

The gradient flow running coupling in $SU(2)$ with 8 flavors

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- A class of gauge theories has a conformal fixed point
- Technicolor requires (nearly) conformal field theories
- Direct lattice calculations of the β -function expensive
 - High accuracy required to distinguish the slow running
 - Large coupling, massless fermions
- Running coupling from gradient flow a promising method
- SU(2) gauge field theory 8 fermions¹
 - Expected to have a fixed point at large coupling

¹H. Ohki, et al., PoS LATTICE **2010**, 066 (2010), [arXiv:1011.0373 [hep-lat]]

The Model

$$S = (1 - c_g)S_G(U) + c_g S_G(V) + S_F(V) + c_{SW} \delta S_{SW}(V)$$

- HEX Smearing:²
 - three sequential stout smearing steps using only orthogonal directions
- Standard gauge action $S_G(U)$
- Smeared Wilson fermion action $S_F(V)$
- Smeared gauge action $S_G(V)$
- Bulk correction term $\delta S_{SW}(U)$
- Here, $c_g = 0.5$ and $c_{SW} = 1$

²S. Capitani, S. Durr and C. Hoelbling, JHEP **0611** (2006) 028

The Model

- Schrödinger Functional boundary conditions:
 - Mass anomalous dimension from the same dataset.
- Trivial time boundaries, periodic spatial boundary conditions

$$U_k(x) = 1, \text{ when } x_0 = 0, L$$

$$U_\mu(x + L\hat{k}) = U_\mu(x)$$

$$\psi(x) = 0, \text{ when } x_0 = 0, L$$

$$\psi(x + L\hat{k}) = \psi(x)$$

Gradient Flow

- Flow along the gradient of the gauge action ³

$$\partial_t B_{t,\mu} = D_{t,\mu} B_{t,\mu\nu},$$

$$B_{0,\mu} = A_\mu$$

$$B_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}]$$

- Correlators of the flow field renormalized, encode physical properties
- A new scale $\sqrt{8t}$

³M. Luscher and P. Weisz, JHEP **1102** (2011) 051 [arXiv:1101.0963 [hep-th]]

Gradient Flow

- The field strength:

$$\begin{aligned}\langle E(t) \rangle &= \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle \\ &= \frac{3(N^2 - 1) g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4)\end{aligned}$$

- Define a renormalized coupling ⁴

$$g_{GF}^2 = \frac{t^2 \langle E(t) \rangle}{N}$$

⁴Z. Fodor *et al.* JHEP **1211** (2012) 007 [arXiv:1208.1051]

Z. Fodor *et al.* PoS LATTICE **2012** (2012) 050 [arXiv:1211.3247]

P. Fritzsch and A. Ramos, arXiv:1301.4388 [hep-lat]

Gradient Flow

- Gradient flow from the Symanzik action

$$P^{11} \rightarrow \frac{5}{3}P^{11} - \frac{1}{20}(P^{12} + P^{21})$$

$$P^{11} = \square, P^{12} = \square \uparrow$$

- The measurable E

$$G = (1 - c_E) \frac{1}{4} C^{11} + c_E \frac{1}{16} (C^{12} + C^{21}),$$

$$C^{11} = \begin{array}{c} \square \uparrow \\ \square \downarrow \end{array}, \quad C^{12} = \begin{array}{c} \square \uparrow \\ \square \downarrow \end{array}$$

$$c_E = -2/3$$

Gradient Flow

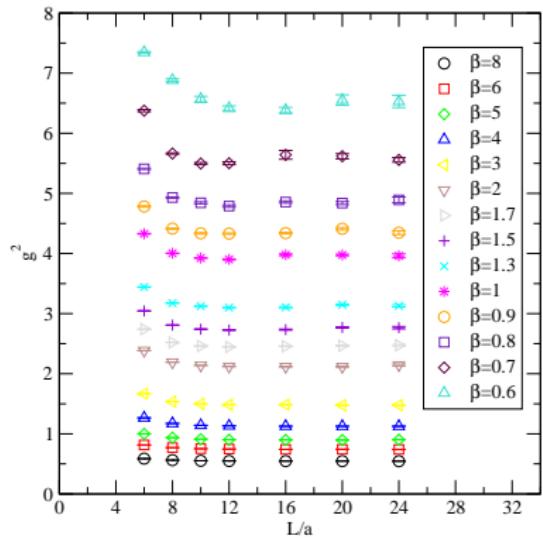
- Fixed boundaries
Use the middle time slice

