Centre Vortex Effects on the Overlap Quark Propagator

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Overview

- The fundamental aspects of the QCD vacuum that are responsible for the dynamical generation of mass through chiral symmetry breaking and confinement are an ongoing source of debate
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- Overlap quark propagator on vortex-free and vortex-only backgrounds
    - Qualitatively different results to previous ASQTAD results
- Effects of cooling on vortex-only backgrounds

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Identifying Centre Vortices on the Lattice

- Transform to Maximal Centre Gauge, where links are brought close to centre elements

$$\begin{aligned} Z_\mu(x) &= z\mathrm{I}, \qquad z^3 = 1 \\ &= \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]\mathrm{I}, \quad m_\mu(x) \in \{-1, 0, 1\} \end{aligned} \tag{1}$$

- Require transformation $\Omega(x)$ maximising overlap between gauge links and centre elements

$$\sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\left[U_\mu^\Omega(x) Z_\mu^\dagger(x)\right] \to \mathrm{Max} \tag{2}$$

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- Implemented through 'mesonic' centre gauge fixing condition

$$R_{mes} = \sum_{x,\mu} |\mathrm{Tr}\, U_\mu^\Omega(x)|^2 \to \mathrm{Max} \tag{3}$$

- Then we project onto Z_3

$$\frac{1}{3}\mathrm{Tr}U_\mu^\Omega(x) = r_\mu(x)\exp(i\phi_\mu(x)) \tag{4}$$

Choose $m_\mu(x) \in \{-1, 0, 1\}$ with $\frac{2\pi m_\mu(x)}{3}$ closest to $\phi_\mu(x)$

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Simulation Details

- We use the overlap operator, which has a lattice-deformed version of chiral symmetry, leading to greater sensitivity to topological effects
- Results calculated on 50 $20^3 \times 40$ gauge-field configurations using Lušcher-Weisz $\mathcal{O}(a^2)$ mean-field improved action with a lattice spacing of $0.125\,\mathrm{fm}$

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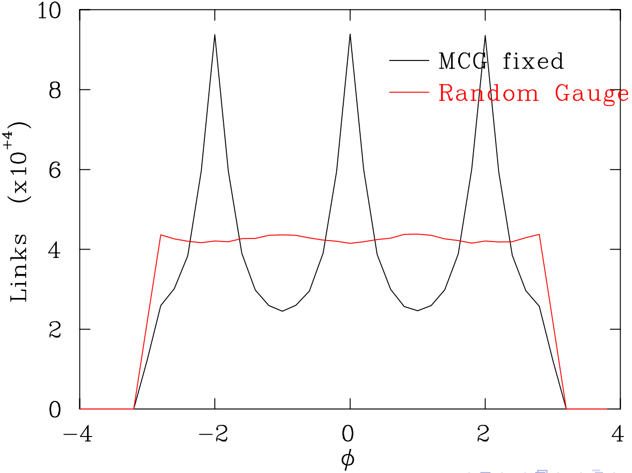
MCG-fixed phases



Identifying Centre Vortices on the Lattice

3 sets of configurations:

- Untouched configurations

$$U_\mu(x) \tag{5}$$

- Vortex-only configurations

$$Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right] \mathrm{I} \tag{6}$$

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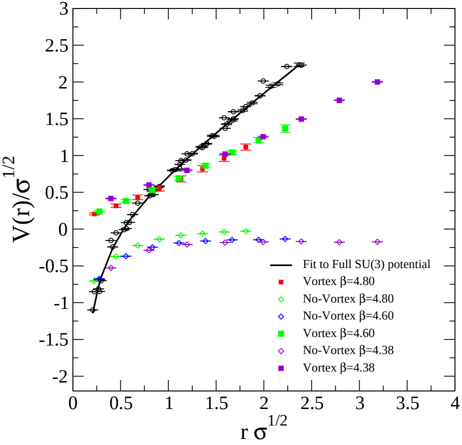
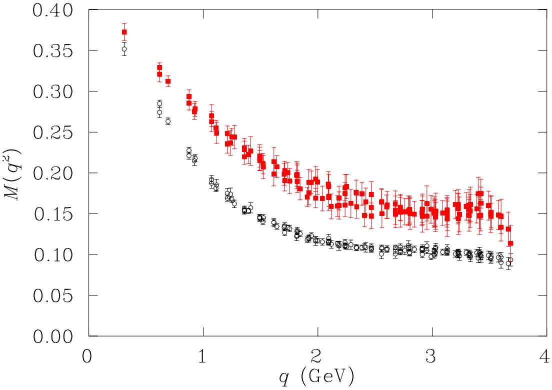
Centre Vortices and Confinement



