Thermodynamics of Gauge Theory from Gradient Flow

For FlowQCD Collaboration

Thermodynamics of Gauge Theory from Gradient Flow

For FlowQCD Collaboration

$$T_{\mu\nu}$$

$$T_{\mu\nu}$$

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Poincare
symmetry

$$T_{\mu\nu}$$

Poincare symmetry

$$T_{\mu\nu}$$

energy momentum

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

stress pressure

Poincare symmetry

$$T_{\mu\nu}$$

energy

momentum

$$\begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

stress

pressure

Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

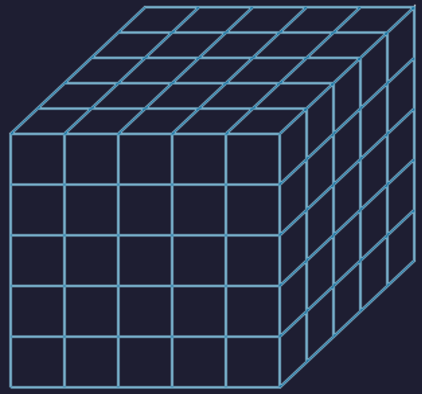
Hydrodynamic Eq.

$$\partial_\mu T_{\mu\nu} = 0$$

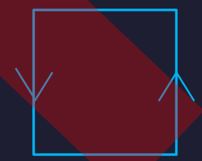
$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

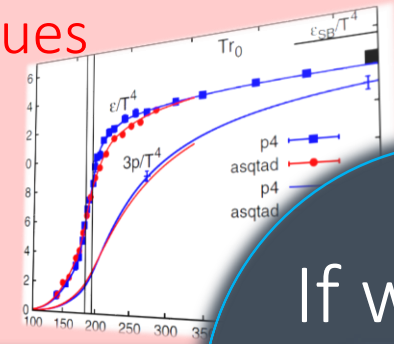
If we have

$$T_{\mu\nu}$$

Thermodynamics

direct measurement of expectation values

$$\langle T_{00}\rangle, \langle T_{ii}\rangle$$



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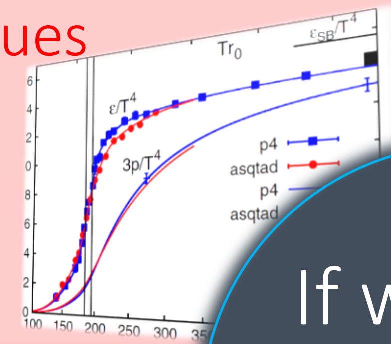

Fluctuations and Correlations

viscosity, specific heat, ...

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

$$\eta = \langle T_{12}; T_{12} \rangle$$

If we have

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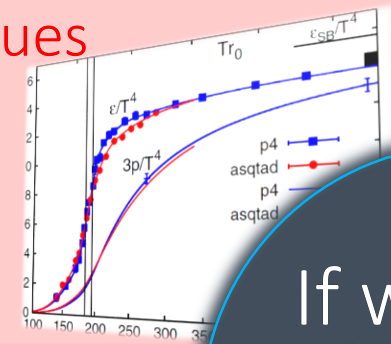

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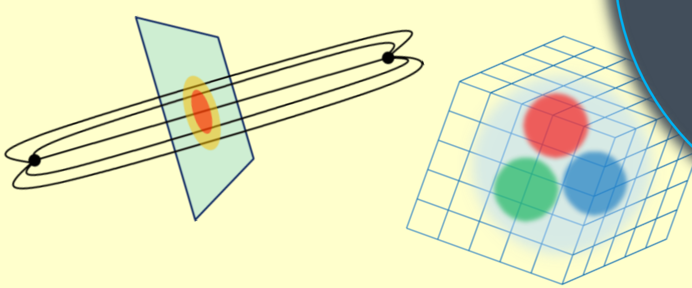
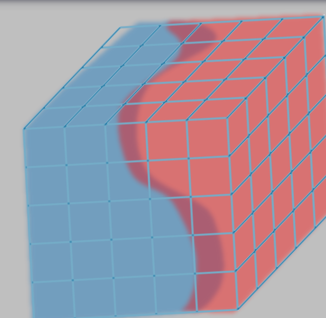


- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Measurement of Thermodynamics using gradient flow

Masakiyo Kitazawa (Osaka U.)

for FlowQCD Collaboration
Asakawa, Hatsuda, Iritani, Itou, MK, Suzuki

FlowQCD, arXiv:1312.7492[hep-lat]; to appear in PRD

LATTICE2014, 28 June 2014, New York

Flow QCD Collaboration

Flow QCD Collaboration

from

Gradient

flow

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

Flow QCD Collaboration

from

Gradient

flow

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

to

Hydrodynamic

flow

$$\partial_\mu T_{\mu\nu} = 0$$

Thermodynamics

direct measurement of expectation values

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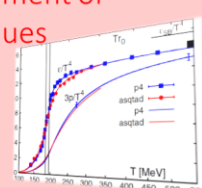

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Gradient Flow

YM Gradient Flow

Luescher, 2010

A. Ramos, plenary, Friday

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

t: “flow time”
dim:[length^2]

$$A_\mu(0,x) = A_\mu(x)$$

YM Gradient Flow

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$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

- transform gauge field like diffusion equation

$$\boxed{\partial_t A_\mu} = D_\nu G_{\mu\nu} = \boxed{\partial_\nu \partial_\nu A_\mu} + \cdots$$

- diffusion length $d \sim \sqrt{8t}$

- This is NOT the standard cooling/smearing
- All composite operators at t>0 are UV finite Luescher,Weisz,2011

Applications of Gradient Flow

A. Ramos, plenary, Friday

① scale setting

② running coupling

③ topology

④ operator relation

⑤ autocorrelation, etc.

Small Flow Time Expansion of Operators and EMT

Operator Relation

Luescher, Weisz, 2011

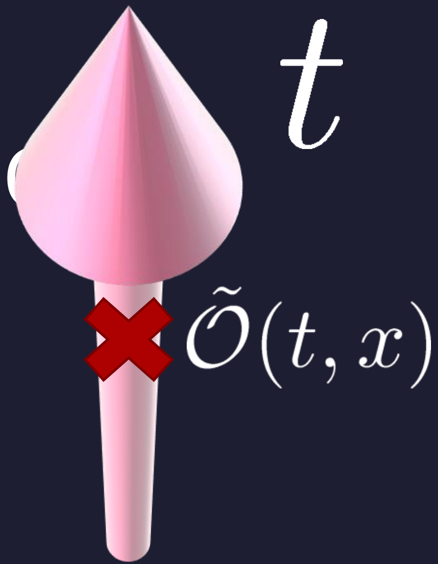
$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

remormalized operators of original theory

Operator Relati

Luescher, Weisz, 2011



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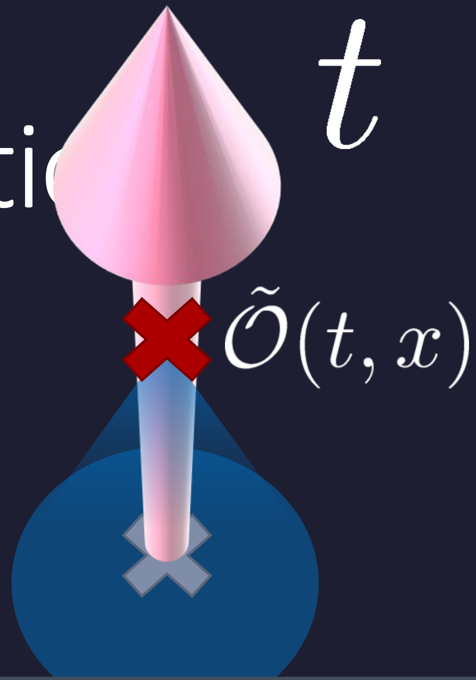


$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t \to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

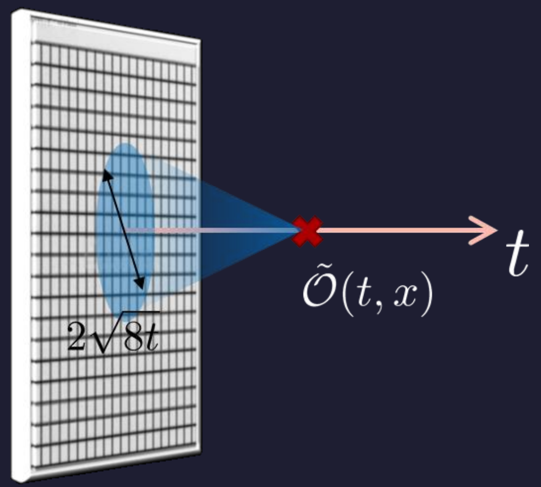
an operator at t>0

remormalized operators of original theory

Constructing EMT

Suzuki, 2013
DelDebbio,Patella,Rago,2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **gauge-invariant dimension 4 operators**

