (relatively weak coupling regime), tunneling phenomena is ubiquitous.

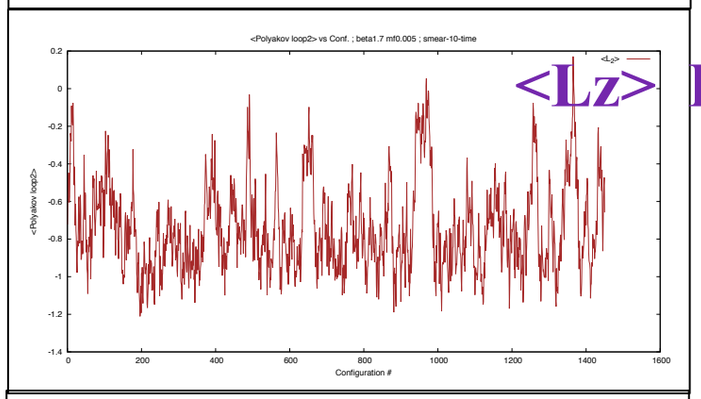
Volume(12×12^3) Mass 0.005



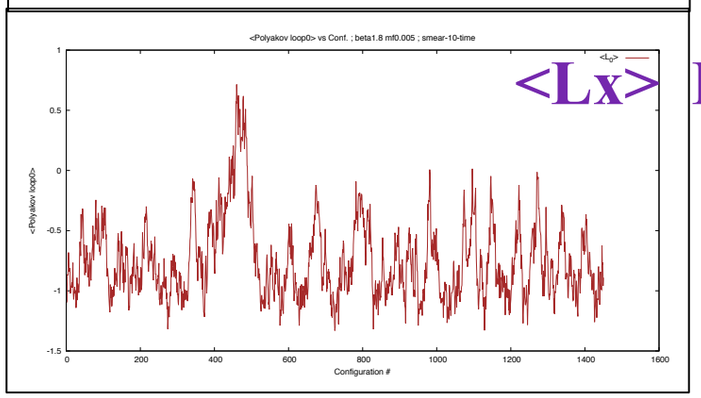
Beta 1.475



Beta 1.6

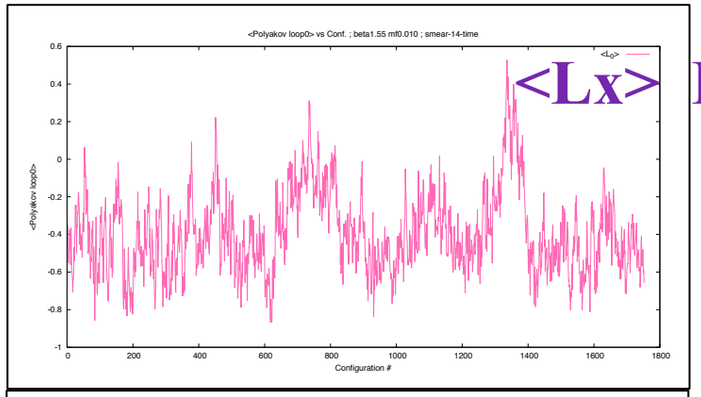


Beta 1.7

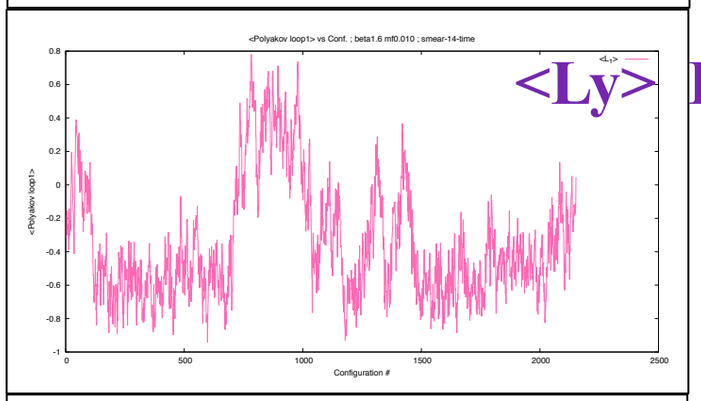


Beta 1.8

Volume(16×16^3) Mass 0.010



Beta 1.55



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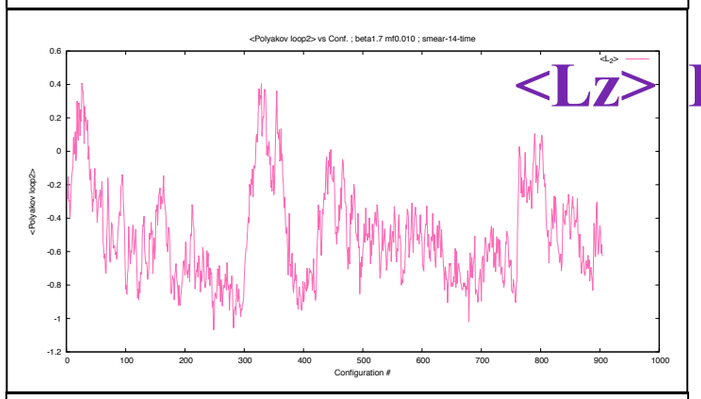

