Results on the disconnected contributions for hadron structure

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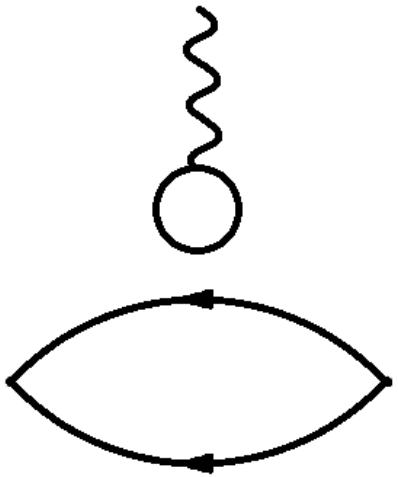
The Cyprus Institute

CaSToRC

Outline

- Disconnected contributions
- The Truncated Solver Method (TSM)
- The one-end trick
- Results
- Conclusions
- Future work

Disconnected contributions



$L(x) = \mathrm{Tr}\left[\Gamma G(x;x)\right]$

- **Closed fermion loops in form factors**
- **Expensive to compute**
- **In flavor-non-singlets they are small or zero**
- **Very important in other cases**
  - Flavor singlets
  - Pure disconnected quantities
  - High precision computations
- **Large effort to address this problem**

Disconnected contributions

- **Require the all-to-all or an estimation of the diagonal of the propagator (probing)**
- **Exact computation unfeasible nowadays**
- **Extremely difficult problem that requires large efforts and new ideas**
- **Combination of computational power and algorithms can give results**
  - GPUs yield large computer power
  - On the algorithmic side
    - Stochastic estimation of the inverse matrix
    - Variance reduction with Truncated Solver Method (TSM)
    - Special variance reduction with the one-end trick

The Truncated Solver Method

- In the inversion for an stochastic estimation, $M\left|s_j\right\rangle = \left|\eta_j\right\rangle$ exactly, we truncate the solver at n_{LP} Bali, Collins, Schäffer 2007
- Introduces a bias we can correct stochastically

$$M_E^{-1} := \frac{1}{N_{HP}} \sum_{j=1}^{N_{HP}} \left(\left|s_j\right\rangle \left\langle \eta_j \right|_{HP} - \left|s_j\right\rangle \left\langle \eta_j \right|_{LP} \right) + \frac{1}{N_{LP}} \sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} \left|s_j\right\rangle \left\langle \eta_j \right|_{LP}$$

- If M is properly conditioned, small correction, we need only a few N_{HP}
- Error should decrease essentially as $1/\sqrt{N_{LP}}$
- Requires loop-dependent fine-tuning of n_{LP} and $\frac{N_{LP}}{N_{HP}}$

The one-end trick

- **General trick that reduces variance, usually applied to 2pt** Foster, Michael 1998; McNeile, Michael 2006
- **In tmQCD, we can apply it as well to disconnected diagrams**
- **The difference/sum of propagators in the twisted basis is**

$$\sum X \left(M_u^{-1} - M_d^{-1}\right) = -2i\mu \sum_r \left\langle s^\dagger X \gamma_5 s \right\rangle_r$$

$$\sum X \left(M_u^{-1} + M_d^{-1}\right) = 2 \sum_r \left\langle s^\dagger \gamma_5 X \gamma_5 D_W s \right\rangle_r$$

- **Errors are considerably reduced**
  - The μ factor suppresses the noise
  - The volume sum enhances statistics
  - Improves signal-to-noise ratio from $\left(\frac{1}{\sqrt{V}}\right)$ to $O(1)$

