Disconnected Contributions to Nucleon Charges

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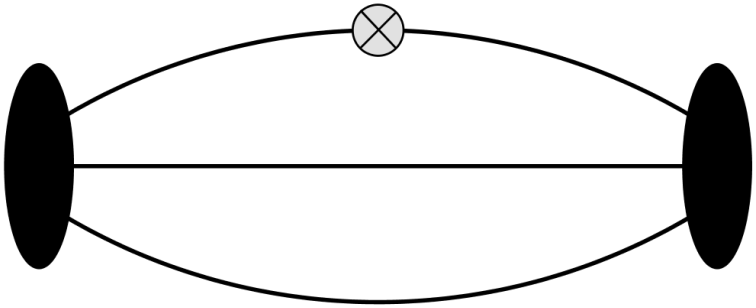
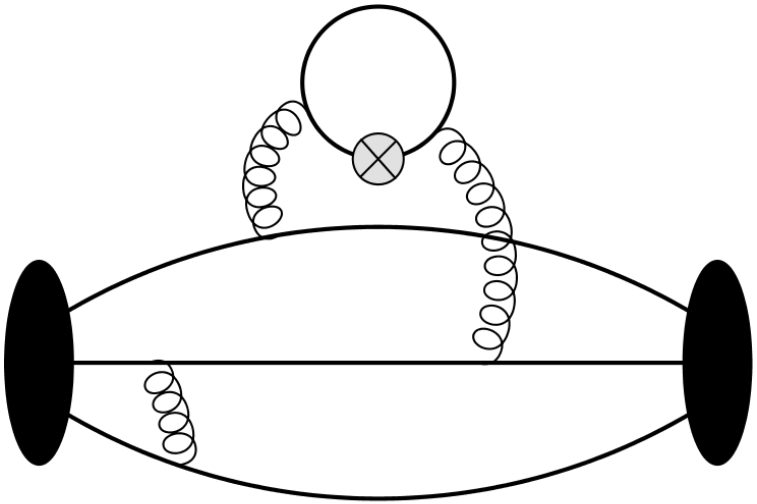
General Three-point Functions

$$\text{ME}_\Gamma \sim \langle N| \, \overline{q}_i \Gamma q_j \, |N\rangle$$





- Isoscalar operators ($q_i = q_j$) have disconnected quark loops

Physics with Disconnected Contributions

Physics	Observables	Diagrams
Neutron β-decay	g_V, g_A, g_S, g_T	Connected No-disconnected
nEDM { qEDM cEDM	g_T $\langle N\|\overline{q}\sigma \cdot Gq\|N\rangle$	Connected & Disconnected
Sigma term Dark-matter search	$\langle N\|\overline{q}q\|N\rangle$	Connected & Disconnected
TMDs	$\langle N\|\overline{q}(0)\Gamma\, \mathcal{U}_0^b\, q(b)\|N\rangle$	Connected & Disconnected

D^{-1} : Improvement & Error Reduction Techniques

- Multigrid Solver [Osborn, *et al.*, 2010; Babich, *et al.*, 2010]
  [Plenary talk: Andreas Frommer, Tue 10:15]
- All-Mode Averaging (AMA) for Two-point Correlators
  [Blum, Izubuchi and Shintani, 2013]
- Hopping Parameter Expansion (HPE)
  [Thron, *et al.*, 1998; McNeile and Michael , 2001]
- Truncated Solver Method (TSM) [Bali, Collins and Schäfer, 2007]
- Dilution [Bernardson, *et al.*, 1994; Viehoff, *et al.*, 1998]

Lattice Setup for Comparison Study

- MILC $N_f = 2 + 1 + 1$ HISQ lattice
  - $a = 0.12\,\mathrm{fm}$
  - $m_\pi = 305\,\mathrm{MeV}$
  - Geometry $= 24^3 \times 64$
  - Number of confs $= 1013$
- Clover valence quarks (Clover on HISQ)
- HYP smeared gauge links

All-Mode Averaging (AMA) for Two-point Correlators

Improved Estimator of Two-point Function

$$C^{\text{2pt, imp}} = \underbrace{\frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} C^{\text{2pt}}_{\text{LP}}(\mathbf{x}_i)}_{\text{LP estimate}} + \underbrace{\frac{1}{N_{\text{HP}}} \sum_{j=1}^{N_{\text{HP}}} \left[C^{\text{2pt}}_{\text{HP}}(\mathbf{x}_j) - C^{\text{2pt}}_{\text{LP}}(\mathbf{x}_j) \right]}_{\text{Crxn term}}$$

- All-mode averaging (AMA) [Blum, Izubuchi and Shintani, 2013] with Multigrid solver for Clover in Chroma [Osborn, *et al.*, 2010]
- Exploiting translation symmetry of lattice, 2pt-func is averaged on multiple source positions
- "LP" term is the low-precision estimate, truncated the inverter with a low accuracy stopping criterion (e.g., $r_{\text{LP}} = 10^{-3}$)
- "HP" (high-precision) correction term (e.g., $r_{\text{HP}} = 10^{-8}$) makes the estimator unbiased; Systematic error $\Rightarrow$ Statistical error
- $N_{\text{LP}} \gg N_{\text{HP}}$ brings computational gain (e.g., N_{LP} = 60, N_{HP} = 4)

Gain in Computational Cost

	4 HP		28 LP + 4 Crxn		60 LP + 4 Crxn	
	g_Γ^{dis}	Gain	g_Γ^{dis}	Gain	g_Γ^{dis}	Gain
g_S^{dis}	1.253(192)	1	1.202(83)	1.4	1.203(71)	1.1
g_A^{dis}	−0.0828(173)	1	−0.0877(62)	2.0	−0.0872(48)	2.0
g_T^{dis}	−0.0141(82)	1	−0.0122(31)	1.8	−0.0136(23)	2.0
Cost	1		3.8		6.4	

- Nucleon charges at $t_{\text{sep}} = 8a$, $t_{\text{ins}} = 4a$
- All sources are distributed on 4 timeslices
- Gain = $\dfrac{\text{Decrease of } \sigma^2}{\text{Increase of Compt. cost}}$
- Between "28 LP + 4 Crxn" and "60 LP + 4 Crxn", cost and error scale in the same way, and the gain stays around 2
- We use "60 LP + 4 Crxn" setup

Hopping Parameter Expansion (HPE)

Hopping Parameter Expansion (Preconditioning)

$$M = \frac{1}{2\kappa}\left(\mathbb{1} - \kappa D\right)$$
$$M^{-1} = 2\kappa\mathbb{1} + 2\kappa^2 D + \kappa^2 D^2 M^{-1}$$

For disconnected quark loops, we need $\mathrm{Tr}\left[M^{-1}\Gamma\right]$

(1) $\mathrm{Tr}\left[2\kappa\Gamma\right]_{\Gamma=\mathbb{1}} = 24\kappa$

(2) $\mathrm{Tr}\left[2\kappa\Gamma\right]_{\Gamma\neq\mathbb{1}} = 0$

(3) $\mathrm{Tr}\left[2\kappa^2 D\Gamma\right] = 0$

- Need to calculate only $\kappa^2 D^2 M^{-1}$
- Removed noise from first two orders (1-3) – reduces error

Truncated Solver Method (TSM)

Truncated Solver Method (TSM)

[Bali, Collins and Schäfer, 2007]

$$M_E^{-1} = \underbrace{\frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} |s_i\rangle_{\text{LP}} \langle \eta_i|}_{\text{LP estimate}} + \underbrace{\frac{1}{N_{\text{HP}}} \sum_{i=N_{\text{LP}}+1}^{N_{\text{LP}}+N_{\text{HP}}} \Big(|s_i\rangle_{\text{HP}} - |s_i\rangle_{\text{LP}} \Big) \langle \eta_i|}_{\text{Crxn term}}$$

- Stochastic estimate of M^{-1}
- Same form as AMA
  - $C^{\text{2pt}} \Longrightarrow M^{-1}$
  - Sum over source positions $\Longrightarrow$ Sum over random noise sources
- $|\eta_i\rangle$: complex random noise vector satisfying

$$\frac{1}{N} \sum_{i=1}^{N} |\eta_i\rangle = \mathcal{O}\left(\frac{1}{\sqrt{N}}\right), \qquad \frac{1}{N} \sum_{i=1}^{N} |\eta_i\rangle\langle\eta_i| = \mathbb{1} + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