From Bowman et al, Phys. Rev. D ***84****, 034501 (2011)*

Previous Results Using an ASQTAD action



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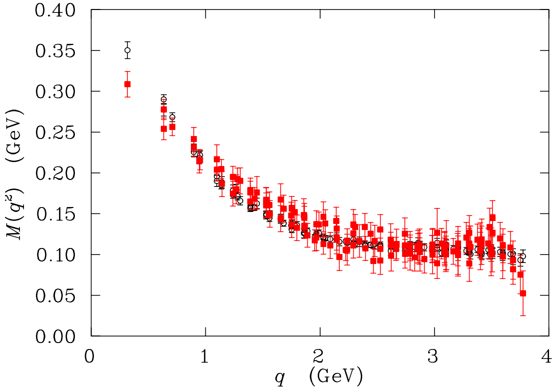

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Overlap Quark Propagator

- Write momentum-space propagator as

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}, \tag{8}$$

with q_μ the tree-level improved kinematic lattice momentum[1]

- Fixed to Landau gauge using a Fourier transform accelerated algorithm [2] to the $\mathcal{O}(a^2)$ improved gauge-fixing functional [3].

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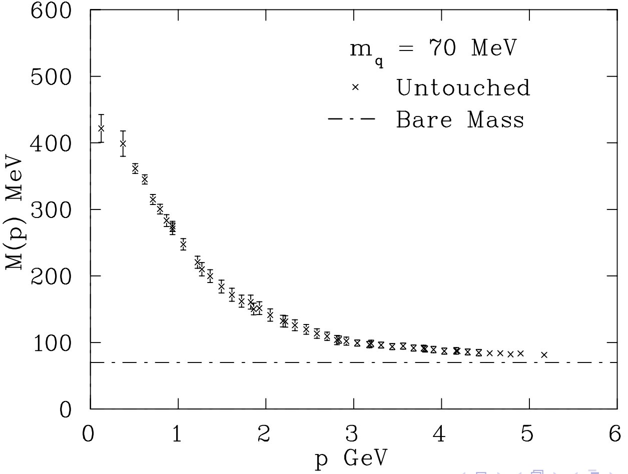
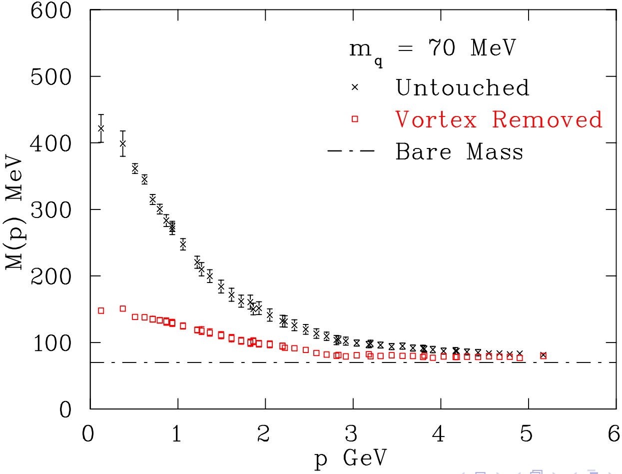
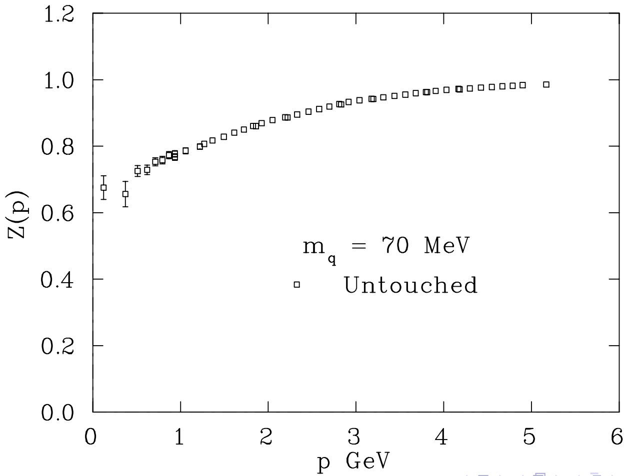
Mass function on Untouched Configurations



