Topological Insulators and the QCD vacuum: the theta parameter as a Berry phase

Lattice 2014

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Numerical and theoretical evidence for topological charge membranes in the QCD vacuum:

Lattice QCD results (Horvath, Thacker, et al; Ilgenfritz,et al,2003):

- Studies of local TC distribution have revealed a "lasagna vacuum" of extended, thin, codimension 1 membranes of coherent TC in a laminated, alternating sign array. Analogous codimension 1 structures are observed in 2D CPN models (Thacker, et al. 2007)

Holographic QCD (Luscher, 1978; Witten, Sakai, Sugimoto,1998):

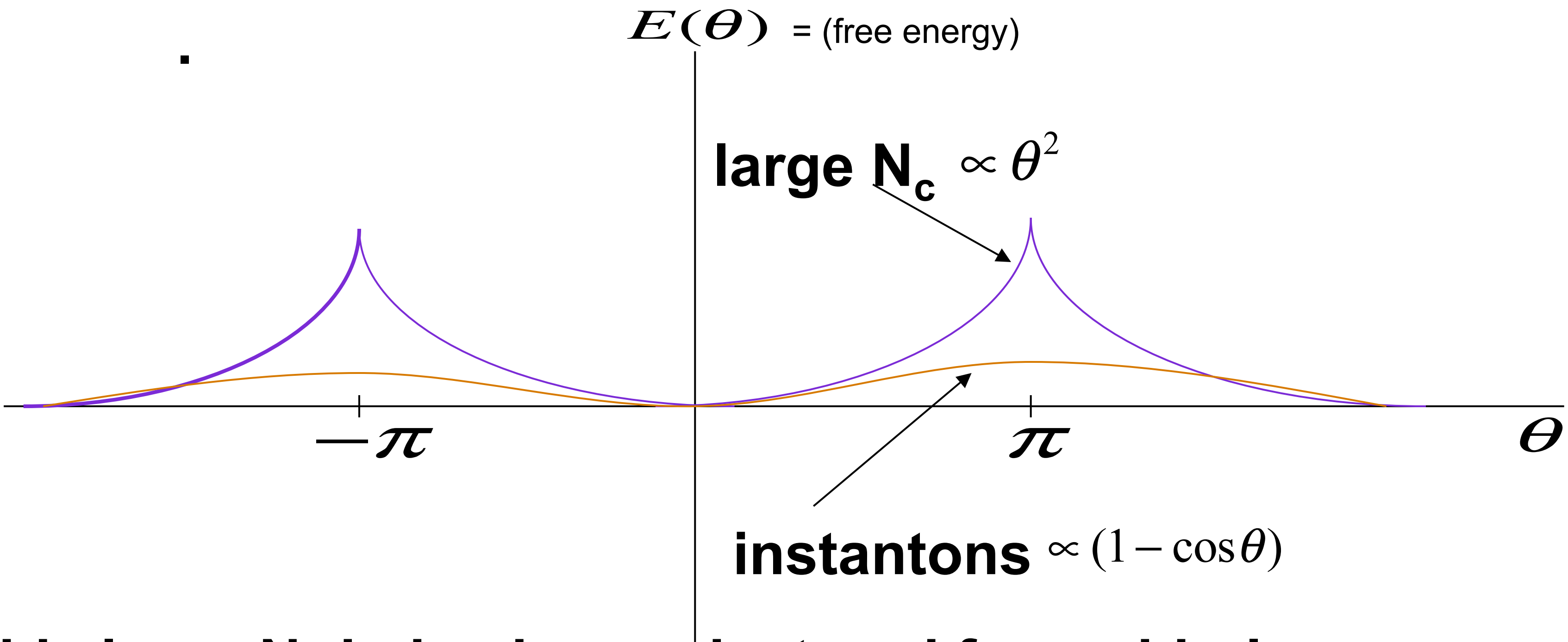
- Topological charge membranes have a natural holographic interpretation as D6 branes of IIA string theory = "domain walls" between k-vacua with $\theta_{loc} = \theta + 2\pi k$

Large-N_c Chiral Dynamics (Witten, Veneziano,1979):

- Chiral Lagrangian form of axial U1 anomaly is an eta-prime mass term $\propto (\log Det U)^2 \sim (\eta')^2$. Periodic under $\eta' \to \eta' + 2\pi$, but <u>only</u> by a change of branch for log $\Longrightarrow$ multiple discrete k-vacua separated by domain walls. (Here $(\eta' / f_\pi) \to \eta'$ is the chiral U(1) phase.)