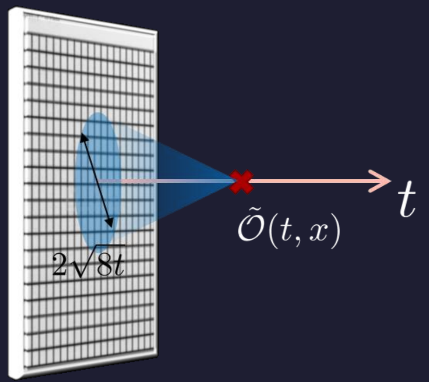
$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

Constructing EMT 2

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$



Suzuki coeffs. $\begin{cases} \alpha_U(t) = g^2\left[1 + 2b_0 s_1 g^2 + O(g^4)\right] \\ \alpha_E(t) = \dfrac{1}{2b_0}\left[1 + 2b_0 s_2 g^2 + O(g^4)\right] \end{cases}$

$g = g(1/\sqrt{8t})$

$s_1 = 0.03296\ldots$

$s_2 = 0.19783\ldots$

See also, Patella, Parallel7E, Thu.

Constructing EMT 2

Suzuki, 2013

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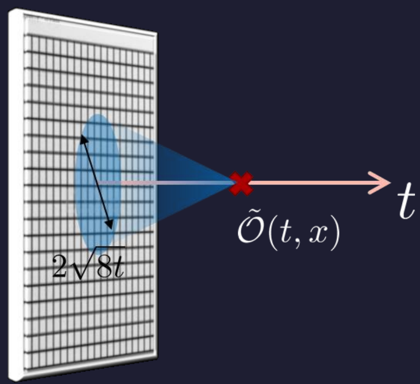

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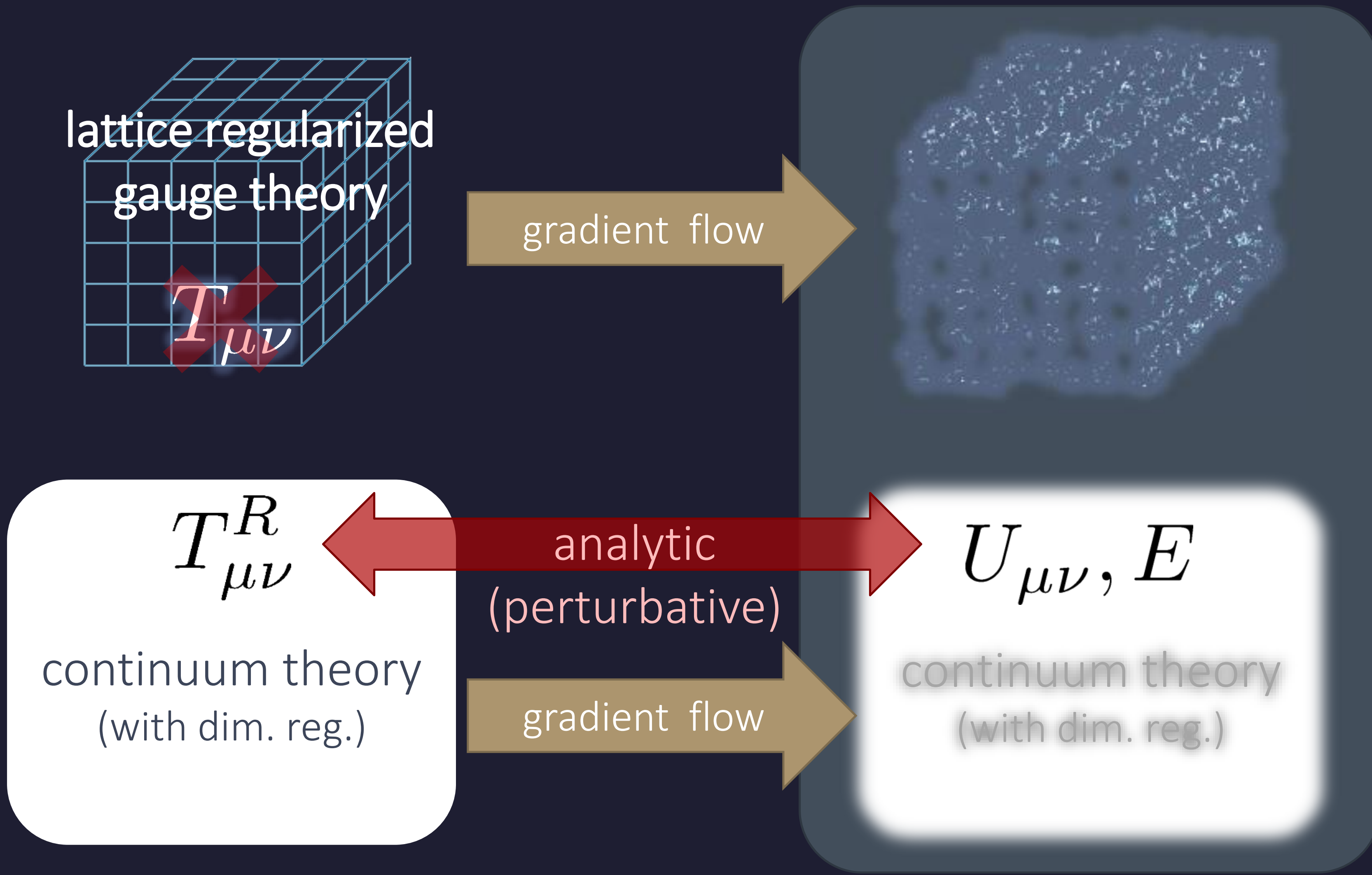
Remormalized EMT

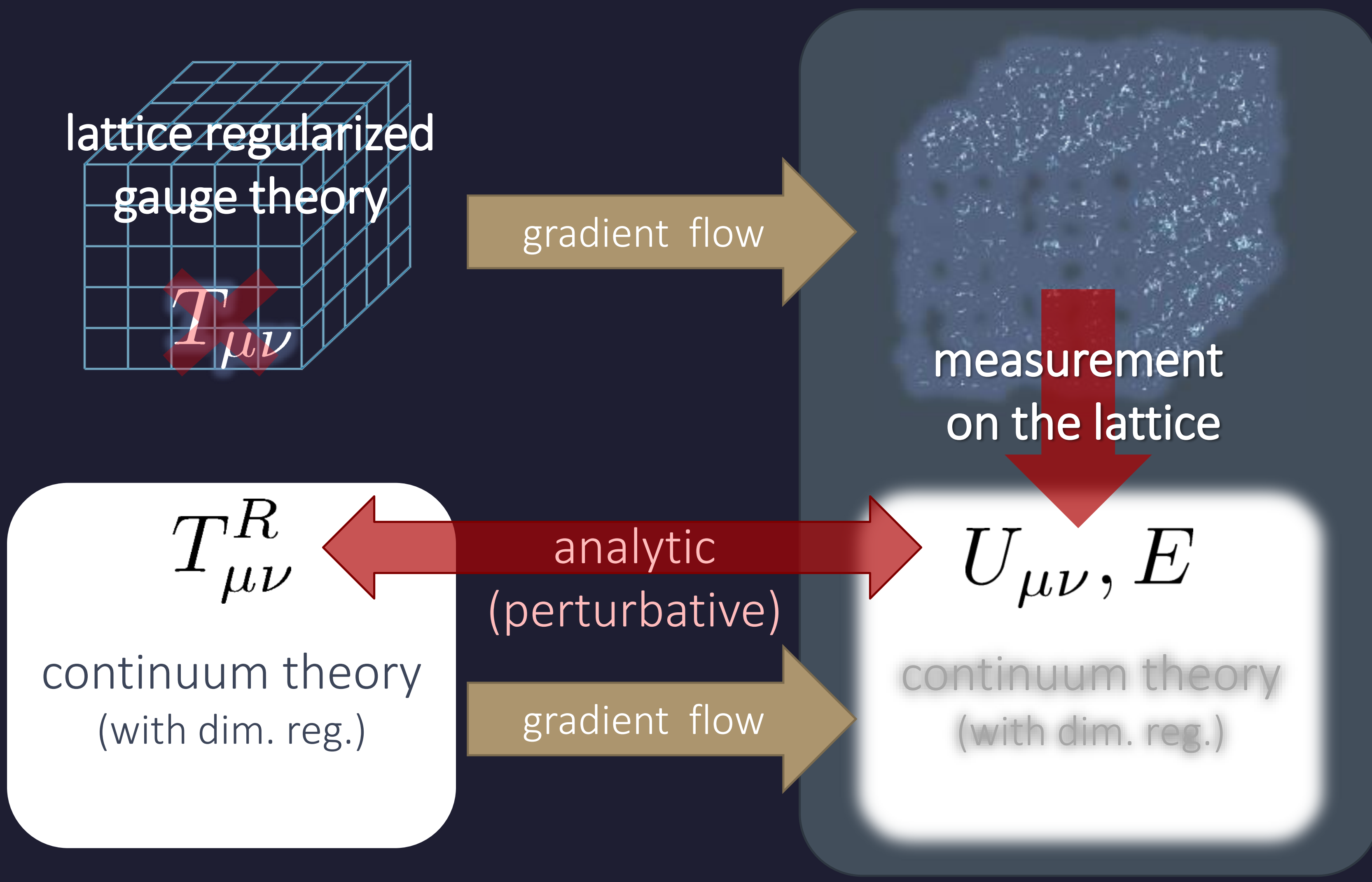
$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\text{subt.}}\right]$$