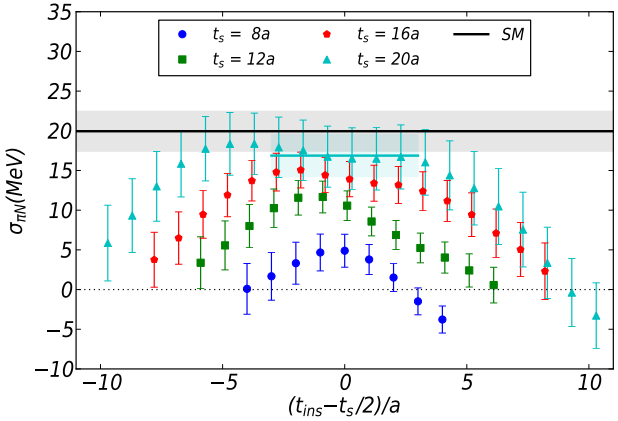
Our ensemble

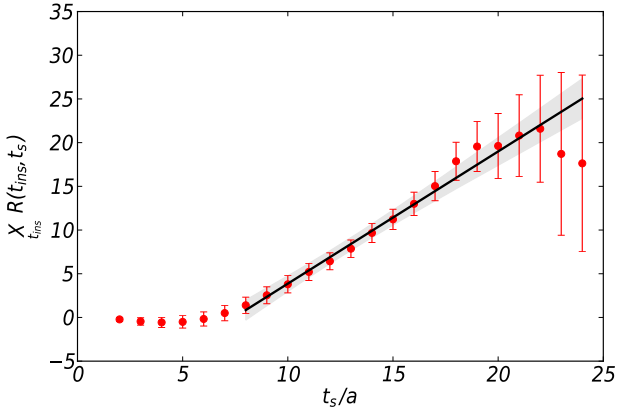
Name	B55.32
Volume	$32^3 \times 64$
a(fm)	0.0823
m_π (MeV)	371.6
$m_\pi L$	4.97
m_s (MeV)	92.4(6)(2.0)
m_c (MeV)	1186.0(4.6)
Confs	4698
Flavors	2+1+1

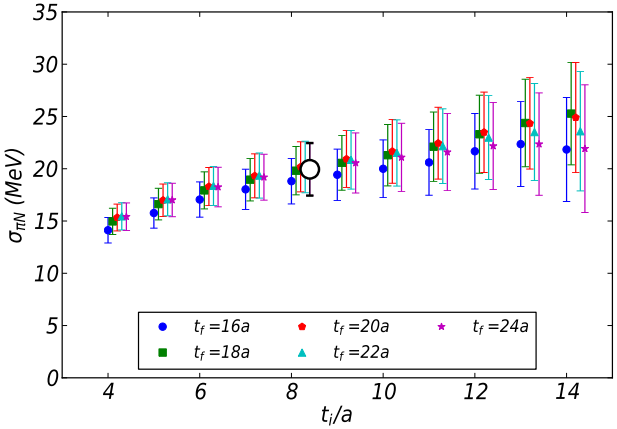
Baron *et al.* ETMC 2010

- **We calculated disconnected loops with an arbitrary gamma structure**
- **Also we included contractions with a covariant derivative**
- **First time such a large high precision study of disconnected has been attempted**