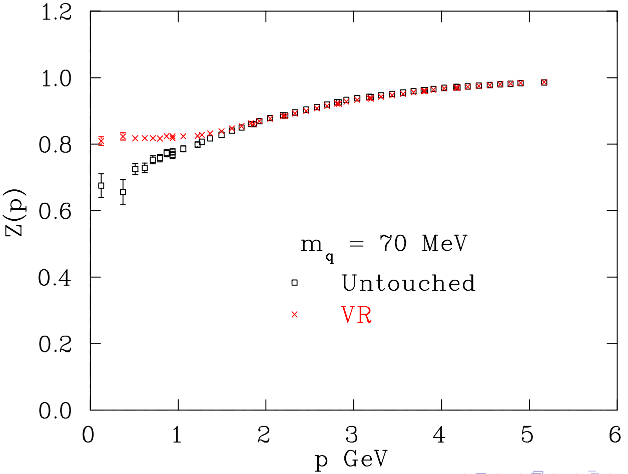
Mass function with Vortex Removed Configurations



Renormalization function on UT Configurations



Renormalization function with VR Configurations



Quark Propagator on Vortex Removed Configurations

- ASQTAD propagator unable to show loss of dynamical mass generation with vortex removal
- Overlap propagator shows loss of dynamical mass generation coincident with vortex removal
- Loss of confinement on vortex removed backgrounds using overlap

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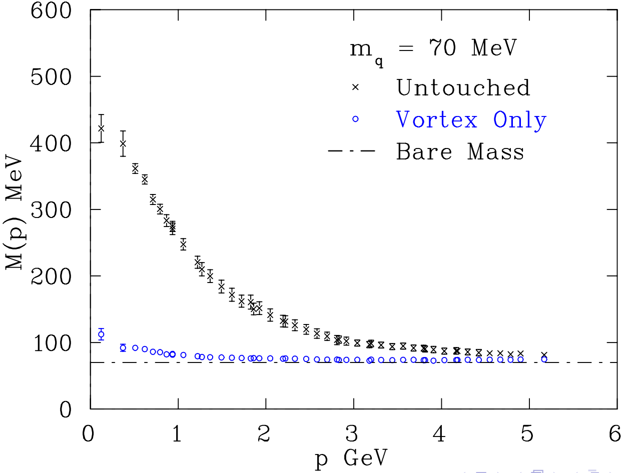
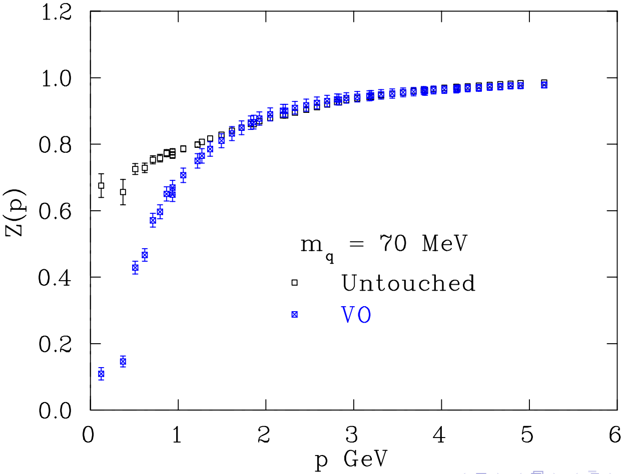
Mass function on Vortex Only Configurations



Renormalization function on VO Configurations



The story so far...