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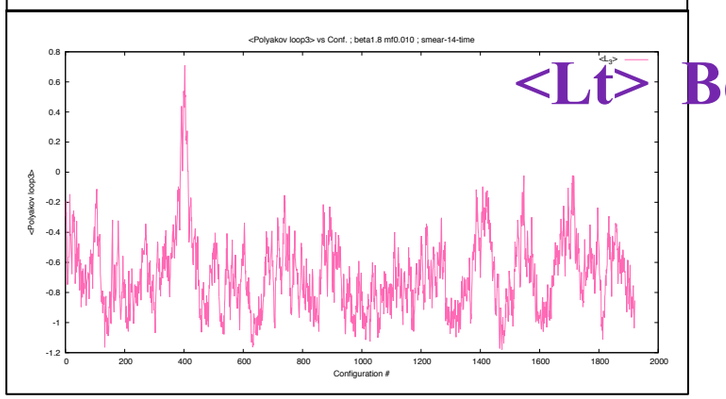

Beta 1.8

Backup: Polyakov loop— The Constraint Effective potential

- In quantum field theory, taking a single scalar field ϕ as an example, the Euclidean partition function is

$$\boxed{Z[J] = \int D\phi \ \mathbf{e}^{-\{S[\phi]+\int d^4x J(x)\phi(x)\}}.}$$

- We can thereby define the Schwinger function

$$\boxed{W[J] = -lnZ[J].}$$

$$\boxed{W[J] = -lnZ[J].}$$

- The effective action Γ is the Legendre transformation of the Schwinger function

$$\boxed{\Gamma[\Phi] = W[J] - \int d^4x J(x)\Phi(x),}$$

→ J(x) is an external source.

→ $\Phi(x) \equiv \frac{\delta W[J]}{\delta J(x)} = < 0|\phi(x)|0 >_J.$

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$$\Gamma[\Phi] = W[J] - \int d^4x J(x)\Phi(x),$$

- When $J(x) \to 0$, $\Phi(x) \to <0|\phi(x)|0> = \phi_c$, and

$$\Gamma[\phi_c] = -\Omega V(\phi_c),$$

where Ω is the total volume of spacetime.

→ V is the effective potential.

Backup: Polyakov loop— The Constraint Effective potential

- Constructing the effective potential without introducing external sources:

Introduce delta functions in the functional integrals,

making constraints on the fields.

Introduce delta functions in the functional integrals,

making constraints on the fields

- Consider a scalar field $\phi(\mathrm{x})$, expressing as

$$\boxed{\phi(x) = \phi_0 + \phi'(x),}$$

→ ϕ_0: Zero four-momentum component of the field.

→ $\phi'(\mathrm{x})$: The rest part of the field,

satisfying $$\boxed{\int d^4x \phi'(x) = 0.}$$

Introduce delta functions in the functional integrals,

making constraints on the fields

- Now treat the constraint that $<0|\,\phi(x)|0> = \phi_c$ when $J(x) \to 0$ to be equivalent to introducing $\delta(\phi_0 - \phi_c)$ in the functional integral:

[R. Fukuda and E. Kyriakopoulos, Nucl. Phys. B85 (1975) 354-364]

We have $$\mathbf{e}^{\Omega V_c(\phi_c)} = const. \int D\phi \mathbf{e}^{-S[\phi]}[\delta(\phi_0 - \phi_c)],$$

→ $V_c(\phi_c)$ is the constraint effective potential.

→ *const*. does not depend on ϕ_c.

→ $\Omega \equiv \int d^4x$.

$$\mathbf{e}^{\Omega V_c(\phi_c)} = const. \int D\phi \mathbf{e}^{-S[\phi]}[\delta(\phi_0 - \phi_c)],$$

- Connect this realization and the usual one, where Legendre transformation is introduced :

→ Replace $\delta(\phi_0 - \phi_c)$ by its integral representation.

$$\mathbf{e}^{\Omega V_c(\phi_c)} = C \int_{-\infty}^{\infty} dJ \int D\phi \mathbf{e}^{-S[\phi] - \int d^4x[\phi_0 J]} \mathbf{e}^{\phi_c J \Omega}$$

$$= C' \int_{-\infty}^{\infty} dJ \mathbf{e}^{-(W'[J] - \phi_c J)\Omega},$$

→ *C* and *C'* are different constants.

→ $W'[J]$: The vacuum energy density with an opposite sign in the presence of a constant external source J.

$$\mathbf{e}^{\Omega V_c(\phi_c)} = C\int_{-\infty}^{\infty} dJ \int D\phi \mathbf{e}^{-S[\phi]-\int d^4x[\phi_0 J]}\mathbf{e}^{\phi_c J\Omega}$$

$$= C'\int_{-\infty}^{\infty} dJ \mathbf{e}^{-(W'[J]-\phi_c J)\Omega},$$

- When $\Omega \to \infty$ the prime contribution to the integral comes from the stationary point of the phase of integrand :

→ We have $V_c(\phi_c) \xrightarrow{\Omega\to\infty} V(\phi_c) = (\phi_c J - W'[J])_{\phi_c = \frac{\partial W'}{\partial J}},$

being equivalent to our usual definition of the effective potential ($\Gamma[\phi_c] = -\Omega V(\phi_c)$).