Results: Nucleon σ–term light

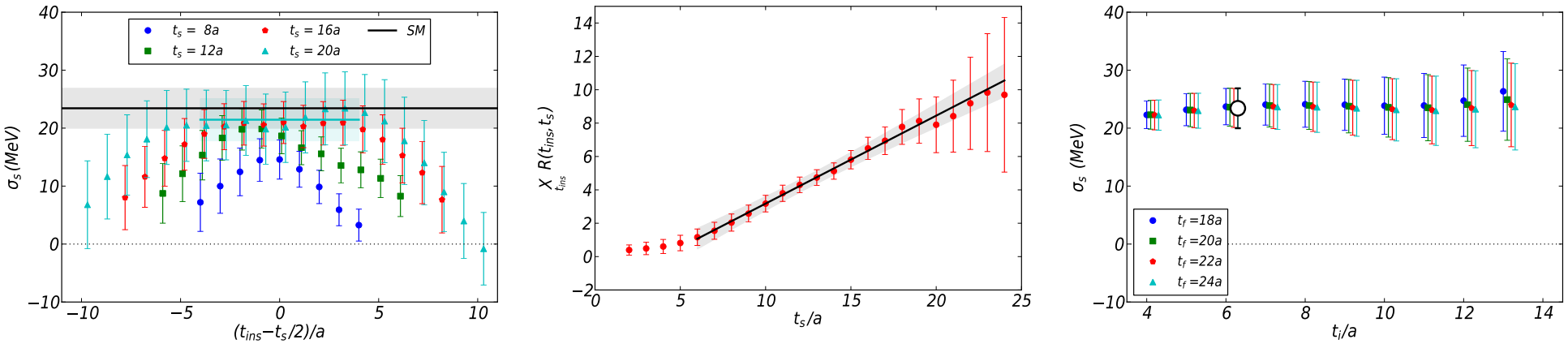






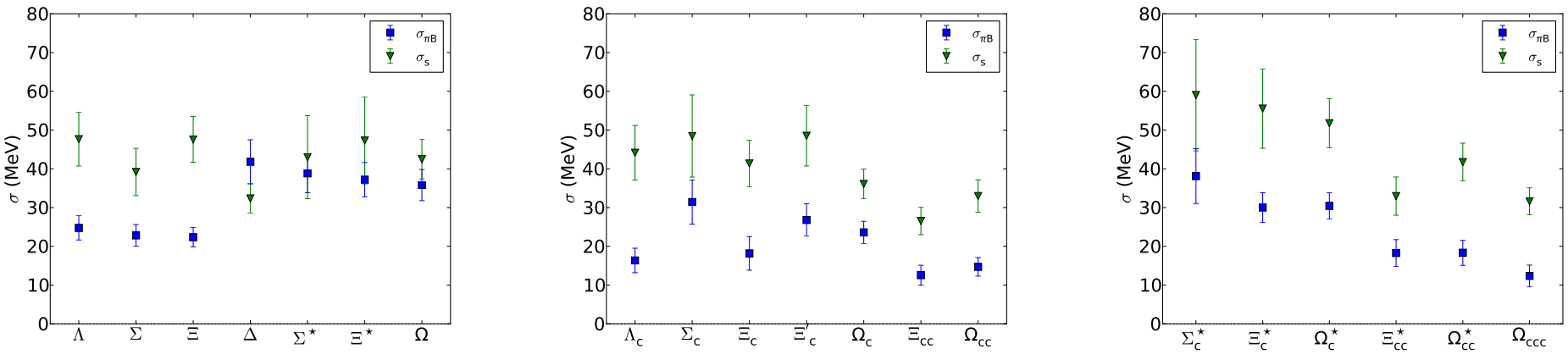
- Describes the scalar light content of the nucleon
- Important for dark matter searches through Higgs interaction
- Used 2298 confs with 16 source positions $\longrightarrow$ 147072 measurements
- Excited states appear, long source sink separations required
- Disconnected value represents $\approx 10\%$ of the total value

Results: Nucleon σ–term strange



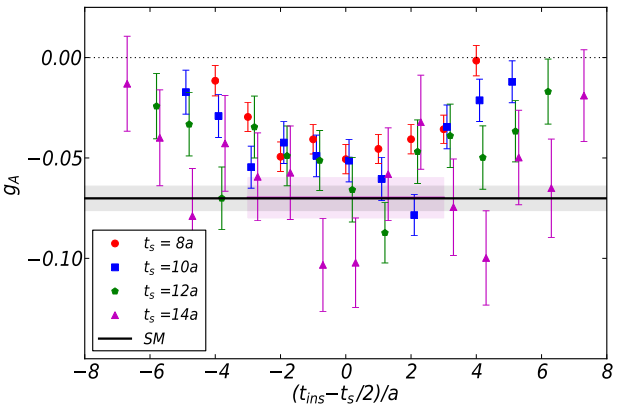
- Describes the strange content of the nucleon
- Statistics 147072 measurements
- Excited states appear, long source sink separations required
- Pure disconnected quantity