Theta dependence, phase transitions:

Witten (1979): Large-N_c arguments require a phase transition (cusp) at $\theta = \pi$ Contradicts instanton expansion, which gives smooth θ - dependence.



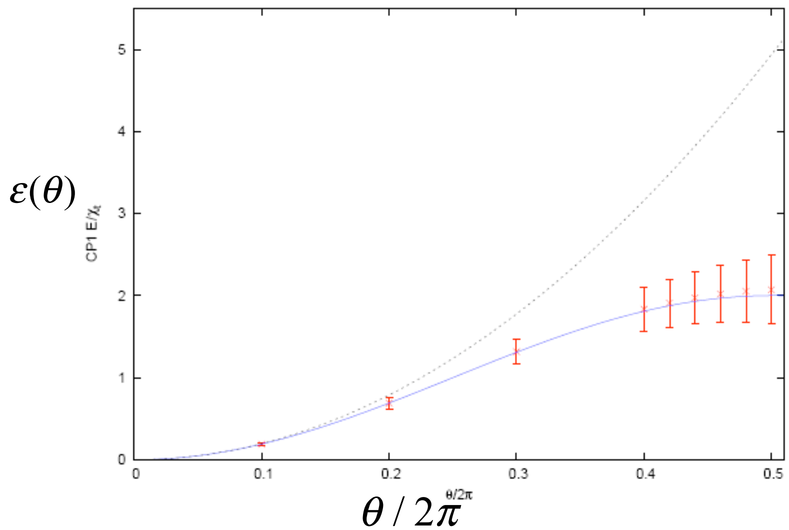
This large N_c behavior conjectured from chiral Lagrangian arguments (Witten, 1979)

Clarified by AdS/CFT duality (Witten, 1998)

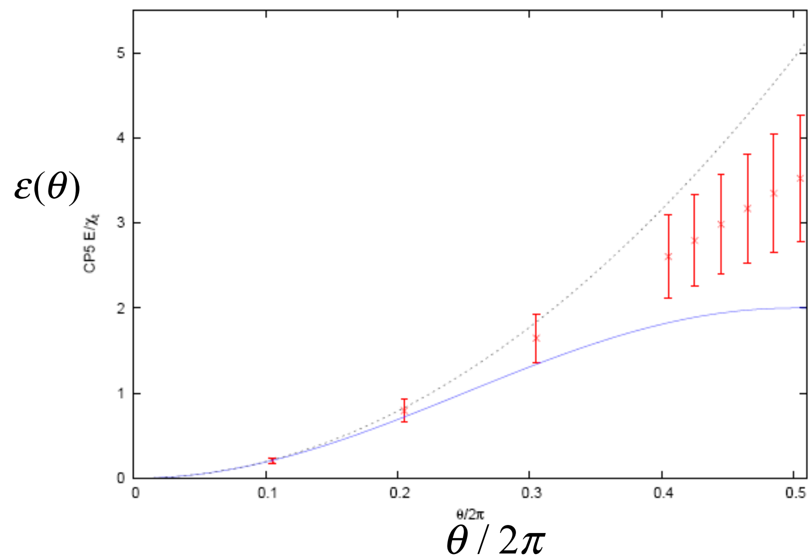
$\varepsilon(\theta)$ from fractionally charged Wilson loops

(Keith-Hynes and HT, PRD 2008)