- Vortex-only backgrounds cannot reproduce dynamical mass generation
- Vortex-only backgrounds not trivial; evidence of confinement
- The question: what information about the original configurations do vortex-only configurations retain?

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Cooling

- Vortex-only configurations consist only of center elements
  $\Rightarrow$ high action
- We will perform cooling on vortex-only configurations
- Cooling is performed using an $\mathcal{O}(a^4)$-three-loop improved action, and the topological charge density is calculated using an $\mathcal{O}(a^4)$-five-loop improved definition of the field-strength tensor.

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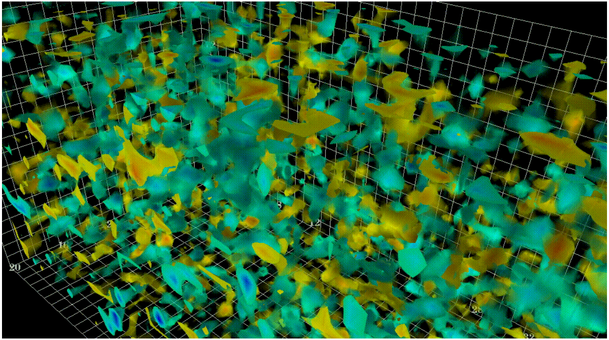
Untouched Configurations with Cooling



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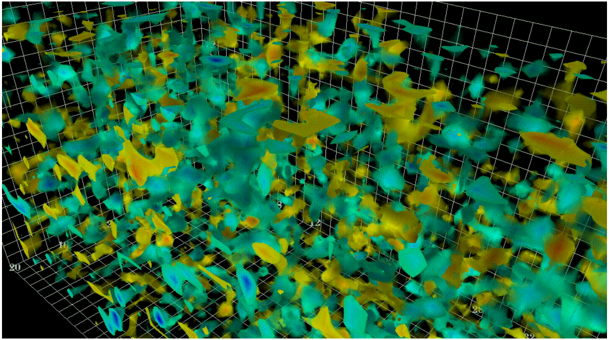
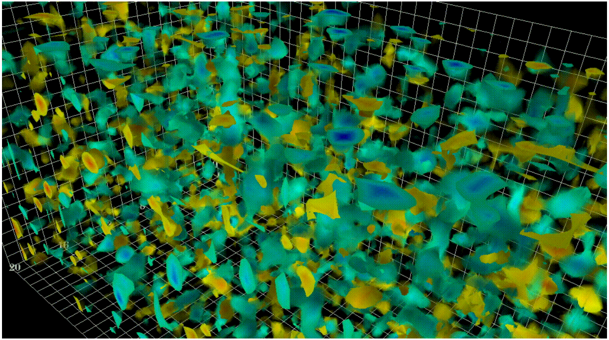


Vortex Only Configurations with Cooling



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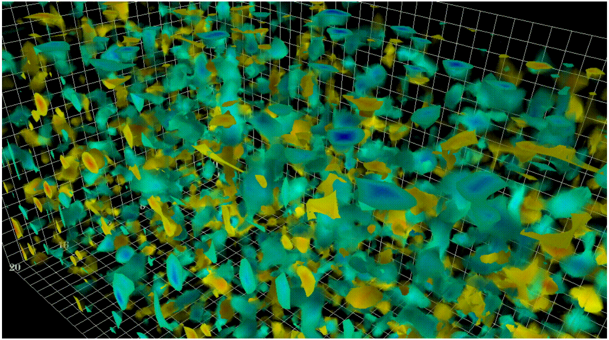
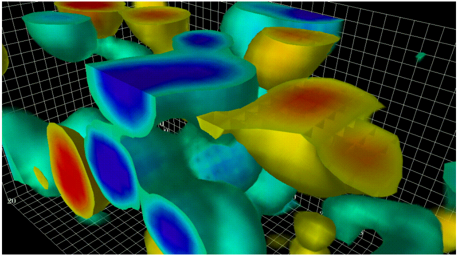
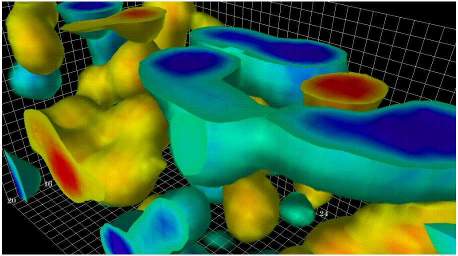