Backup: Polyakov loop— The Constraint Effective potential

- In pursuit of a corresponding equation of

$$\boxed{\mathbf{e}^{\Omega V_c(\phi_c)} = const. \int D\phi \mathbf{e}^{-S[\phi]}[\delta(\phi_0 - \phi_c)],}$$

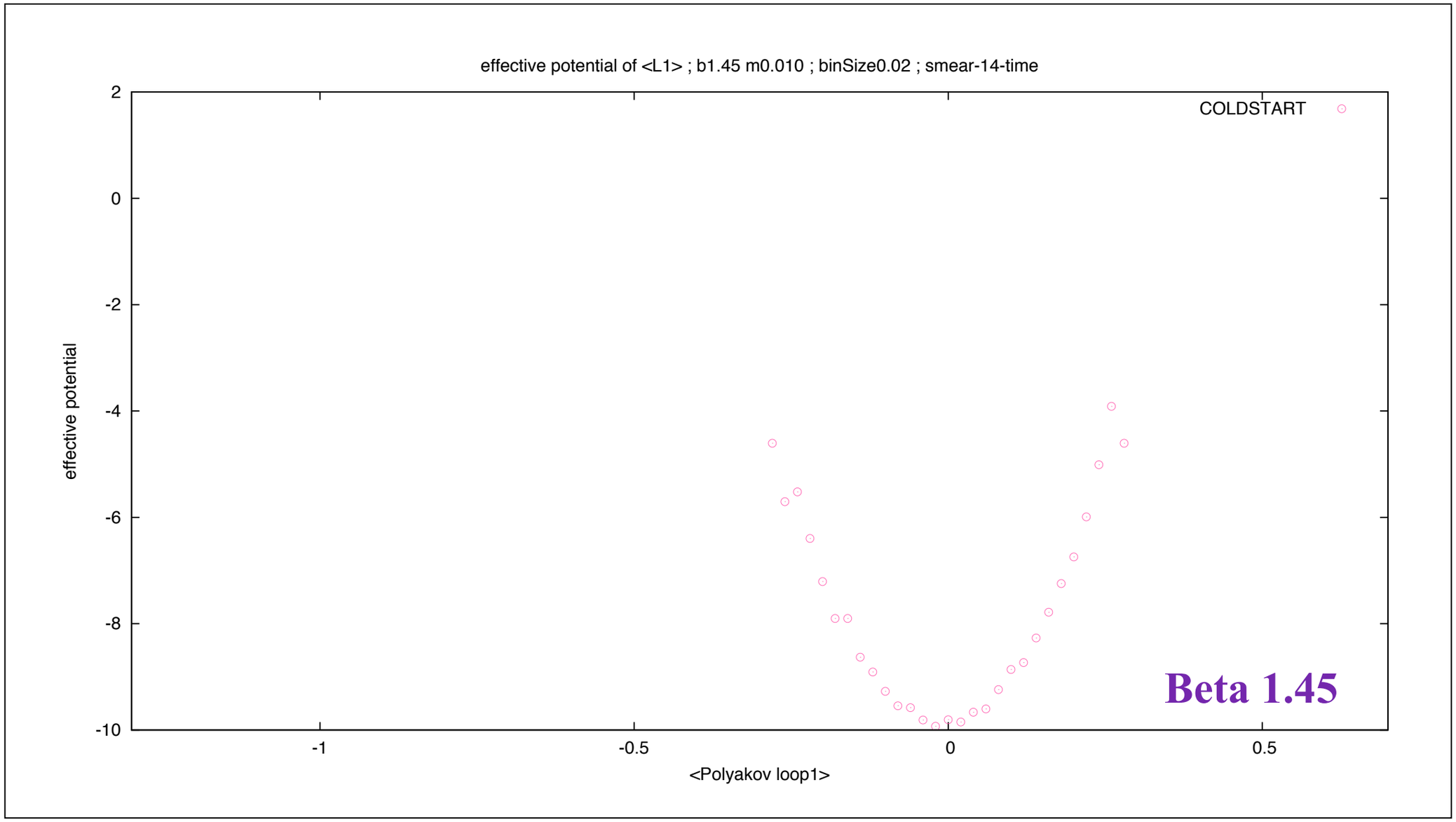
for constructing the constraint effective potential for Polyakov loop, write

$$\mathbf{e}^{\Omega V_c(<\mathbf{L}>)} = const.'' \int_{-\infty}^{\infty} dJ \mathbf{e}^{-(W'[J]-<\mathbf{L}>J)\Omega}, \; with \begin{cases} W'[J] = ln \int d\mathbf{L}\, \mathbf{e}^{-V_c(\mathbf{L})+\mathbf{L}J} \\ V_c(<\mathbf{L}>) \xrightarrow{\Omega\to\infty} V(<\mathbf{L}>) = (<\mathbf{L}>J - W'[J])_{<\mathbf{L}>=\frac{\partial W'}{\partial J}} \end{cases}$$

→ $V_c(<\mathbf{L}>)$ is the constraint effective potential.

Backup: Polyakov loop—The Constraint Effective Potential

- Plots of the constraint effective potential for smeared <**L**> (take <Ly> in volume(16×16^3), mass 0.010 as an example):



