SU(3) gauge theory with 12 flavours in a twisted box

C.-J. David Lin

National Chiao-Tung University, Hsinchu, Taiwan

Lattice 2014, Columbia U., NYC
25/06/2014

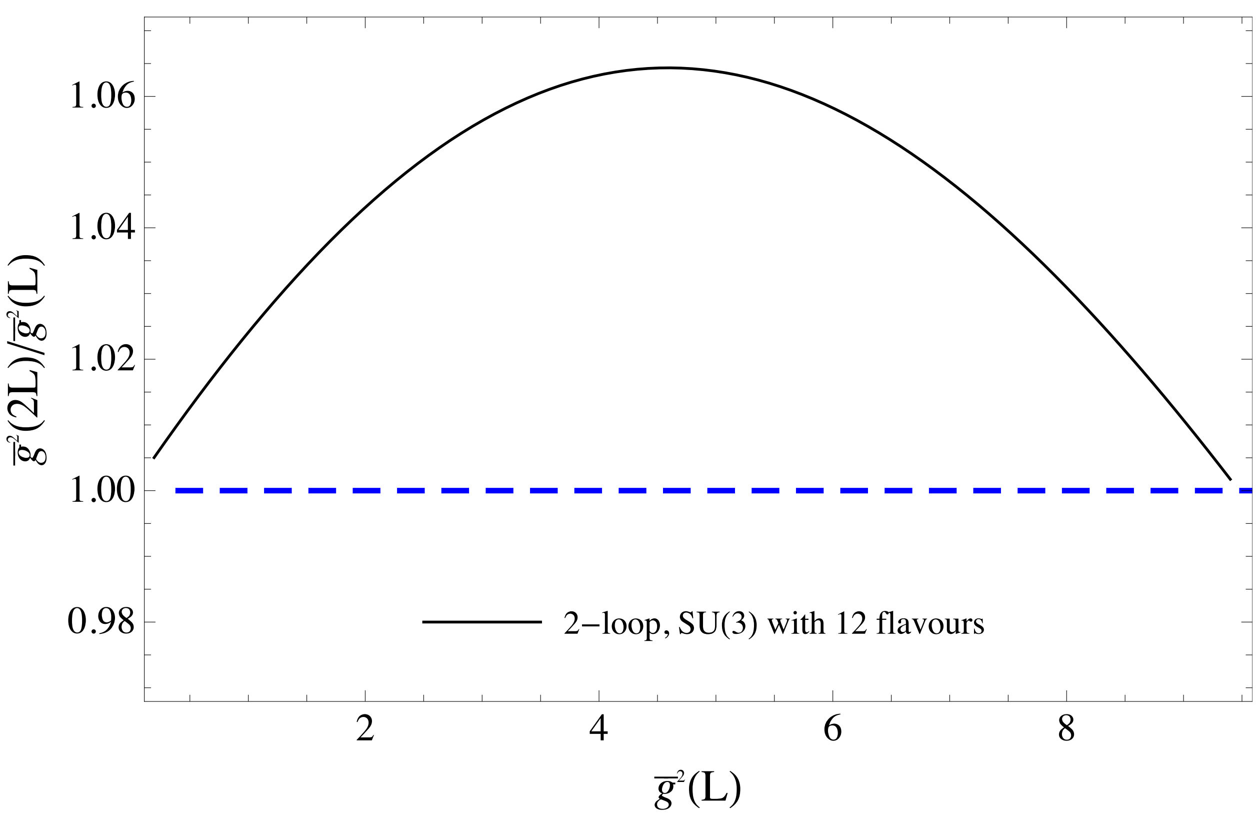
Collaborators

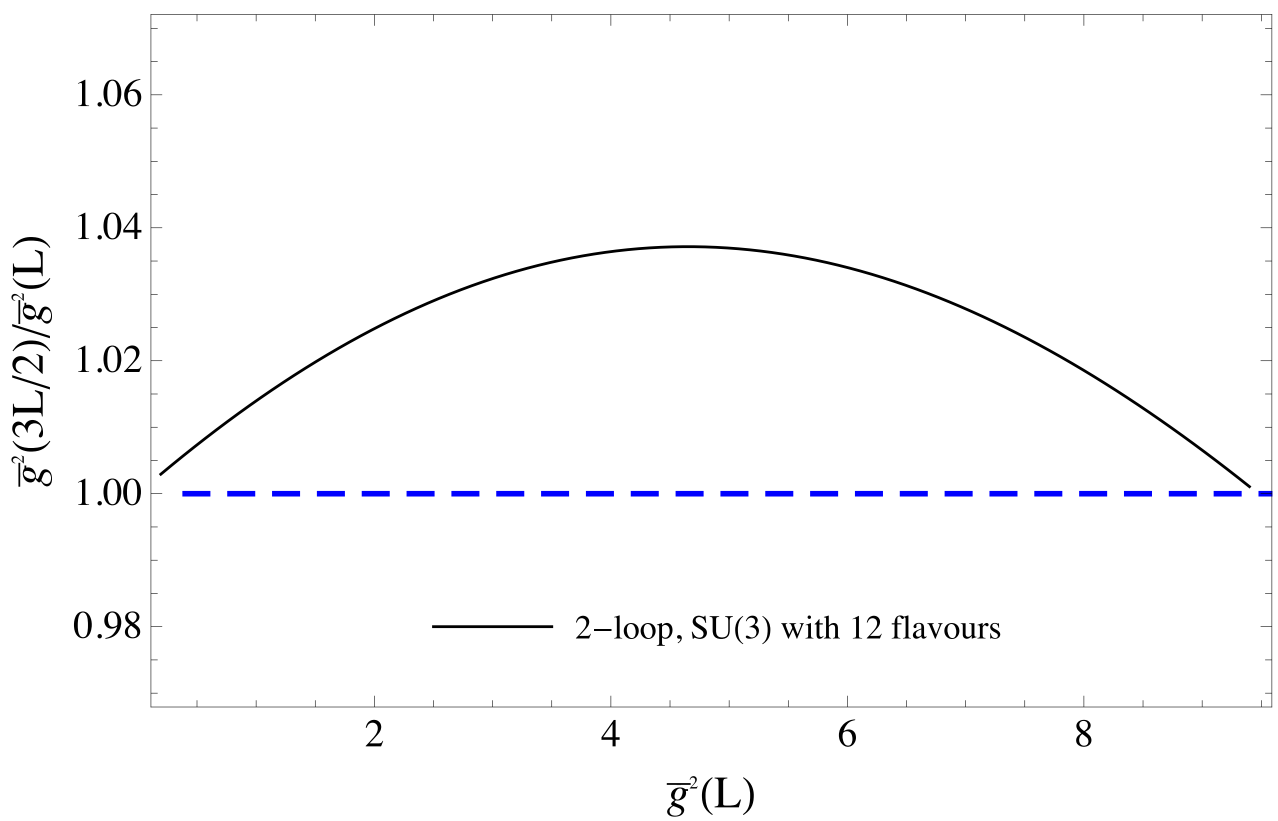
- Kenji Ogawa (NCTU, Taiwan*)
- Hiroshi Ohki (KMI, Nagoya U., Japan)
- Alberto Ramos (DESY Zeuthen, Germany)
- Eigo Shintani (U. of Mainz, Germany)

* Till november 2013

Strategy/challenges for the lattice search of IRFP

- Spectrum: Large finite-volume effects.
- Finite-size scaling *a'la* M. Fisher : universal curves.
- Running coupling: (slow) running within error.