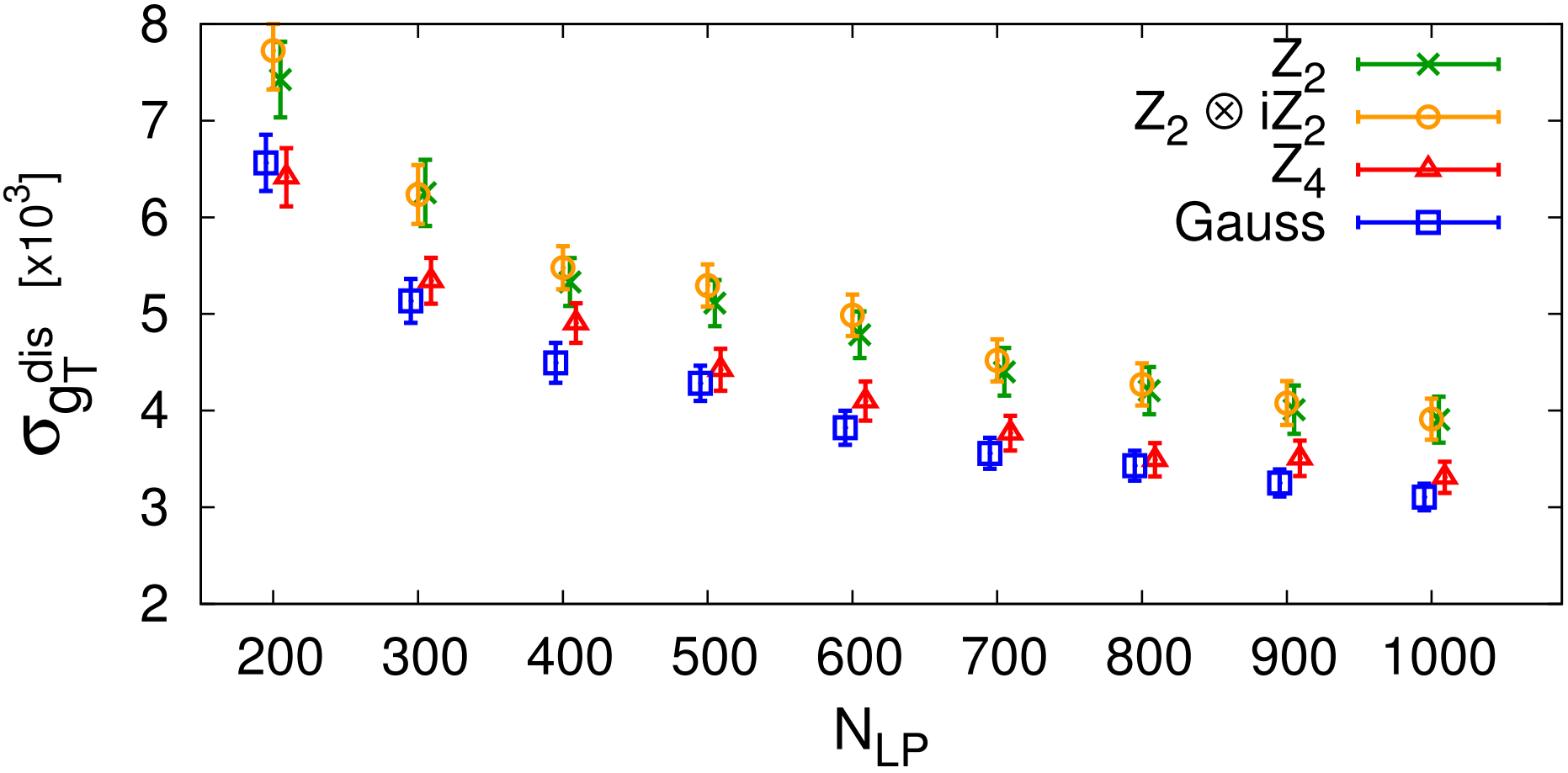
- $|s_i\rangle$: solution vector; $M|s_i\rangle = |\eta_i\rangle$

Random Noise

- Type of random noises investigated

Type	Noise	Condition
$\mathbb{Z}_2$	r_r	$r_r \in \{1, -1\}$
$\mathbb{Z}_2 \otimes i\mathbb{Z}_2$	$r_r + ir_i$	$r_{r,i} \in \{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$
$\mathbb{Z}_4$	r_c	$r_c \in \{1, i, -1, -i\}$
Gaussian	$r_r + ir_i$	$r_{r,i} \sim$ Gaussian $/\sqrt{2}$

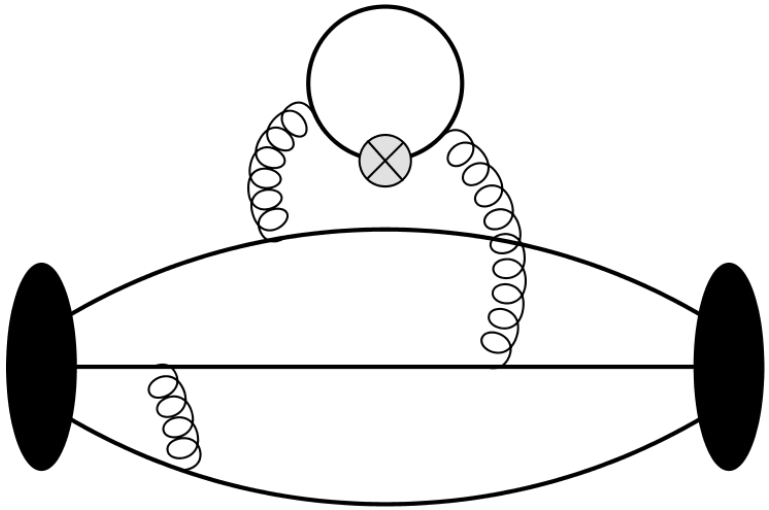
Comparing $\sigma_{g_T^{\text{dis}}}$ for Random Noises



- Statistical error in g_T at $t_{\text{sep}} = 8a$, $t_{\text{ins}} = 4a$ (mid point)
- Random noise sources are placed only on 8 timeslices (dilution)
- Gaussian random noise is (marginally) better than others

Required Number of Random Sources

$$M^{-1} \approx \frac{1}{N_{\mathsf{LP}}} \sum_{i=1}^{N_{\mathsf{LP}}} |s_i\rangle_{\mathsf{LP}} \langle \eta_i|$$

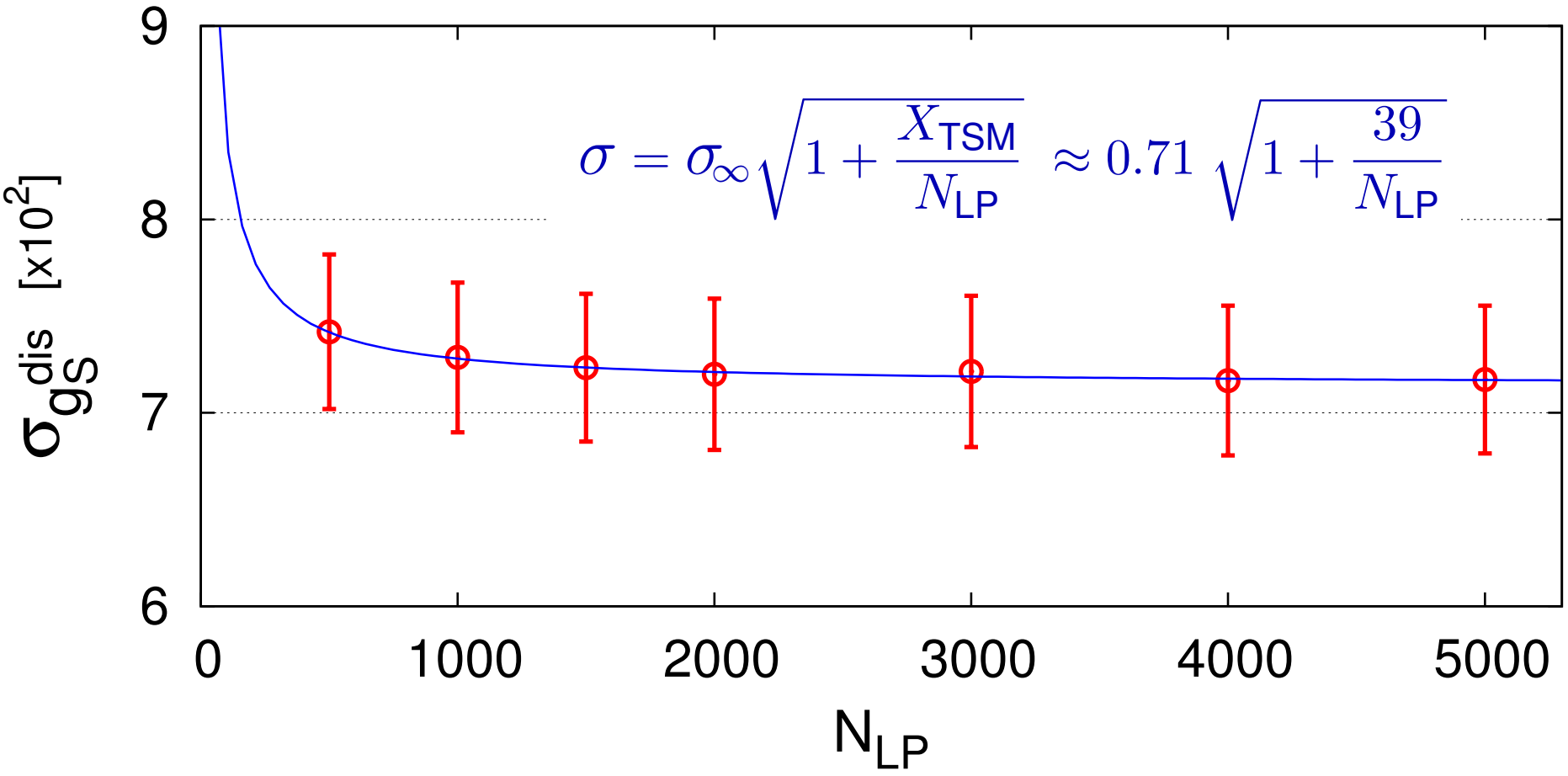


- Total statistical error scales as

$$\sigma_{\mathsf{tot}} = \sigma_\infty \sqrt{1 + \frac{X_{\mathsf{TSM}}}{N_{\mathsf{LP}}}}$$