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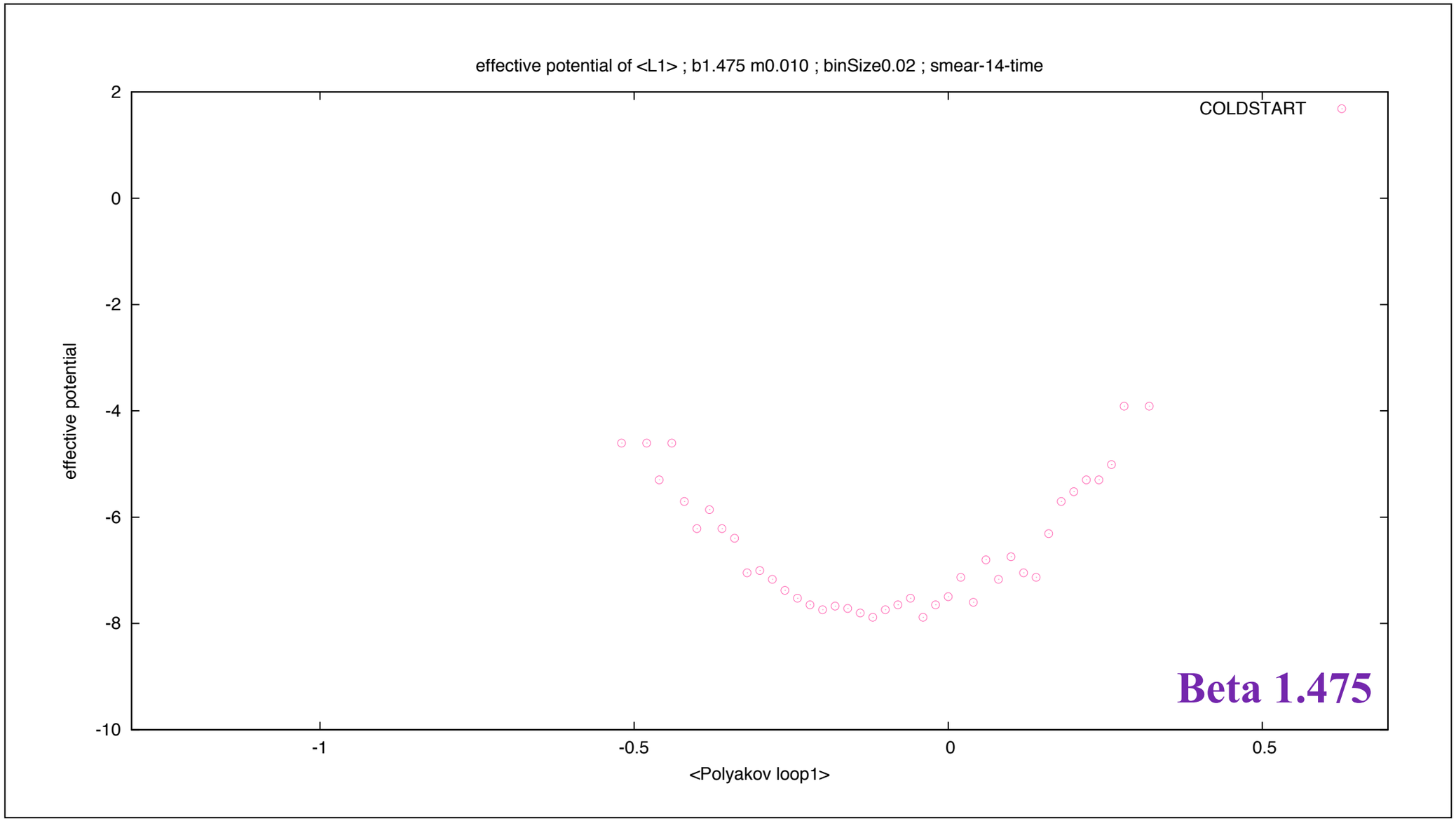

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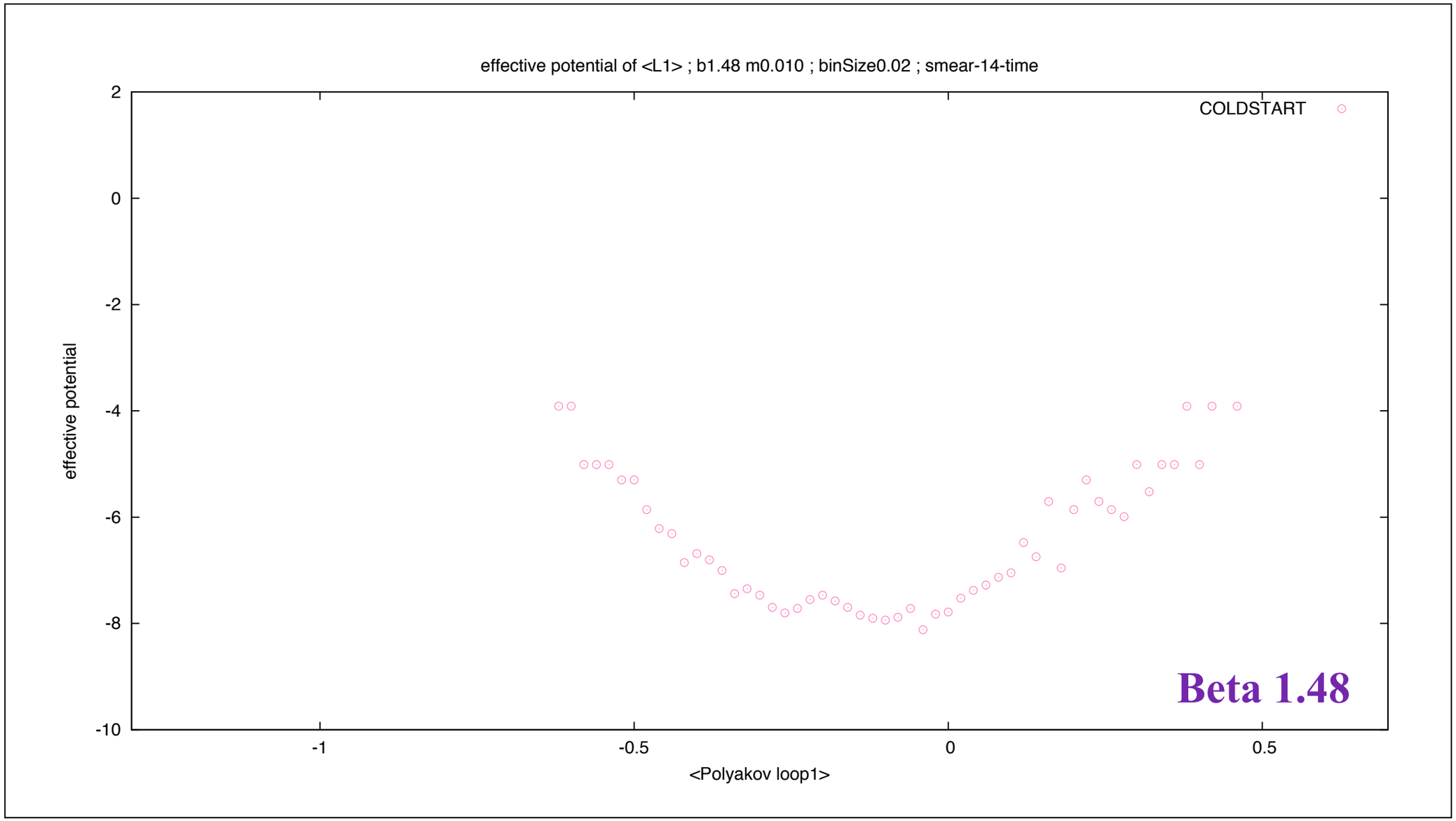