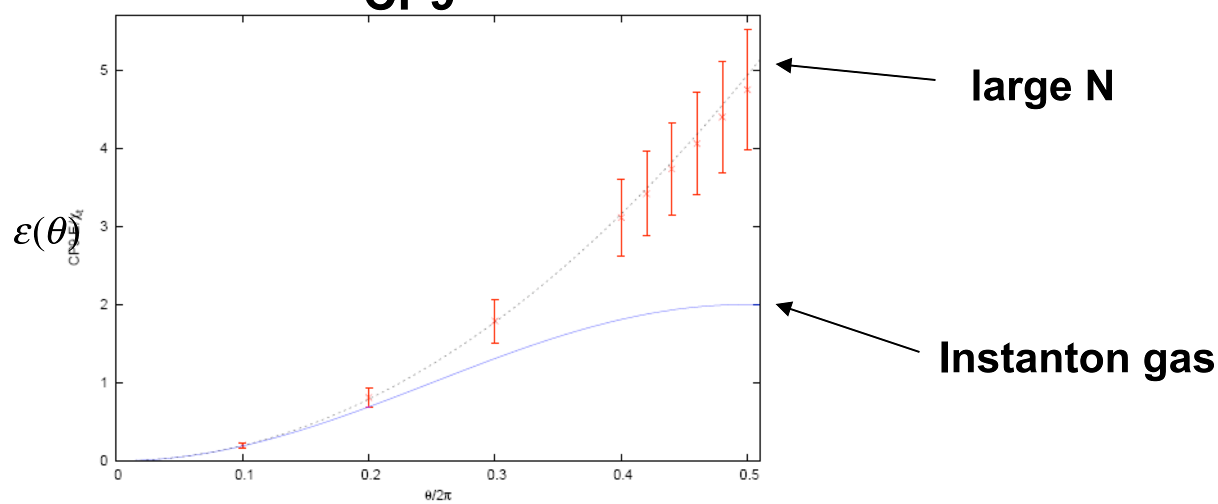
CP1



CP5

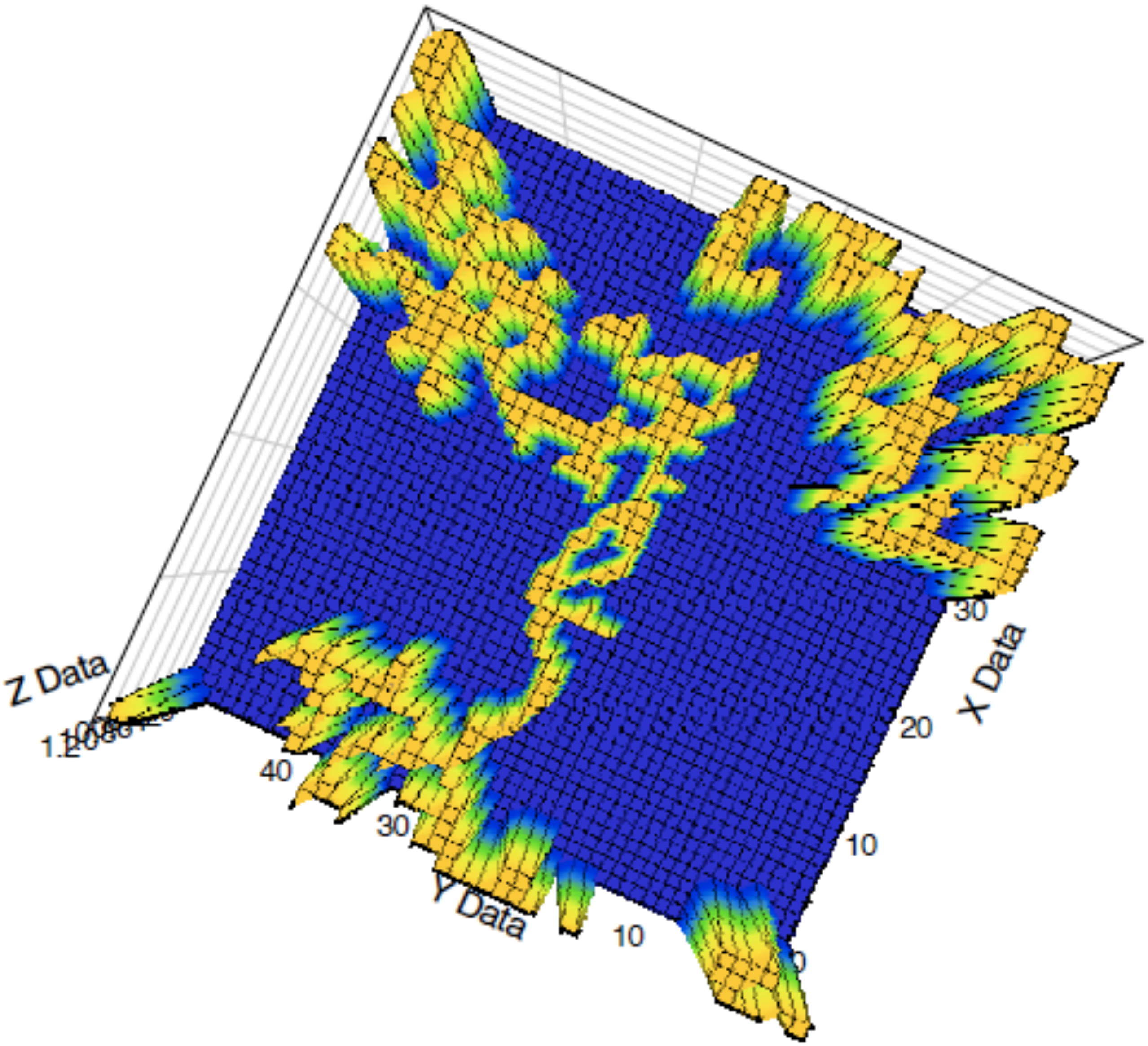


CP9



$\theta / 2\pi$

Coherent structure: CP^3 50×50 $\beta = 1.2$ $(\xi \approx 20)$

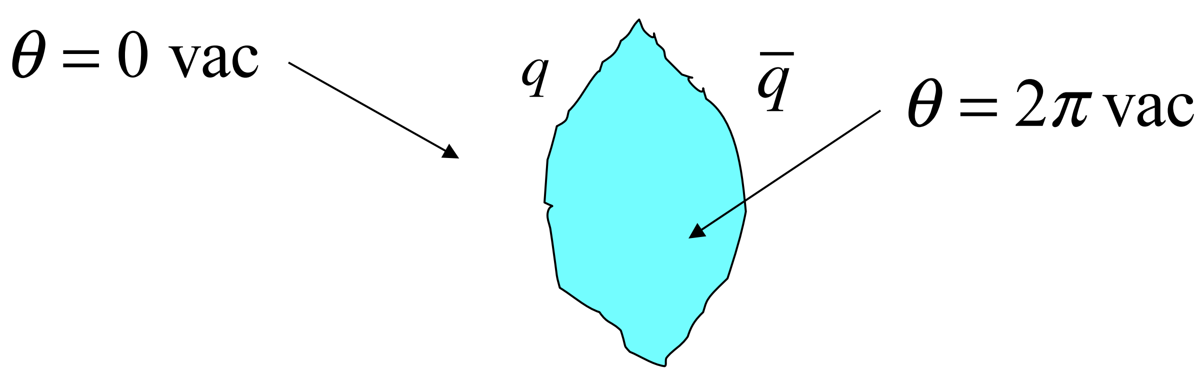


--Multiple discrete k-vacua characterized by an effective value of θ which differs from the θ in the action by integer multiples of 2π= number of units of background Ramond-Ramond flux in the holographic framework.