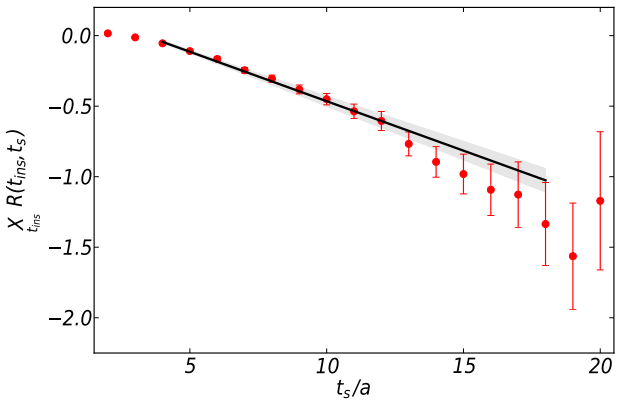
Results: Other particles σ–terms

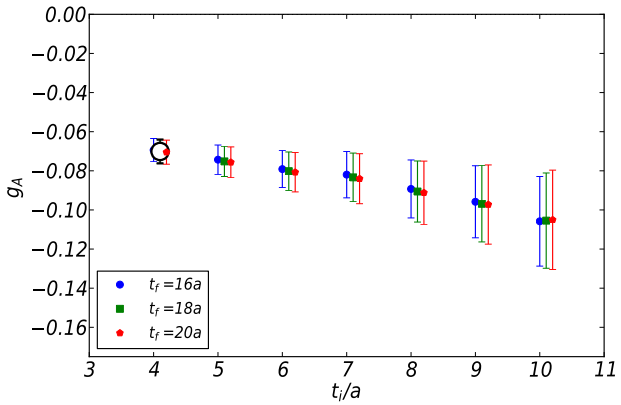


- Statistics 4644 confs$\times$ 2 directions $\times$ channels
- Channels vary from particle to particle
- First time the disconnected σ–term has been computed for such a range or particles
- All the particles receive contributions from the disconnected

Results: Nucleon g_A light

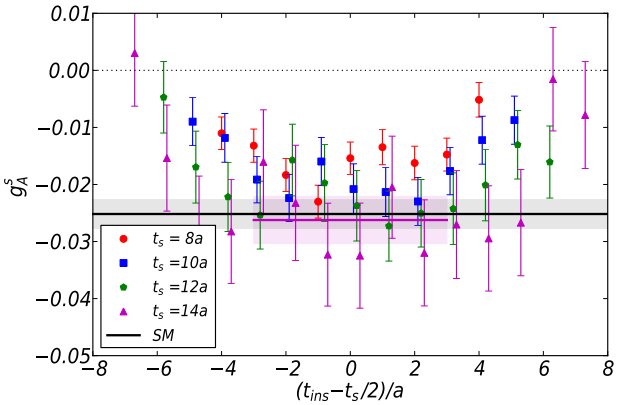


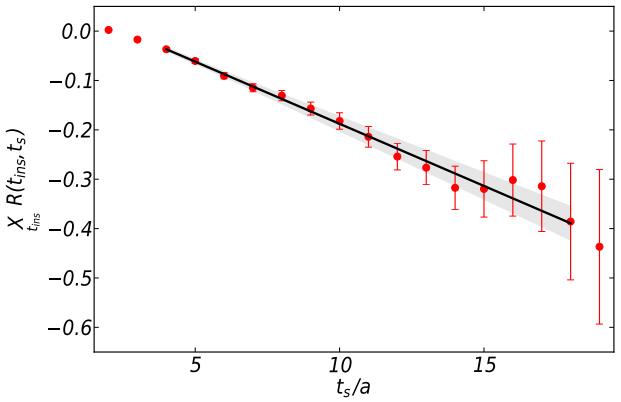


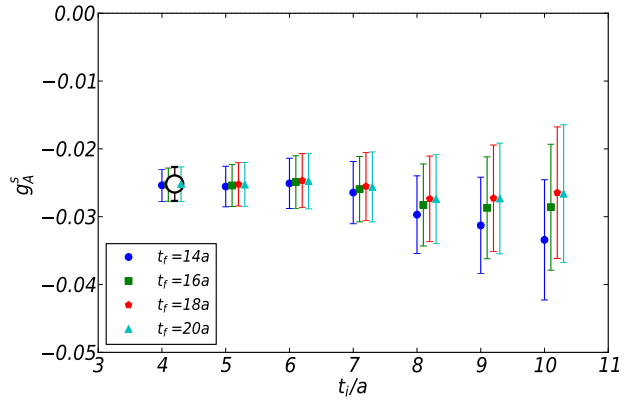


- Nucleon spin fraction carried by the light quarks
- Strongly motivated by the proton spin crisis
- Statistics 147072 measurements
- Less contaminated than σ–terms
- Disconnected value represents $\approx 10\%$ of the total value