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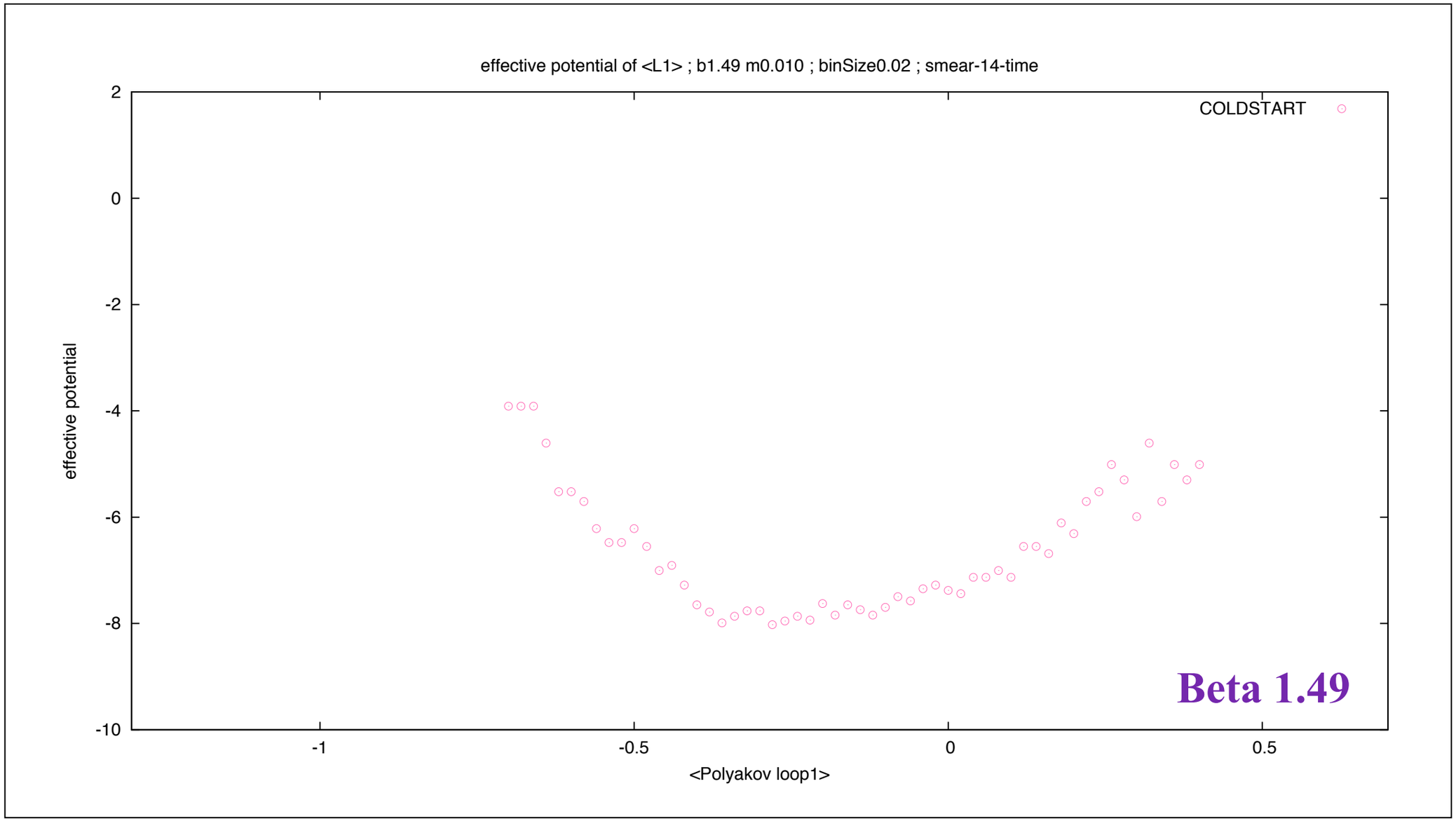

