SU(3) gauge theory with 12 flavours in a twisted box

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Strategy/challenges for the lattice search of IRFP

- Spectrum: Large finite-volume effects.
- \textbf{Running coupling}: (slow) running within error.

![Graph 1: \(g^2(L)\) vs. \(g^2(L)\)]

![Graph 2: \(g^2(2L)\) vs. \(g^2(L)\)]

- 2-loop, SU(3) with 12 flavours
The Twisted Polyakov Loop scheme

without (L/a=12 $\rightarrow$ L/a = 24)

systematics severely underestimated...

It is challenging to draw any conclusion from such a “noisy scheme”.


K.Ogawa, lattice 2013
Outline

- Step scaling.
- Twisted box.
- Wilson flow and numerical results.
- Outlook.
The step-scaling method
The practice

- Massless unimproved staggered fermions with Wilson’s plaquette gauge action.
- Compute $g^2_{\text{lat}}$ at many $g_0^2$ for each volume, and then interpolate volume by volume.
- Very challenging to pin down percentage-level effects in $r_\sigma = \frac{\sigma(u)}{u}$.

\[ \sigma(u) = \lim_{L \to \infty} \sum_{u} \sigma(u, L) \]

\[ g^2_{\text{lat}}(g_0, L) \text{ is given by interpolation} \]

\[ g^2_{\text{lat}}(g_0, L) = \begin{cases} 
  g^2_{12} & L = 12 \\
  g^2_{10} & L = 10 \\
  g^2_{8} & L = 8 \\
  g^2_{6} & L = 6 
\end{cases} \]

\[ g^2_{\text{lat}}(g_0', L) = \begin{cases} 
  g^2_{12} & L = 24 \\
  g^2_{10} & L = 20 \\
  g^2_{8} & L = 16 \\
  g^2_{6} & L = 12 
\end{cases} \]
Twisted box
removing the torons, no odd powers in g.

- **Gauge field:**
  \[ U_\mu(x + \hat{v} L) = \Omega_\nu U_\mu(x) \Omega_\nu^\dagger, \; \nu = 1, 2, \]
  where the twist matrices \( \Omega_\nu \) satisfy
  \[ \Omega_1 \Omega_2 = e^{2i\pi/3} \Omega_2 \Omega_1, \; \Omega_\mu \Omega_\mu^\dagger = 1, \; (\Omega_\mu)^3 = 1, \; \text{Tr}(\Omega_\mu) = 0. \]

- **Fermion:** If \( \psi(x + \hat{v} L) = \Omega_\nu \psi(x) \)
  \[ \Rightarrow \psi(x + \hat{v} L + \hat{\rho} L) = \Omega_\rho \Omega_\nu \psi(x) \neq \Omega_\nu \Omega_\rho \psi(x) \]

- **The fermion “smell” dof:** \( N_s = N_c \)
  \[ \psi^a_\alpha(x + \hat{v} L) = e^{i\pi/3} \Omega^a_\nu \psi^b_\beta(x) (\Omega_\nu)^\dagger_{\beta\alpha}. \]

G.‘t Hooft, 1979
G. Parisi, 1983
The Gradient Flow scheme

- The quantity, \( \langle E(t) \rangle = \frac{1}{4} \langle G_{\mu \nu}(t) G_{\mu \nu}(t) \rangle \), is finite when expressed in terms of renormalised coupling at positive flow time.
- In a colour-twisted box, can define,

\[
\bar{g}_{GF}^2(L) = \mathcal{N}^{-1} t^2 \langle E(t) \rangle = \bar{g}_{MS}^2 + \mathcal{O}(\bar{g}_{MS}^4),
\]

with tree-level improvement.
- Use the clover operator, as well as the plaquette, to extract \( \langle E(t) \rangle \).
- Step scaling at fixed \( c_\tau = \frac{\sqrt{8t}}{L} \).
Bare-coupling interpolation