Results: Nucleon g_A strange

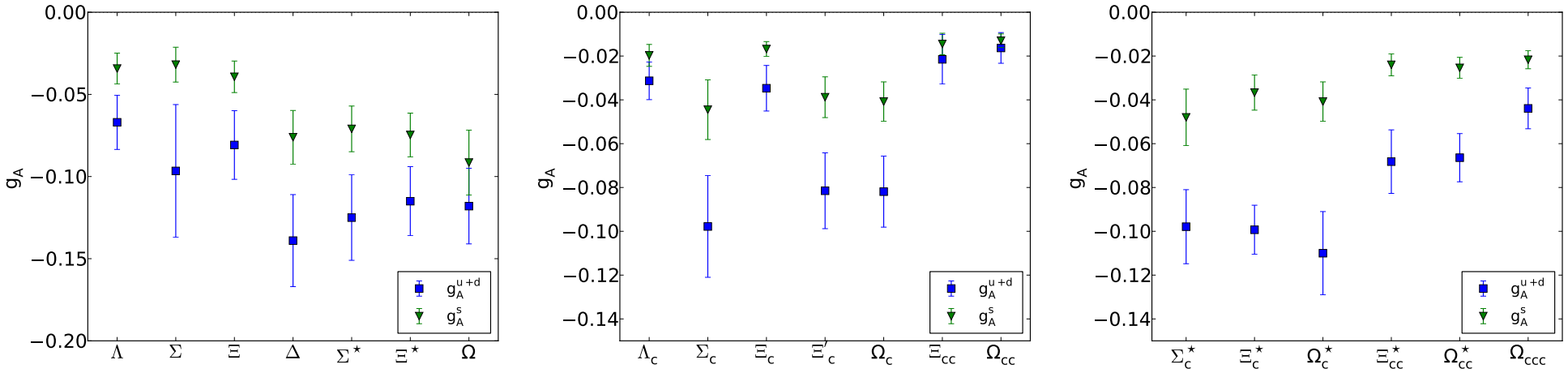






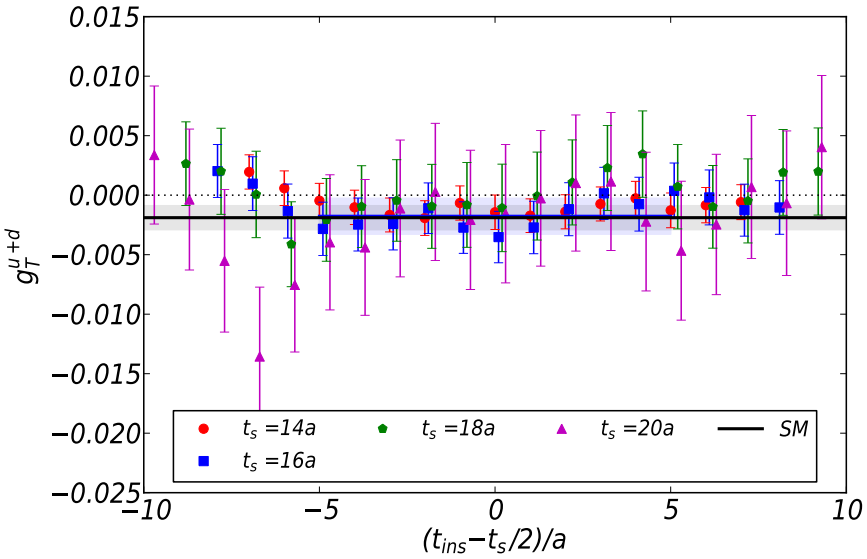
- Nucleon spin fraction carried by the strange quark
- Statistics 147072 measurements
- Pure disconnected quantity

