Effects of an external magnetic field on the QGP

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Motivation

The EOS of QGP in an external magnetic field is of interest to:

- Cosmology — Shortly after the big bang, strong magnetic fields of $O(10^{16}\,\mathrm{T})$ and higher may have existed as a result of the nonequilibrium dynamics of the electroweak phase transition, generation of topological defects and other phenomena. Effects on structure formation?

- Astrophysics — Very strong magnetic fields are also generated in the vicinity of magnetars $[O(10^{11}\,\mathrm{T})]$ and if these fields permeate the interior of such a star, they may affect the state of the high-density hadronic matter in its core and thus potentially influence the star's properties such as its temperature and diameter-to-mass ratio.

- Heavy-ion Collisions — The developing quark-gluon plasma is immersed in an external magnetic field, which is estimated to be of $O(10^{15}\,\mathrm{T})$. If such a strong magnetic field modifies the properties of the plasma, the particle spectra produced might also be affected.

The Taylor expansion method

- Expand the pressure in Taylor series with respect to the magnetic field.

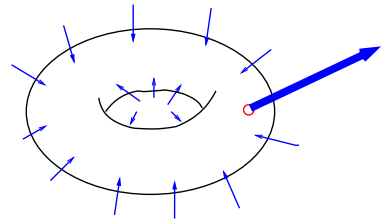
$$\frac{p(T)}{T^4} = \frac{\ln Z(B)}{T^3 V} = \sum_{n=0}^{\infty} C_n(T)(|e|B/T^2)^n,$$

$$C_n(T) = \frac{L_t^3}{L_s^3}\frac{1}{n!}\frac{\partial^n \ln Z(B)}{\partial(|e|B/T^2)^n}\bigg|_{B=0}.$$

Due to CP symmetry only even terms are nonzero.

- Advantages: Computationally cheaper than other methods (to 4th order at least) — no need to generate ensembles with explicit magnetic fields for each B studied. Can be applied to other observables as well. Needs $T = 0$ subtraction only for the C_2 coefficient. No pressure anisotropy (G.S. Bali et al. arXiv:1303.1328).
- But: Need to study the radius of convergence. Very large magnetic fields may require high orders in the expansion.

Quantized magnetic field on a torus



- Let $\vec{B} = B\hat{z}$. The magnetic field quantization (quark charge $|q| = |e|/3$) is:

$$|q|B = 2\pi b/(L_x L_y a^2), \qquad 0 < b < L_x L_y/2.$$

- Chose a continuum vector potential:

$$A_y = Bx, \qquad A_\mu = 0 \ \text{ for } \ \mu = x, z, t.$$

- Lattice $U(1)$ links choice:

$$\begin{aligned} u_y(B, q, X) &= e^{ia^2 qBx} \\ u_{x,z,t}(B, q, X) &= \begin{cases} 1 & \text{for } x \in [0, L_x - 2] \\ e^{-ia^2 qBL_x y} & \text{for } x = L_x - 1 \end{cases} \end{aligned}$$

- Quantization is problematic for a Taylor expansion.

The half-and-half field configuration

- Instead of working with a quantized magnetic field we change the field configuration so that quantization is not necessary:

$$B\hat{z}(x) = \begin{cases} +B & \text{for } x < L_x/2 \\ -B & \text{for } x \geq L_x/2 \end{cases}$$

- The flux through half of the hypersurface comes out from the other half. The configuration is CP-invariant.

- The changes in the field direction induce a surface effect $\sim O(1/L_s)$.

- Generally, for suitably large volumes, the pressure should not be strongly affected by the surface effect (need more extensive tests for this statement). However, the size of the finite volume effects may differ for the different Taylor coefficients; the expectation is that higher orders are affected more strongly.

Choosing the vector potential

- We have freedom in how to chose the vector potential for the same resulting magnetic field.

- The statistical noise in the measured observables is strongly influenced by the choice of vector potential. The minimal noise configuration we work with (up to a lattice translation) is:

$$
\begin{aligned}
u_y(B,q,X) &= e^{ia^2qB(x-L_x/4)}, \quad x\leq L_x/2 \\
u_y(B,q,X) &= e^{ia^2qB(3L_x/4-x)}, \quad x>L_x/2 \\
u_{x,z,t}(B,q,X) &= 1
\end{aligned}
$$

  Statistical noise is reduced by a sizable factor when compared with other choices.

