Spectrum of the SU(4) lattice gauge theory with fermions in the anti-symmetric two index representation

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Content of the talk

- Why SU(4) sextet?
- Lattice setup
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Why SU(4) sextet?

- The representation is real $\rightarrow$ no sign problem at finite density (like in QC_2D).
- It is an interesting generalization of QCD, from the large-N_c point of view.
- Do meson and baryon states follow the large-N_c scaling? How about diquarks and tetraquarks?
- Here is the first large N_c calculation with dynamical fermions.

Let's take a look!

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Lattice setup

- Wilson plaquette gauge action + clover fermions actions with nHYP smeared links as the gauge connections.
- SU(4) sextet with $N_f = 2$; compared with SU(N_c) fundamental with N_c = 3, 5, and 7 quenched; SU(3) fundamental with $N_f = 2$; and SU(4) partially quenched (PQ) points ($\kappa = 0.129$ configs).
- The parameters used: $V = 16^3 \times 32$; $a \approx 0.1 fm$ ($r_1 \approx 0.31$ fm)

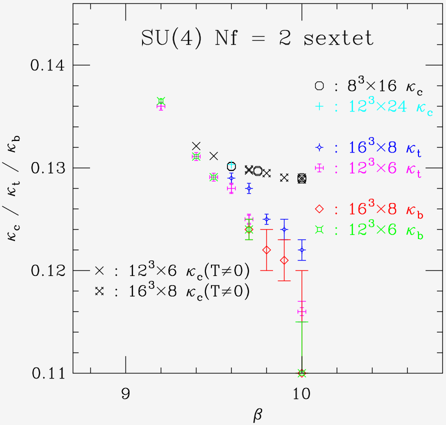
κ	0.128	0.1285	0.129	0.1292
configurations	146	140	200	161
r_1/a	2.50(1)	2.78(2)	2.97(2)	3.22(3)

Table: Parameters of the $SU(4)$ simulations. $\beta = 9.6$.

κ	0.125	0.126	0.1265	0.127	0.1272
configurations	100	100	100	100	100
r_1/a	2.95(3)	3.09(4)	3.03(3)	3.18(5)	3.34(4)

Table: Parameters of the $SU(3)$ simulations. $\beta = 5.4$.

Phase diagram for $N_f = 2$ SU(4) sextet



κ_t: thermal phase transition points determined from the averaged Wilson line.

κ_b: bulk phase transition points determined from the averaged plaquette.

κ_c: critical κ determined from vanishing am_q.

κ dependence of the lattice spacing a

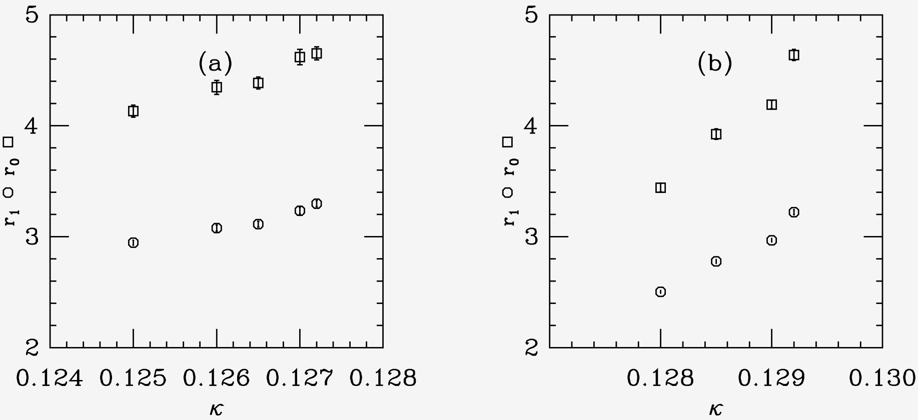


Figure: Sommer parameters r_0 and r_1 for dynamical SU(3) (panel (a)) and SU(4) (panel (b)) data sets.

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Mesons and decay constant

- Meson masses should not depend on N_c.
- Pseudoscalar decay constant f_π:

$$\langle 0|\bar{u}\gamma_0\gamma_5 d|\pi\rangle = m_\pi f_\pi$$

- The continuum quantity:

$$f_\pi = f_\pi^L Z_A(1 - \frac{3\kappa}{4\kappa_c})$$

- The expected scaling behavior:

$$f_\pi \sim \begin{cases} \sqrt{N_c} & \text{fundamental} \\ N_c & \text{sextet} \end{cases}$$

- The real world value:

$$f_\pi \approx 0.31\text{fm} \times 132\text{MeV}/(197.3\text{fm MeV}) \approx 0.21$$