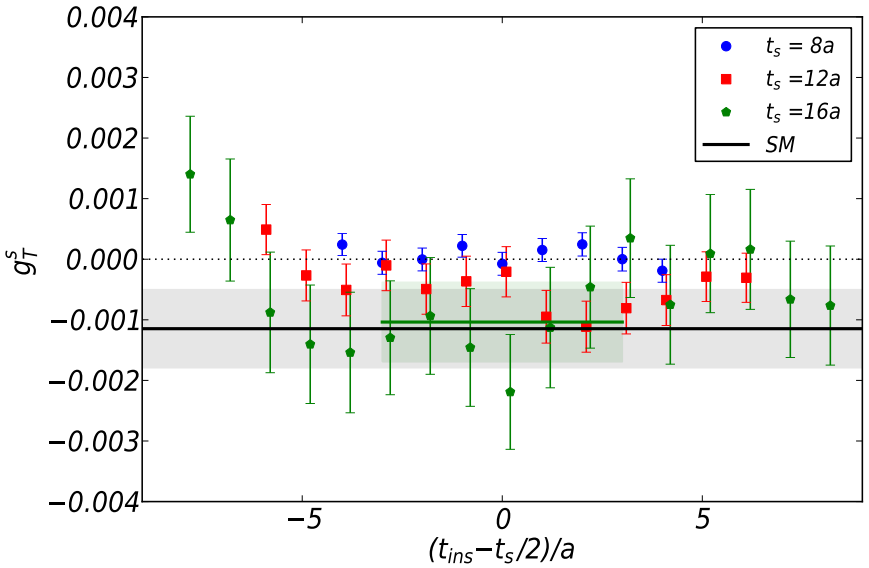
Results: Other baryons g_A



- **Statistics 4644 confs$\times$ 2 directions $\times$ channels**
- **Channels vary from particle to particle**
- **First time g_A disconnected is calculated for all these particles, and it's non-zero for all of them**

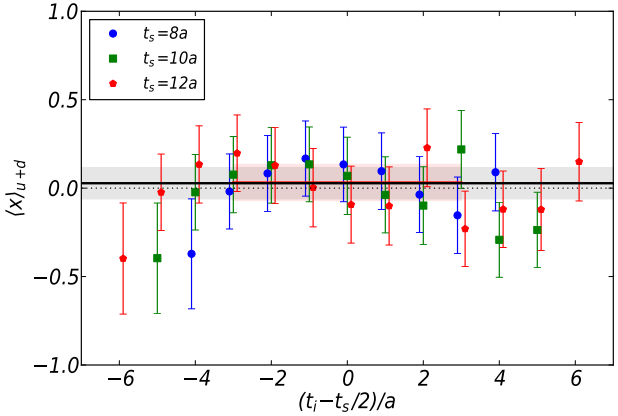
Results: Nucleon g_T light and strange

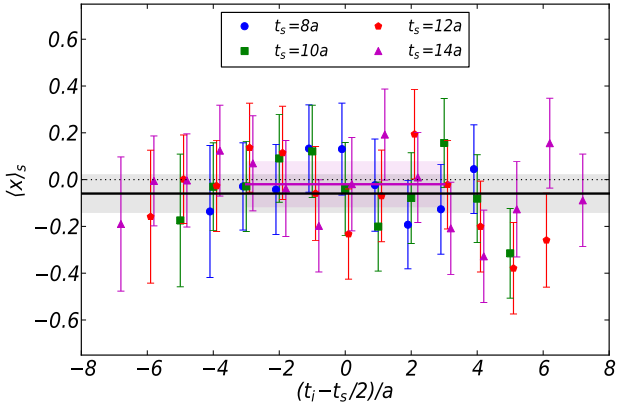


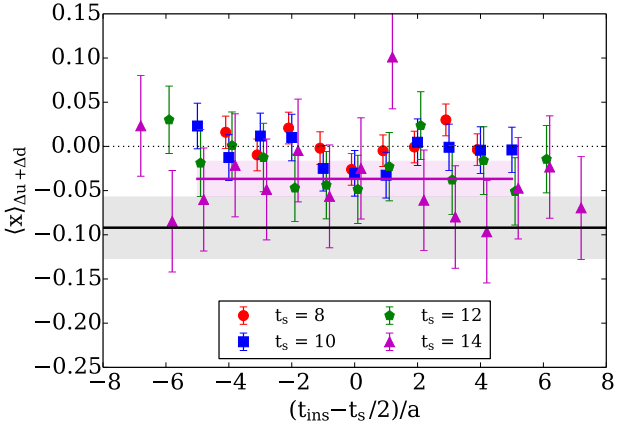


- Gives information about the strength of the coupling of the particle to tensor operators
- Statistics 147072 measurements
- For the light, is a small correction $\approx 1\%$
- The strange quantity is pure disconnected

Results: Nucleon $\langle x \rangle_{u+d}$, $\langle x \rangle_s$ and $\langle x \rangle_{\Delta u+\Delta d}$