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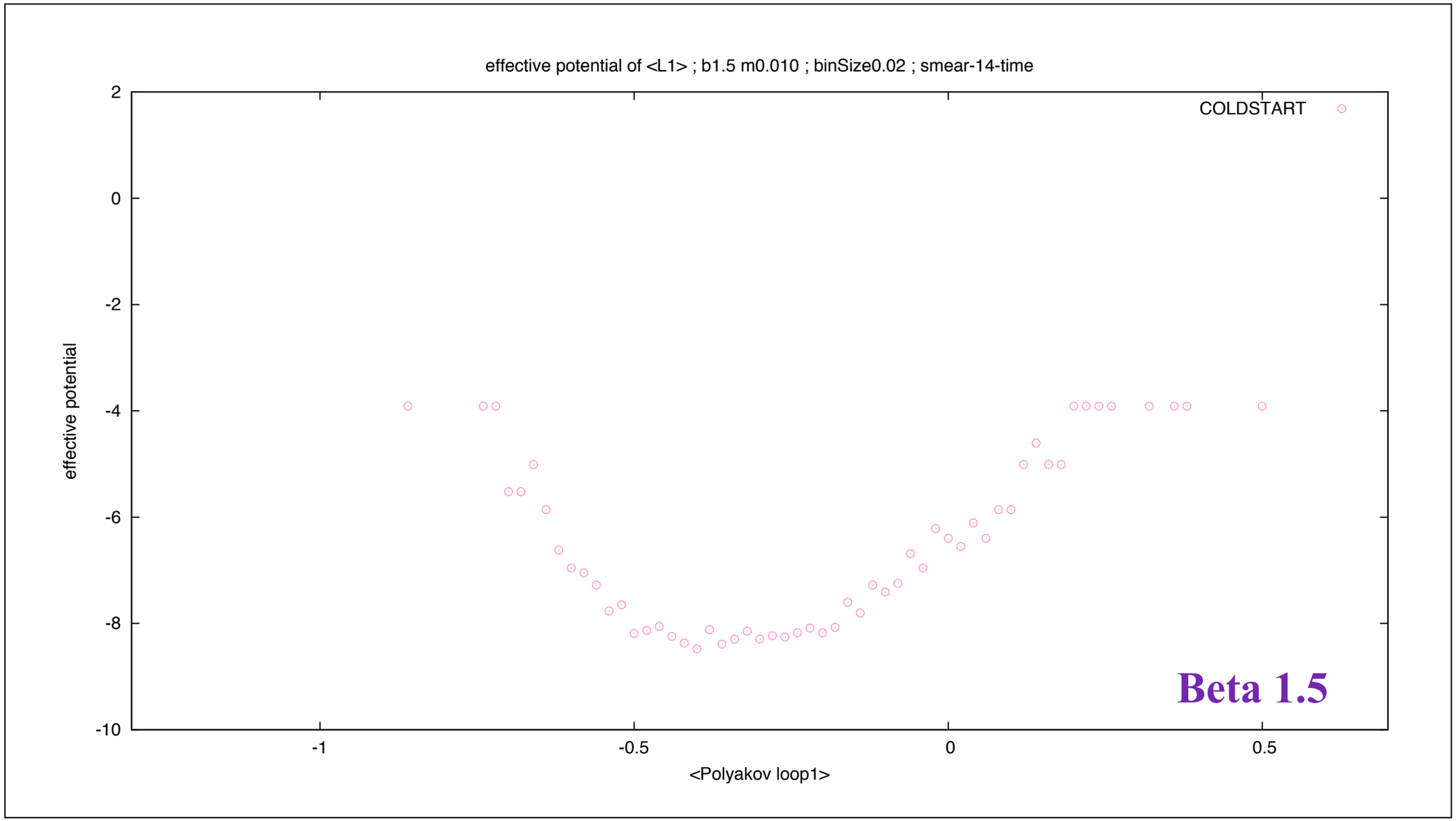