Numerical Analysis on the Lattice

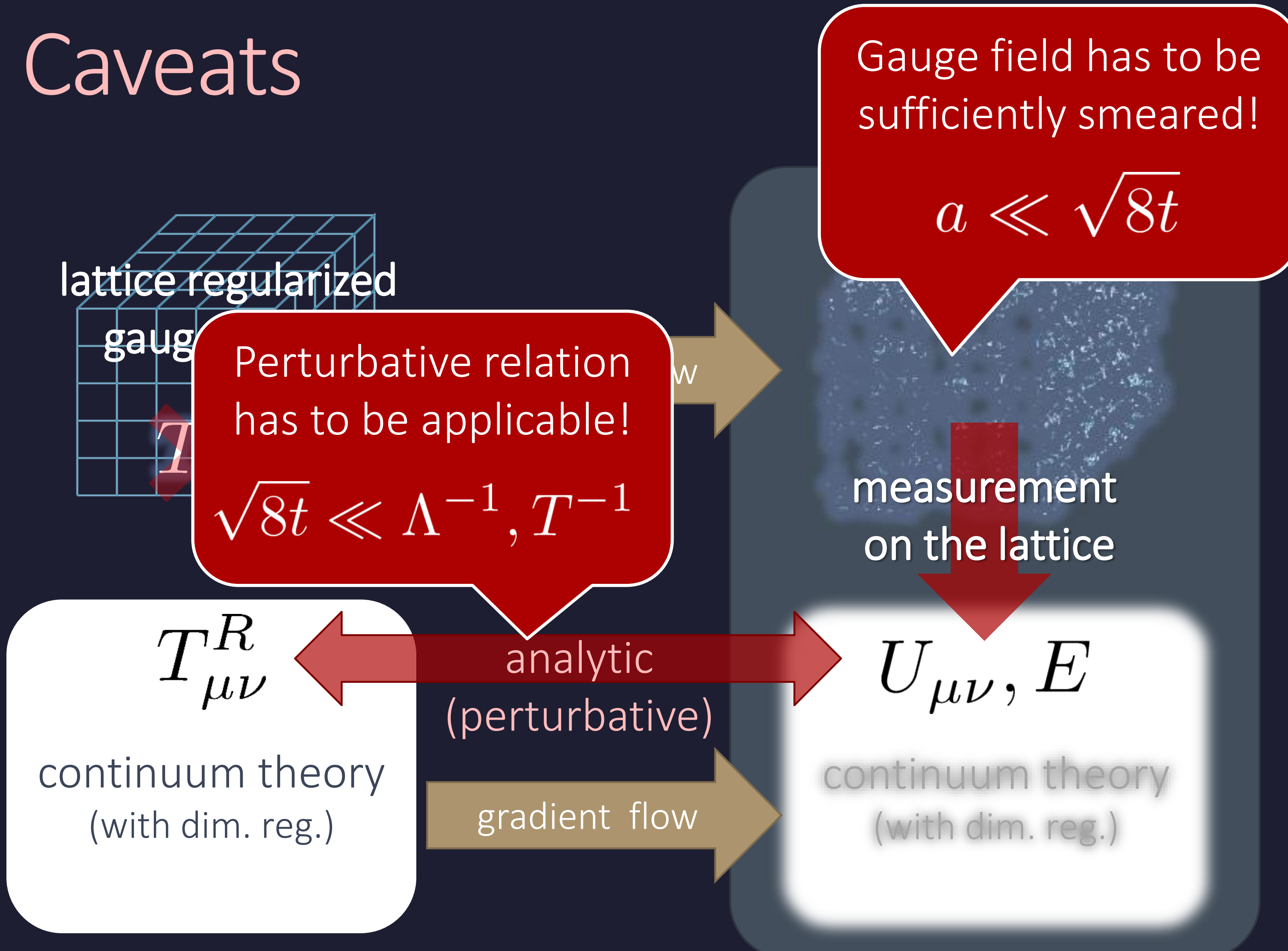
Gradient Flow Method



Gradient Flow Method



Caveats



Gauge field has to be sufficiently smeared!
$a \ll \sqrt{8t}$
lattice regularized
gaug
Perturbative relation has to be applicable!
$\sqrt{8t} \ll \Lambda^{-1}, T^{-1}$
measurement on the lattice
$T^R_{\mu\nu}$
analytic (perturbative)
$U_{\mu\nu}, E$
continuum theory
(with dim. reg.)
gradient flow
continuum theory
(with dim. reg.)

Caveats

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w

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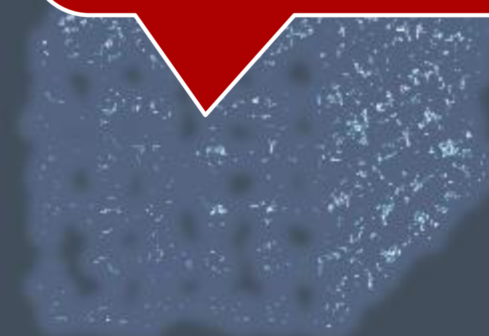


$$T^R_{\mu\nu}$$

continuum theory

(with dim. reg.)

analytic

(perturbative)

$$U_{\mu\nu}, E$$

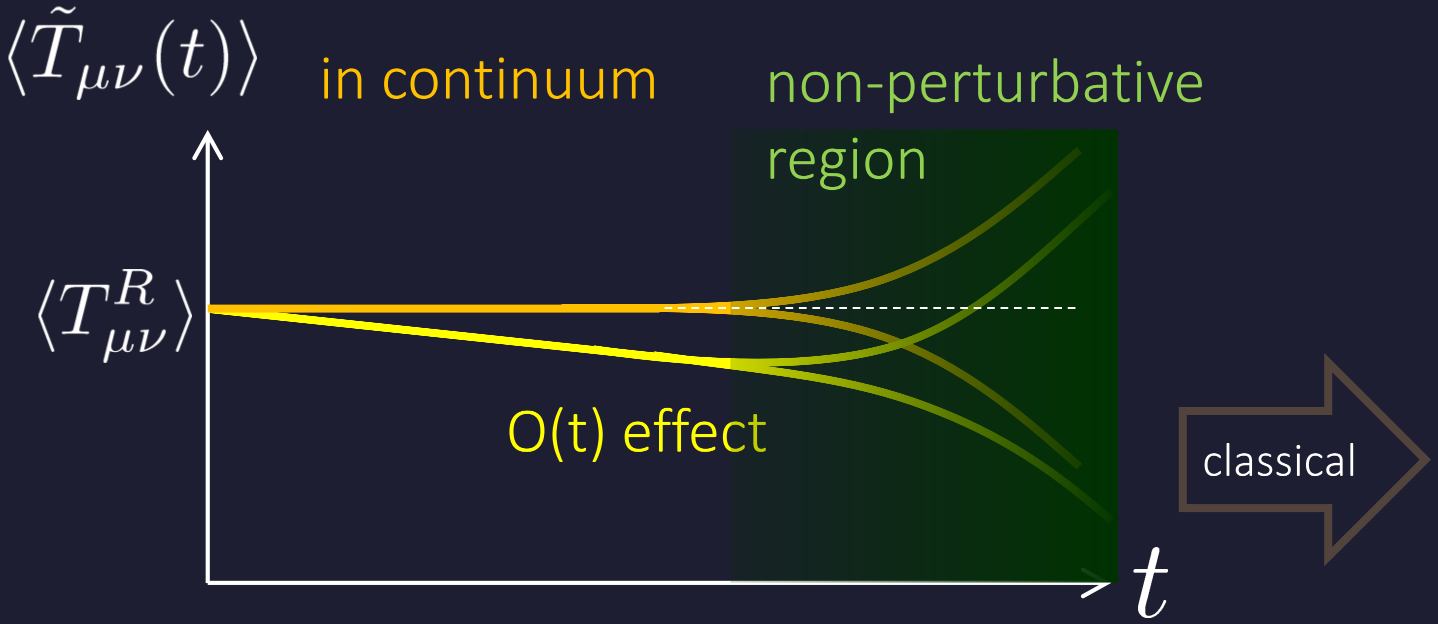
continuum theory

(with dim. reg.)