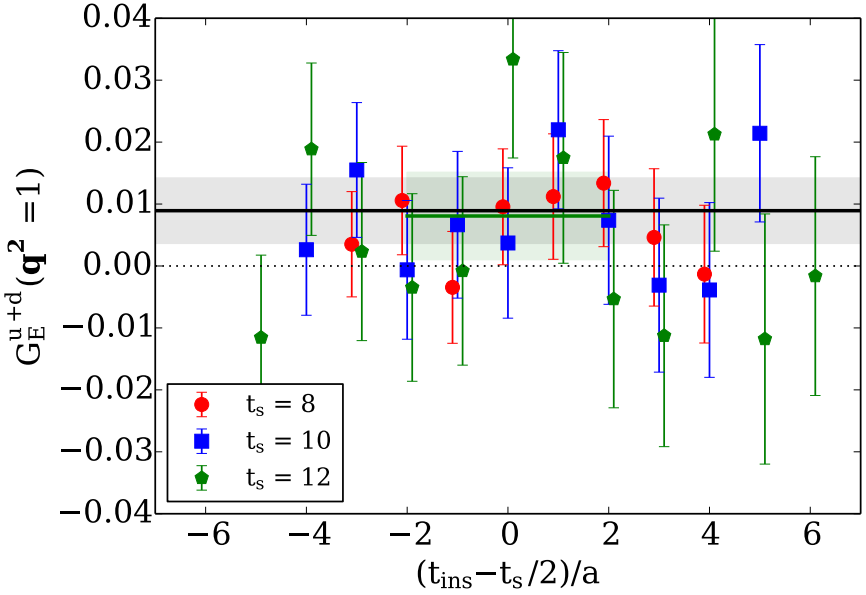


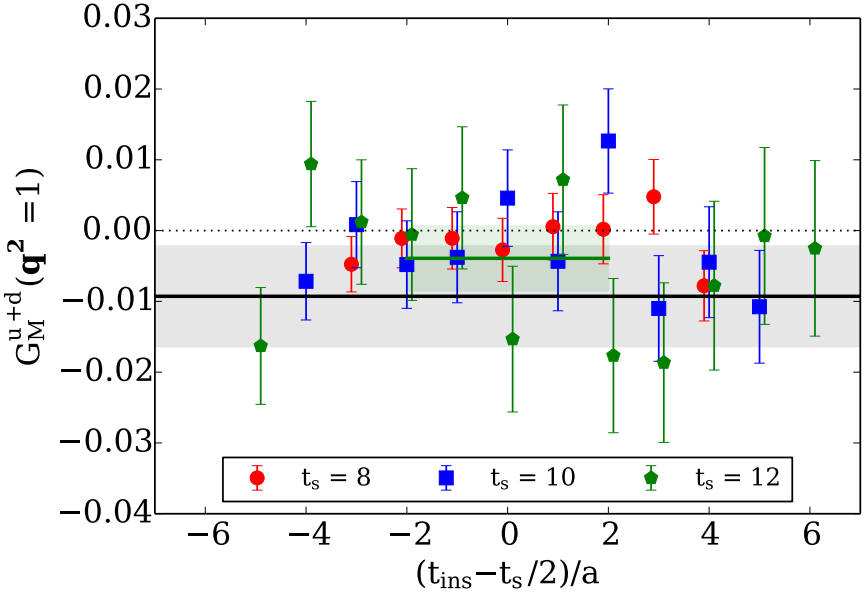




- $\langle x \rangle_q$ **Gives information about the fraction of nucleon momentum carried by the quark** q
- $\langle x \rangle_{\Delta q}$ **is the nucleon helicity fraction carried by** q
- **Statistics 147072 measurements**
- **For the light, the disconnected part represents a** $\approx 2 - 4\%$ **correction**
- **The strange quantity is pure disconnected**

Results: Nucleon electromagnetic form factors





- Can be used to extract information about the distribution of the *u*-*d* quarks inside the nucleon
- Statistics 147072 measurements
- Few percent correction to the connected part

Conclusions

- **We achieved high precision results for disconnected diagrams**
- **First time such a broad study has been carried out**
  - We calculated ALL local and one-derivative insertions in a single run
- **Extremely difficult problem which requires a large effort**
  - Unprecedented computer power with GPUs
  - Huge statistics ($\approx$150000 for the nucleon)
  - State-of-the-art methods
- **We can aim to remove the systematics for many quantities**

Future work

- **Currently working on high-statistics studies of other ensembles**
- **Plan to tackle the physical point**
  - Twisted-clover regularization included in QUDA
  - Unfortunately our tests reveal that TSM alone is not enough for such low-masses
  - Requires change of strategy
    - Exact deflation to remove the low eigenmodes
    - Probing, hierarchical probing

Stathopoulos, Laeuchli, Orginos 2013

ΔΕΣΜΗ 2009-2010

Θέλω στο μέλλον.