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Meson spectrum scaling

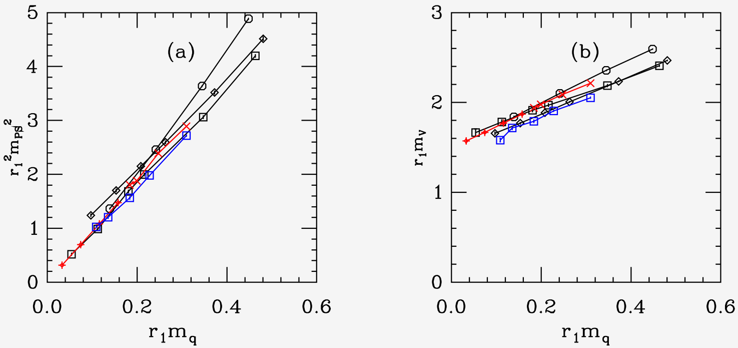

Figure: Mesons. On the left, the squared pseudoscalar mass scaled by r_1^2, on the right, r_1 times the vector meson mass. The abscissa is r_1 times the AWI quark mass. The data sets are: black squares for quenched $SU(3)$ fundamentals, black diamonds for quenched $SU(5)$ fundamentals, black octagons for quenched $SU(7)$ fundamentals, red crosses for $SU(4)$ sextet with $N_f = 2$; the fancy diamonds are the PQ data. Finally, the blue squares are $SU(3)$ fundamentals with $N_f = 2$.

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Pseudoscalar decay constant scaling

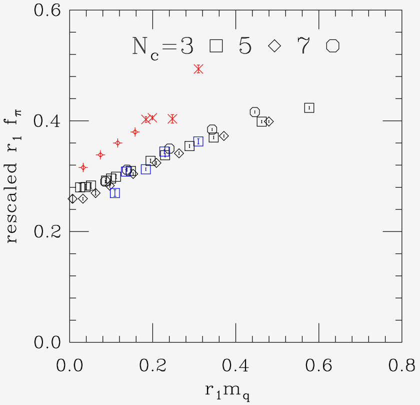

Figure: Pseudoscalar decay constant. The abscissa is r_1 times the AWI quark mass. The data sets are: black squares for quenched $SU(3)$ fundamentals, black diamonds for quenched $SU(5)$ fundamentals, black octagons for quenched $SU(7)$ fundamentals, red crosses for $SU(4)$ sextet with $N_f = 2$; the fancy diamonds are the PQ data. Finally, the blue squares are $SU(3)$ with$N_f = 2$.

Diquark and meson scaling

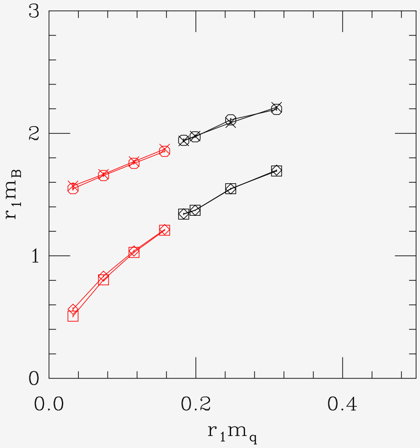

Figure: $SU(4)$ mesons and diquarks: octagon the $I = 0, J = 1$ diquark, squares the $I = 1, J = 0$ diquark, diamonds the pseudoscalar meson and crosses the vector meson. Black data points are with dynamical fermions(not partially quenched) and the red points are partially quenched.

Diquark and meson scaling

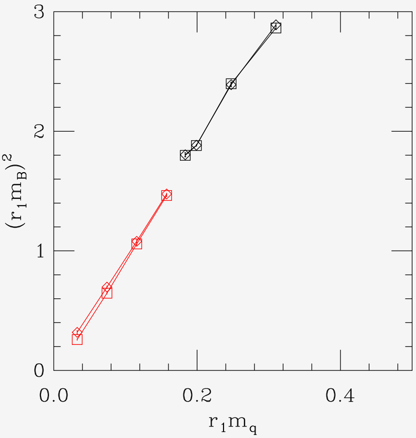

Figure: Squared masses of $SU(4)$ pseudoscalar mesons and diquarks: squares the $I = 0$, $J = 1$ diquark, diamonds the pseudoscalar meson. Black data points are with dynamical fermions (not partially quenched) and the red points are partially quenched.