gradient flow

$$a \ll \sqrt{8t} \ll \Lambda^{-1}$$

$$\tilde{T}_{\mu\nu}(t) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} E(t)_{\rm subt.} \qquad T^R_{\mu\nu} = \lim_{t\to 0} \tilde{T}_{\mu\nu}(t)$$



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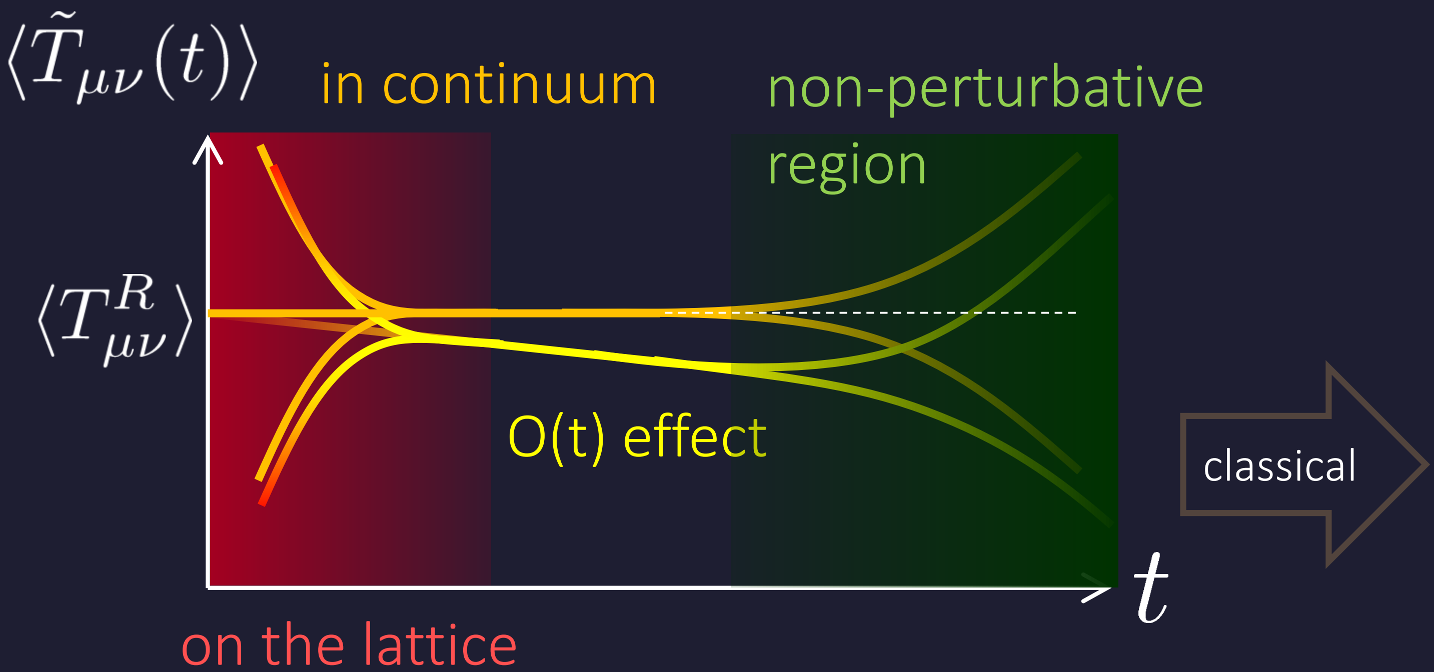
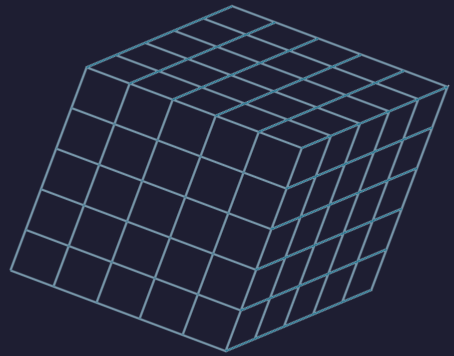


☐ t→0 limit with keeping t>>a^2

Numerical Simulation



- ➢ SU(3) YM theory
- ➢ Wilson gauge action

twice finer lattice!

Simulation 1
(arXiv:1312.7492)

- lattice size: 32^3xN_t
- Nt = 6, 8, 10
- β = 5.89 – 6.56
- ~300 configurations

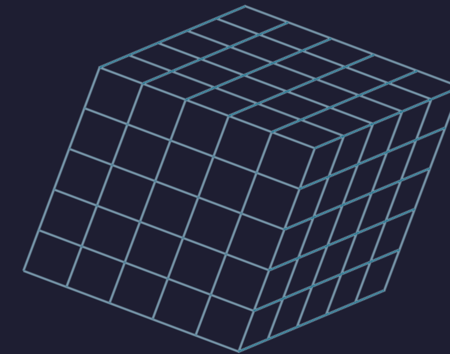
using SX8 @ RCNP
SR16000 @ KEK

Simulation 2
(new, preliminary)

- lattice size: 64^3xN_t
- Nt = 10, 12, 14, 16
- β = 6.4 – 7.4
- ~2000 configurations

using BlueGeneQ @ KEK
efficiency ~40%

Numerical Simulation



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ε-3p at T=1.65T$_c$

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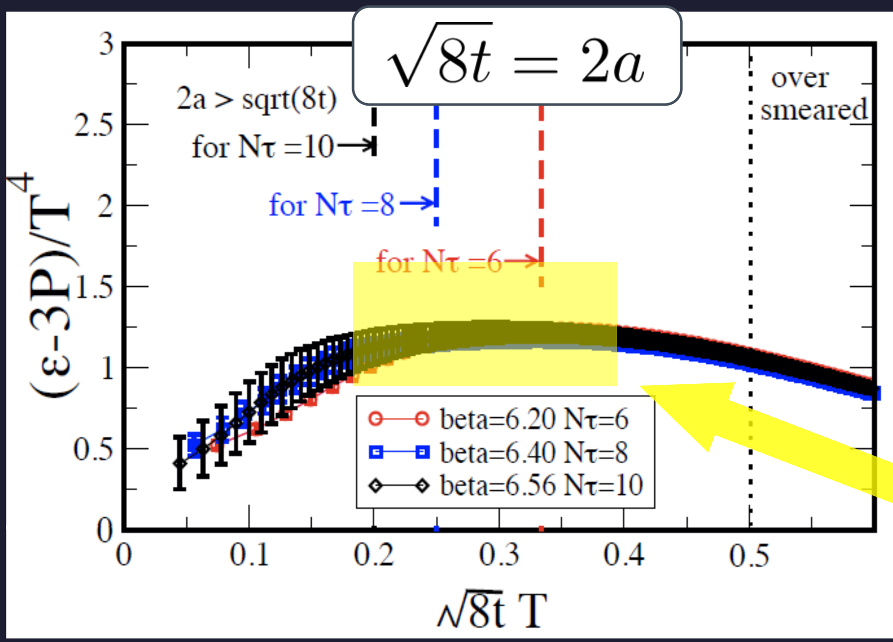

Emergent plateau!

$$2a \lesssim \sqrt{8t} \lesssim 0.4T^{-1}$$

the range of t where the EMT formula is successfully used!

Nt=6,8,10

~300 confs.