$$Ng_{GF}^2 = t^2 \langle E(t, x_0) \rangle$$

$$x_0 = L/2, t = c_t^2 L^2 / 8$$

$$c_t = 0.4$$

- $N(a/L)$ is unknown,
measure it at $\beta = 80$,
 $\kappa = 0.125$



Gradient Flow

- Measure the running by the step scaling function

$$\Sigma(u, a/L) = g_{GF}^2(g_0, 2L/a)/u \Big|_{g_{GF}^2(g_0, L/a)=u}$$

$$\sigma(u) = \lim_{a \rightarrow 0} \Sigma(u, a/L)$$

- For step scaling, define coupling with a single scale
- Fix the flow time t to the lattice size L

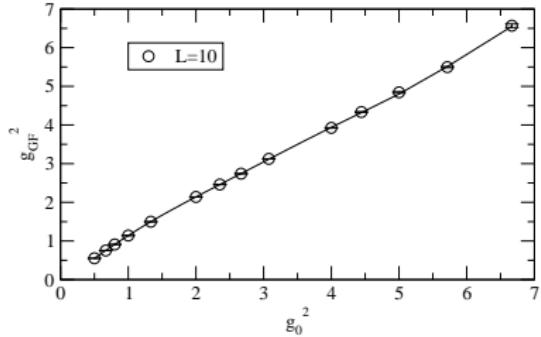
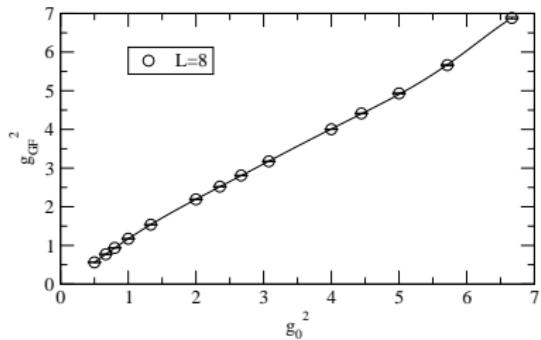
$$\sqrt{8t} = c_t L$$