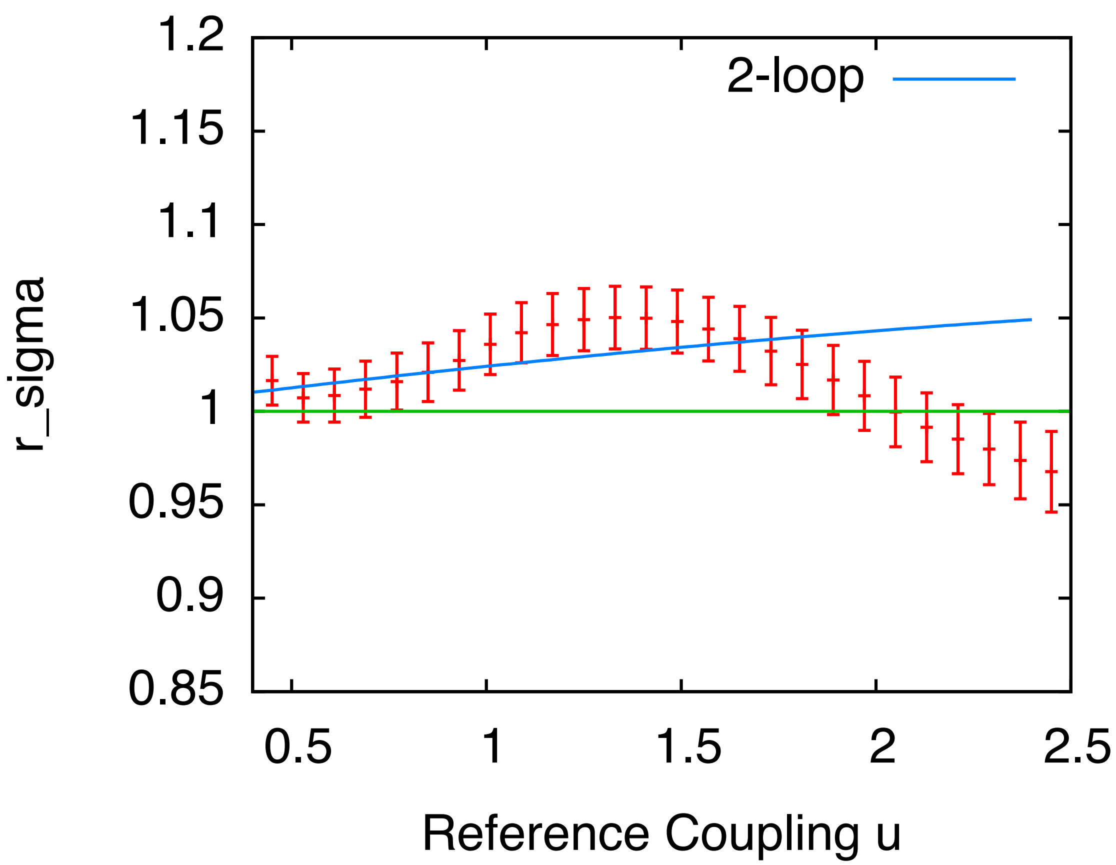




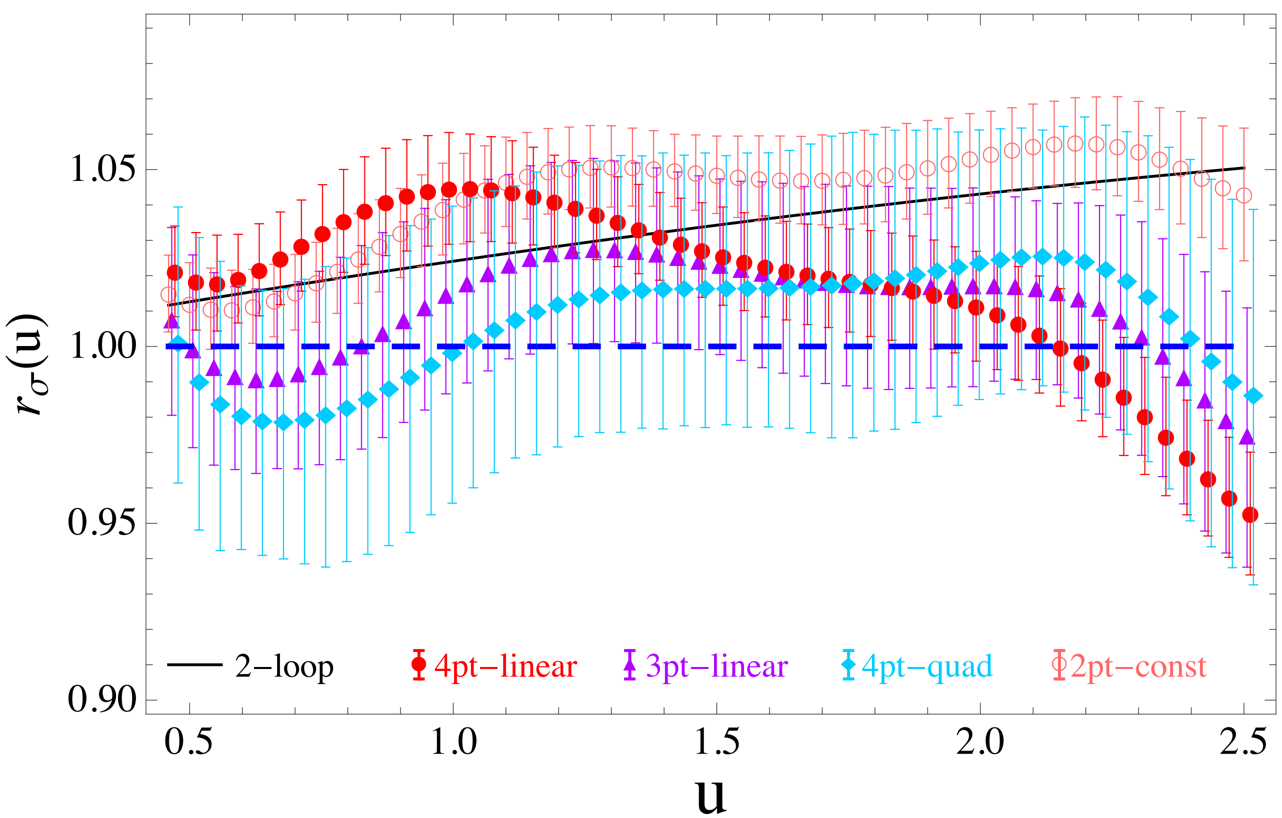
The Twisted Polyakov Loop scheme

C.-J.D.L., K.Ogawa, H.Ohki, E.Shintani, 2012

K.Ogawa, lattice 2013



without (L/a=12 ⟶ L/a = 24)
systematics severely underestimated...



with (L/a=12 ⟶ L/a=24)

