Spectrum of the SU(4) lattice gauge theory with fermions in the anti-symmetric two index representation

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The 32nd International Symposium on Lattice Field Theory
June 27, 2014
Content of the talk

- Why SU(4) sextet?
- Lattice setup
- Phase diagram
- Mesons and decay constant scaling
- Diquarks?
- Baryons and rotor spectrum
- Summary
Why SU(4) sextet?

- The representation is real $\rightarrow$ no sign problem at finite density (like in QC$_2$D).
- It is an interesting generalization of QCD, from the large-$N_c$ point of view.
- Do meson and baryon states follow the large-$N_c$ scaling? How about diquarks and tetraquarks?
- Here is the first large $N_c$ calculation with dynamical fermions.

Let’s take a look!
Lattice setup

- Wilson plaquette gauge action + clover fermions actions with nHYP smeared links as the gauge connections.

- SU(4) sextet with $N_f = 2$; compared with SU($N_c$) fundamental with $N_c = 3, 5, 7$ quenched; SU(3) fundamental with $N_f = 2$; and SU(4) partially quenched (PQ) points ($\kappa = 0.129$ configs).

- The parameters used: $V = 16^3 \times 32$; $a \approx 0.1\text{ fm}$ ($r_1 \approx 0.31\text{ fm}$)

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Table: Parameters of the $SU(4)$ simulations. $\beta = 9.6$.

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<td>$r_1/a$</td>
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Table: Parameters of the $SU(3)$ simulations. $\beta = 5.4$. 

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Phase diagram for $N_f = 2$ SU(4) sextet

$\kappa_t$: thermal phase transition points determined from the averaged Wilson line.

$\kappa_b$: bulk phase transition points determined from the averaged plaquette.

$\kappa_c$: critical $\kappa$ determined from vanishing $am_q$. 
dependence of the lattice spacing $a$

Figure: Sommer parameters $r_0$ and $r_1$ for dynamical SU(3) (panel (a)) and SU(4) (panel (b)) data sets.
Mesons and decay constant

- Meson masses should not depend on $N_c$.
- Pseudoscalar decay constant $f_\pi$:

  $$\langle 0 | \bar{u} \gamma_0 \gamma_5 d | \pi \rangle = m_\pi f_\pi$$

- The continuum quantity:

  $$f_\pi = f_\pi^L Z_A (1 - \frac{3\kappa}{4\kappa_c})$$

- The expected scaling behavior:

  $$f_\pi \sim \begin{cases} \sqrt{N_c} & \text{fundamental} \\ N_c & \text{sextet} \end{cases}$$

- The real world value:

  $$f_\pi \approx 0.31 \text{ fm} \times 132 \text{ MeV} / (197.3 \text{ fm MeV}) \approx 0.21$$
Meson spectrum scaling

Figure: Mesons. On the left, the squared pseudoscalar mass scaled by $r_1^2$, on the right, $r_1$ times the vector meson mass. The abscissa is $r_1$ times the AWI quark mass. The data sets are: black squares for quenched SU(3) fundamentals, black diamonds for quenched SU(5) fundamentals, black octagons for quenched SU(7) fundamentals, red crosses for SU(4) sextet with $N_f = 2$; the fancy diamonds are the PQ data. Finally, the blue squares are SU(3) fundamentals with $N_f = 2$. 
**Pseudoscalar decay constant scaling**

**Figure:** Pseudoscalar decay constant. The abscissa is $r_1$ times the AWI quark mass. The data sets are: black squares for quenched $SU(3)$ fundamentals, black diamonds for quenched $SU(5)$ fundamentals, black octagons for quenched $SU(7)$ fundamentals, red crosses for $SU(4)$ sextet with $N_f = 2$; the fancy diamonds are the PQ data. Finally, the blue squares are $SU(3)$ with $N_f = 2$. 
Diquark and meson scaling

Figure: SU(4) mesons and diquarks: octagon the $I = 0, J = 1$ diquark, squares the $I = 1, J = 0$ diquark, diamonds the pseudoscalar meson and crosses the vector meson. Black data points are with dynamical fermions (not partially quenched) and the red points are partially quenched.
Diquark and meson scaling

