Computing the nucleon sigma terms at the physical point

Christian Torrero
for the Budapest-Marseille-Wuppertal collaboration

Centre de Physique Théorique
Aix-Marseille Université

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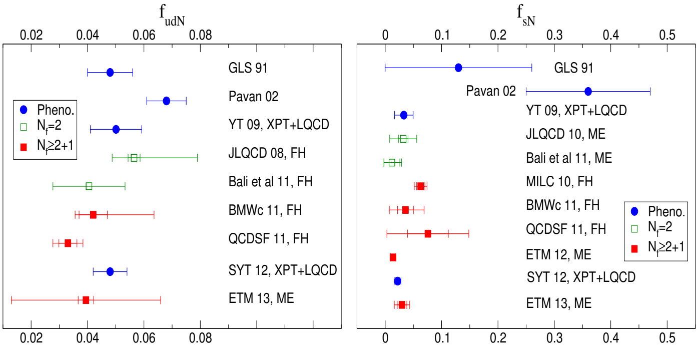
Outline

The nucleon sigma terms $\sigma_{\pi N}$ and $\sigma_{\bar{s}sN}$ are observables of great interest given their relation to

- the quark-mass ratio m_{ud}/m_s;
- $\pi - N$ and $K - N$ scattering;
- counting rates in Higgs-Boson searches;
- direct detection of dark matter (DM).

Estimates from phenomenology do not agree with each other and have large uncertainties $\longrightarrow$ **need for ab-initio computations of strong-interaction effects**.

Computations of $f_{udN} \equiv \sigma_{\pi N}/M_N$ and $f_{sN} \equiv \sigma_{\bar{s}sN}/(2M_N)$ already exist:



However, most calculations employ model assumptions and/or have incomplete error analyses.

The nucleon sigma terms $\sigma_{\pi N}$ and $\sigma_{\bar{s}sN}$ are defined as

$$\sigma_{\pi N} \equiv m_{ud} \langle N|\bar{u}u + \bar{d}d|N\rangle \ , \qquad \sigma_{\bar{s}sN} \equiv 2m_s \langle N|\bar{s}s|N\rangle \ .$$

A possible strategy to compute them consists of relying on the Feynman-Hellman theorem, i.e.,

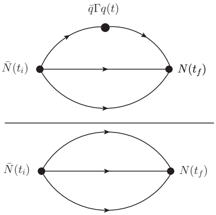
$$\sigma_{\pi N} = m_{ud} \left. \frac{\partial M_N}{\partial m_{ud}} \right|_{\Phi} \ , \qquad \sigma_{\bar{s}sN} = 2\, m_s \left. \frac{\partial M_N}{\partial m_s} \right|_{\Phi} \ ,$$

where derivatives have to be computed at the physical point (Φ).

Main advantages of this approach:

- need for 2-point functions only;
- no disconnected contributions.

Given that derivatives above are small (in particular for the s-case), an alternative is represented by a direct computation.



$$\xrightarrow{(t-t_i),(t_f-t)\to\infty} \langle N(\vec{0})|\bar{q}\Gamma q|N(\vec{0})\rangle$$

- ✓ Desired quantity appears at leading order
- ✗ Complex renormalization
- ✗ Compute challenging quark-connected diagrams

We begin with the indirect approach.

As it is well known, the mass of a given particle p can be extracted from a time correlator $C_p(t, t_S)$

$$C_p(t, t_S) = a^3 \sum_{\vec{x}} G_p(x, x_S) \ ,$$

being a the lattice spacing, t the time component of point $x = (t, \vec{x})$ in the $4D$ discretized spacetime and with $G_p(x, x_S)$ given by

$$G_p(x, x_S) = \langle O_p(x) O_p(x_S) \rangle \ ,$$

where $O_p(x)$ is an interpolating operator capable of creating a hadron p out of the vacuum.

Masses are eventually extracted from the asymptotic behaviour of $C_p(t, 0)$ with suitable fit functions for mesons and baryons.

Some technical details:

- tree-level improved Symanzik gauge action (S_G) and clover-improved Wilson action (S_F);
- 2-HEX link-smearing in S_F;
- $N_f = 2 + 1$;
- 47 ensembles corresponding to about 13000 overall configurations with
  - $0.054\ fm \lesssim a \lesssim 0.093\ fm$;
  - pion mass M_π down to $\lesssim 120\ MeV$;
  - box size up to $\approx 6\ fm$;
- full non-perturbative renormalization and running of quark masses in RGI (as in BMWc, JHEP 1108);
- derivatives computed in terms of quark masses instead of corresponding pseudoscalar masses squared.

An example of functional form — with **experimental input** and **fit parameters** — employed in the fit reads

$$
\begin{aligned}
aM_X &= a\left\{ M_X^{(\Phi)} + \sum_i c_{X,ud,i}\left[\frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}a^2)} - m_{ud}^{(\Phi)}\right]^i + \right.\\
&+ \left. \sum_j c_{X,s,j}\left[\frac{am_s Z_s^{-1}(\beta)}{a(1+d_s a^2)} - m_s^{(\Phi)}\right]^j \right\},
\end{aligned}
$$

with $X = \Omega$ (for scale setting), π, K^χ and N.