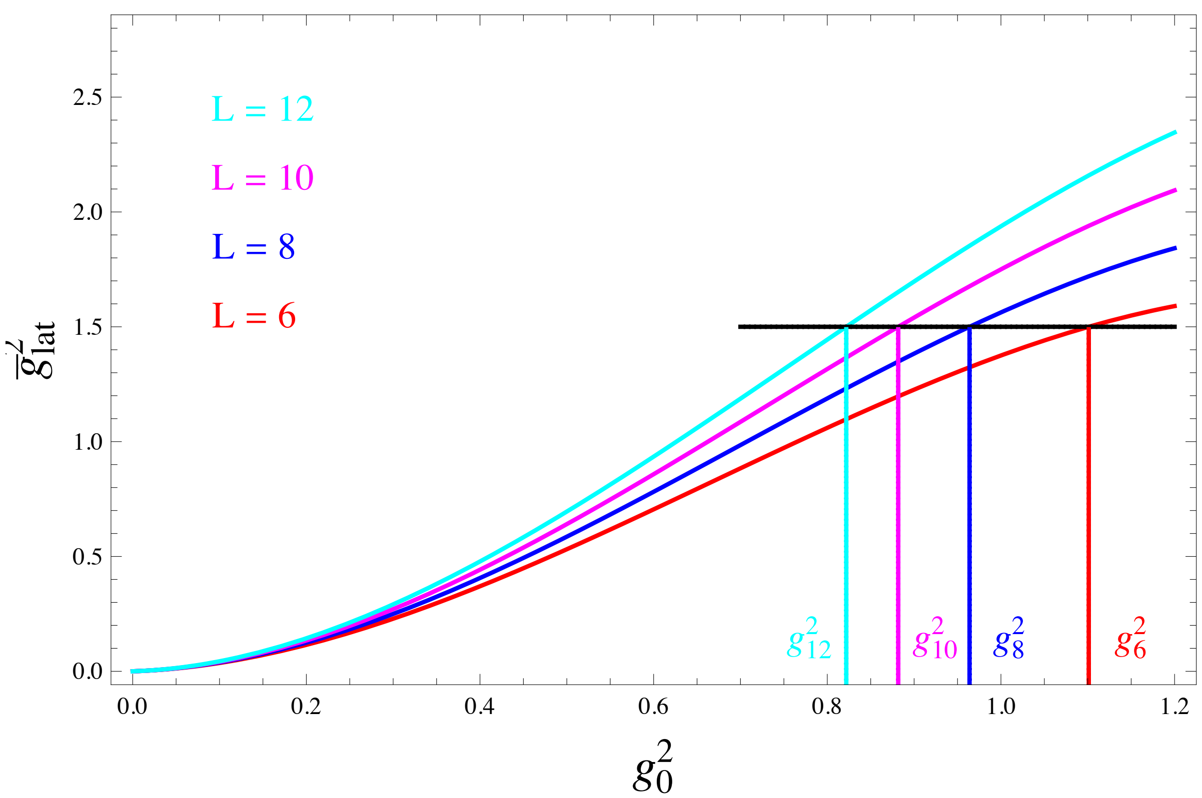
It is challenging to draw any conclusion from such a “noisy scheme”.

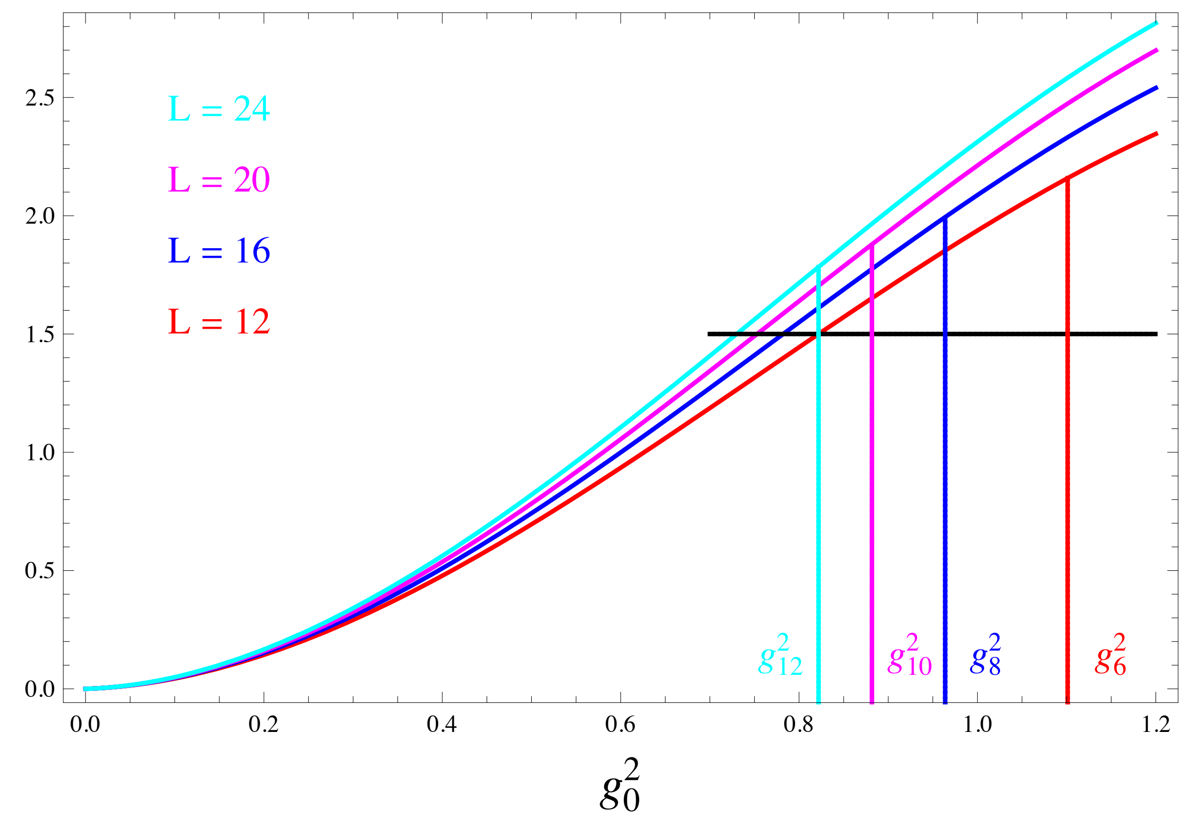
Outline

- Step scaling.
- Twisted box.
- Wilson flow and numerical results.
- Outlook.

The step-scaling method

The practice





- Massless unimproved staggered fermions with Wilson's plaquette gauge action.
- Compute $\bar{g}^2_{\rm lat}$ at many g_0^2 for each volume, and then interpolate volume by volume.
- Very challenging to pin down percentage-level effects in $r_\sigma = \dfrac{\sigma(u)}{u}$.

Twisted box

removing the torons, no odd powers in g.

- **Gauge field:** G. 't Hooft, 1979

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger, \;\; \nu = 1,2,$$

where the twist matrices Ω_ν satisfy

$$\Omega_1\Omega_2 = \mathrm{e}^{2i\pi/3}\Omega_2\Omega_1, \;\; \Omega_\mu\Omega_\mu^\dagger = 1, \;\; (\Omega_\mu)^3 = 1, \;\; \mathrm{Tr}(\Omega_\mu) = 0.$$

- **Fermion:** If $\psi(x+\hat{\nu}L) = \Omega_\nu\psi(x)$

$$\Rightarrow \psi(x+\hat{\nu}L+\hat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$$

- **The fermion "smell" dof:** $N_s = N_c$ G. Parisi, 1983

$$\psi_\alpha^a(x+\hat{\nu}L) = \mathrm{e}^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(x)(\Omega_\nu)_{\beta\alpha}^\dagger.$$

The Gradient Flow scheme

- The quantity, $\langle E(t) \rangle = \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.

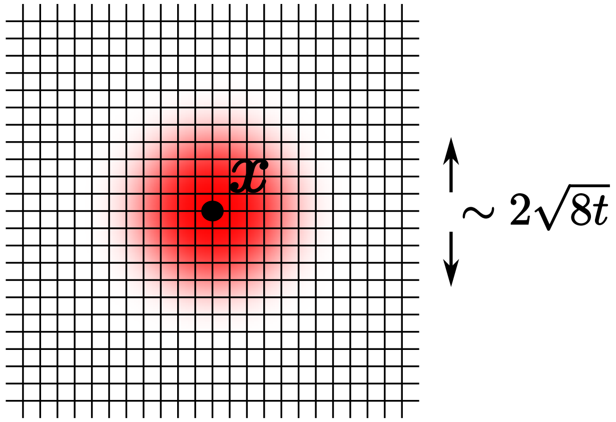
- In a colour-twisted box, can define,

$$\bar{g}^2_{\rm GF}(L) = \mathcal{N}^{-1} t^2 \langle E(t) \rangle = \bar{g}^2_{\rm MS} + \mathcal{O}(\bar{g}^4_{\rm MS}),$$