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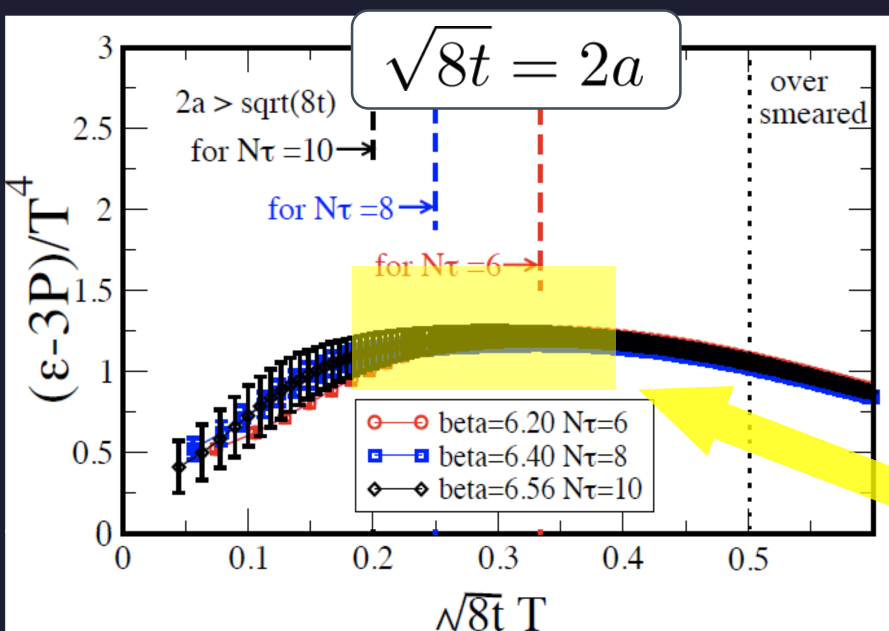

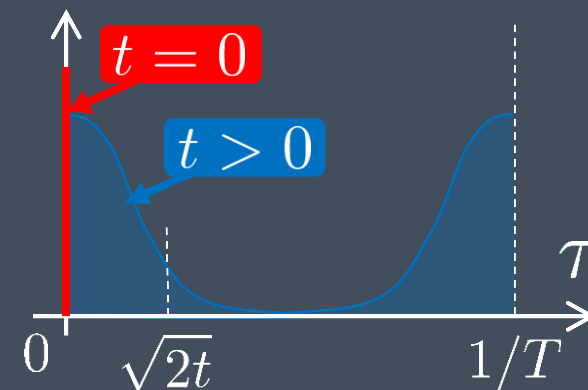

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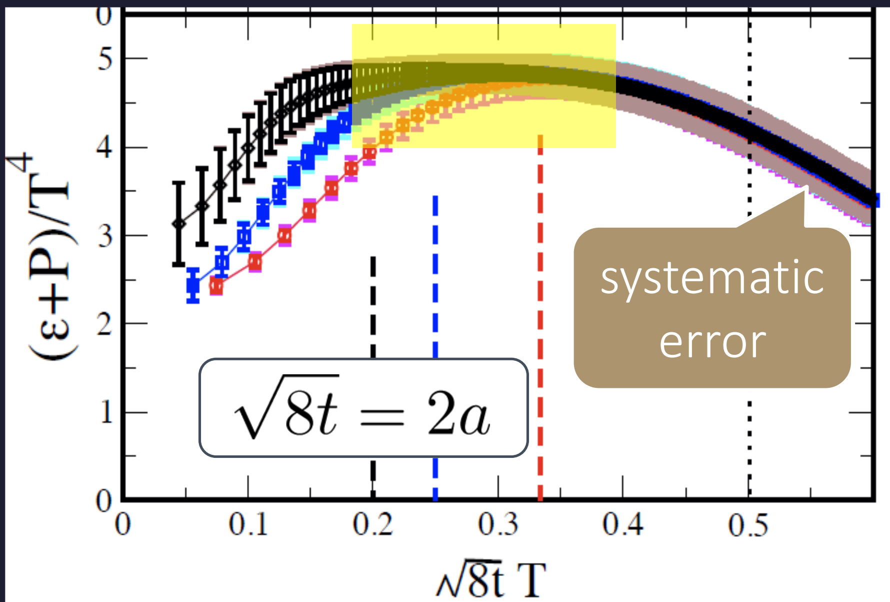
Entropy Density at T=1.65Tc



Emergent plateau!

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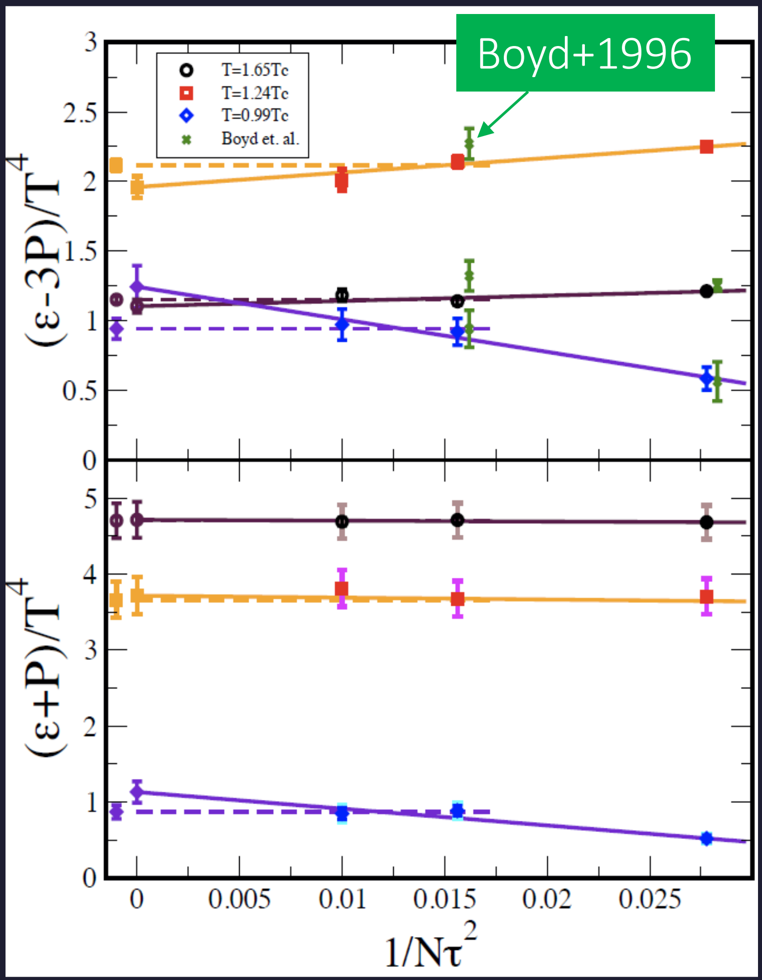
Nt=6,8,10
~300 confs.

Direct measurement of e+p on a given T!