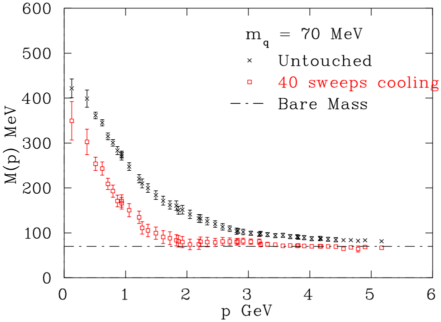
40 sweep comparison





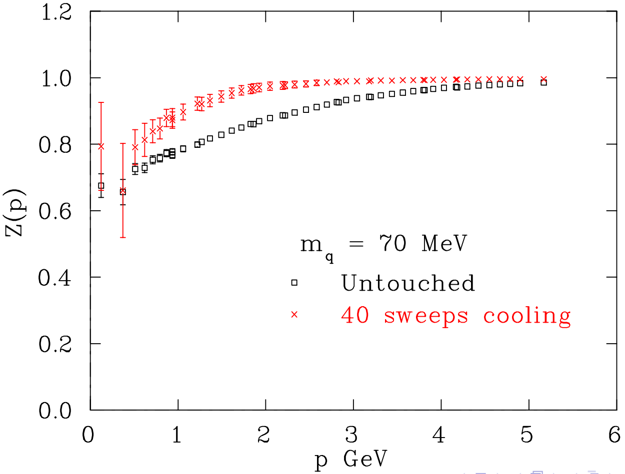
Mass function with cooling

- Under a UV filter, the overlap mass function retains its form qualitatively, with some loss of dynamical mass generation[1]



[1] *D. T, W. Kamleh, D. Leinweber and D. S. Roberts, Phys. Rev. D* **88**, *034501 (2013) [arXiv:1306.3283 [hep-lat]].*

Renormalization function with cooling



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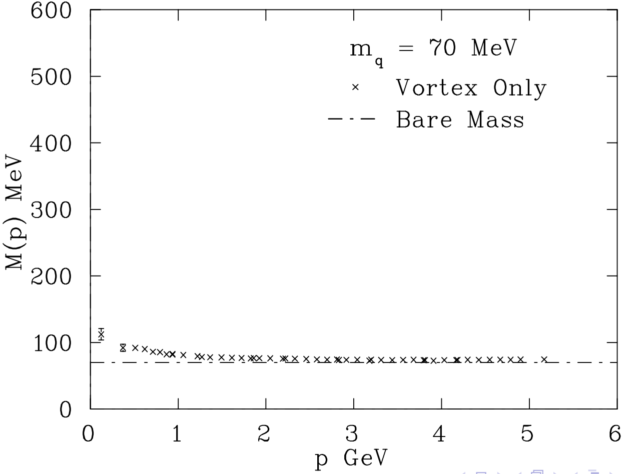

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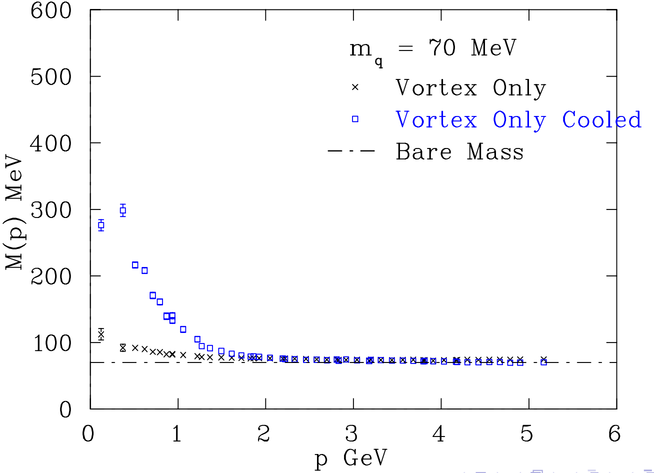