Analytic framework

- Partition function for $2+1$ flavors

$$Z(B) = \int dU e^{-S_g} e^{\frac{1}{4}\ln\det M_u(B,q_u)}\, e^{\frac{1}{4}\ln\det M_d(B,q_d)}\, e^{\frac{1}{4}\ln\det M_s(B,q_s)}.$$

- HISQ/asqtad fermion matrix for flavor f:

$$M^f_{X,Y}(B,q_f) = am_f\delta_{X,Y} + D^{z,t,x}_{X,Y} + D^y_{X,Y}(B,q_f),$$

where the B-independent term $D^{z,t,x}_{X,Y}$ is a sum of the Dirac operators in the x, z and t directions at all points.

- It is convenient to define the observable:

$$A_{nml} = \frac{1}{q_u^n q_d^m q_s^l}\left\langle e^{-U}e^{-D}e^{-S}\frac{\partial^n e^U}{\partial(a^2B)^n}\frac{\partial^m e^D}{\partial(a^2B)^m}\frac{\partial^l e^S}{\partial(a^2B)^l}\right\rangle,$$

with $U = \ln\det M^u(q_u,B)/4$, $D = \ln\det M^d(q_d,B)/4$, $S = \ln\det M^s(q_s,B)/4$.

Analytic framework continued

- Then

$$C_2 = \frac{1}{2L_tL_s^3}\left[(q_u^2+q_d^2)A_{200}+q_s^2A_{002}+2q_uq_dA_{110}+2(q_u+q_d)q_sA_{101}\right]$$

$$\begin{aligned} C_4 &= \frac{1}{4!}\frac{1}{L_t^5L_s^3}\left[(q_u^4+q_d^4)A_{400}+q_s^4A_{004})+12q_uq_dq_s(q_sA_{112}+(q_u+q_d)A_{121})+6(q_u^2q_d^2A_{220}+(q_u^2+q_d^2)qs^2A_{022})\right. \\ &+ \left.4((q_u^3q_d+q_d^3q_u)A_{310}+(q_u^3+q_d^3)q_sA_{301}+(q_u+q_d)q_s^3A_{103})-3(2L_tL_s^3C_2)^2\right]. \end{aligned}$$

- To calculate A_{mnl} we need the derivatives $\partial^n \ln\det M_f/(\partial a^2B)^n$, which are computed in terms of derivatives of the fermion matrix:

$$\begin{aligned} \left.\frac{\partial^n M^f(B,q_f)}{\partial a^2B^n}\right|_{B=0} &= \frac{1}{2}\eta_y(X)\left[(iq_fx')^nU_y^{(F)}(X)\delta_{X+\hat{y},Y}\right. \\ &+ \left.(3iq_fx')^nU_y^{(L)}(X)\delta_{X+3\hat{y},Y}-h.c.\right]. \end{aligned}$$

- The derivatives are calculated using stochastic estimators.

Issues of renormalization and vacuum pressure

- An external magnetic field results in the appearance of vacuum pressure. Considering that the HIC plasma is probably in an environment where the B field is nonzero both in- and outside the plasma, the vacuum pressure may not play a large role in the plasma expansion. Thus we consider only the thermal contribution of the magnetic field to the pressure.

$$\begin{aligned}\Delta p(B,T) &= p(B,T) - p(0,T) - p(B,0) + p(0,0) \\ &= C_2^r(T)(eB)^2 + C_4^r(T)(eB)^4/T^4 + \dots\end{aligned}$$

  where $C_n^r(T) = C_n(T) - C_n(0)$.

- For the C_2^r coefficient, the subtraction renormalizes the electric charge. There is no vacuum pressure contribution. C_2^r is entirely of thermal origin.