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Research Promotion Foundation

Tuning the physical point

- M not properly conditioned at the physical point, n_{LP} must be revised
- We write the variance of the loop as

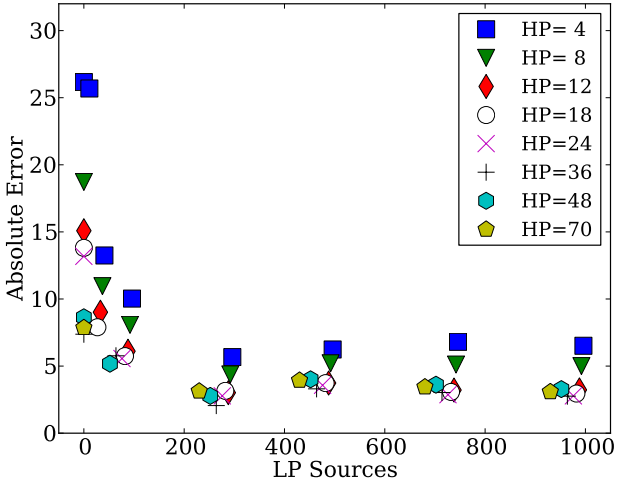
$$\sigma^2 = \frac{f_{HP}(n_{LP})}{N_{HP}} + \frac{f_{LP}(n_{LP})}{N_{LP}}$$

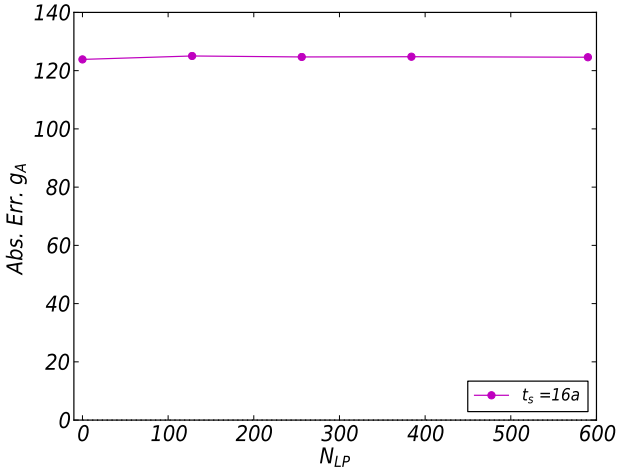
- Try to minimize the variance at fixed cost (C)

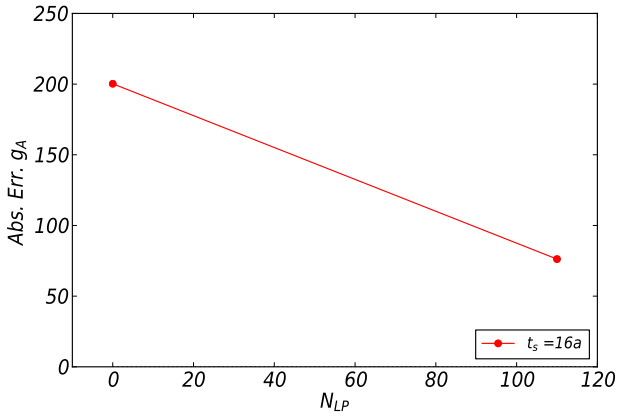
$$C = N_{HP} \times n_{HP} + N_{LP} \times n_{LP}$$

- Try to minimize the variance at fixed cost (C)
- We solve for n_{LP} and $\frac{N_{LP}}{N_{HP}}$

Tuning the physical point







- **B55 ensemble**
- $n_{HP} \approx 100 n_{LP}$
- $\rho_{LP} \approx 10^{-2}$
- $N_{HP} = 24$

- **Physical point**
- $n_{HP} \approx 1000 n_{LP}$
- $\rho_{LP} \approx 10^{-2}$
- $N_{HP} = 24$

- **Physical point**
- $n_{HP} \approx 4 n_{LP}$
- $\rho_{LP} \approx 10^{-6}$
- $N_{HP} = 4$

- **TSM not efficient for our regularization (twisted mass + clover) at the physical point**