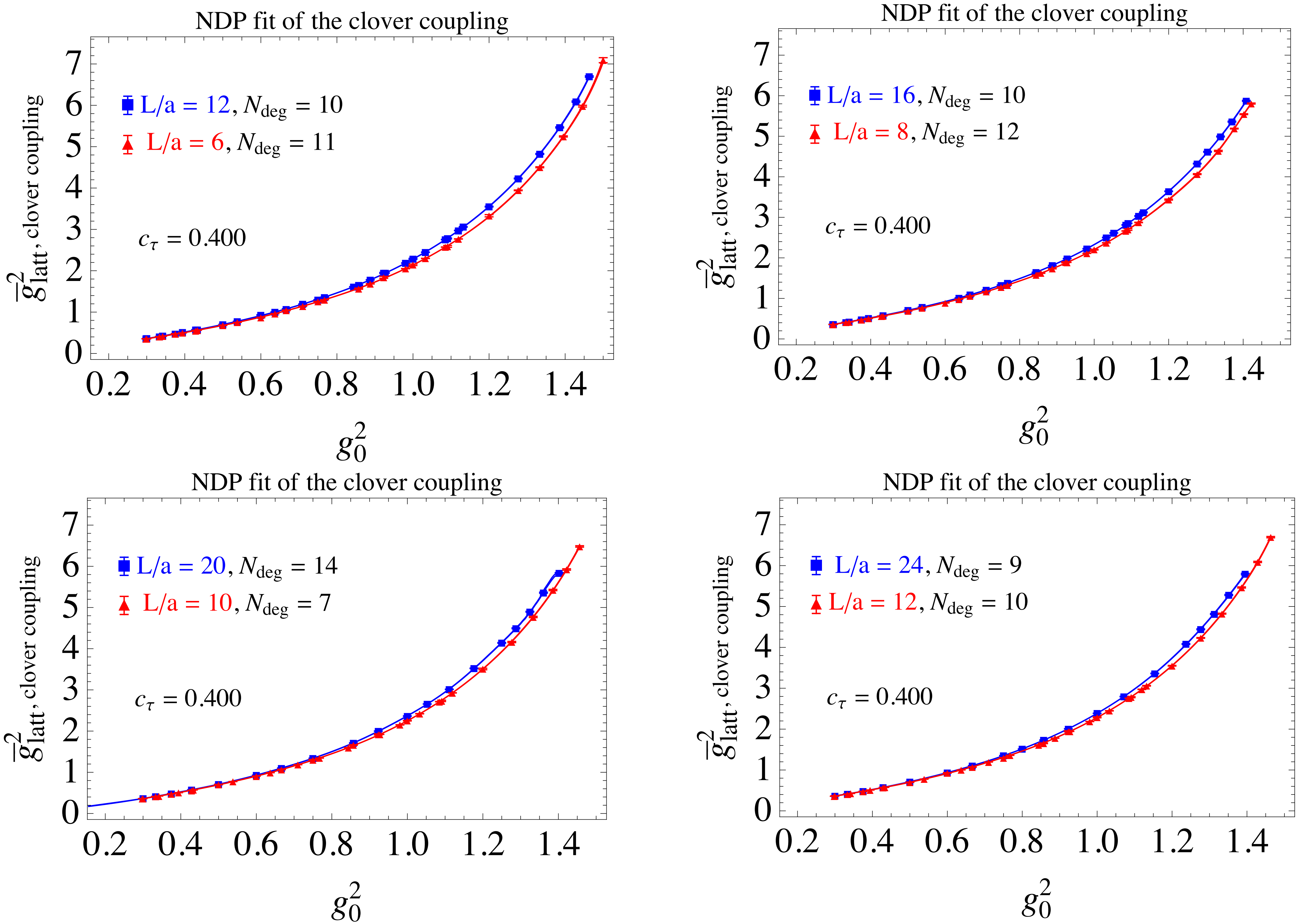
  with tree-level improvement.

- Use the clover operator, as well as the plaquette, to extract $\langle E(t) \rangle$.

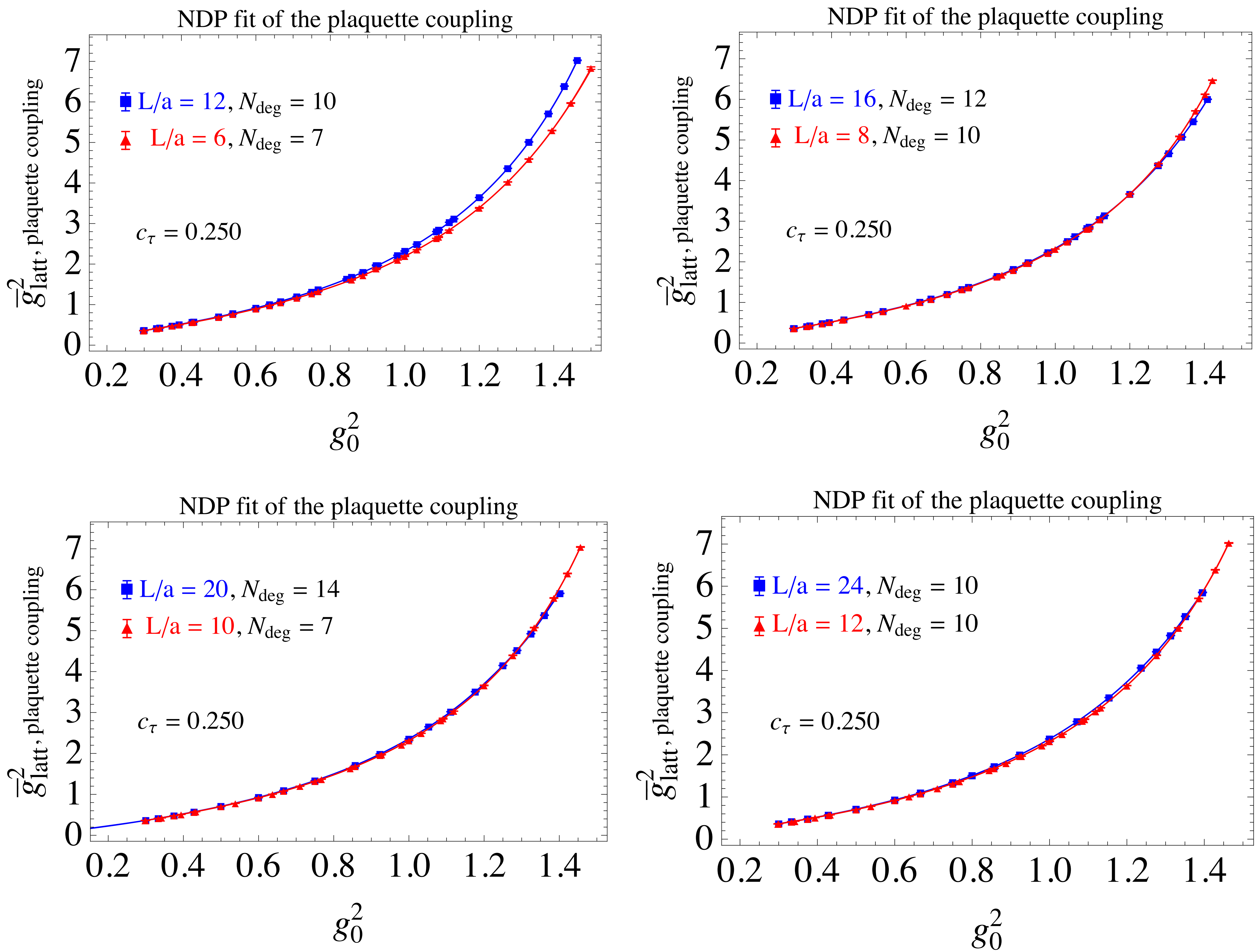
- Step scaling at fixed $c_\tau = \frac{\sqrt{8t}}{L}$.