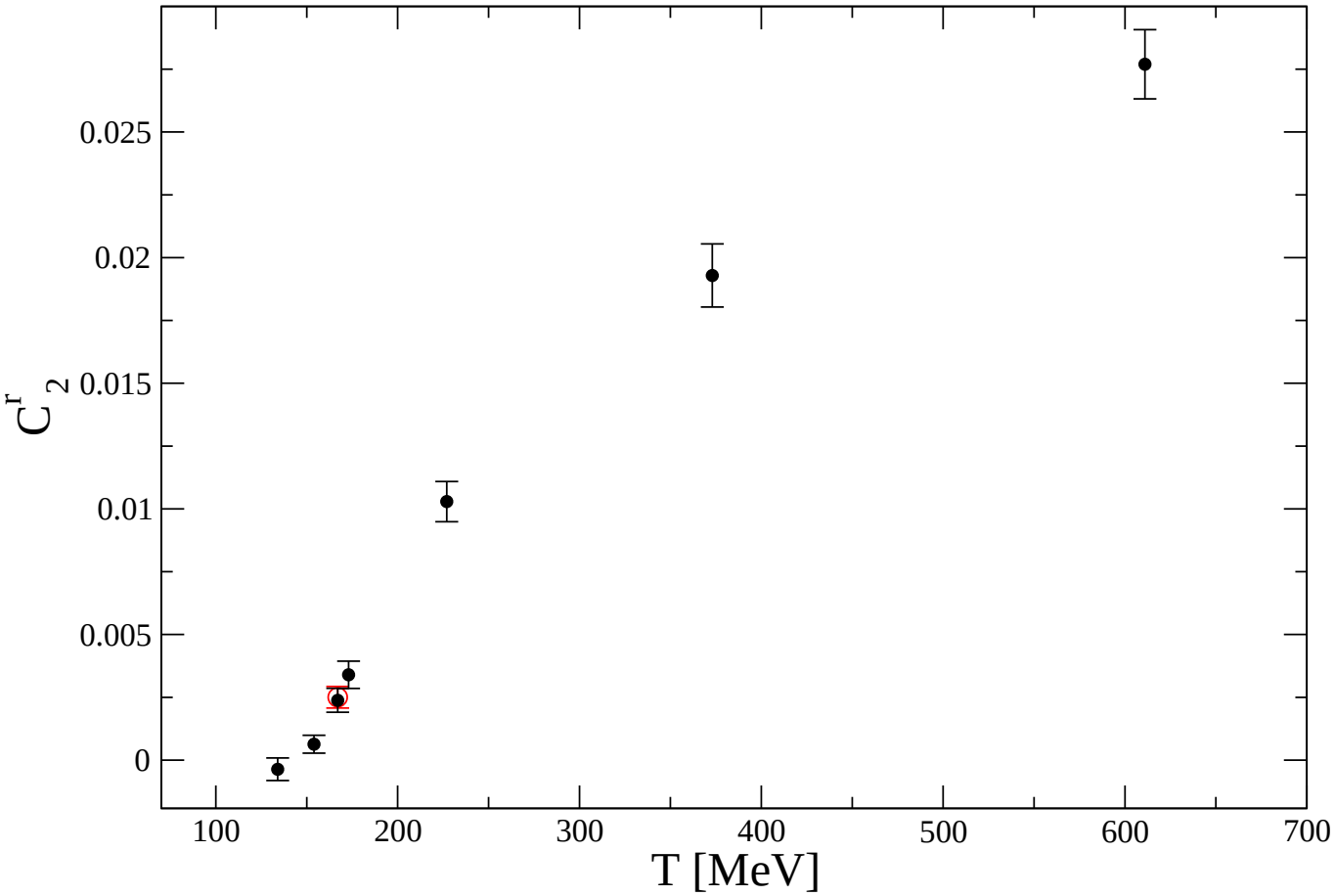
- The C_4^r coefficient doesn't need renormalization; the subtraction removes the 4th order contribution of the vacuum pressure.

Simulation details

- We use $2+1$ flavor HISQ plus tree-level Symanzik gauge action (HotQCD project).
- Follow the $m_l = 0.05 m_s$ line of constant physics at fixed $N_t = 8$.
- Prelimanary results to $O(2)$ were published in PRL arXiv:1309.1142.
- We extended the study to $O(4)$ and increased the statistics at $T \neq 0$.
- Cost so far: ∼440 000 GPU-hours using QUDA. (20% went into checking for finite volume effects.)