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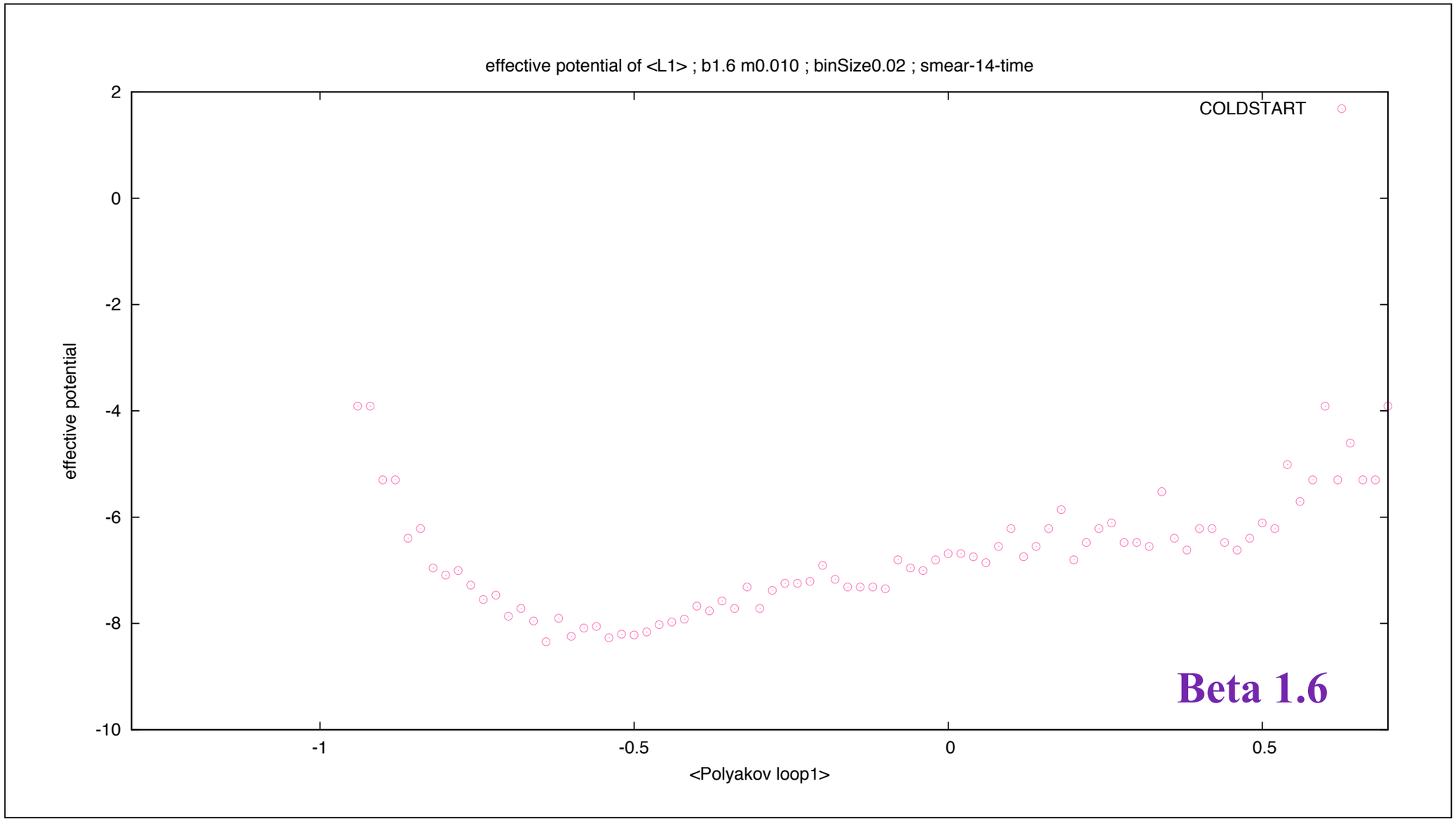
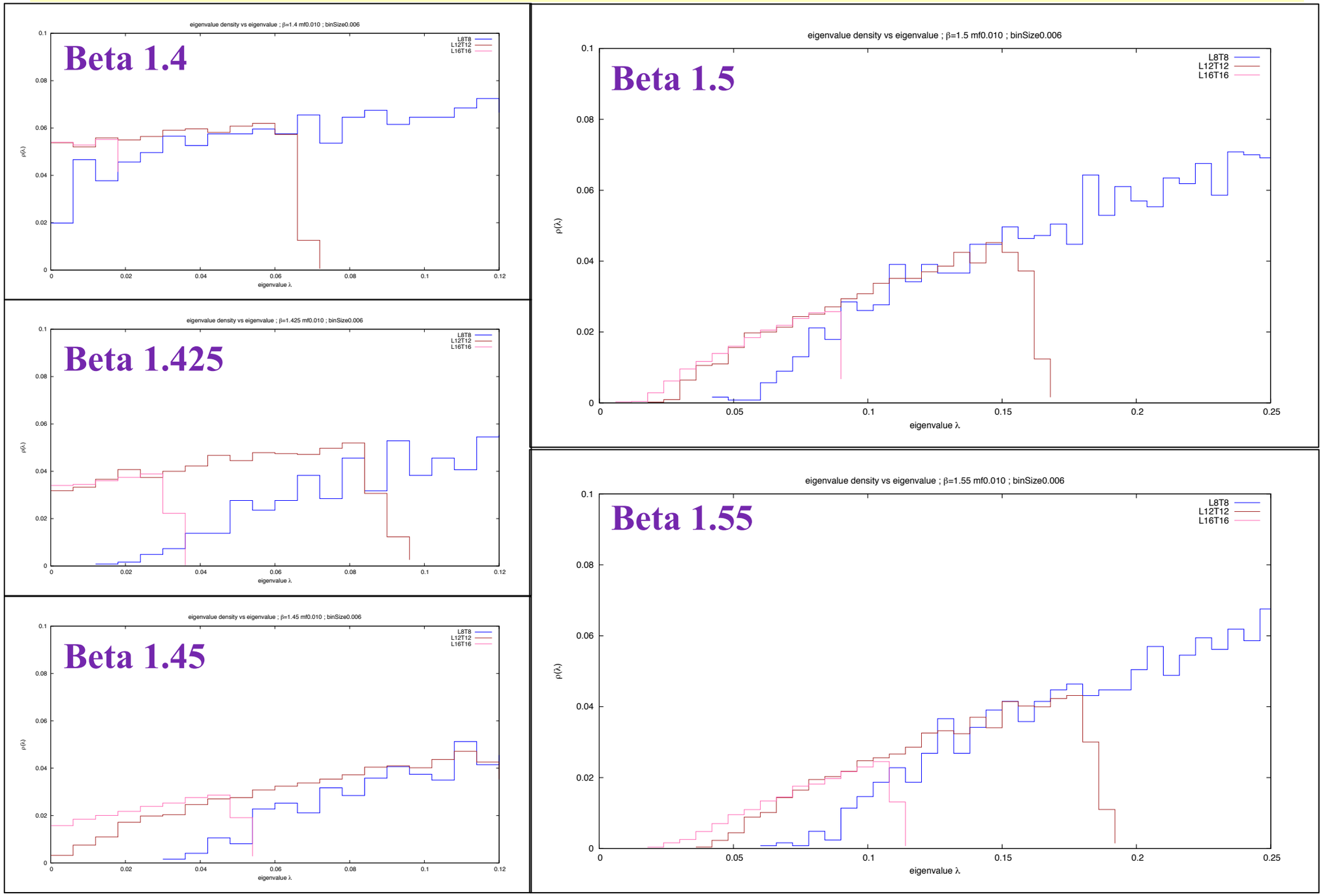