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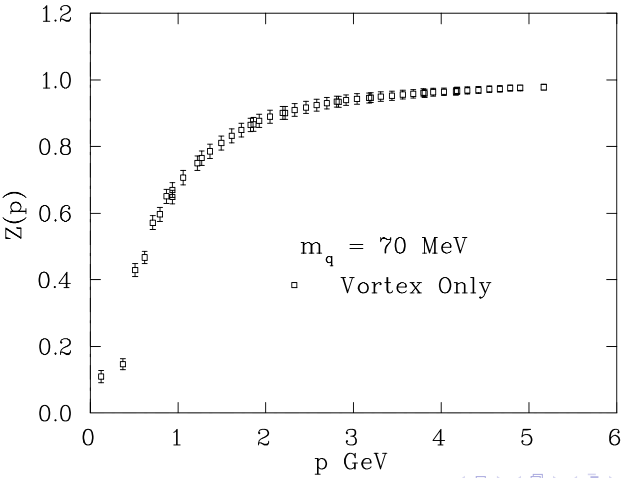

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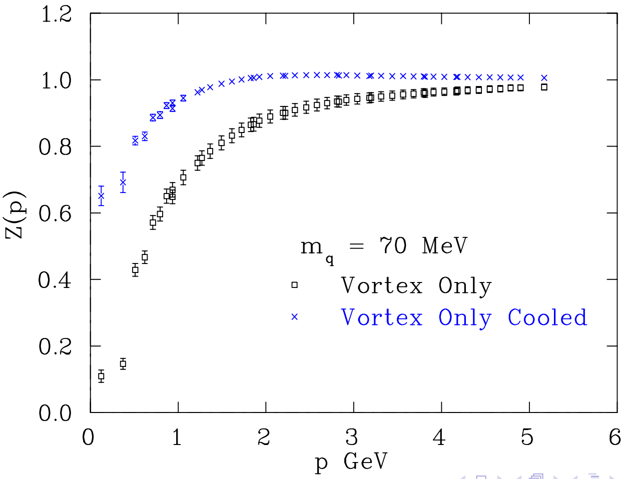

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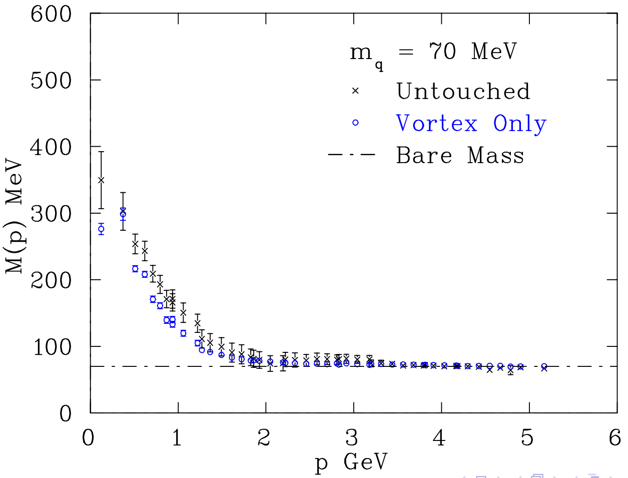

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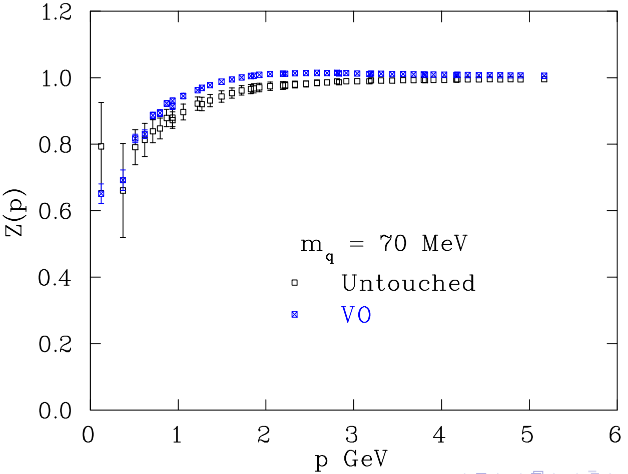