-- Interpretation of effective θ similar to Coleman's discussion of 2D massive Schwinger model (Coleman, 1976), where

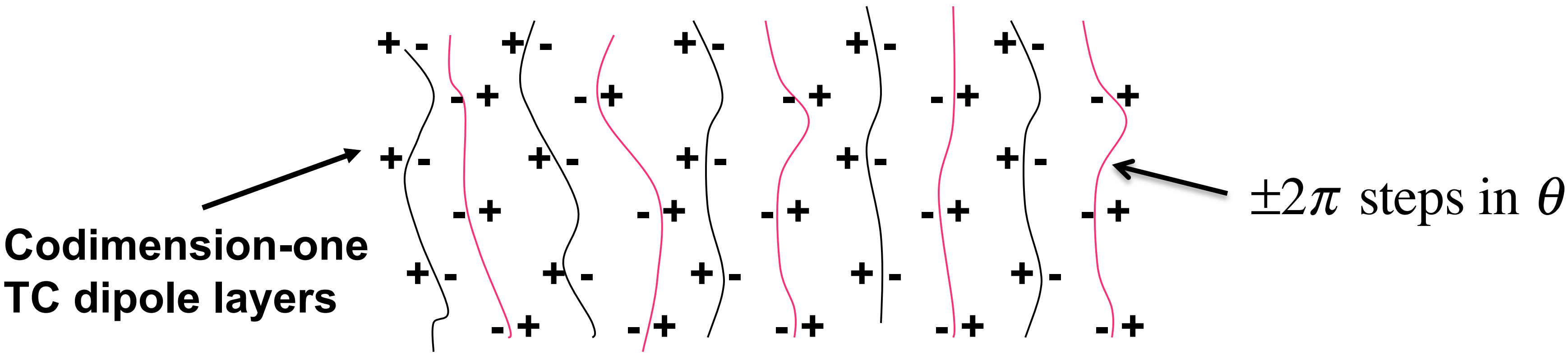
$$\theta = \textbf{ background E field}$$

In 2D U(1) models (CP(N-1) or Schwinger model): Domain walls between k-vacua are world lines of charged particles:



The emerging picture -- A "laminated" vacuum:

❖ Vacuum is filled with alternating sign dipole layers of topological charge. Confinement scale set by correlation length of surface orientation vector $\partial_\mu \theta$.



The nature of the chiral condensate in this picture follows from the observation that, in the presence of quarks, θ becomes the U1 chiral field, so the axial vector current is given by $J^5_\mu = \partial_\mu \theta$. This gives a delta-function on the brane surface.

Conclusion: condensate consists of surface quark modes on the membranes, with left and right chiral densities $\bar{\psi}(1 \pm \gamma_5)\psi$ living on topological charge sheets on opposite sides of each membrane.

Topological insulators and the QCD vacuum:The Chiral Goldstone field as a Berry phase:(arXiv:1311.7104)

QCD vacuum is analogous to a topological insulator:

(1) A bulk mass gap (= large glueball mass)

(2) Long range propagation of Goldstone bosons (= chiral currents) via surface modes on Chern-Simons domain wall boundaries (analogous to chiral edge currents in topological insulator or quantum hall states).

A central idea in topological insulator theory is a Berry connection = gauge connection over momentum space instead of coordinate space.

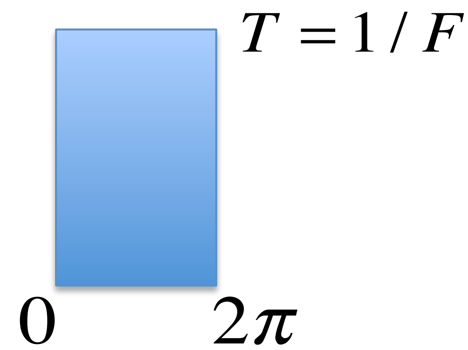
--- Berry phase is the phase acquired by a Hamiltonian eigenstate under transport around a closed loop in momentum space (typically a Bloch wave transported around a Brillouin zone).

--- The local theta parameter (= topological order parameter) can be defined as a Berry phase of a quark eigenstate under transport around the BZ. (Compactified BZ = lattice cutoff). (Berry phase interpretation of theta first suggested by R. Jackiw in 1986.)