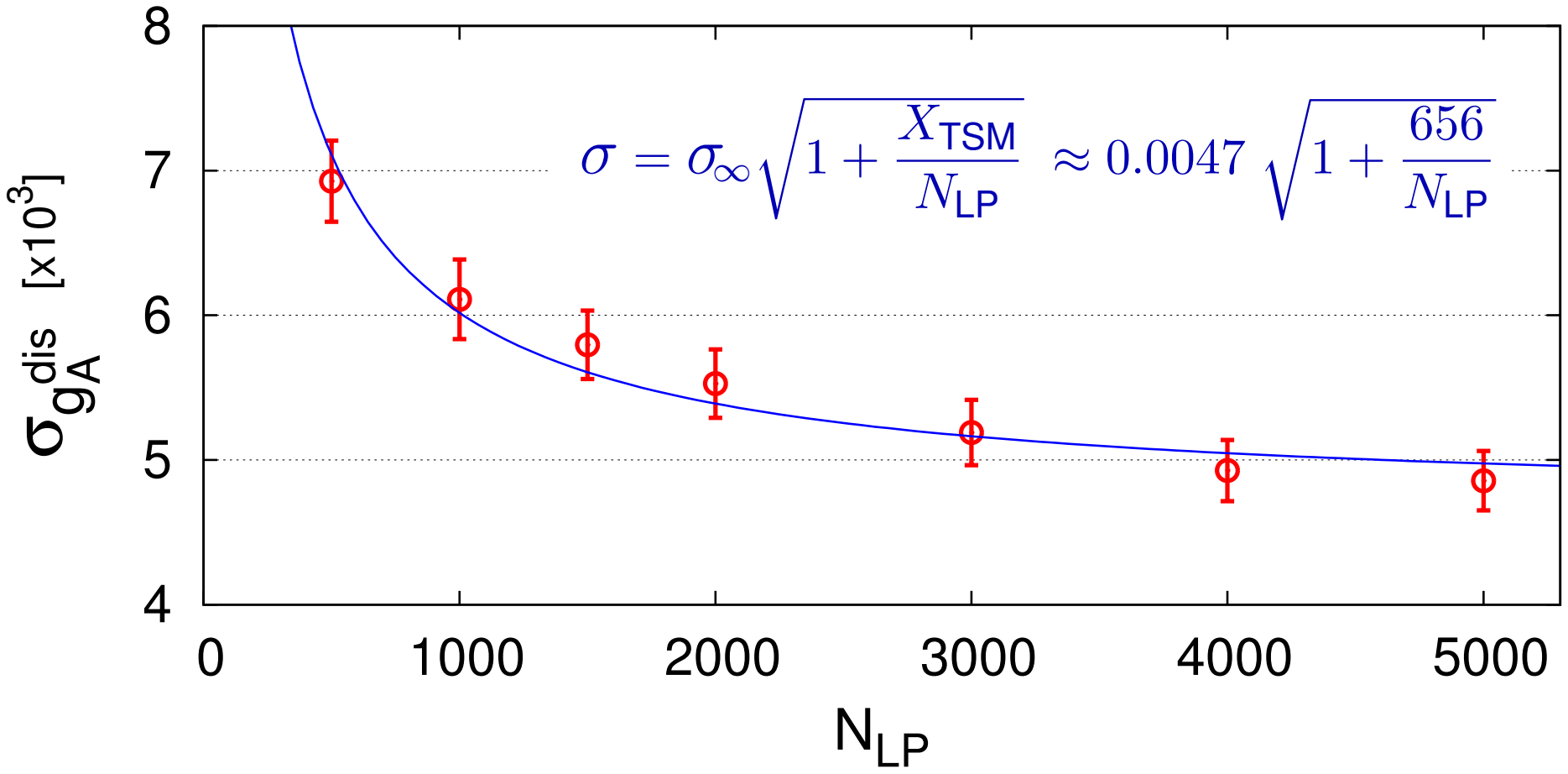
- X_{TSM} depends on TSM parameters, $N_{\mathsf{LP}}/N_{\mathsf{HP}}$ and r_{LP}

Required N_{LP} : g_S



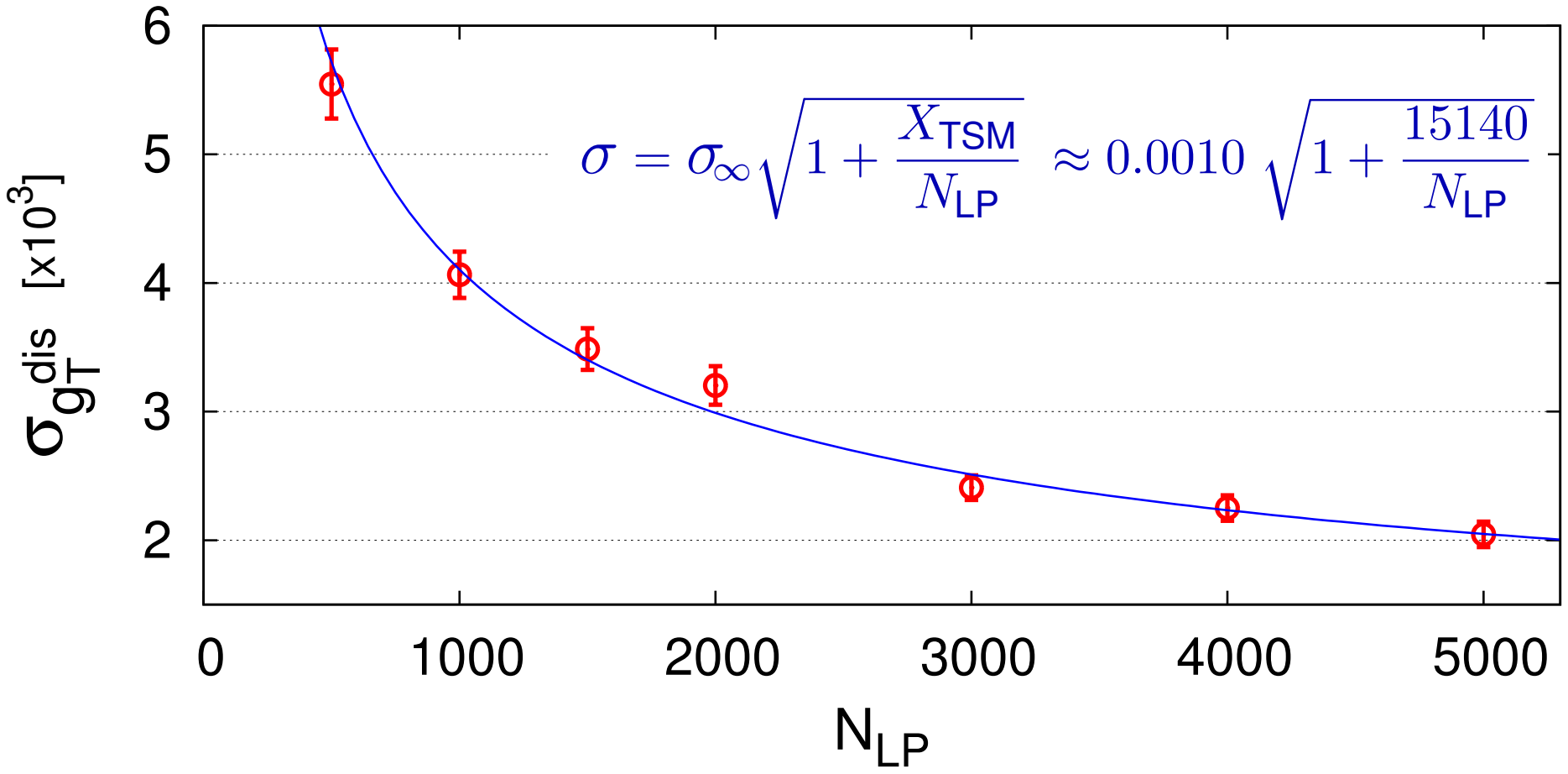
- Statistical error in g_S at $t_{\text{sep}} = 8a$, $t_{\text{ins}} = 4a$ (mid point)
- Noise srcs are placed on 44 timeslices that satisfy $|t_{\text{src}} - t| \geq 3a$
- Conclusion : $N_{\text{LP}} \approx 500$ is enough