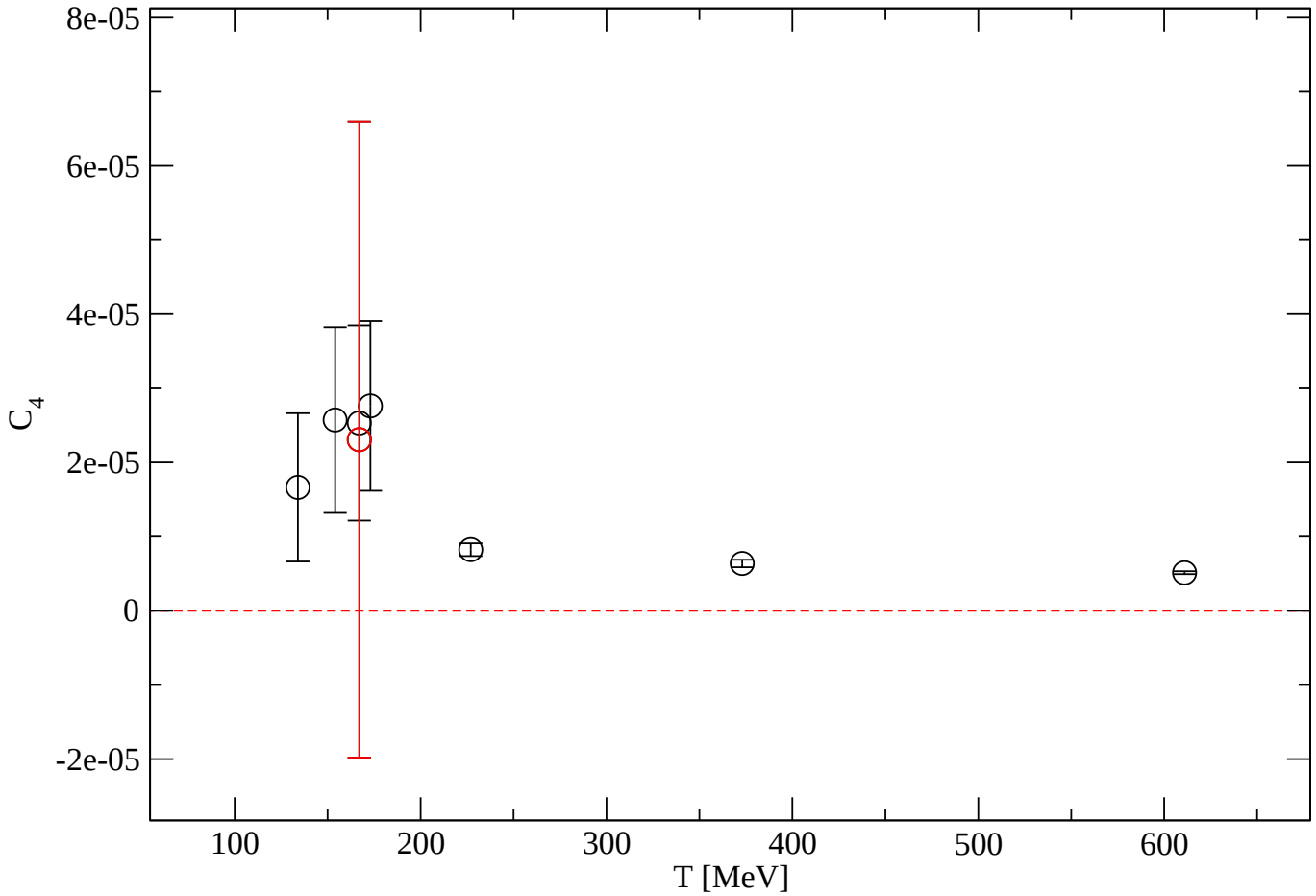
T [MeV]	β	m_l/m_s	$V_{T\neq 0}$	$V_{T=0}$	Random sources		Configurations		$C_2^r \times 10^{-3}$	$C_4 \times 10^{-5}$
					$T \neq 0$	$T = 0$	$T \neq 0$	$T = 0$		
134	6.195	0.00440/0.0880	$32^3 \times 8$	$32^3 \times 32$	4800	400	210	50	−0.4(4)	1.7(1.0)
154	6.341	0.00370/0.0740	$32^3 \times 8$	$32^3 \times 32$	4800	500	200	50	0.6(4)	2.6(1.3)
167	6.423	0.00335/0.0670	$32^3 \times 8$	$32^3 \times 32$	2400	200	420	50	2.4(5)	2.5(1.3)
167	6.423	0.00335/0.0670	$48^3 \times 8$	$48^3 \times 48$	4800	400	150	70	2.5(4)	2.3(4.2)
173	6.460	0.00320/0.0640	$32^3 \times 8$	$32^3 \times 64$	2400	200	100	60	3.4(5)	2.8(1.1)
227	6.740	0.00238/0.0476	$32^3 \times 8$	$48^3 \times 48$	2400	200	50	50	10.3(8)	0.82(9)
373	7.280	0.00142/0.0284	$32^3 \times 8$	$48^3 \times 64$	1200	40	50	50	19.3(1.3)	0.64(5)
611	7.825	0.00082/0.0164	$32^3 \times 8$	$64^3 \times 64$	1200	40	50	50	27.7(1.4)	0.51(2)