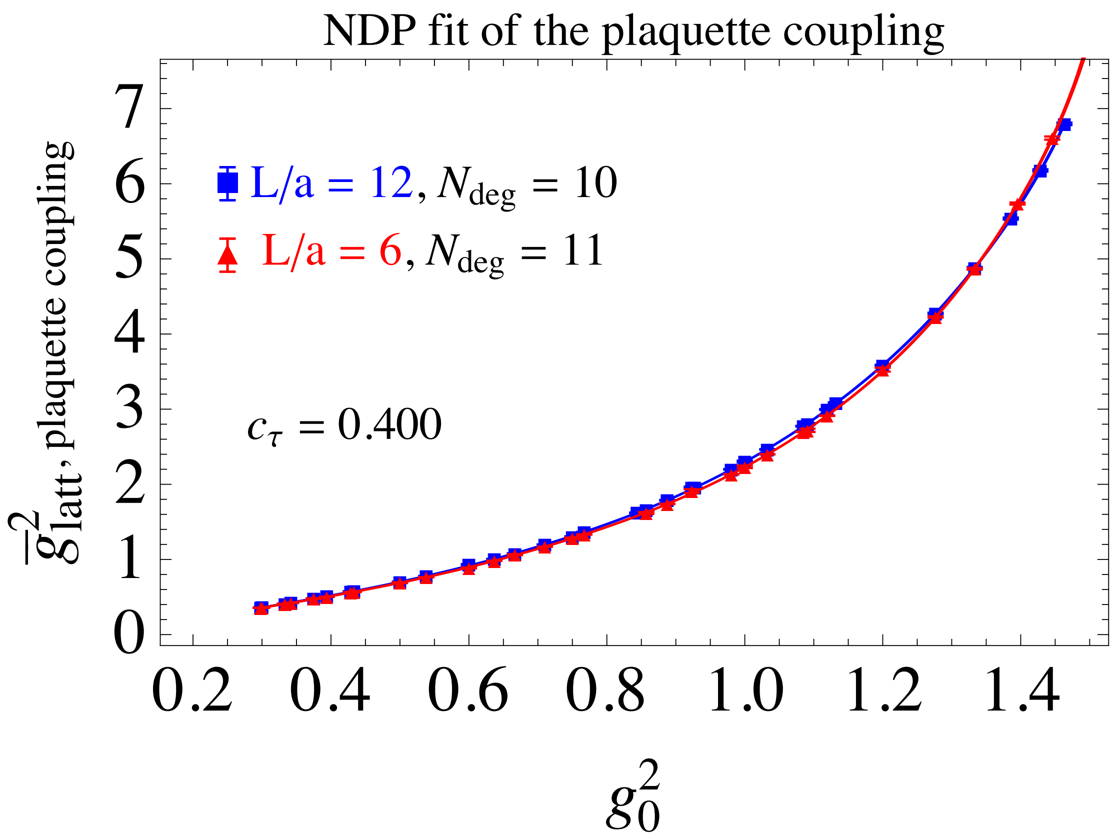
Bare-coupling interpolation

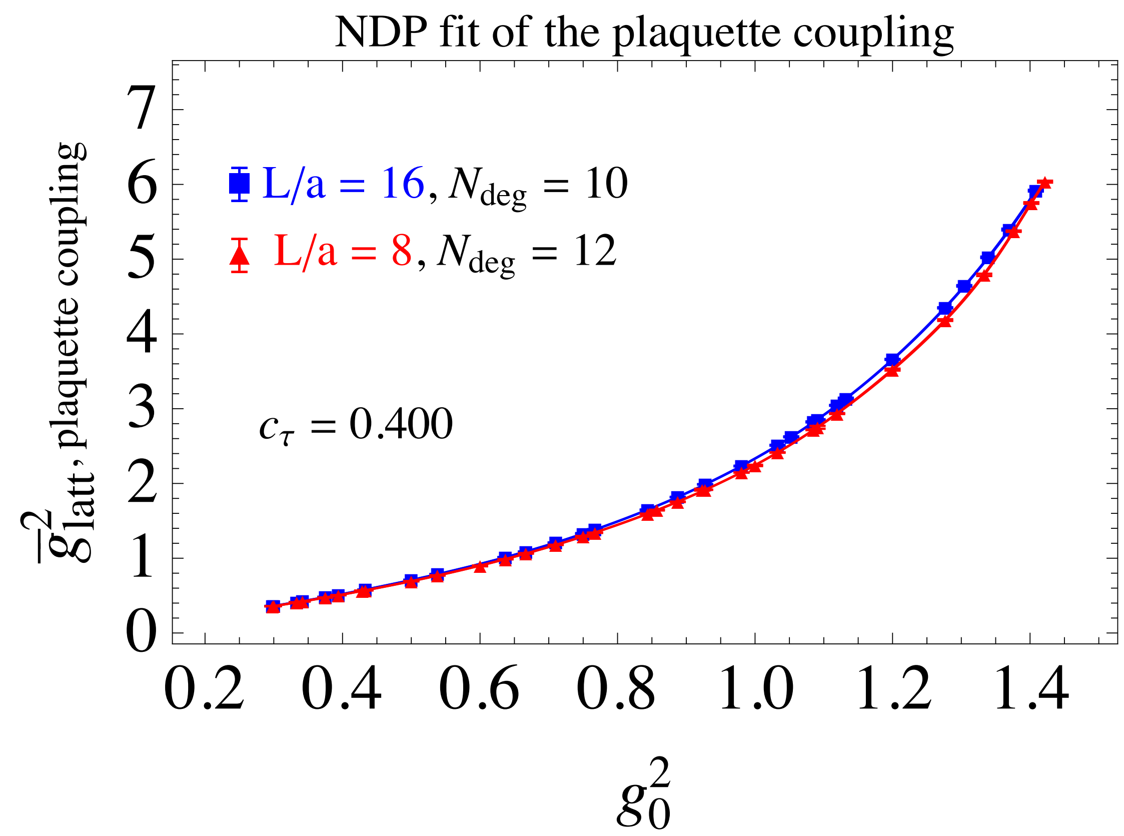


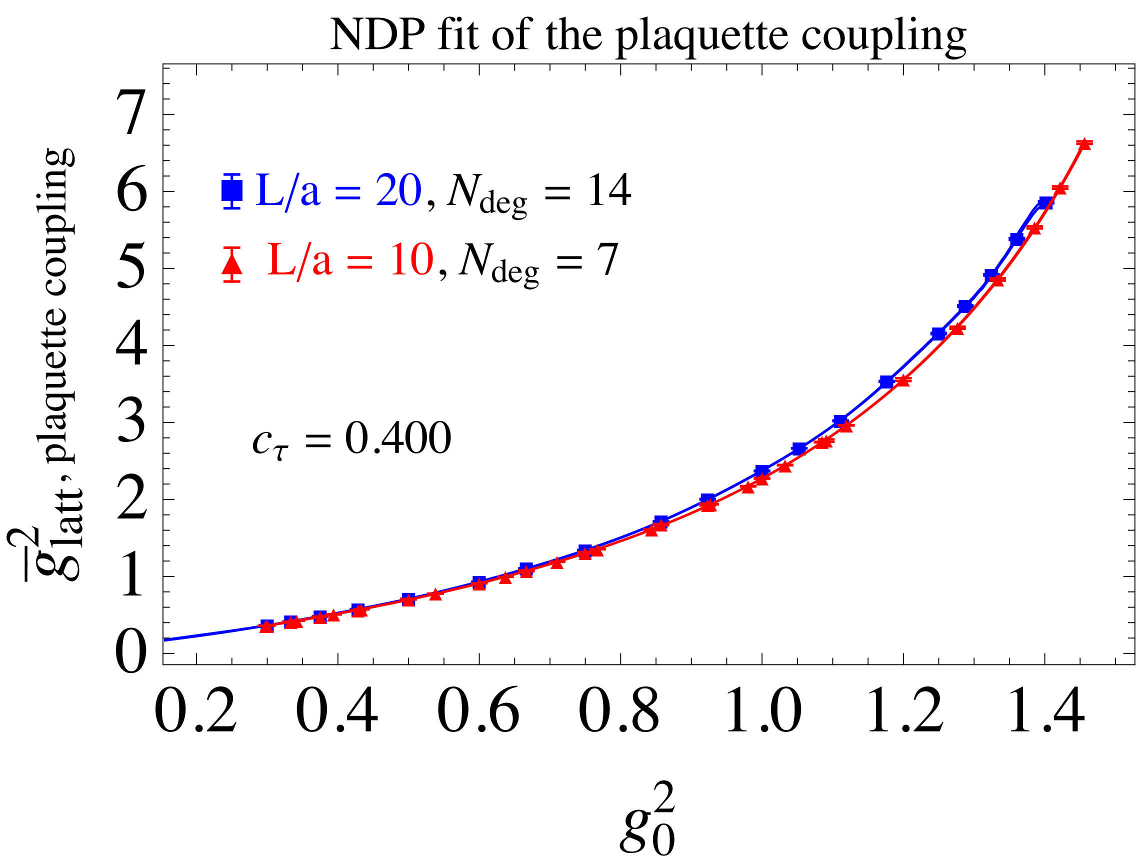
Bare-coupling interpolation

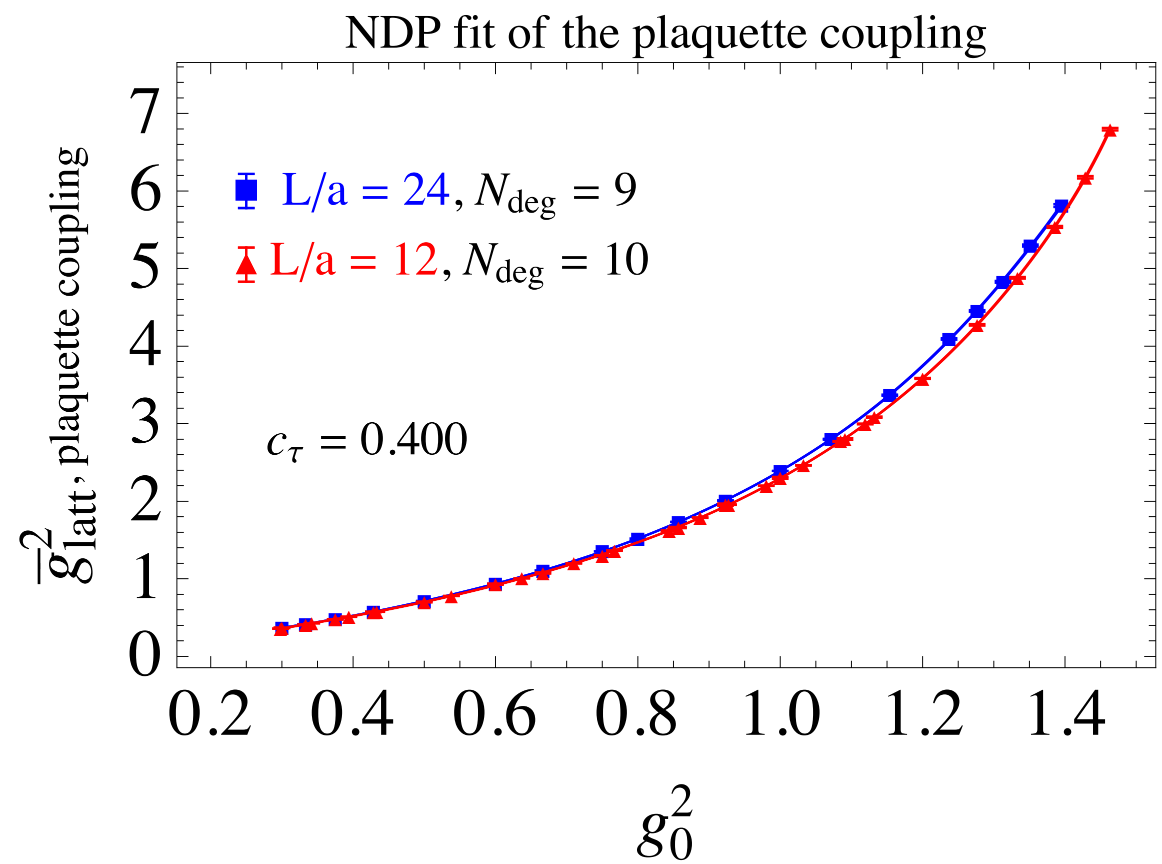


Bare-coupling interpolation

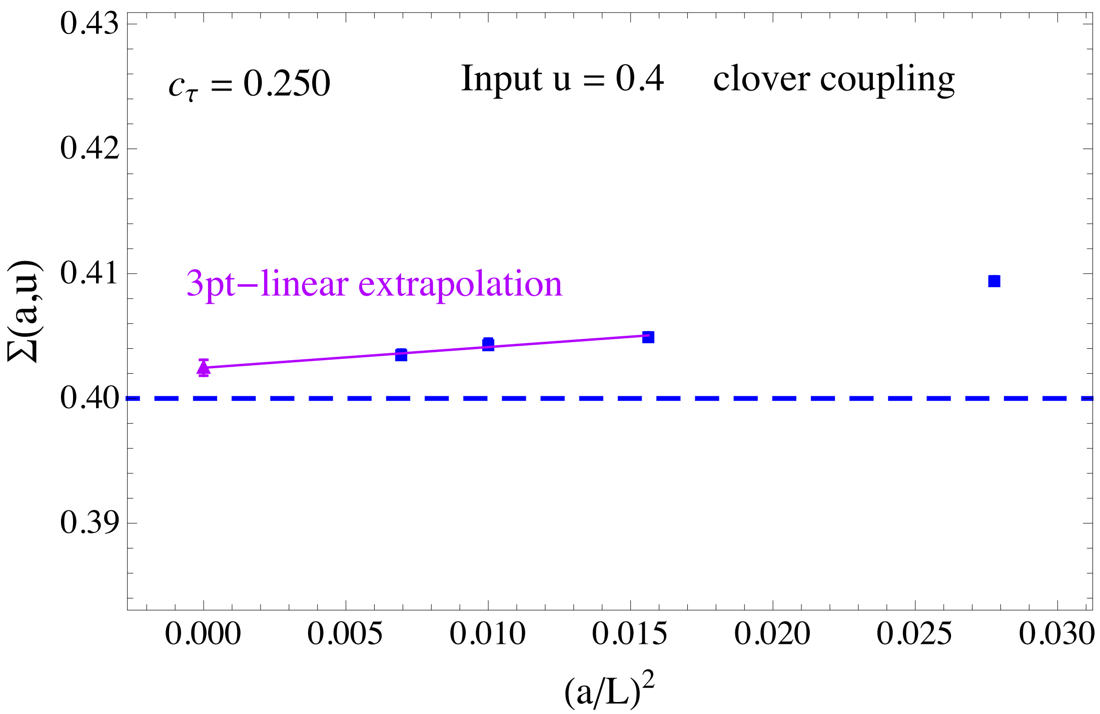


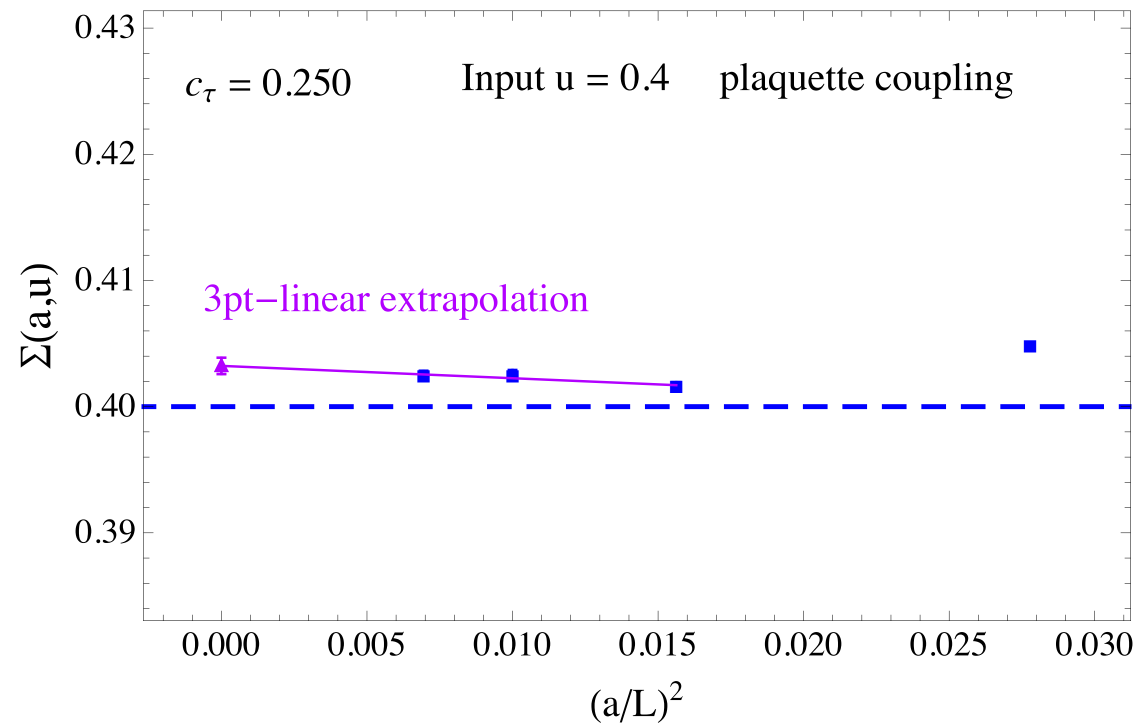