Required N_{LP} : g_A



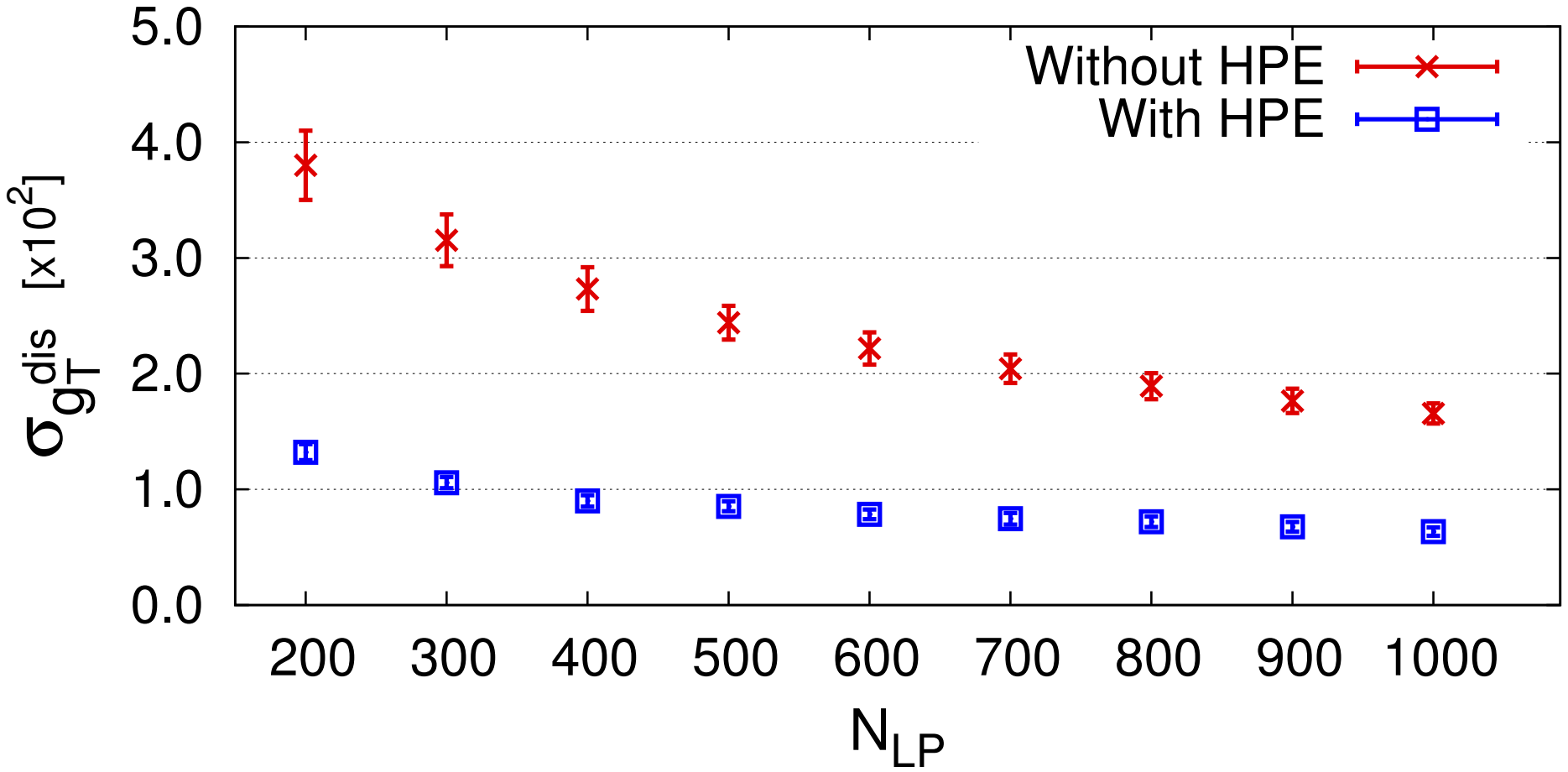
- Statistical error in g_A at $t_{\text{sep}} = 8a$, $t_{\text{ins}} = 4a$ (mid point)
- Noise srcs are placed on 44 timeslices that satisfy $|t_{\text{src}} - t| \geq 3a$
- Conclusion : $N_{\text{LP}} \approx 3000$ is enough

Required N_{LP} : g_T



- Statistical error in g_T at $t_{\text{sep}} = 8a$, $t_{\text{ins}} = 4a$ (mid point)
- Noise srcs are placed on 44 timeslices that satisfy $|t_{\text{src}} - t| \geq 3a$
- Conclusion : $N_{\text{LP}} \gtrsim 5000$ is needed

Effect of Hopping Parameter Expansion



Dilution

Dilution Technique

- Divide $\mathcal{R} = \{\text{spacetime} \otimes \text{color} \otimes \text{spin}\}$ into m subspaces $\mathcal{R}_j$

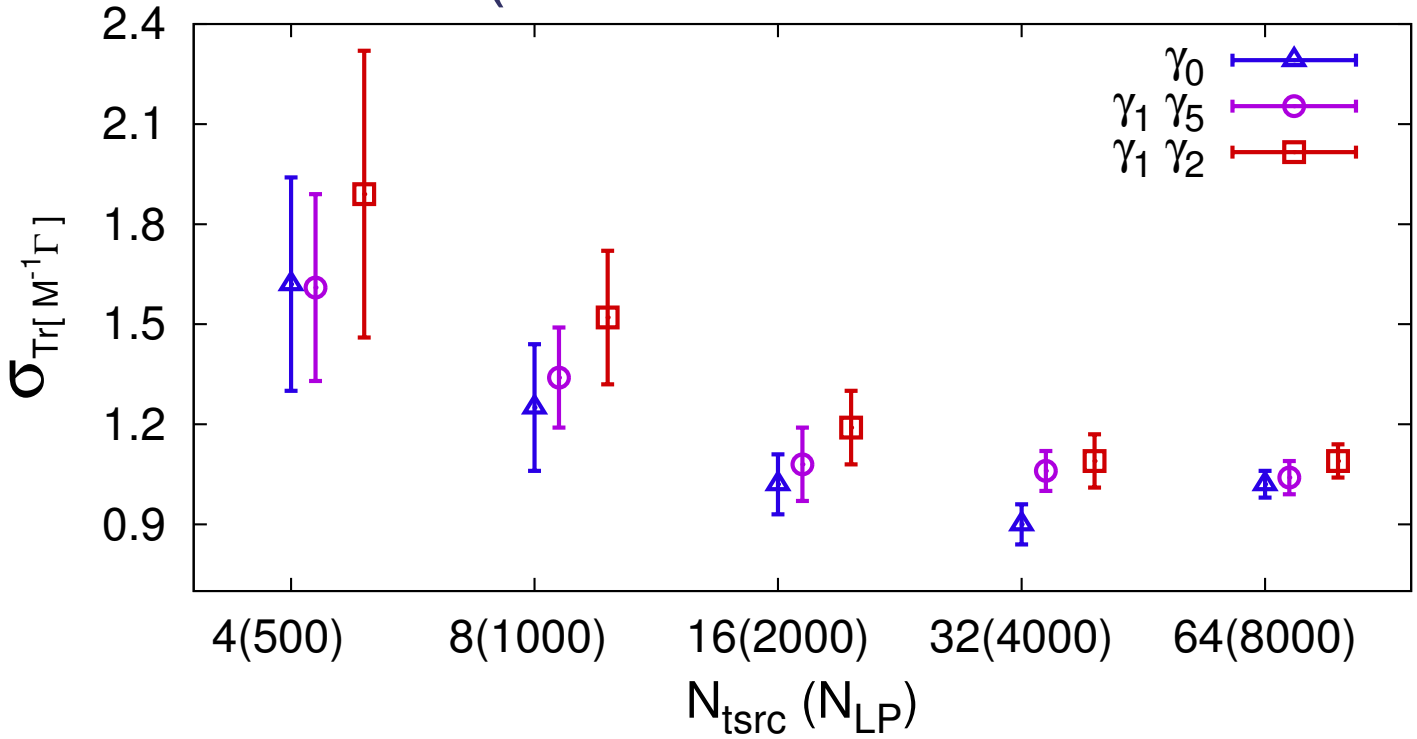
$$\mathcal{R} = \sum_{j=1}^{m} \mathcal{R}_j$$

- Sum M^{-1} evaluated on each subspaces

$$M^{-1} \approx \sum_{j=1}^{m} \left[\frac{1}{N} \sum_{i=1}^{N} |s_i\rangle\langle\eta_i| \right]$$

- If noise is reduced by more than $\sqrt{m}$, worth doing dilution

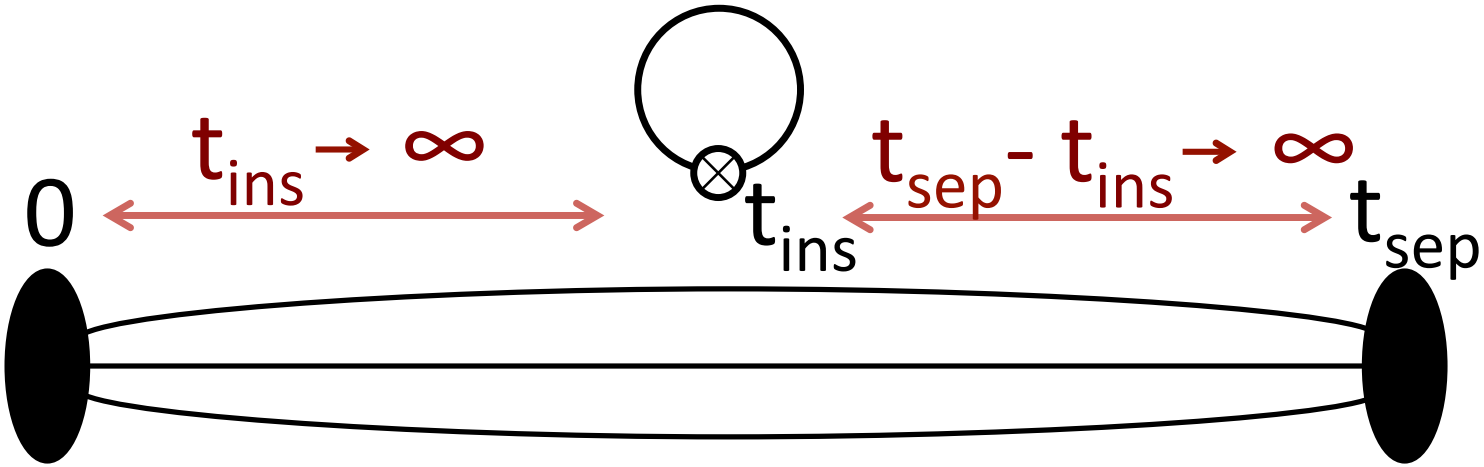
Testing Time Dilution (Gaussian Random Source)



- Statistical error of a disconnected quark loop with time Dilution for different number of source timeslices (N_{tsrc})
- e.g., if $N_{\text{tsrc}} = 32$, random noises are on 32 timeslices; cover all timeslices with two applications for a $24^3 \times 64$ Lattice
- Total computational cost is fixed ($N_{\text{LP}} \times 64/N_{\text{tsrc}}$ =fixed)
- Time dilution is not efficient!