NO integral / **NO** vacuum subtraction

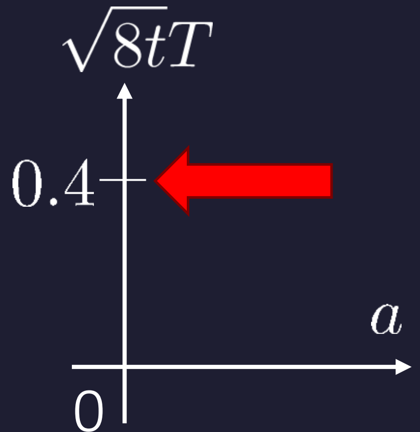
Continuum Limit



32^3xNt

Nt = 6, 8, 10

T/Tc=0.99, 1.24, 1.65



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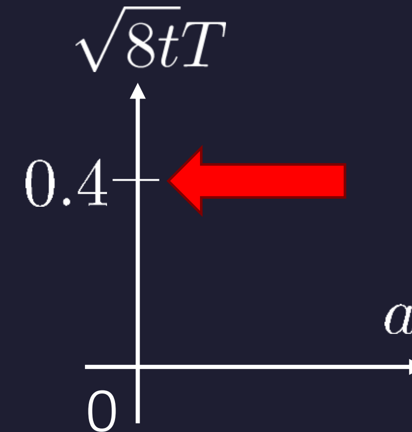
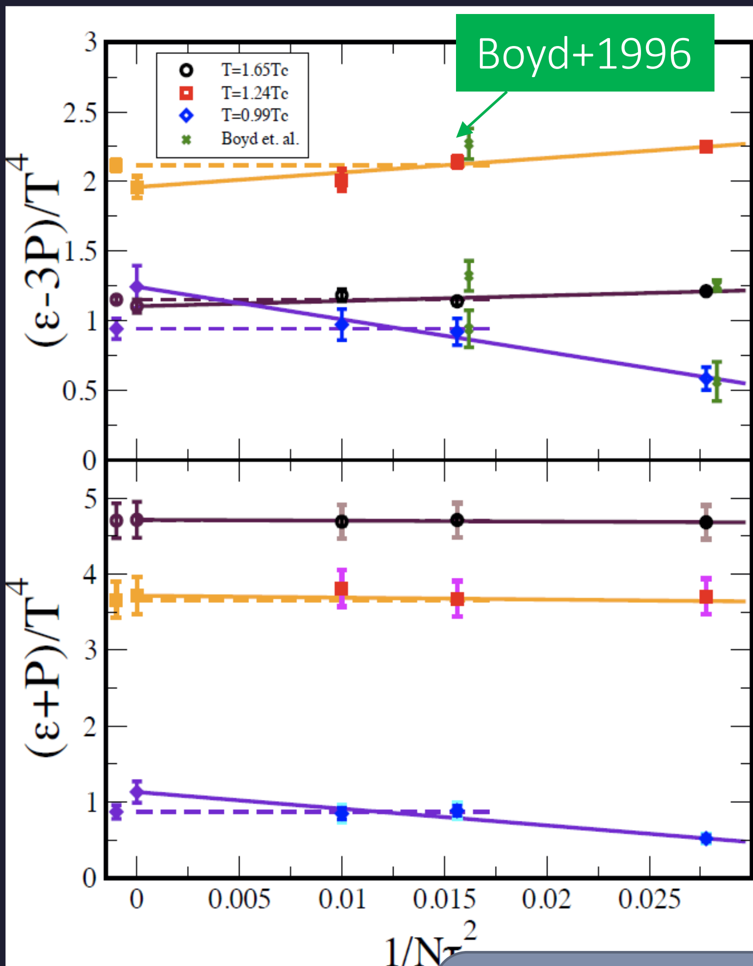
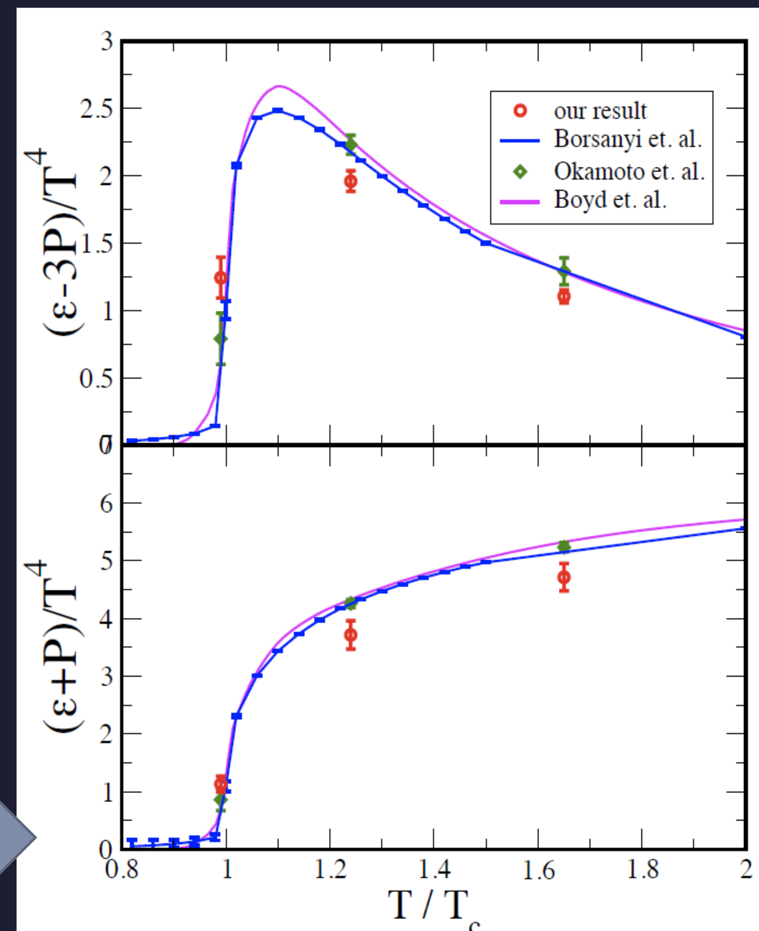


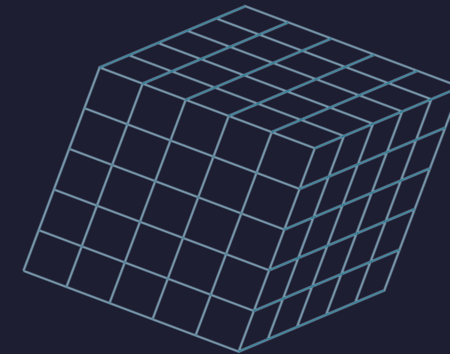


32^3xNt
Nt = 6, 8, 10
T/Tc=0.99, 1.24, 1.65

Comparison with previous studies



Numerical Simulation



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(new, preliminary)

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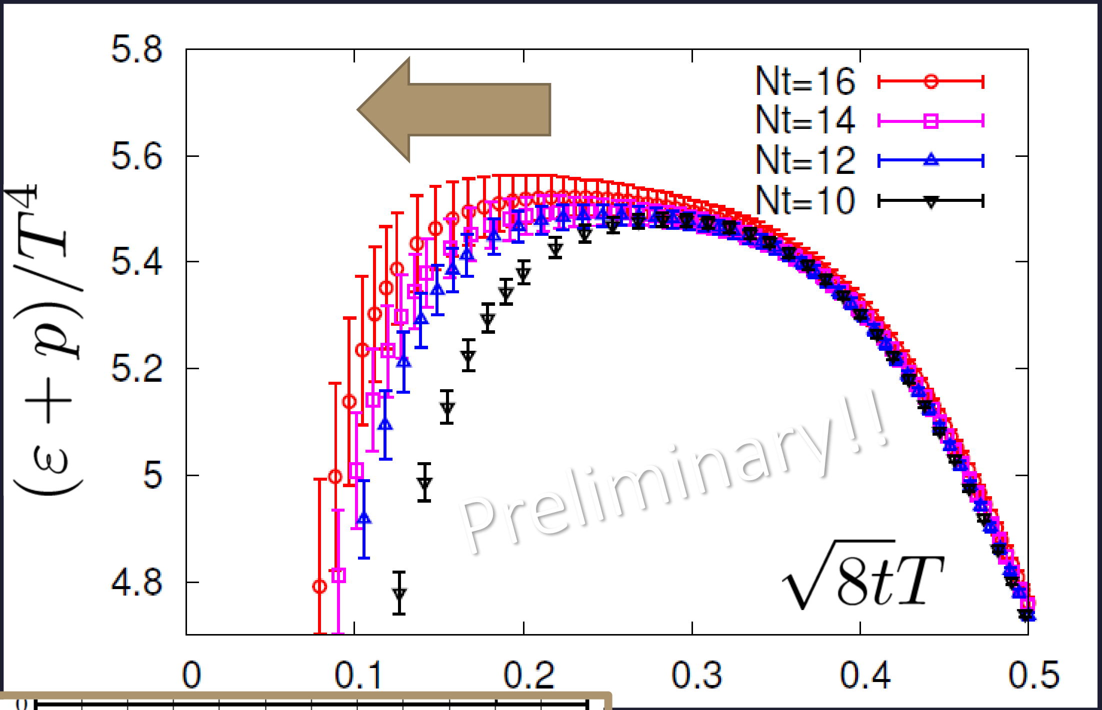
efficiency ~40%