$$c_t = 0.4$$

Gradient Flow

- For step scaling
interpolate

$$g_{GF}^2(g_0, a/L) = g_0^2 \left[1 + \sum_{i=1}^7 a_i g_0^{2i} \right]$$



Improved Continuum Limit

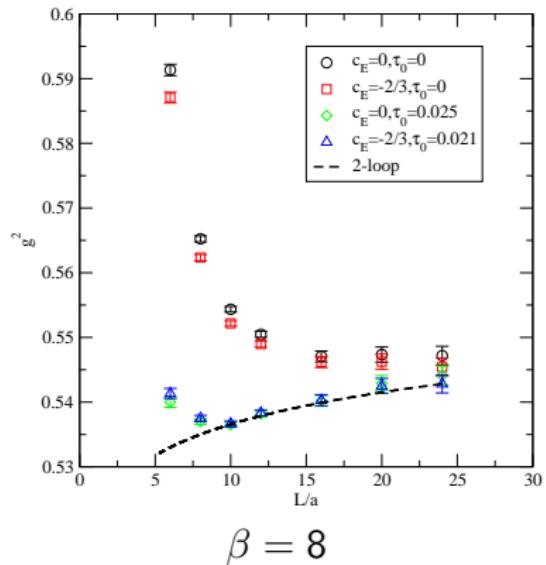
- Correction to flow time ⁵

$$E(t) \rightarrow E(t + \tau_0 a^2)$$

$$g_{GF}^2 = \frac{t^2}{N} \langle E(t) \rangle$$

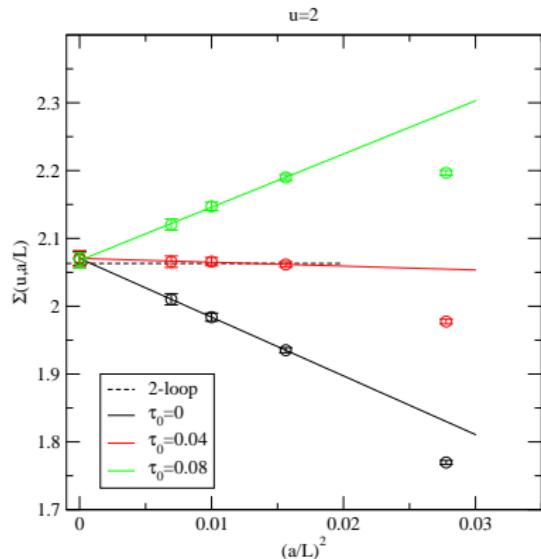
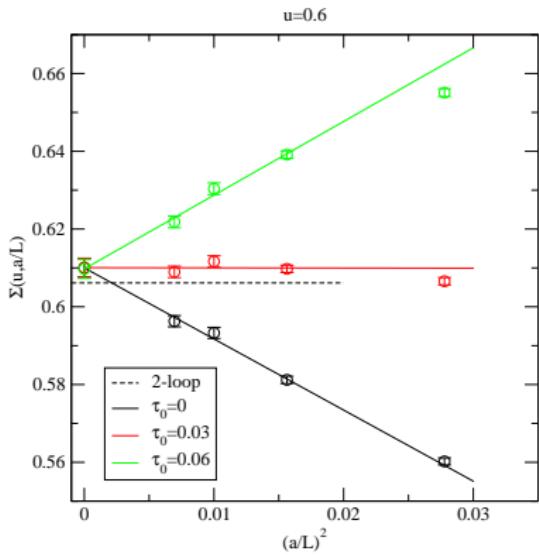
$$+ \frac{t^2}{N} \left\langle \frac{\partial E(t)}{\partial t} \right\rangle \tau_0 a^2$$

- A large correction
- Easily changed without additional cost



⁵A. Cheng, A. Hasenfratz, Y. Liu, G. Petropoulos and D. Schaich, JHEP **1405**, 137 (2014) [arXiv:1404.0984 [hep-lat]]