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Baryons in Large N_c with N_R quarks

- Isospin-spin locked:

$$I = J = N_R/2, N_R/2 - 1, \cdots, 1/2.$$

- Rotor formula:

$$M_B(N_R, J) = N_R m_0 + BJ(J+1)/N_R + \cdots$$

  - Fundamental: $N_R = N_c$ (of course)
  - Antisymmetric: $N_R = N_c(N_c - 1)/2$
- m_0 and B have $1/N_c$ corrections
- m_0 and B depend on m_q

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Baryon spectrum scaling

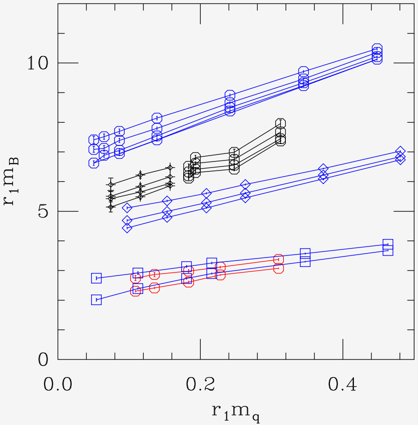

Figure: Baryons. The blue data are from the top quenched $SU(7)$,SU(5) and $SU(3)$ data. The red octagons are $SU(3)$ with dynamical fermions. The black lines are the six quark baryons in $SU(4)$ sextet, octagons for dynamical and fancy diamonds for partially quenched.

Fit to rotor formula

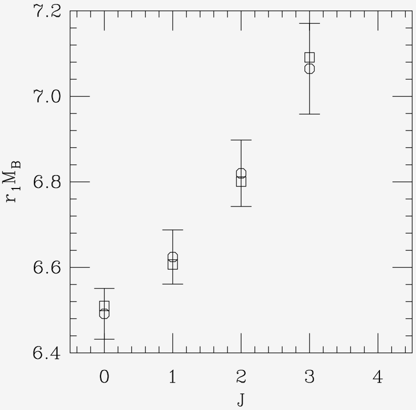

Figure: Fit to rotor formula. SU(4) sextet; $\kappa = 0.1285$. Octagons are the data points; squares the best fit values.

$$M_B(N_R, J) = N_R m_0 + BJ(J+1)/N_R + \cdots$$

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Fit to rotor formula

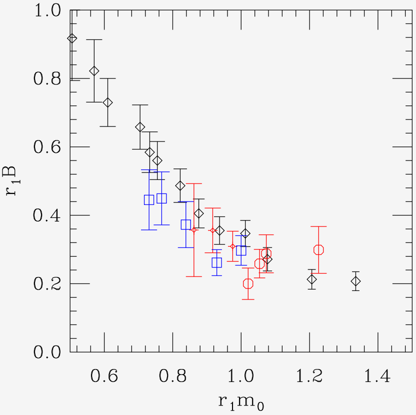

Figure: B vs. m_0 from the rotor formula; black diamonds from quenched $SU(3)$, red squares from full $SU(3)$. The $SU(4)$ data are shown as blue octagons for the dynamical sets and fancy diamonds for the partially quenched set.

$$M_B(N_R, J) = N_R m_0 + BJ(J+1)/N_R + \cdots$$

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Summary

- Large N_c scaling works amazingly well for both $SU(N_c)$ fundamental and sextet representation.
  - – Meson masses show expected large N_c scaling (no N_c dependence).
  - – f_π scales with $\sqrt{N_c}$ for fundamental and N_c for sextet.
  - – Baryons obey the rotor spectrum.
- What will happen to the spectrum when the chemical potential is turned on? How does the phase diagram look like?
- Can we get useful information about our real world QCD? Locating the tri-critical point in the $\beta - \mu$ plane?
- Need improved actions to go to stronger coupling region.

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References

Thank you for your attention!

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Backup slides

Diquarks

- Diquark color wave functions are symmetric, which is different from normal QCD.
- Therefore its space-spin-isospin wave function is totally antisymmetric.
- Two kinds of diquarks: a spin-zero isotriplet and a spin-1 isosinglet.
- Diquark state are degenerate with mesonic spin partners.
- There are nine Goldstone bosons but only three pseudoscalar $q\bar{q}$ isospin states. The other six states are the isotripled of J = 0 diquarks and their antiparticles.

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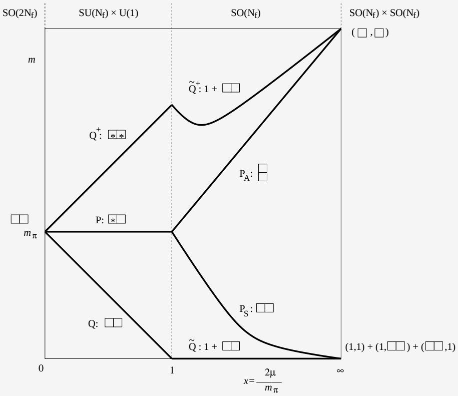

Figure: Spectrum of two-color QCD ($\beta = 1$) at finite μ and m (schematic).

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