Entropy Density on Finer Lattices

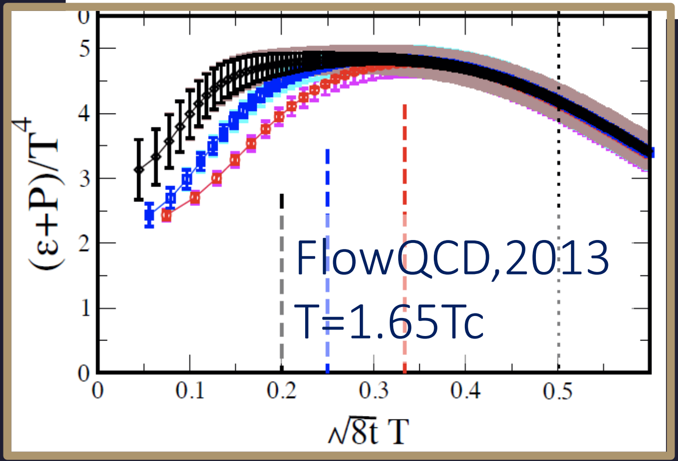


T = 2.31Tc

64^3xNt

Nt = 10, 12, 14, 16

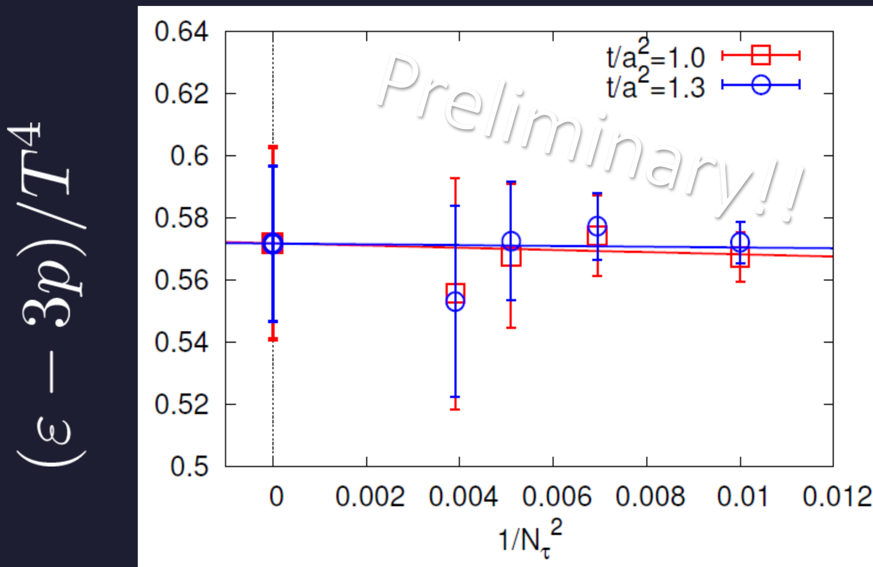
2000 confs.

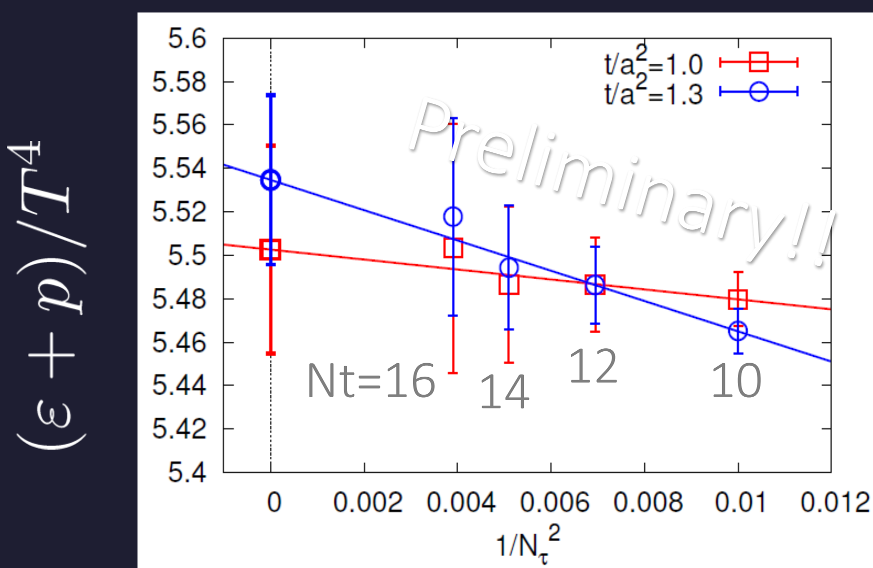


- ☐ The wider plateau on the finer lattices
- ☐ Plateau may have a nonzero slope

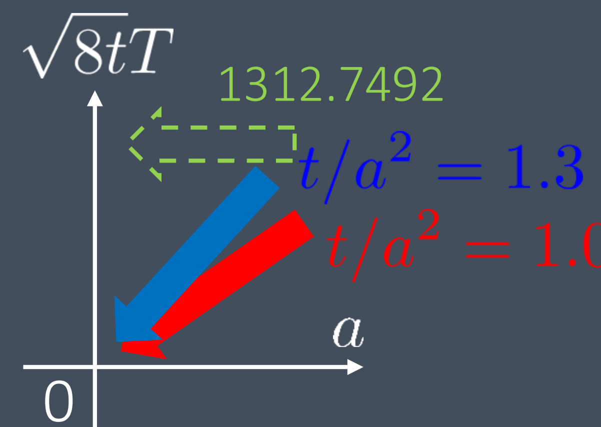
Continuum Extrapolation

- $T=2.31T_c$
- 2000 confs
- Nt = 10 ~ 16





a→0 limit with fixed t/a^2



Continuum extrapolation is stable

Summary

$$T^R_{\mu\nu}(x)$$

Summary

EMT formula from gradient flow

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}E(t,x)_{\mathrm{subt.}}\right]$$

This formula can successfully define and calculate the EMT on the lattice

This operator provides us with novel approaches to measure various observables on the lattice!

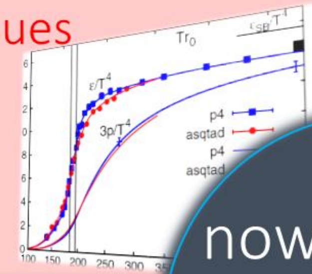
This method is direct, intuitive and less noisy

Many Future Studies!!

Thermodynamics

direct measurement of expectation values

$\langle T_{00} \rangle, \langle T_{ii} \rangle$



Fluctuations and Correlations

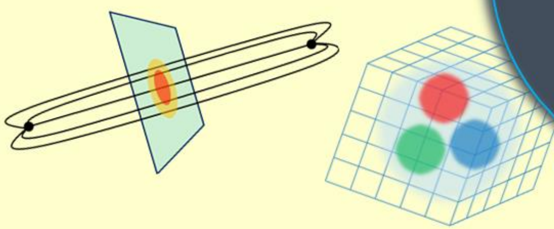
viscosity, specific heat, ...

$c_V \sim \langle \delta T_{00}^2 \rangle$

$\eta = \langle T_{12} ; T_{12} \rangle$

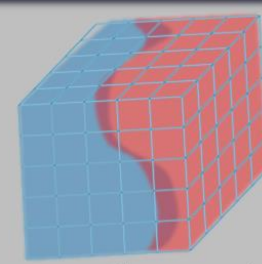
now we have

$T_{\mu\nu}$



- confinement string
- EM distribution in hadrons

Hadron Structure



- vacuum configuration
- mixed state on 1st transition

Vacuum Structure

Other observables

full QCD Makino,Suzuki,2014

non-pert. improvement Patella 7E(Thu)

O(a) improvement

Nogradi, 7E(Thu); Sint, 7E(Thu)

Monahan, 7E(Thu)

and etc.

One More Thing…

One More Thing...

Fluctuations and Correlations

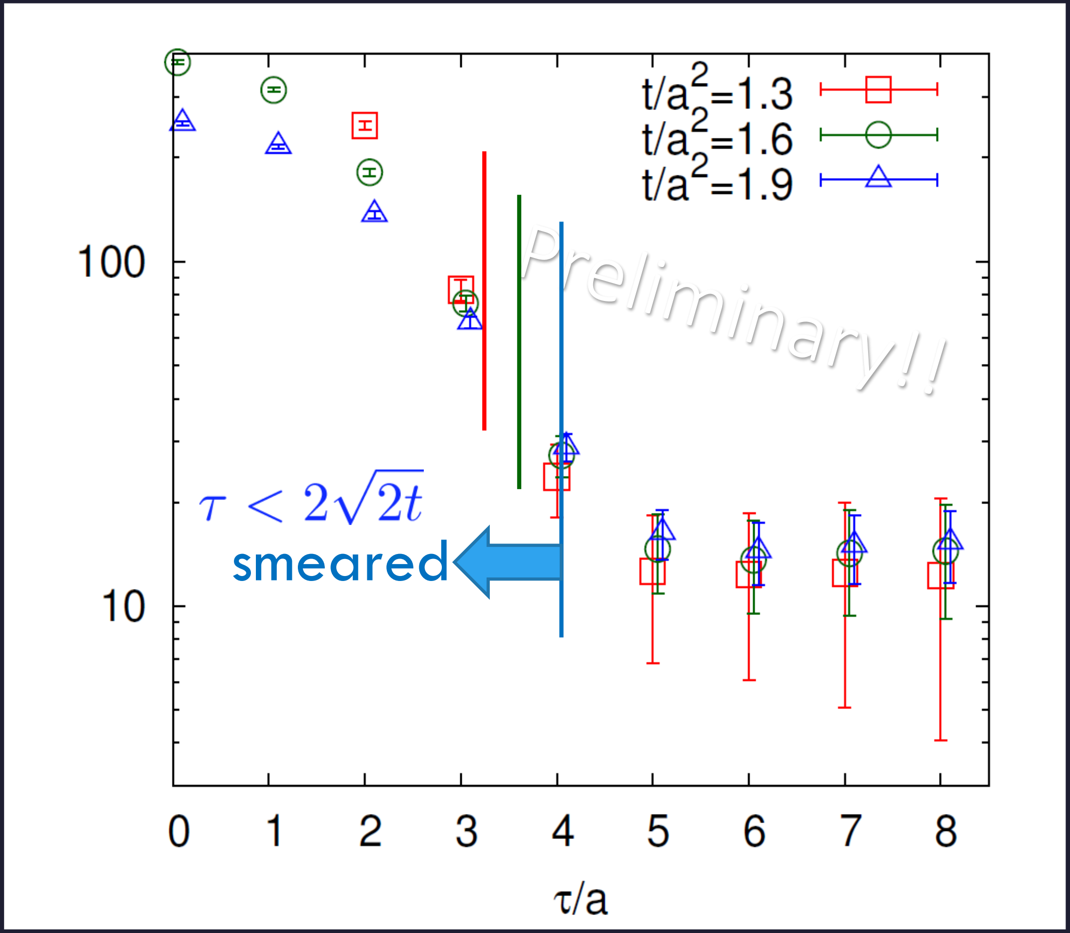
viscosity, specific heat, ...