Improved Continuum Limit

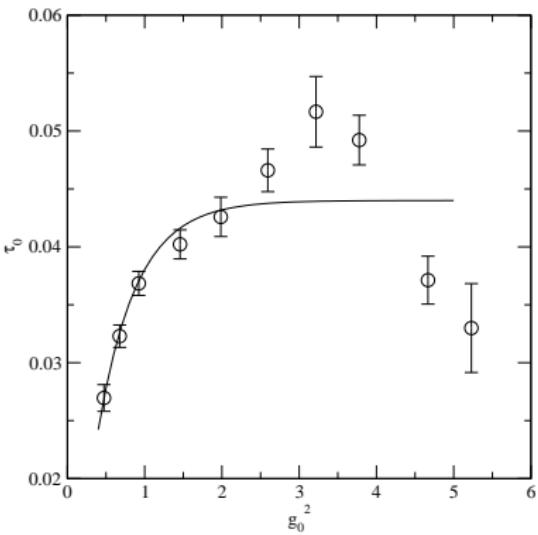


- Continuum limit with $\Sigma(u, a/L) = \sigma(u) + c(a/L)^2$

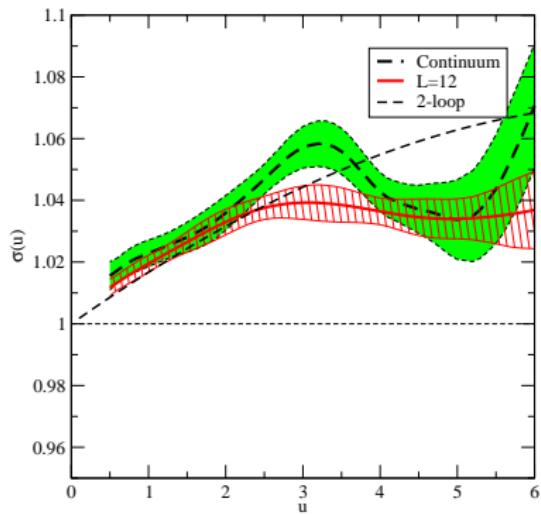
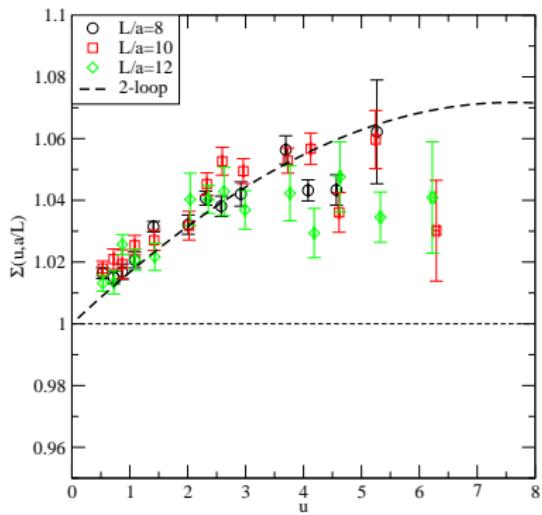
Improved Continuum Limit

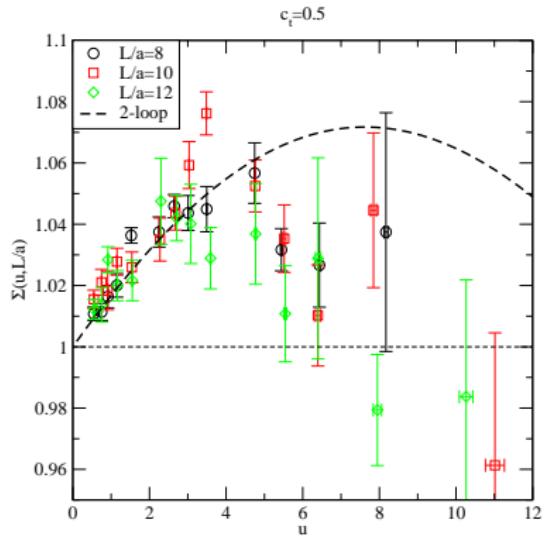
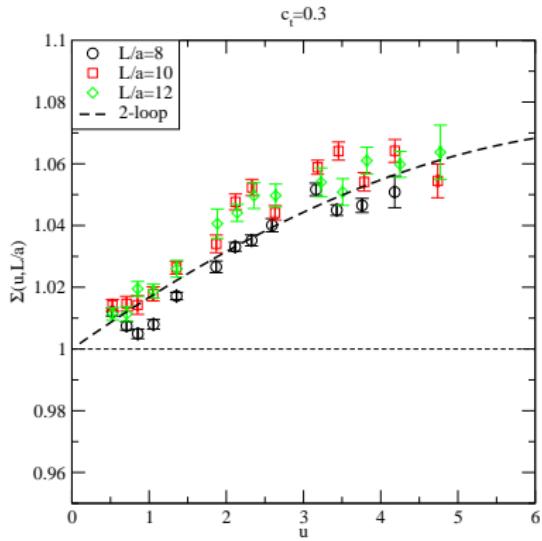
- Quickly sketching the correction
- Approximating with

$$\tau_0 = 0.044 \left(1 - e^{-2g_0^2} \right)$$



Improved Continuum Limit





$$c_t = 0.3 : \tau_0 = 0.04 \left(1 - e^{-2g_0^2} \right)$$

$$c_t = 0.5 : \tau_0 = 0.1 \left(1 - e^{-0.5g_0^2} \right)$$

Mass anomalous dimension

- The mass anomalous dimension γ_m can be measured from the pseudoscalar density renormalization constant

$$Z_P(g_0, L/a) = \frac{\sqrt{3f_1}}{f_P(L/2a)}$$

- Scales like $\frac{1}{\overline{m}(a/L)}$.

Mass anomalous dimension

$$\Sigma_P(u, a/L) = \left. \frac{Z_P(g_0, 2L/a)}{Z_P(g_0, L/a)} \right|_{g^2(g_0, L/a) = u}$$

$$\sigma_P(g^2) = \lim_{a \rightarrow 0} \Sigma_P(g^2, a/L)$$

Mass anomalous dimension