Results

Removing Excited States Contamination



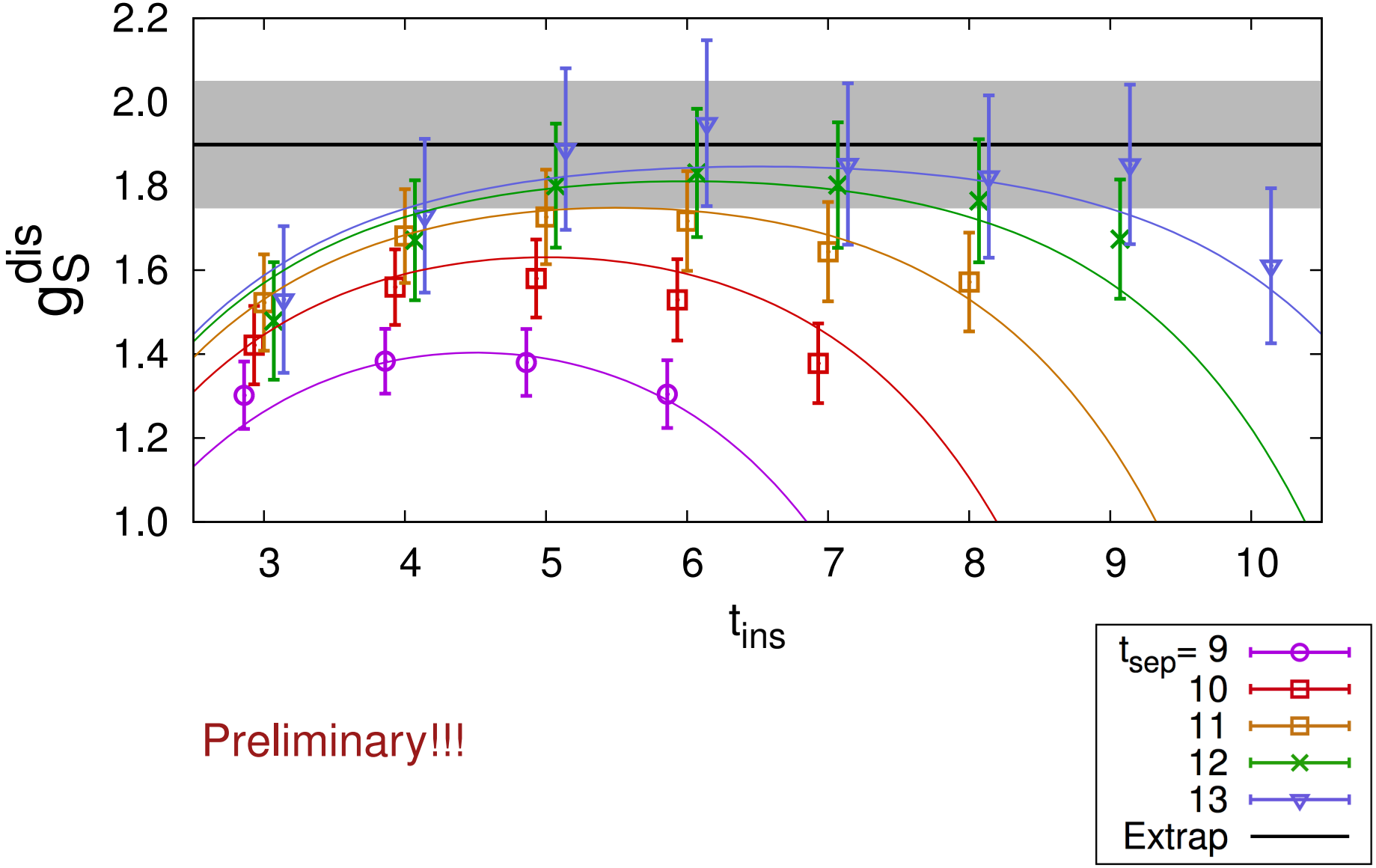
- Fitting functions include one excited state

$$C^{\text{2pt}}(t_{\text{sep}}) = A_1 e^{-M_0 t_{\text{sep}}} + A_2 e^{-M_1 t_{\text{sep}}}$$

$$C^{\text{3pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_1 e^{-M_0 t_{\text{sep}}} + B_2 e^{-M_1 t_{\text{sep}}} + B_{12}\left[e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}} - t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}} - t_{\text{ins}})}\right]$$

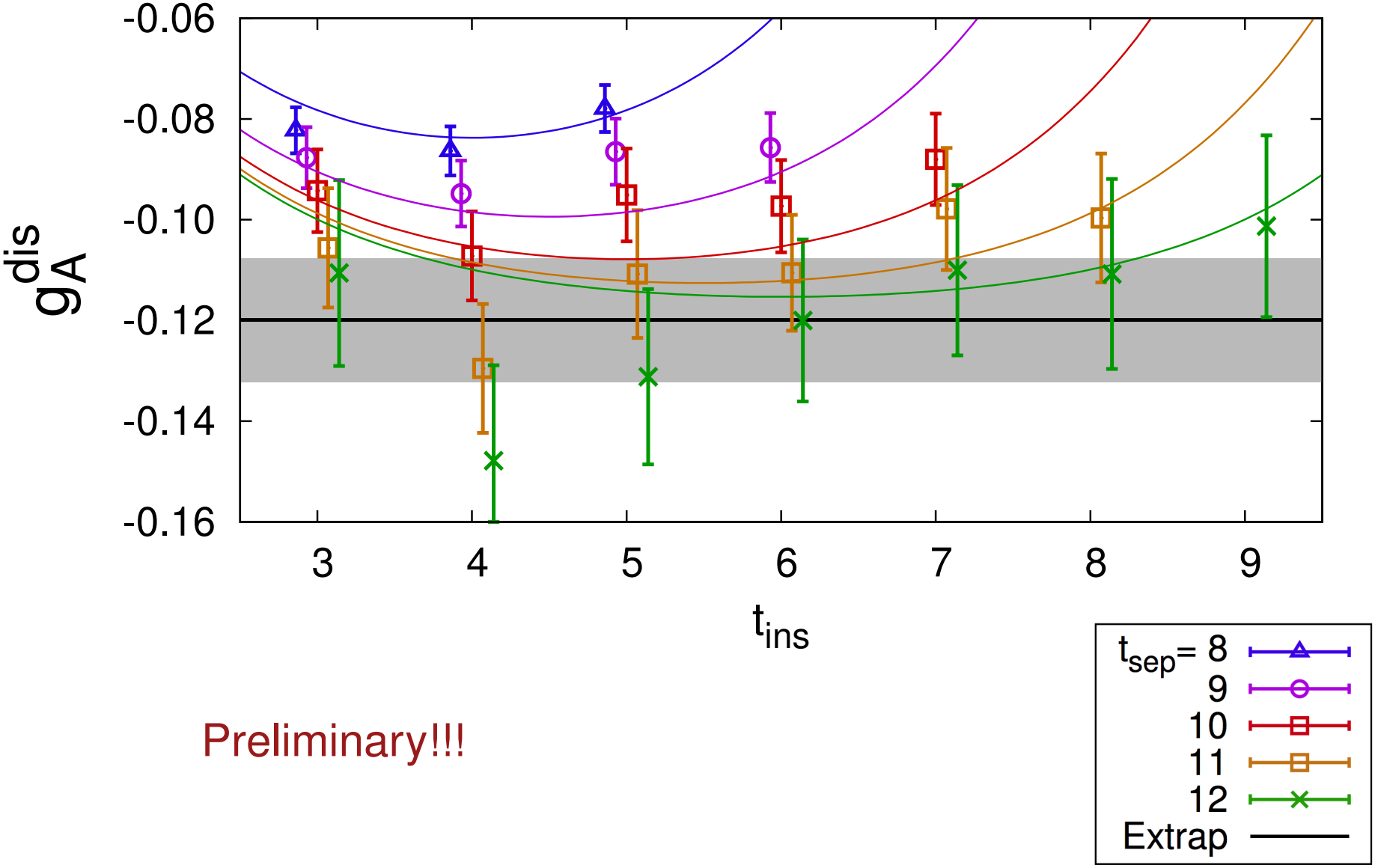
- A_1 and B_1 are the results for the ground state