The masses of these four particles are fitted at the same time, i.e. the corresponding functionals share the same fit parameters - with the exception of the $c_{X,ud,i}$'s and $c_{X,s,i}$'s.

Quark masses in the functional above are obtained through the ratio-difference method (BMWc, JHEP 1108).

Fit parameters $c = \{a, m_{ud}^{(\phi)}, m_s^{(\phi)}, \ldots\}$ of functions $f^{(i)}(c, x)$ — with $i = 1, 2, 3, 4$ and $x = \{am_{ud}, am_s\}$ — are determined by minimizing a χ^2 function defined as

$$\chi^2 = V^T C^{-1} V \ ,$$

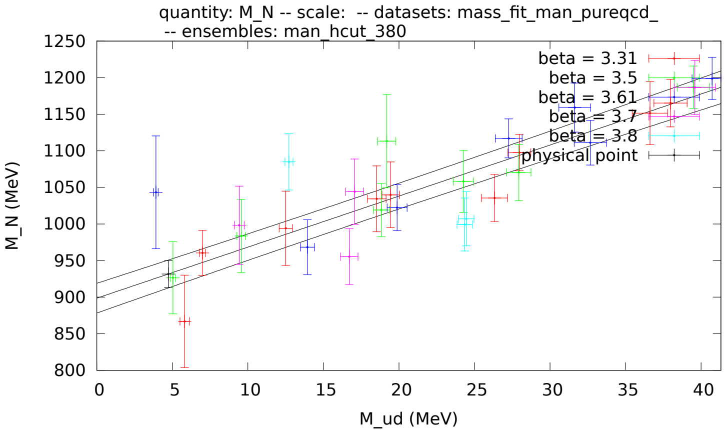
where C is the covariance matrix associated to the entries of the column vector V whose structure reads

$$V = (y_1^{(1)} - \ f^{(1)}(c, x_1) \ , \ \ldots \ , \ y_n^{(4)} - \ f^{(4)}(c, x_n) \ , \ x_1 - q_1 \ , \ x_2 - q_2 \ , \ \ldots \ , \ x_n - q_n) \ ,$$

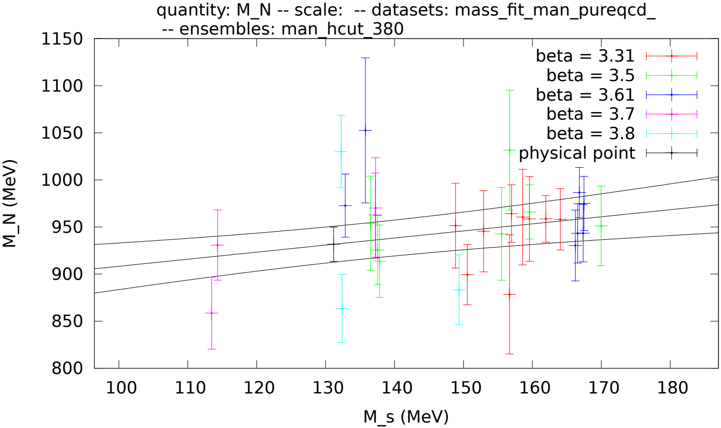
where q_i is the value of variable x_i obtained in simulation i.

Entries of matrix C are obtained via a bootstrap procedure with $n_{boot} = 2000$.

All fits are correlated.



Result from example fit: M_N vs. m_{ud} : $\sigma_{\pi N} = 33.0(2.7)$ MeV, $f_{udN} = 0.035(3)$ (stat. error only).



Result from same example fit: M_N vs. m_s: $\sigma_{\bar{s}sN} = 197(117)$ MeV, $f_{sN} = 0.10(6)$ (stat. error only).

To estimate the systematic uncertainties on results, different strategies will be considered in the fitting procedure:

- choosing two different time intervals for the asymptotic behaviour of $C_p(t,0)$;
- pruning the data with two cuts in the pion mass (at 380 MeV and 480 MeV);
- taking into account six different procedures in computing Z_S (as in BMWc, JHEP 1108);
- relying on ChPT-inspired fitting functions for mesons;
- allowing for different cutoff effects, i.e.

$$\frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}a^2)} \longrightarrow \frac{am_{ud}Z_s^{-1}(\beta)}{a(1+d_{ud}\alpha_s a)} .$$

This will result in $2\cdot 2\cdot 6\cdot 2\cdot 2 = \mathbf{96}$ fitting strategies altogether.

The mean value and systematic error of a generic fit parameter c_i are obtained by computing, respectively, the mean and the standard deviation of the values of c_i resulting from the different fitting procedures.

The bootstrap error on the mean provides the statistical error.

Very preliminary results - with systematic error still to be assessed - read

$$f_{udN} = 0.035(3)\ , \qquad f_{sN} = 0.10(6)\ .$$

Conclusions and outlook

- a first-principle computation of the nucleon sigma terms is being carried out with data all the way down to the physical point;
- preliminary results in the right ballpark;
- a thorough analysis of systematic uncertainties is underway;
- an extensive study of improvements that can be made using the current approach will be carried out to reduce the errorbars.