The magnetic susceptibility C_2^r



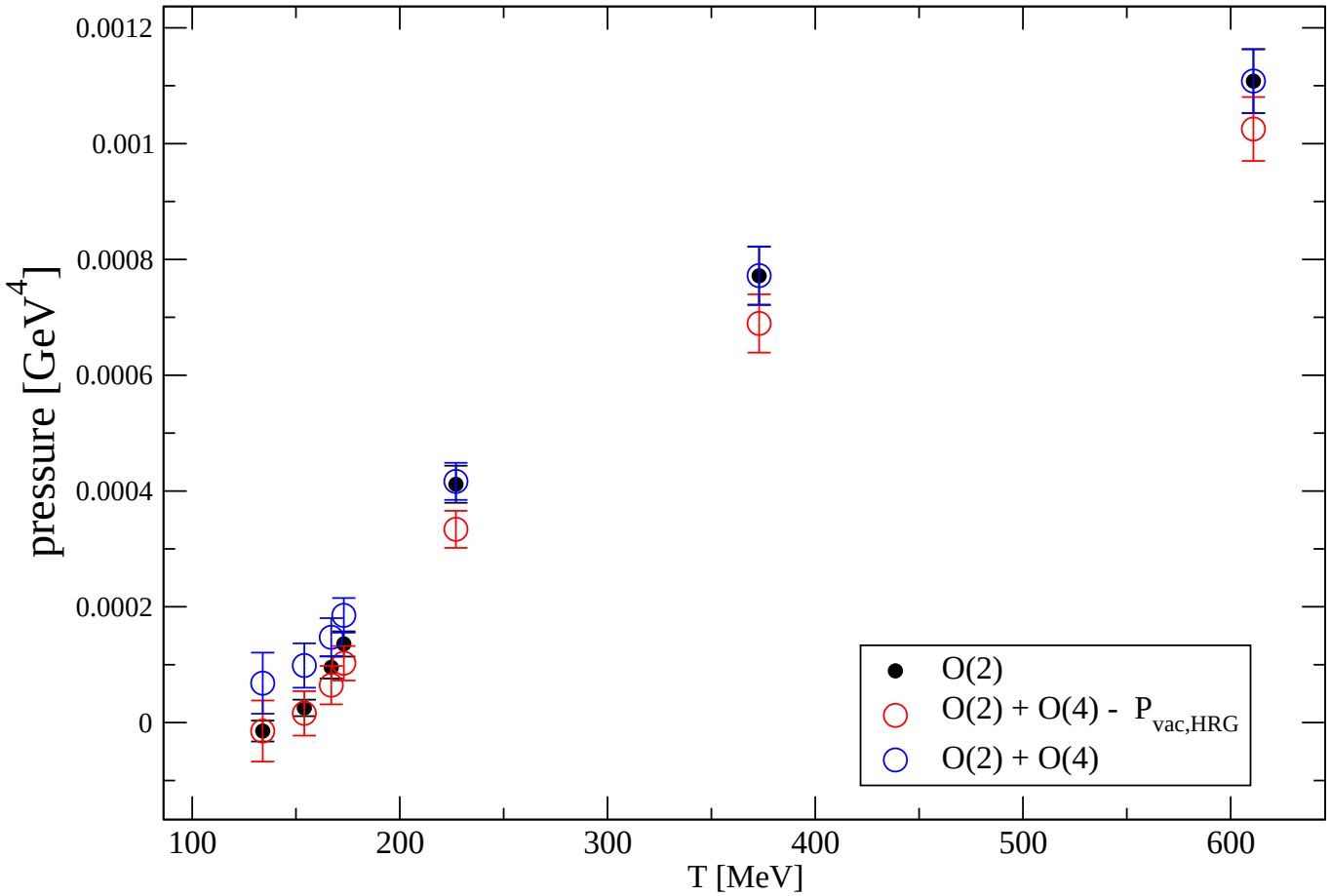
- Red circle shows larger volume result.
- QGP is paramagnetic above the transition ($C_2^r > 0$).

The C_4 coefficient



- Red circle shows larger volume result.
- C_4 shows a small peak in the transition region and large statistical fluctuations.

The pressure at $eB = 0.2$ GeV^2



- $P_{\rm vac,HRG}$ at $B = 0.2$ GeV^2 is taken from Endrödi arXiv:1301.1307.
- With or without the vacuum pressure, the pressure contribution due to the magnetic field is within a few percent of the full QGP pressure for field strengths relevant for HIC.
- But, the $O(4)$ contribution grows as $(eB/T)^4$ when $eB >> T$.

Conclusions

- The Taylor expansion combined with the half-and-half magnetic field configuration is a computationaly cheaper alternative to the current methods using quantized magnetic fields.

- We have performed a calculation to $O(4)$ on 2+1 flavor HISQ/tree ensembles with $m_l = 0.05 m_s$.

- At magnetic fields relevant for HIC [$eB = 0.2\ \mathrm{GeV}^2$] the 4th order contribution is small (within statistical errors for higher temperatures). The $O(4)+O(2)$ contribution to the QGP pressure is within a few percent.

- Still need larger statistics to determine the finite volume effects for the C_4 coefficient.