Spectral flow and the Berry phase in 2D:

-- Consider Dirac Hamiltonian on a Euclidean 2-torus with a constant background F field. Topological charge is quantized, so

$$\int_T F\, dx\, dt = 2\pi \times \text{integer}$$



In gauge $A_x = Ft,\ \ A_t = 0$ the Dirac zero mode is a modular function – periodic in x and quasi-periodic in t (periodic up to a gt)

$$u(x,k) = \sum_n e^{-\frac{T}{2}(n+k)^2} e^{inx}, \quad k \equiv Ft = A_x$$

In Hamiltonian formulation, quasiperiodicity corresponds to quantized spectral flow.

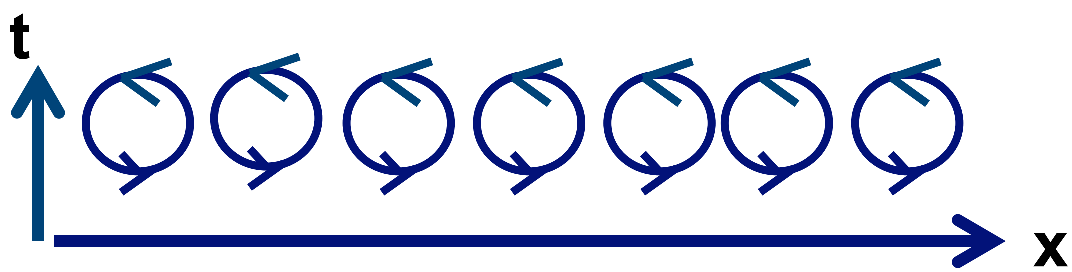
$$H_0 \to H(k) = g^{-1}(k) H_0 g(k), \quad g(k) = e^{ikx}$$

$$H(k) = \gamma_5 \left(-i\partial_x + k\right)$$

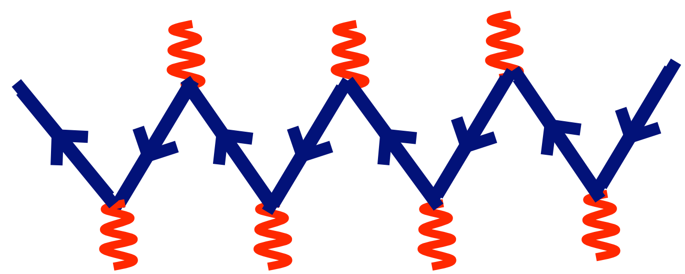
Transforming to Coulomb gauge and performing a Poisson transformation, wave function is periodic in time and quasiperiodic in space direction. This give a Bloch wave state:

$$\Psi(x,k) = e^{ikx}u(x,k) = e^{ikx}\sum_n e^{-\frac{1}{2T}(x-x_n)^2} e^{-ik(x-x_n)}, \quad x_n = 2\pi n$$