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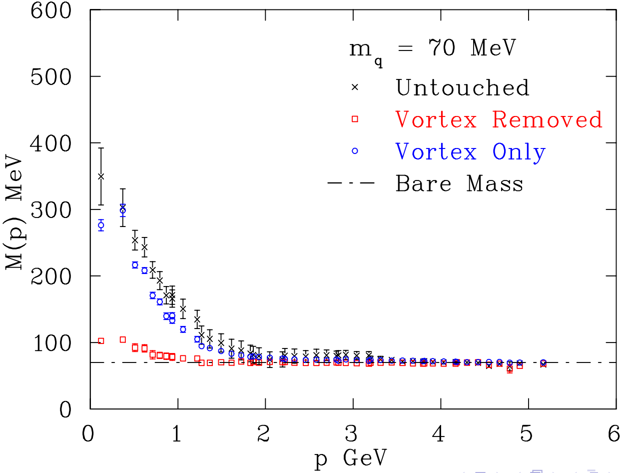

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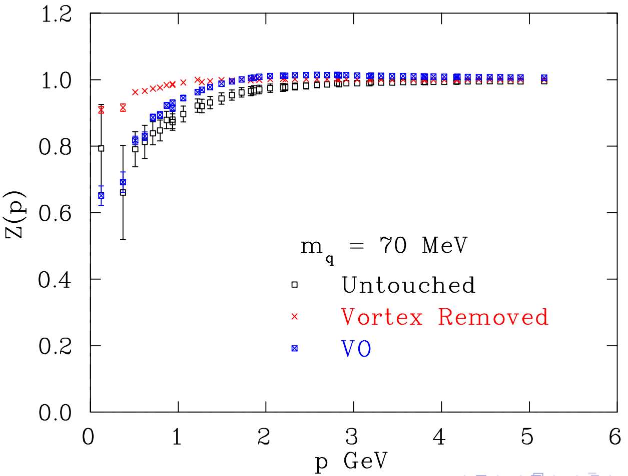

Conclusion

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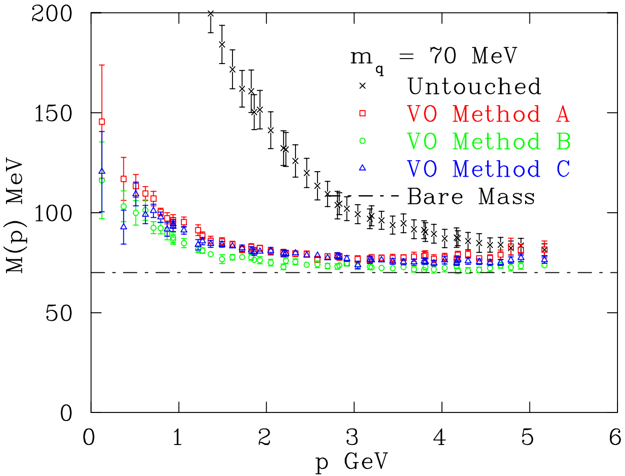
Additional Slides

Preconditioning Landau-gauge fixing



MCG fixing

- Wish to maximise the local quantity

$$R_x = \sum_\mu |\mathrm{Tr}\{G(x)U_\mu(x)\}|^2 + \sum_\mu |\mathrm{Tr}\{U_\mu(x-\hat{\mu})G^\dagger(x)\}|^2 \tag{9}$$

- Use an $SU(2)$ matrix $g = g_4\mathrm{I} - ig_i\sigma_i$ embedded in one of the 3 $SU(2)$ subgroups of $SU(3)$
- Can be re-written as

$$R_x = g_i A_{ij} g_j + g_i b_i + c, \tag{10}$$

with A real, symmetric 4×4 matrix, b a real 4-vector, c a real constant.