Backup: (2) ρ with fixed β & M in different V

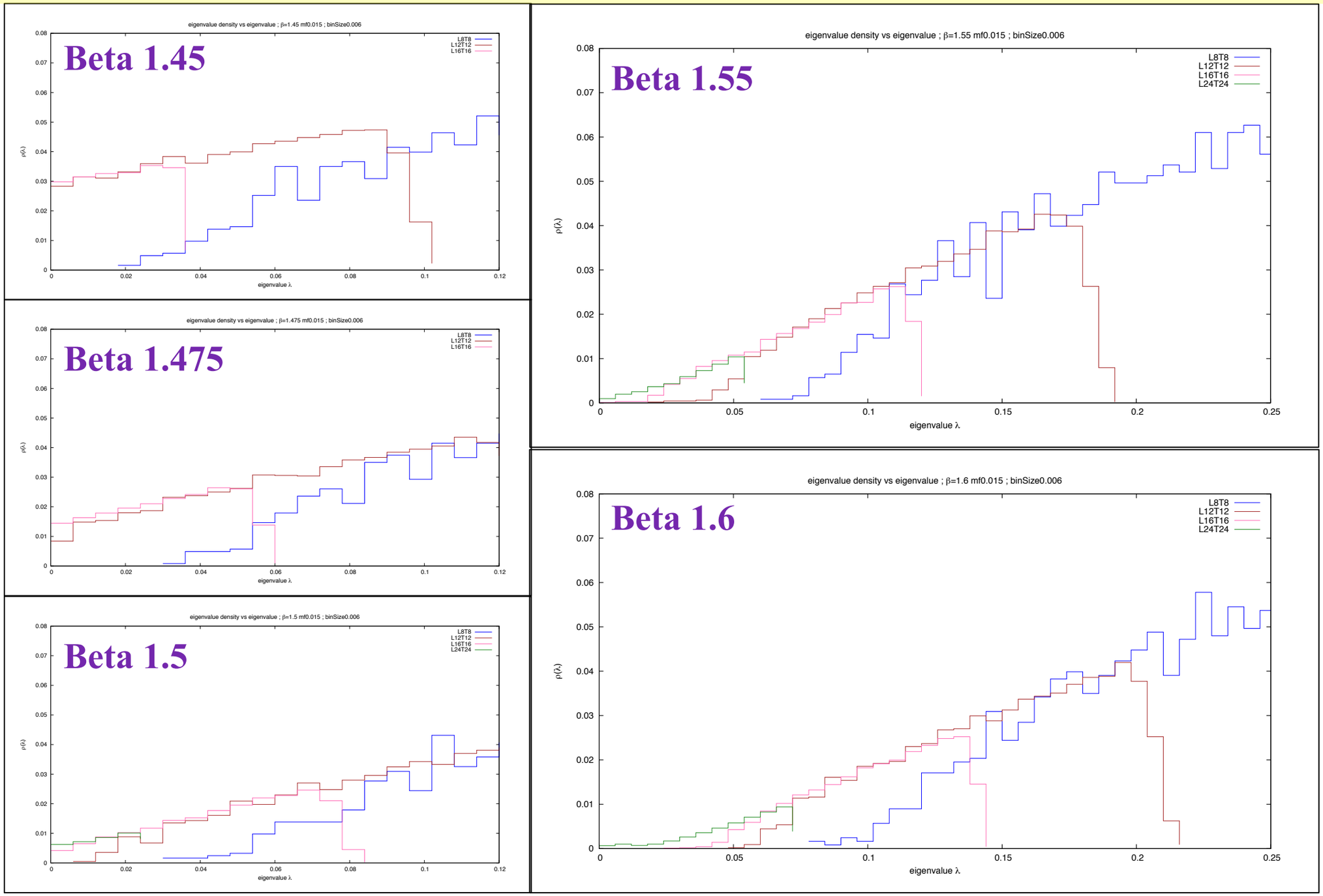
Mass 0.010 [Volume(8×8^3), Volume(12×12^3), Volume(16×16^3)]:



→ The volume effect for these three volumes increases as β increases.

Backup: (2) ρ with fixed β & M in different V

Mass 0.015 [Volume(8×8^3), Volume(12×12^3), Volume(16×16^3), Volume(24×24^3)]:



→ **For each β, the volume effect gradually decreases in larger volumes, which means the chiral phase transition is possible to be bulk.**

Backup: The Dirac Eigenvalue Spectrum

- If the chiral phase transition observed is bulk, we may conclude that: The phase boundary determined here *overlaps* with the one determined by the measurements of plaquette and smeared Polyakov loops.

- However, it is also possible that the phase transition observed here is a *crossover* from the p-regime to the ϵ-regime, which results in zero eigenvalue density.