= a superposition of localized Landau levels

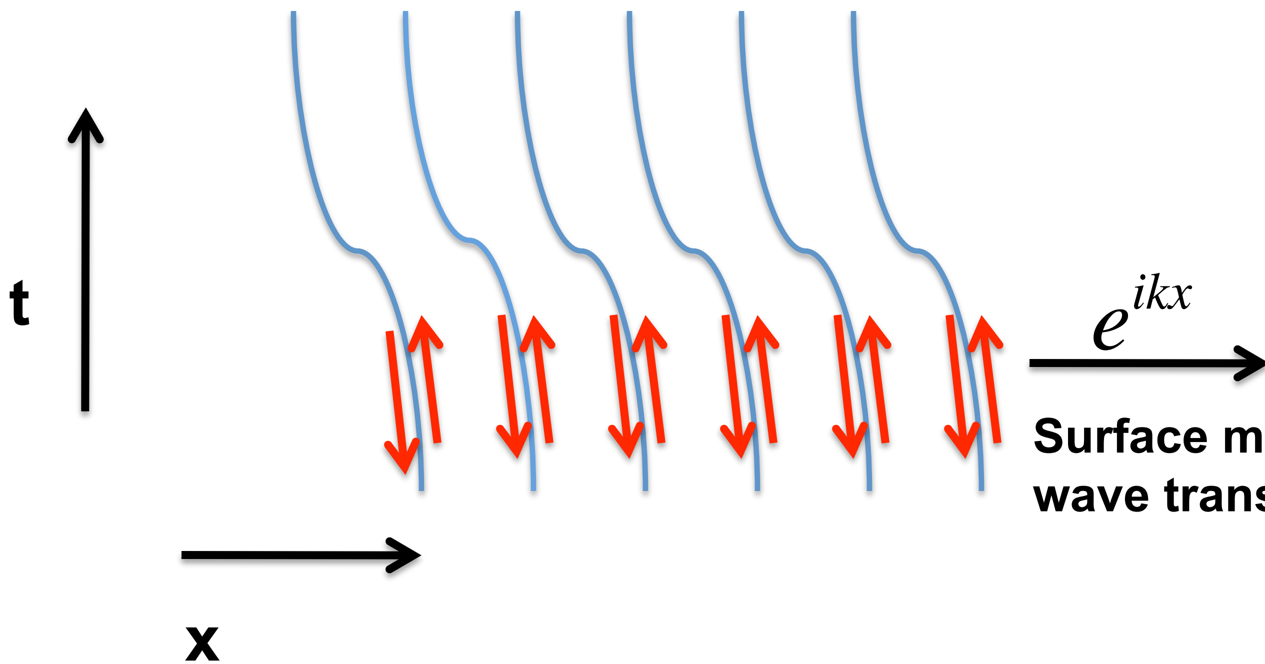


Berry phase around BZ represents transport of charge from left to right boundary:



In 2D electrodynamics, Berry construction is effectively the King-Smith-Vanderbilt theory of electric polarization in topological insulators. Witten's transition between theta vacua corresponds to transport of a unit of current from one side of sample to the other:

$\Rightarrow$ In 4D QCD long range propagation of Goldstone pions ($\approx$ charge transport in topological insulators) occurs by "topologically protected" Dirac surface modes on Chern-Simons membranes.



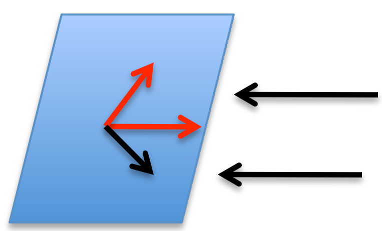
Surface modes form a Bloch wave transverse to branes.

Goldstone pions as brane polarization waves:

-- In 1D topological insulators, θ represents electric polarization. Mod 2π jumps of $\theta \Leftrightarrow$ integer transport of charge between boundaries (as in quantum hall effect).

-- Taking 4D QCD vacuum as a “brane sandwich” of polarizable Chern-Simons brane-antibrane pairs, θ is an order parameter of the pure glue vacuum describing brane polarization. When quarks are introduced, the condensate resides in brane surface modes, so θ becomes the U(1) chiral field.

-- Pion propagates along branes via Dirac surface modes. Transverse propagation is a polarization wave. These are linked together by gauge invariance, via “anomaly inflow” or “bulk-boundary” constraints (Kamat,Thacker, and Xiong, PRD 2012).



in-brane gauge transform of CS tensor

Gauge variation of transverse link represents brane fluctuation

Summary:

◆ The topological structure of the QCD vacuum is an alternating-sign sandwich of membranes of Chern-Simons charge. (In 1+1-D the membranes are pointlike charged particles, and vacuum = BCS-type condensate of charged pairs)

◆ The chiral condensate in QCD consists of Dirac surface modes on CS membranes. The topological order parameter changes by $\pm 2\pi$ across membrane.

◆ The "antiferromagnetic" layered membrane structure appears at the cutoff scale. What survives in the continuum limit is the bulk polarizability of the membranes = topological susceptibility of QCD vacuum.

◆ Berry phase construction of theta parameter identifies the relation between the chiral U(1) Goldstone field and the ``topologically protected" surface modes of quarks propagating on the membranes.