Figure: Squared masses of $SU(4)$ pseudoscalar mesons and diquarks: squares the $I = 0, J = 1$ diquark, diamonds the pseudoscalar meson. Black data points are with dynamical fermions (not partially quenched) and the red points are partially quenched.
Baryons in Large $N_c$ with $N_R$ quarks

- Isospin-spin locked:

$$I = J = \frac{N_R}{2}, \frac{N_R}{2} - 1, \ldots, 1/2.$$ 

- Rotor formula:

$$M_B(N_R, J) = N_R m_0 + B J (J + 1) / N_R + \cdots$$

- Fundamental: $N_R = N_c$ (of course)

- Antisymmetric: $N_R = N_c(N_c - 1)/2$

- $m_0$ and $B$ have $1/N_c$ corrections

- $m_0$ and $B$ depend on $m_q$
Figure: Baryons. The blue data are from the top quenched $SU(7), SU(5)$ and $SU(3)$ data. The red octagons are $SU(3)$ with dynamical fermions. The black lines are the six quark baryons in $SU(4)$ sextet, octagons for dynamical and fancy diamonds for partially quenched.
Fit to rotor formula

Figure: Fit to rotor formula. SU(4) sextet; $\kappa = 0.1285$. Octagons are the data points; squares the best fit values.

\[ M_B(N_R, J) = N_R m_0 + BJ(J+1)/N_R + \cdots \]
**Fit to rotor formula**

**Figure:** $B$ vs. $m_0$ from the rotor formula; black diamonds from quenched $SU(3)$, red squares from full $SU(3)$. The $SU(4)$ data are shown as blue octagons for the dynamical sets and fancy diamonds for the partially quenched set.

$$M_B(N_R, J) = N_R m_0 + BJ(J + 1)/N_R + \cdots$$
Summary

- Large $N_c$ scaling works amazingly well for both $SU(N_c)$ fundamental and sextet representation.
  - Meson masses show expected large $N_c$ scaling (no $N_c$ dependence).
  - $f_\pi$ scales with $\sqrt{N_c}$ for fundamental and $N_c$ for sextet.
  - Baryons obey the rotor spectrum.

- What will happen to the spectrum when the chemical potential is turned on? How does the phase diagram look like?

- Can we get useful information about our real world QCD? Locating the tri-critical point in the $\beta - \mu$ plane?

- Need improved actions to go to stronger coupling region.
Thank you for your attention!

E. Witten

*Baryons in the 1/N expansion.* (Nice place to start with large $N$)

E. Corrigan, and P. Ramond

*A note on the quark content of large color groups.* (Large $N$ in AS2)

G. S. Adkins, C. R. Nappi, and E. Witten

*Static properties of nucleons in the Skyrme model.* (Rotor formula)

E. E. Jenkins

*Baryon hyperfine mass splittings in large N QCD.* (Rotor formula)

S. Bolognesi

*Baryons and Skyrmions in QCD with quarks in higher representations.* (Baryons construction in AS2)

T. DeGrand

*Lattice baryons in the 1/N expansion.*

T. DeGrand

*Lattice calculations of the spectroscopy of baryons with broken flavor SU(3) symmetry and 3, 5, or 7 colors.*
Backup slides
Diquarks

- Diquark color wave functions are symmetric, which is different from normal QCD.
- Therefore its space-spin-isospin wave function is totally antisymmetric.
- Two kinds of diquarks: a spin-zero isotriplet and a spin-1 isosinglet.
- Diquark state are degenerate with mesonic spin partners.
- There are nine Goldstone bosons but only three pseudoscalar $q\bar{q}$ isospin states. The other six states are the isotripled of $J = 0$ diquarks and their antiparticles.
Figure: Spectrum of two-color QCD ($\beta = 1$) at finite $\mu$ and $m$ (schematic).

Kogut et al. NPB582 (2000) 477-513