NDP fit of the clover coupling

$\bar{g}_{\text{latt}}^2$, clover coupling

- $L/a = 12$, $N_{\text{deg}} = 10$
- $L/a = 6$, $N_{\text{deg}} = 11$

$c_{\tau} = 0.400$

NDP fit of the clover coupling

$\bar{g}_{\text{latt}}^2$, clover coupling

- $L/a = 16$, $N_{\text{deg}} = 10$
- $L/a = 8$, $N_{\text{deg}} = 12$

$c_{\tau} = 0.400$

NDP fit of the clover coupling

$\bar{g}_{\text{latt}}^2$, clover coupling

- $L/a = 20$, $N_{\text{deg}} = 14$
- $L/a = 10$, $N_{\text{deg}} = 7$

$c_{\tau} = 0.400$

NDP fit of the clover coupling

$\bar{g}_{\text{latt}}^2$, clover coupling

- $L/a = 24$, $N_{\text{deg}} = 9$
- $L/a = 12$, $N_{\text{deg}} = 10$

$c_{\tau} = 0.400$
Bare-coupling interpolation

NDP fit of the plaquette coupling

\[ \frac{1}{2} g_{\text{latt}}^2, \text{plaquette coupling} \]

- \[ L/a = 12, N_{\text{deg}} = 10 \]
- \[ L/a = 6, N_{\text{deg}} = 7 \]

\[ c_T = 0.250 \]

NDP fit of the plaquette coupling

\[ \frac{1}{2} g_{\text{latt}}^2, \text{plaquette coupling} \]

- \[ L/a = 16, N_{\text{deg}} = 12 \]
- \[ L/a = 8, N_{\text{deg}} = 10 \]

\[ c_T = 0.250 \]

NDP fit of the plaquette coupling

\[ \frac{1}{2} g_{\text{latt}}^2, \text{plaquette coupling} \]

- \[ L/a = 20, N_{\text{deg}} = 14 \]
- \[ L/a = 10, N_{\text{deg}} = 7 \]

\[ c_T = 0.250 \]

NDP fit of the plaquette coupling

\[ \frac{1}{2} g_{\text{latt}}^2, \text{plaquette coupling} \]

- \[ L/a = 24, N_{\text{deg}} = 10 \]
- \[ L/a = 12, N_{\text{deg}} = 10 \]

\[ c_T = 0.250 \]
Bare-coupling interpolation

NDP fit of the plaquette coupling

- $L/a = 12$, $N_{\text{deg}} = 10$
- $L/a = 6$, $N_{\text{deg}} = 11$

$c_r = 0.400$

NDP fit of the plaquette coupling

- $L/a = 16$, $N_{\text{deg}} = 10$
- $L/a = 8$, $N_{\text{deg}} = 12$

$c_r = 0.400$

NDP fit of the plaquette coupling

- $L/a = 20$, $N_{\text{deg}} = 14$
- $L/a = 10$, $N_{\text{deg}} = 7$

$c_r = 0.400$

NDP fit of the plaquette coupling

- $L/a = 24$, $N_{\text{deg}} = 9$
- $L/a = 12$, $N_{\text{deg}} = 10$

$c_r = 0.400$
Continuum extrapolation

$c_\tau = 0.250$  
Input $u = 0.4$  
clover coupling

$c_\tau = 0.250$  
Input $u = 5.6$  
clover coupling

$c_\tau = 0.250$  
Input $u = 0.4$  
plaquette coupling

$c_\tau = 0.250$  
Input $u = 5.6$  
plaquette coupling
Results

$g^2(L)/g^2(L)$ vs $g^2(L)$ for $(8 \to 16, 10 \to 20, 12 \to 24)$ linear continuum extrapolation

- $c_\tau = 0.250$
- 2-loop

- clover
- plaquette

$g^2(2L)/g^2(L)$ vs $g^2(L)$ for $(8 \to 16, 10 \to 20, 12 \to 24)$ linear continuum extrapolation

- $c_\tau = 0.300$
- 2-loop

- clover
- plaquette

$g^2(2L)/g^2(L)$ vs $g^2(L)$ for $(8 \to 16, 10 \to 20, 12 \to 24)$ linear continuum extrapolation

- $c_\tau = 0.400$
- 2-loop

- clover
- plaquette

$g^2(2L)/g^2(L)$ vs $g^2(L)$ for $(8 \to 16, 10 \to 20, 12 \to 24)$ linear continuum extrapolation

- $c_\tau = 0.450$
- 2-loop

- clover
- plaquette
Remarks and outlook

• Wilson Flow offers a very nice tool to perform the difficult task of the search for IRFP.

• In our work, we have to go to $c \sim 0.4$ to have the continuum extrapolation under control.

• From our work, it is still inclusive whether SU(3) gauge theory is QCD-like or conformal in the IR, although the running is very slow.

• Better data on the way...