$$c_V \sim \langle \delta T_{00}^2 \rangle$$

$$\eta = \langle T_{12}; T_{12} \rangle$$

Energy Correlation Function

$$\langle T_{00}(\tau) T_{00}(0) \rangle$$



T=2.31Tc

b=7.2, Nt=16

2000 confs

p=0 correlator

Energy Correlation Function

$$\langle T_{00}(\tau) T_{00}(0) \rangle$$

T=2.31Tc
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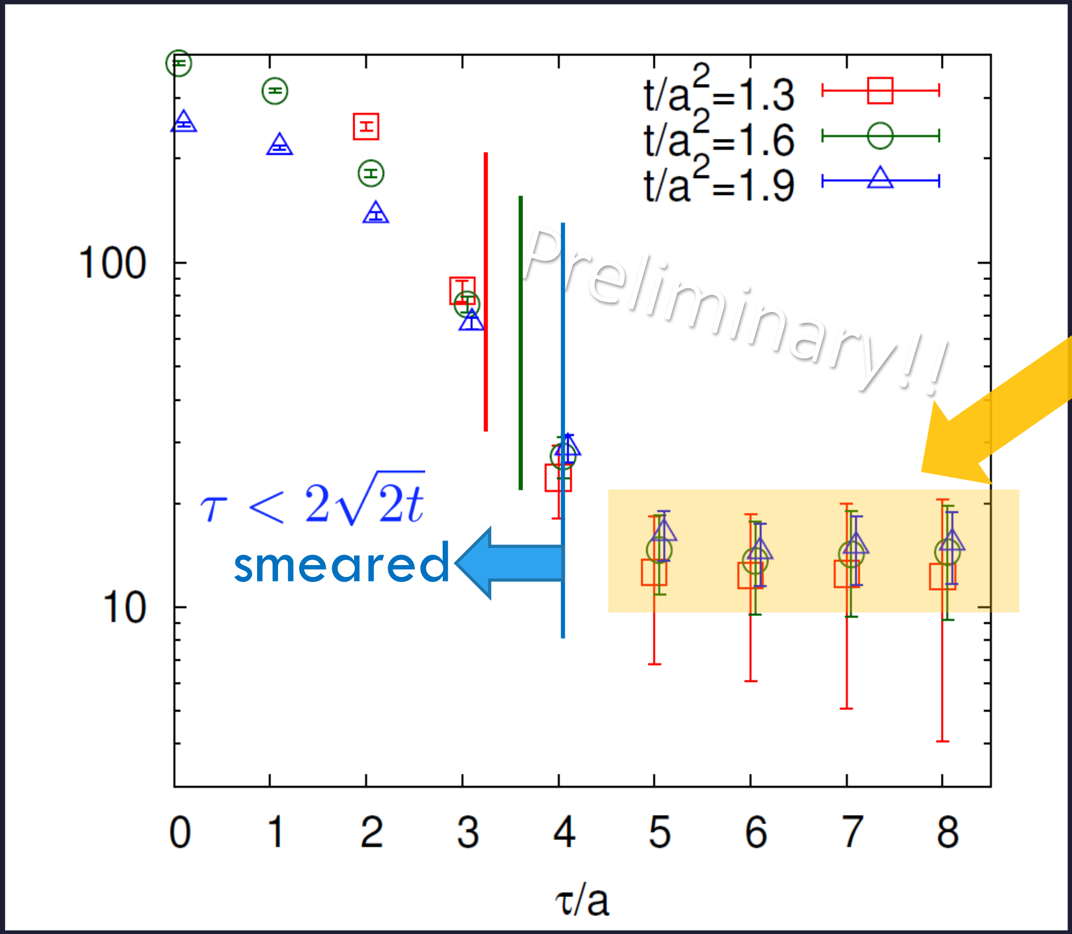
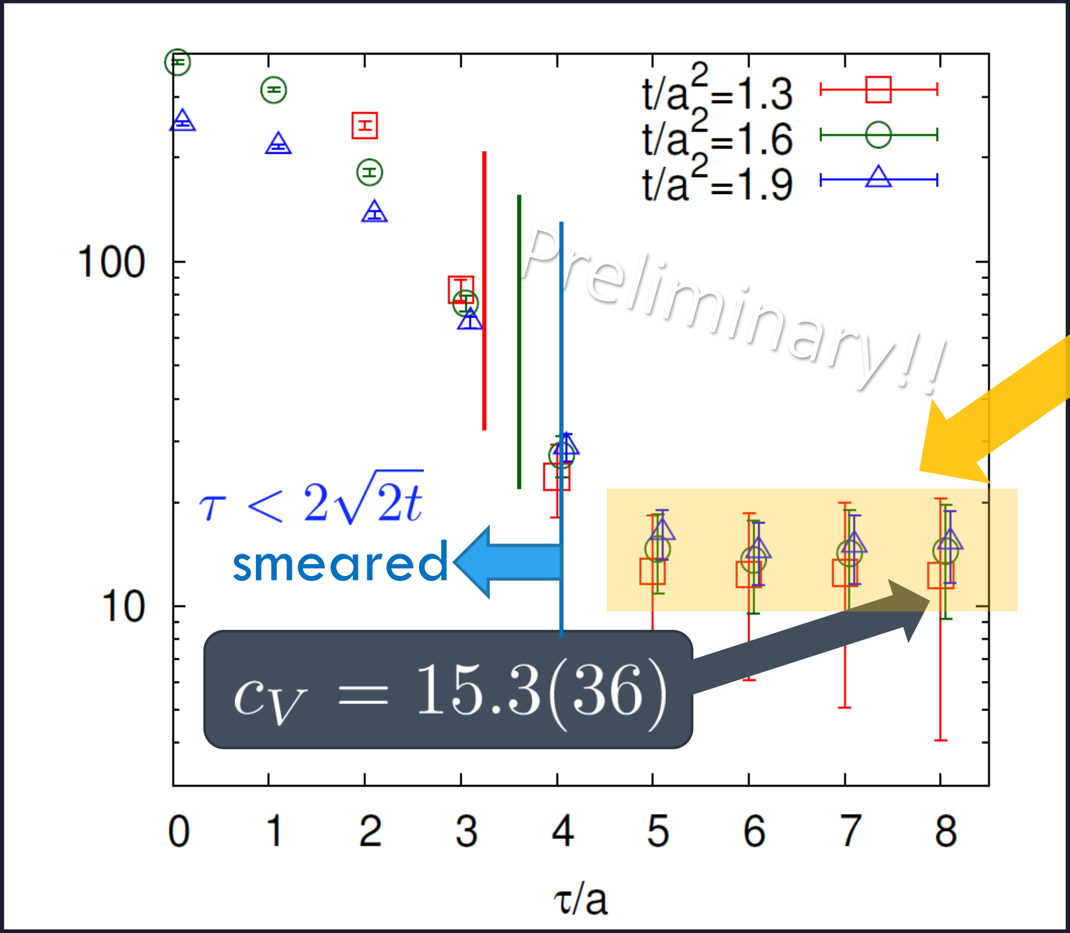


- τ independent const.
  → energy conservation

Energy Correlation Function

$$\langle T_{00}(\tau) T_{00}(0) \rangle$$



T=2.31Tc
b=7.2, Nt=16
2000 confs
p=0 correlator

- τ independent const.
  → energy conservation
- specific heat

$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2}$$

→ Novel approach to measure specific heat!

Keep your attention to this new flow

just like…

Keep your attention to this new flow

just like...

Thank you for your attention!