g_S : t_{sep} and t_{ins} dependence



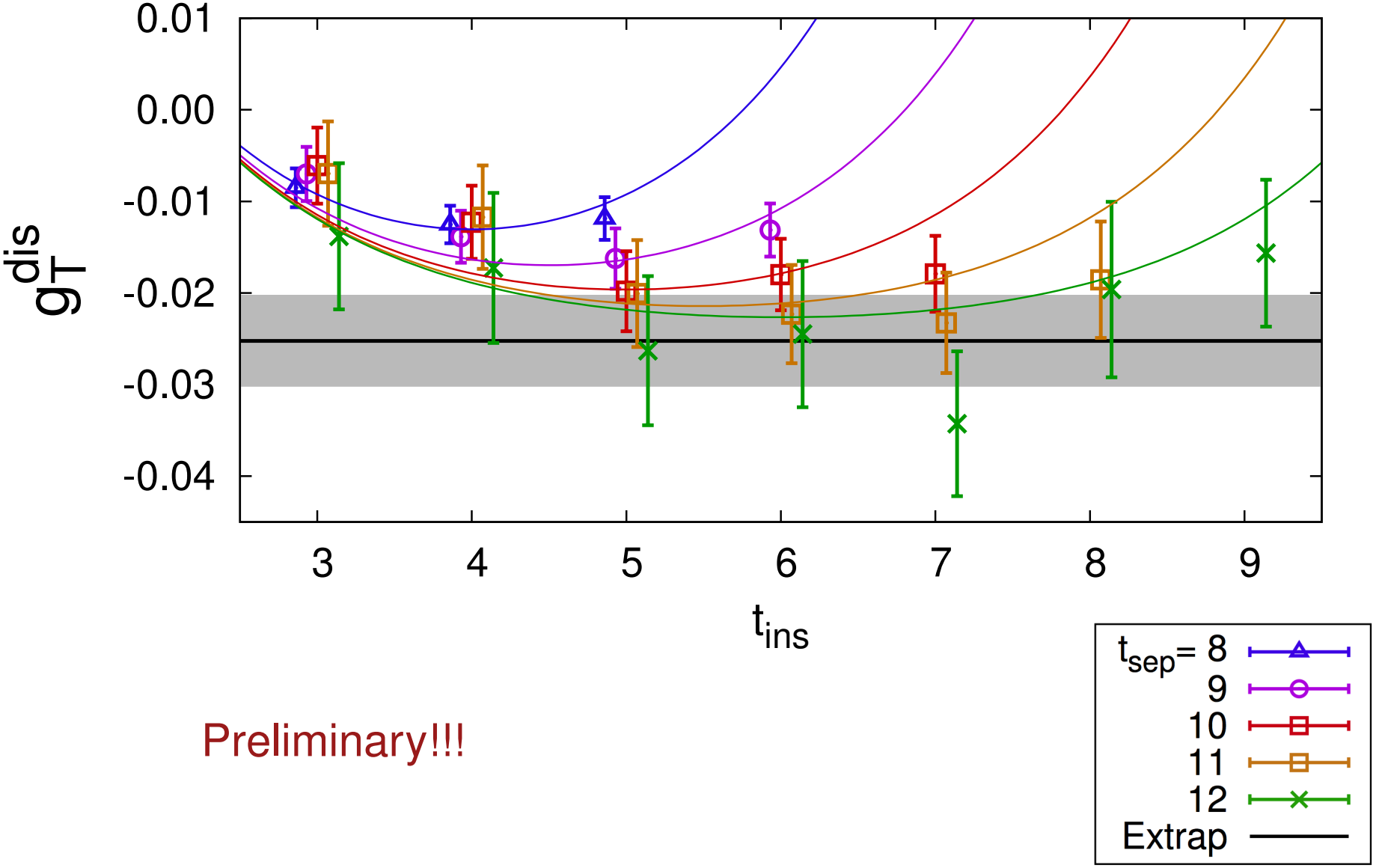
Preliminary!!!

g_A : t_{sep} and t_{ins} dependence



Preliminary!!!

g_T : t_{sep} and t_{ins} dependence



Preliminary!!!

Unrenormalized Isoscalar Nucleon Charges on $a = 0.12\,\mathrm{fm}$ and $m_\pi = 305\,\mathrm{MeV}$ Lattice

Preliminary!!!

	Disconnected	Connected[1]
g_S	1.90(15)	5.79(22)
g_A	−0.120(12)	0.632(36)
g_T	−0.0252(49)	0.650(24)

[1] PRD89 094502 (2014)

Renormalized Isoscalar Nucleon Charges

Preliminary!!!

	PNDME, LANL $a = 0.12\,\mathrm{fm}$, $m_\pi = 305\,\mathrm{MeV}$ Clover on HISQ		**Abdel-Rehim, *et al.*[2]** $a = 0.082\,\mathrm{fm}$, $m_\pi = 375\,\mathrm{MeV}$ Twisted Mass	
	Con.	Discon.	Con.	Discon.
g_S	5.16(24)	1.69(13)	6.30(27)	0.639(95)
g_A	0.610(37)	−0.116(12)	0.576(13)	−0.0699(89)
g_T	0.613(26)	−0.0238(46)	0.673(13)	−0.0016(14)

- Renormalization is done using only connected diagrams at $2\,\mathrm{GeV}$

[2] PRD89 034501 (2014)

Glance at the $a = 0.0888\,\mathrm{fm}$ and $m_\pi = 313\,\mathrm{MeV}$

Preliminary!!!

a	$0.09\,\mathrm{fm}$	$0.12\,\mathrm{fm}$
g_S^{dis}	1.43(12)	1.69(13)
g_A^{dis}	-0.107(16)	$-$0.116(12)
g_T^{dis}	-0.0114(51)	$-$0.0238(46)

- Renormalization is done using only connected diagrams at $2\,\mathrm{GeV}$

Isoscalar Nucleon Charges (con + disc)

Preliminary!!!

a	$0.09\,\mathrm{fm}$	$0.12\,\mathrm{fm}$
g_S	7.44(28)	6.91(26)
g_T	0.612(26)	0.594(27)

- Renormalization is done using only connected diagrams at $2\,\mathrm{GeV}$
- $m_\pi \approx 310\,\mathrm{MeV}$
- Statistical error only

Summary and Outlook

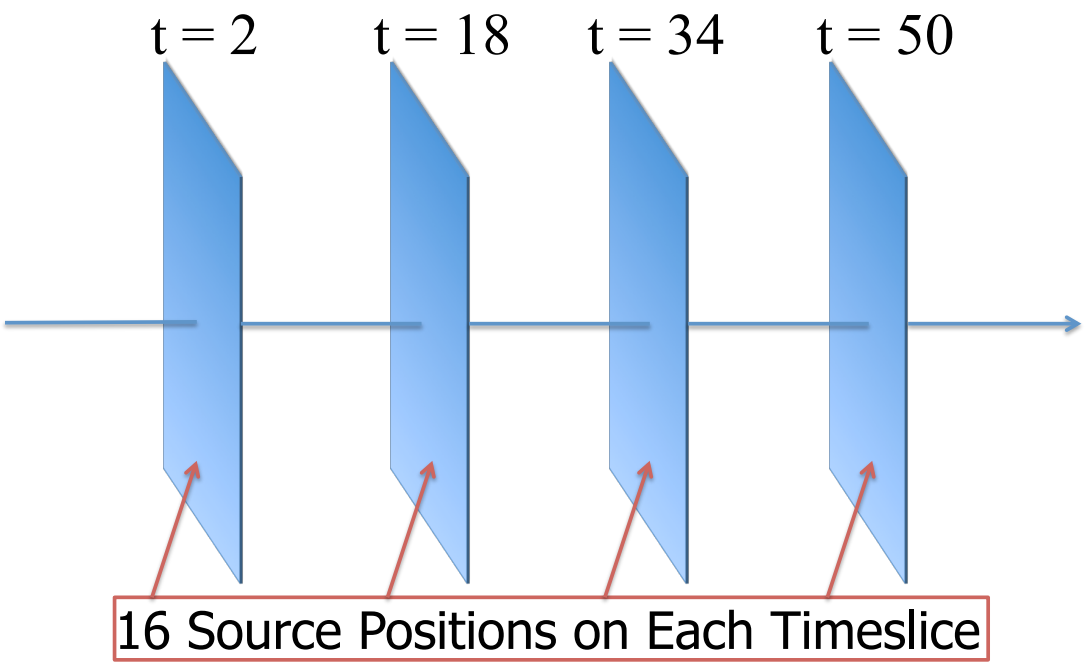
- We calculate disconnected contribution to nucleon charges
- Disconnected contribution is about
  30%, 20% and 4% for g_S, g_A and g_T, respectively
- All-mode averaging, Truncated solver method and Hopping parameter expansion are applied
- Disconnected contribution to the renormalization constants is under investigation
- Techniques can be applied to other physical objects

Computational Cost

2pt (37%)	64 LP (31%)	Src/Sink prep.	(18%)
		Inversion	(11%)
		etc.	(2%)
	4 HP (6%)	Src/Sink prep.	(2%)
		Inversion	(3%)
		etc.	(1%)
Disc. (63%)	5000 LP estimate (53%)	Inversion	(35%)
		Src prep/Trace/HPE	(18%)
	166 Correction (8%)	LP Inversion	(1%)
		HP Inversion	(6%)
		Src prep/Trace/HPE	(1%)
	etc. (2%)		

- Inverter Improvements: BiCGStab $\rightarrow$ Multigrid [x4 ; $m_\pi \approx 310\,\mathrm{MeV}$]
  HP $\rightarrow$ LP [x3 $\sim$ x5 ; $r_{\mathrm{LP}} = 0.001 \sim 0.005$]