Method of A. Montero, Phys. Lett. B ***467****, 106 (1999)*

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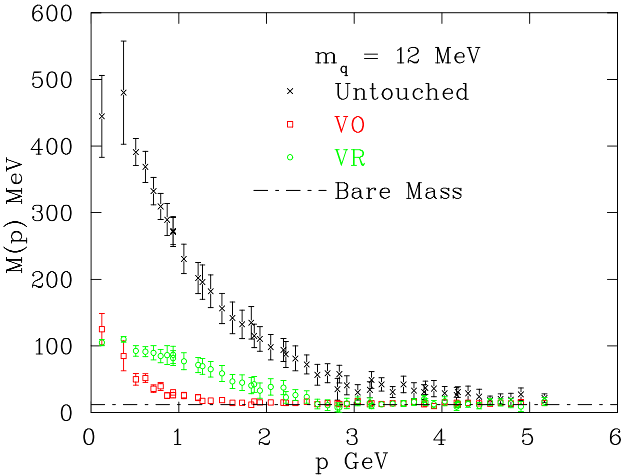

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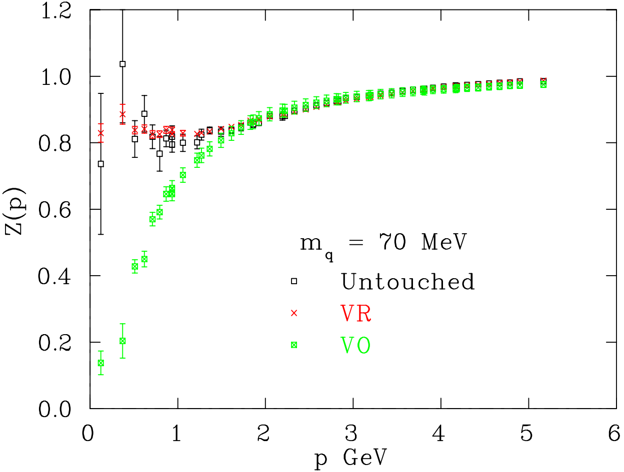
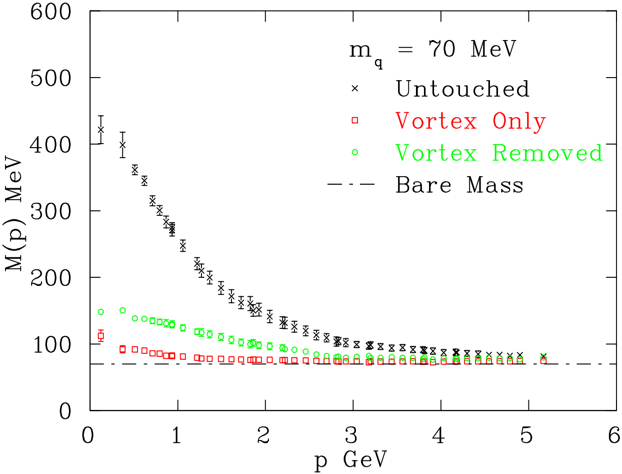
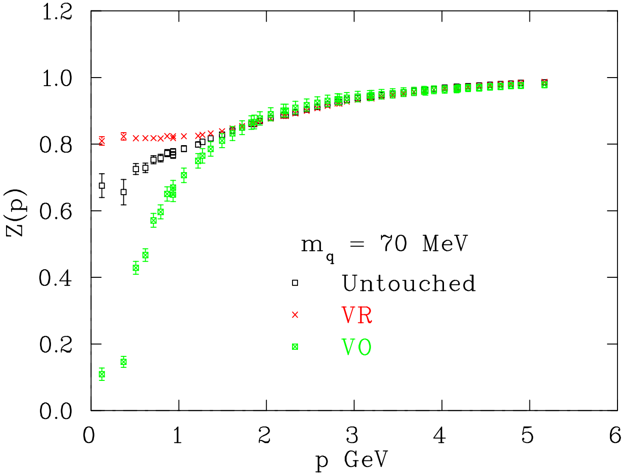




600
500
400
300
200
100
0
M(p) MeV
0
1
2
3
4
5
6
p GeV
m q = 70 MeV
Untouched
Vortex Only
Vortex Removed
Bare Mass


1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z(p)
0
1
2
3
4
5
6
p GeV
m_q = 70 MeV
Untouched
VR
VO