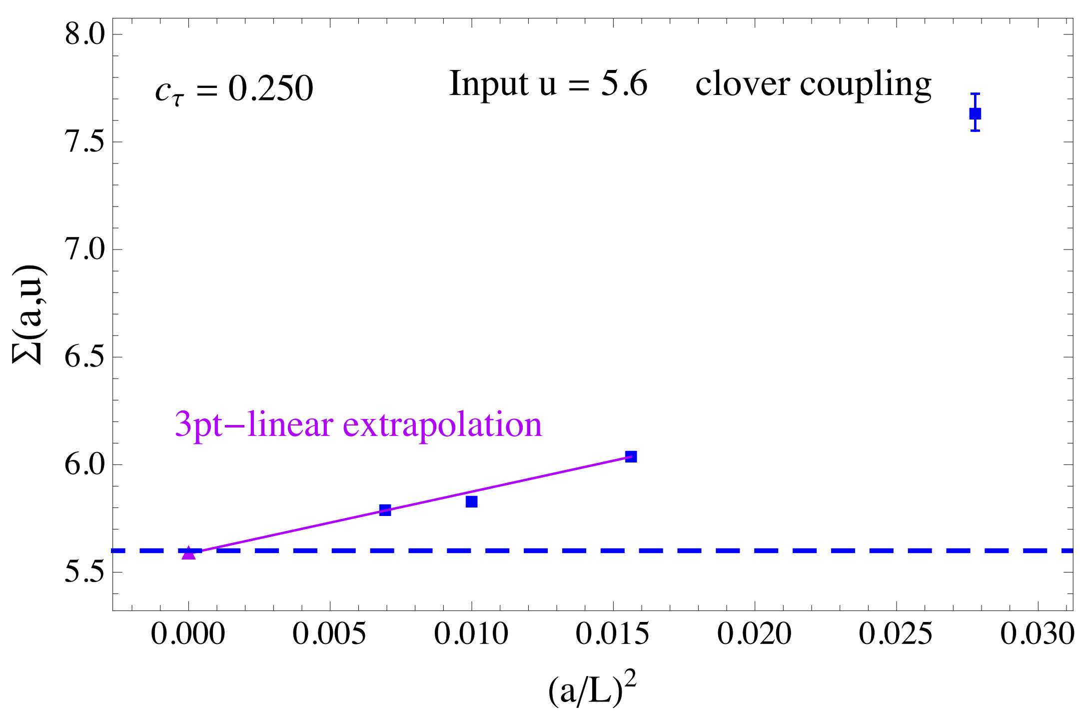


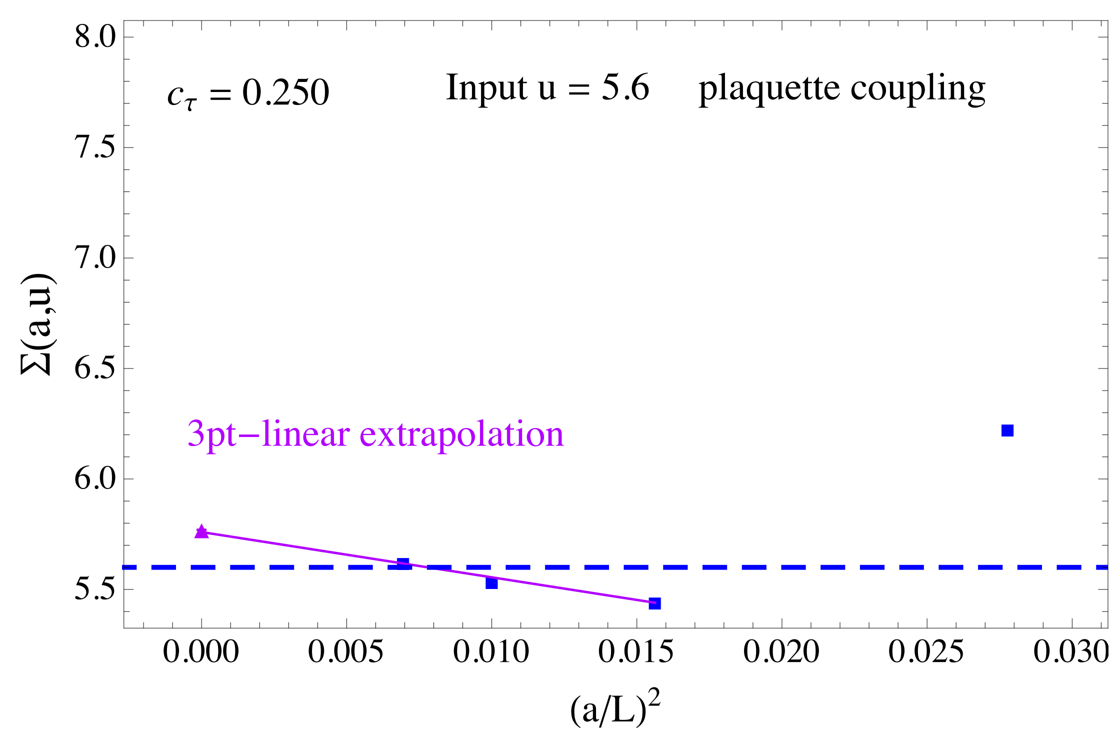


Continuum extrapolation

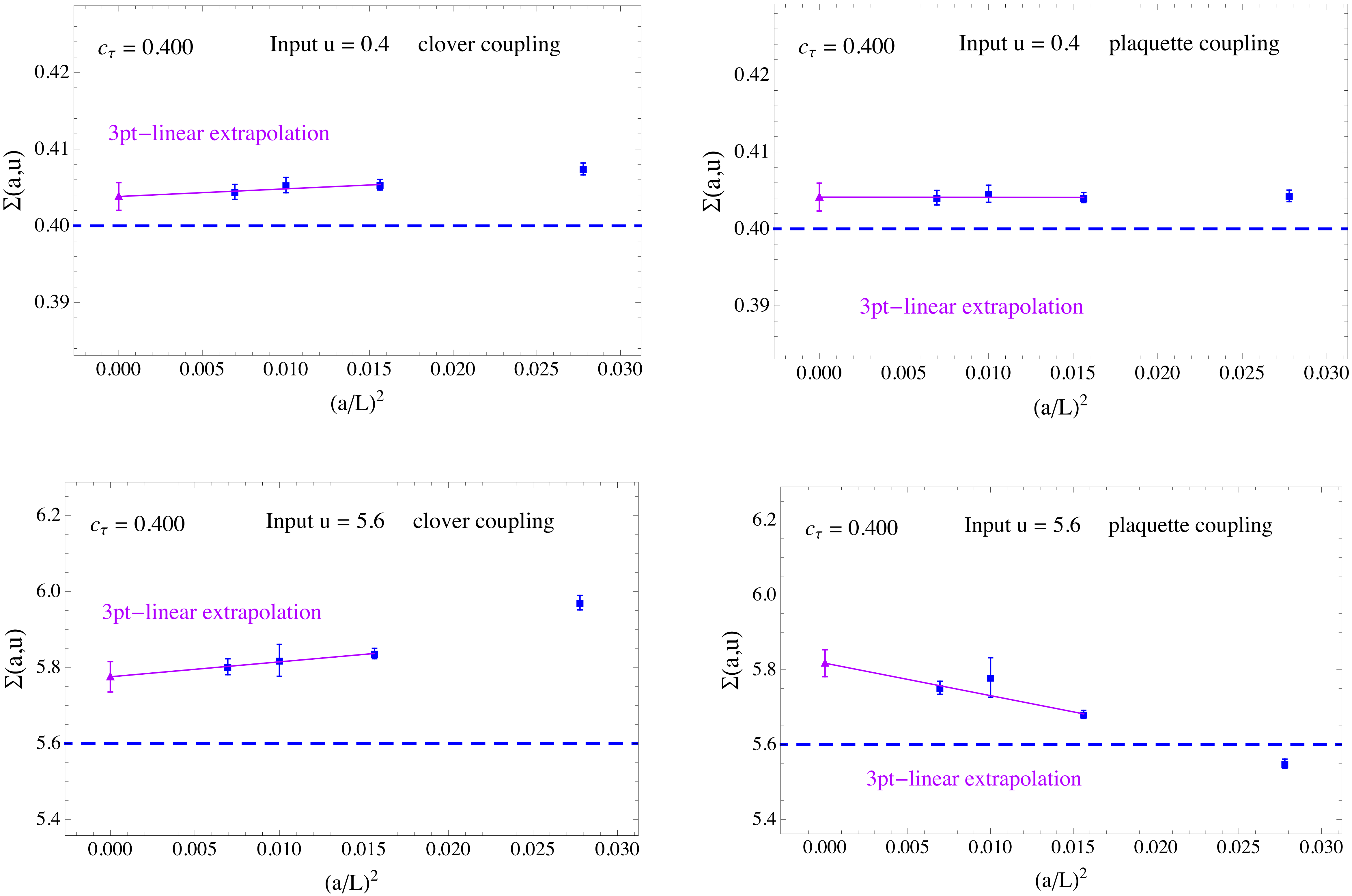




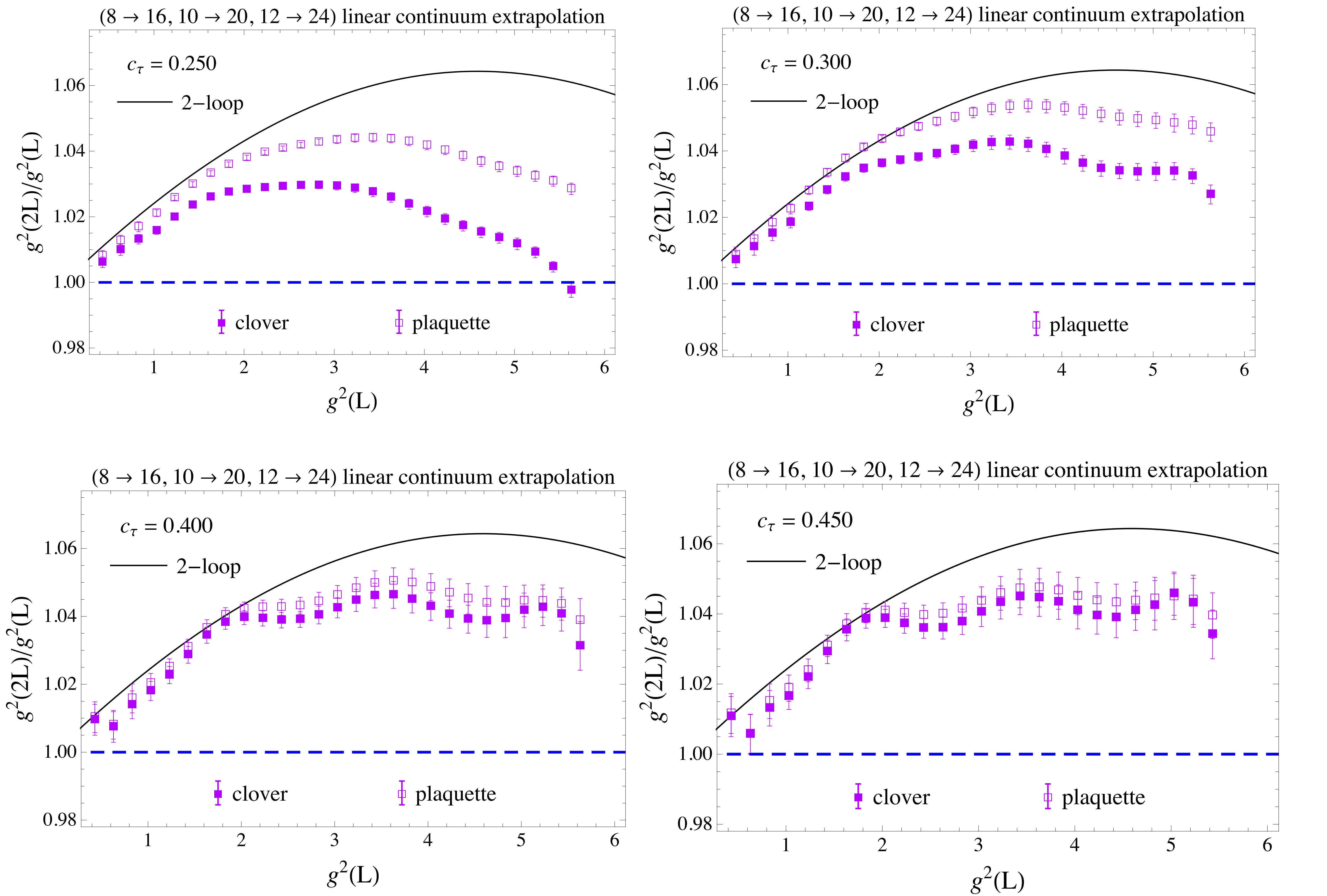




Continuum extrapolation



Results



(8 → 16, 10 → 20, 12 → 24) linear continuum extrapolation
$c_\tau = 0.250$
2−loop
clover
plaquette
$g^2(2L)/g^2(L)$
$g^2(L)$
(8 → 16, 10 → 20, 12 → 24) linear continuum extrapolation
$c_\tau = 0.300$
2−loop
clover
plaquette
$g^2(2L)/g^2(L)$
$g^2(L)$
(8 → 16, 10 → 20, 12 → 24) linear continuum extrapolation
$c_\tau = 0.400$
2−loop
clover
plaquette
$g^2(2L)/g^2(L)$
$g^2(L)$
(8 → 16, 10 → 20, 12 → 24) linear continuum extrapolation
$c_\tau = 0.450$
2−loop
clover
plaquette
$g^2(2L)/g^2(L)$
$g^2(L)$

Remarks and outlook

- Wilson Flow offers a very nice tool to perform the difficult task of the search for IRFP.
- In our work, we have to go to c ~ 0.4 to have the continuum extrapolation under control.
- From our work, it is still inclusive whether SU(3) gauge theory is QCD-like or conformal in the IR, although the running is very slow.
- Better data on the way...