64 source positions on a $24^3 \times 64$ Lattice

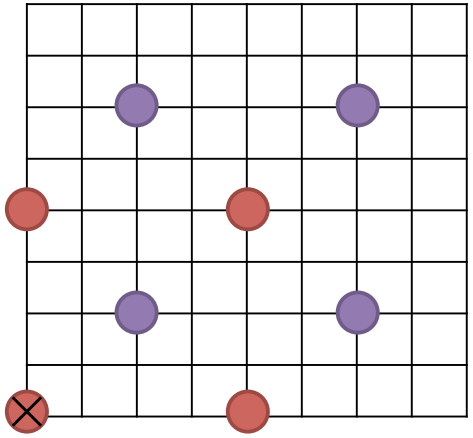


- On each timeslice t, 16 srcs are placed on

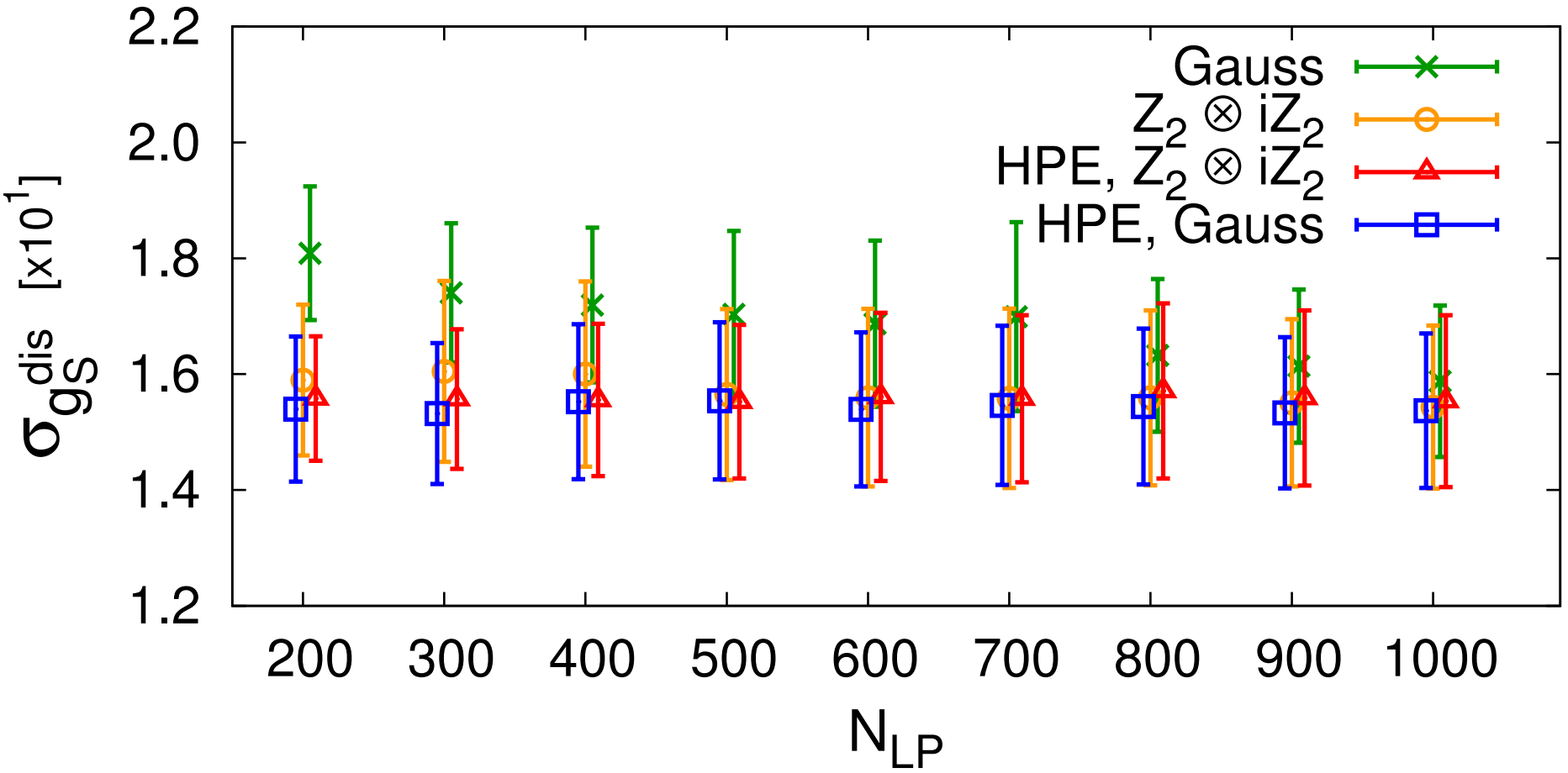
$$(x_1, y_1, z_1, t) \oplus (x_2, y_2, z_2, t)$$

$$x_1, y_1, z_1 \in \{0, \tfrac{L}{2}\}, \qquad x_2, y_2, z_2 \in \{\tfrac{L}{4}, \tfrac{3L}{4}\}$$

- On one of 16 source positions on a timeslice, calculate both HP and LP for correction term



Effect of Hopping Parameter Expansion



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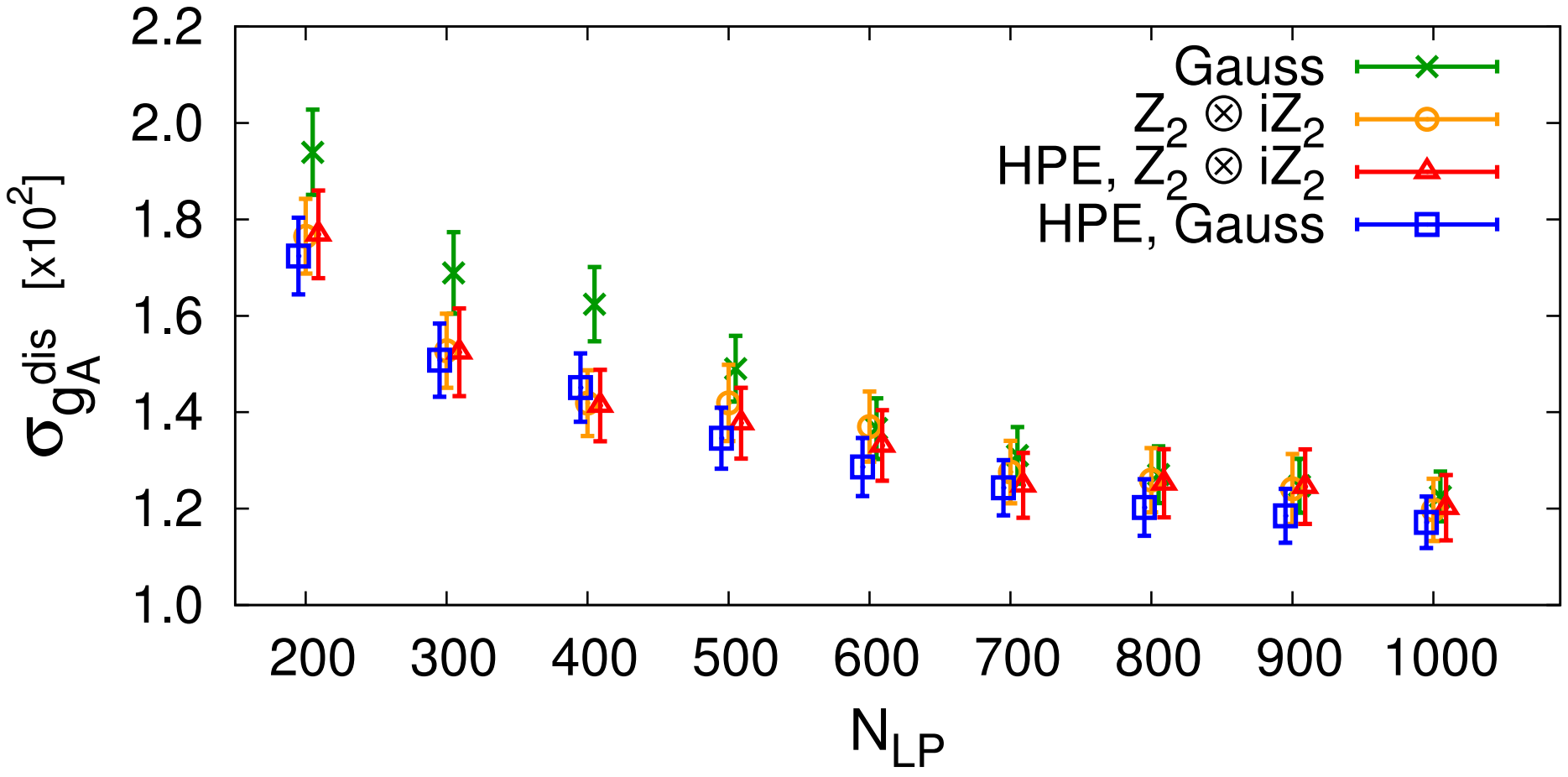

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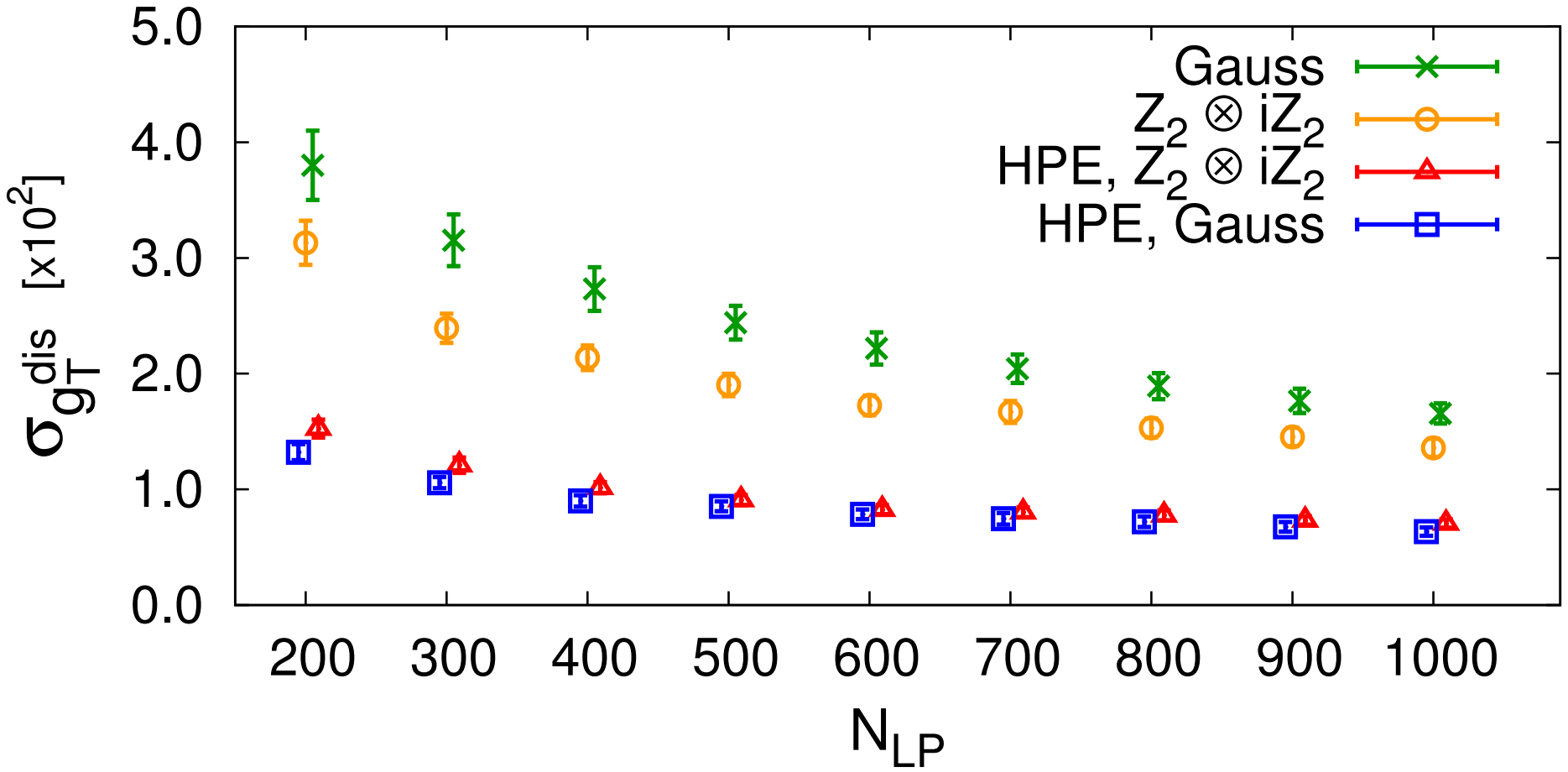
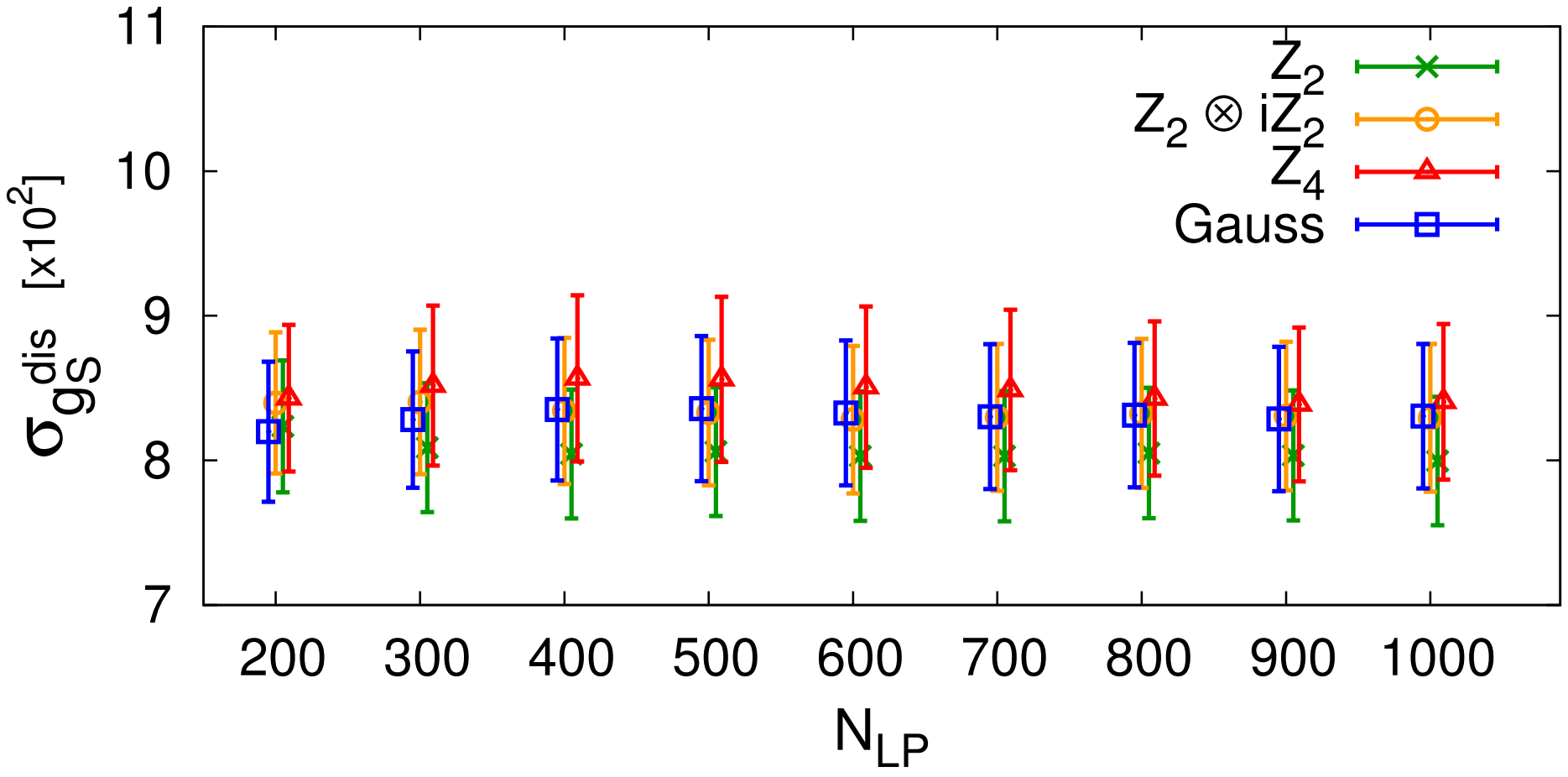
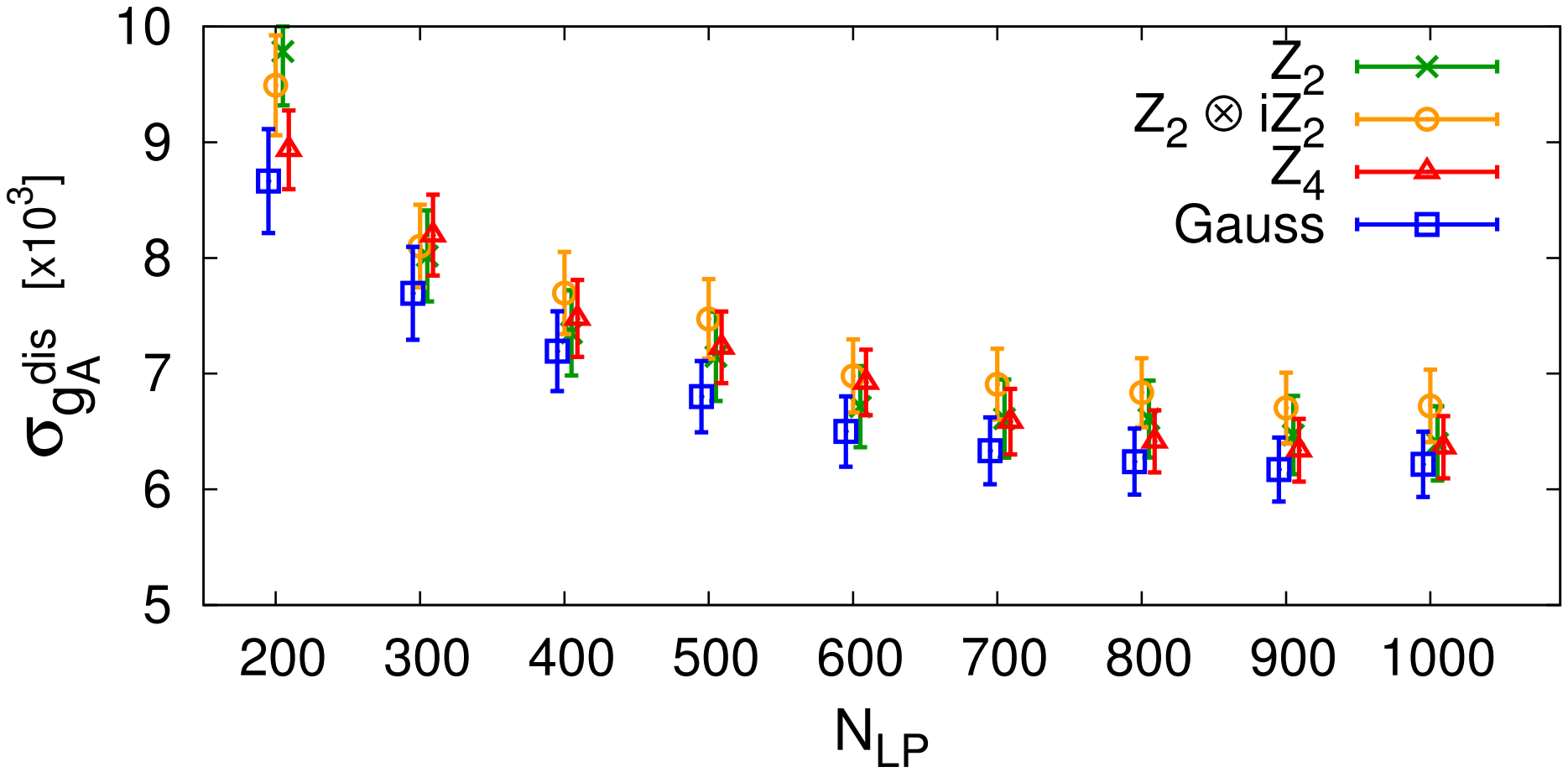


Random Noise Type Dependence



Random Noise Type Dependence



Random Noise Type Dependence