The Kaon Semileptonic Form Factor from Domain Wall QCD at the Physical Point

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for the RBC/UKQCD Collaboration

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Outline

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Motivation

- $K \to \pi l \nu$ decay gives most precise direct experimental constraint on CKM matrix element V_{us}
- Measurements of decay rate constrain [M. Antonelli et al., Eur.Phys.J. C69 (2010)]

$$f_+^{K\pi}(0)|V_{us}| = 0.2163(5)$$

- Requires lattice calculation of non-perturbative form factor $f_+^{K\pi}(0)$
  - Largest source of uncertainty
- Precision test of Standard Model unitarity

$$\delta_{CKM} = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$$

- Goal: $\sim 0.3\%$ lat. error on $f_+^{K\pi}(0)$
- With recent physical point MILC result error from lattice calculation is approaching experimental error [A. Bazavov et al., Phys.Rev.Lett. 112 (2014)]

$K \to \pi l \nu$ Decay in the Standard Model

Standard Model Results:

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 |V_{us}|^2 m_K^5}{128\pi^3} \left| f_+^{K\pi}(0) \right|^2 I_{Kl} S_{EW} \left(1 + \delta_{SU(2)} + \delta_{EM}\right) \quad (1)$$

$$\langle \pi(p_\pi) | V_\mu | K(p_K) \rangle = f_+^{K\pi}(q^2) \left(p_K + p_\pi\right)_\mu + f_-^{K\pi}(q^2) \left(p_K - p_\pi\right)_\mu \quad (2)$$

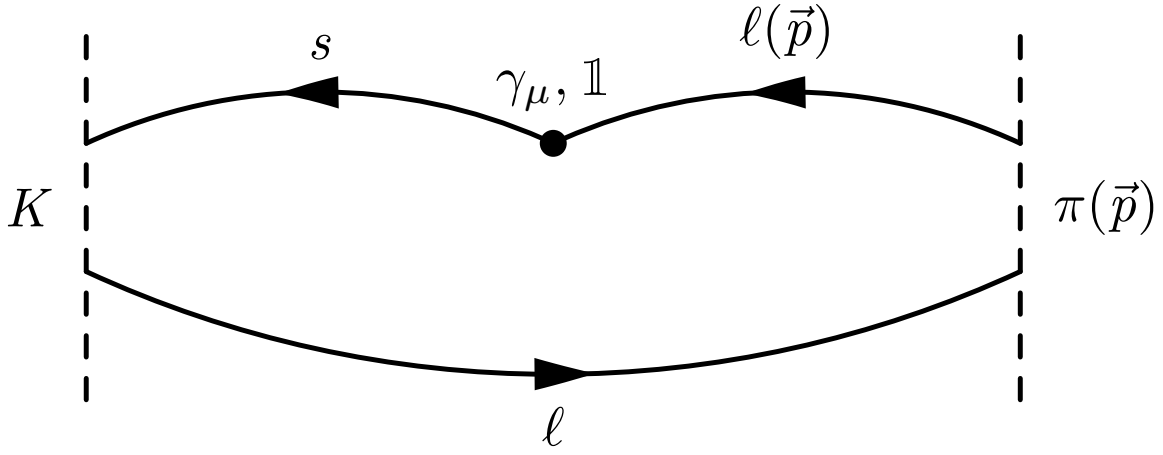
$$\langle \pi(p_\pi) | S | K(p_K) \rangle |_{q^2=0} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_+^{K\pi}(0) \quad (3)$$

- Lattice: $f_+^{K\pi}(0)$
- Theory: $\delta_{SU(2)}$, δ_{EM}, S_{EW} (SU(2)-breaking, electromagnetic, and short-distance electroweak corrections)
- Experiment: I_{Kl} (phase-space integral)
- $q = p_K - p_\pi$ (momentum transfer)
- $V_\mu = \bar{s}\gamma_\mu u$ (flavor-changing vector current)
- $S = \bar{s}u$ (flavor-changing scalar density)

Outline

Lattice Calculation for Physical Point Ensembles

- Focus on calculation for new RBC/UKQCD physical pion mass ensembles:
  - $48^3 \times 96 \times 24$ (48I): $a^{-1} = 1.729(4)$ GeV
  - $64^3 \times 128 \times 16$ (64I): $a^{-1} = 2.358(7)$ GeV
- Lattice setup:
  - $N_f = 2 + 1$ Möbius Domain Wall fermions
  - Iwasaki gauge action
  - All-mode averaging
  - Twisted boundary conditions
  - Coulomb gauge-fixed wall source/sink for K_{l3} 3-point correlation functions



- ℓ: light quark(s)
- s: strange quark
- $\vec{p}$: twist momentum

Vector Current Renormalization

- In DWF simulations we have
  - Exactly conserved, point-split 5-d vector current $\mathscr{V}_\mu^a(x)$
  - Local 4-d vector current $V_\mu^a(x)$
- Latter is computationally cheaper, but needs to be renormalized
- We calculate Z_V from the pion and kaon electromagnetic form factors
- Observe $\sim 0.5\%$ dependence on flavor content of current

Ensemble	Z_V^π	Z_V^K
48I	0.71094(20)	0.71407(9)
64I	0.74295(18)	0.74517(5)

- Must agree in $a^2 \to 0$ limit

All-Mode Averaging [Blum et al., Phys.Rev. D88 (2013)]

- Most expensive part of calculation is light quark propagator inversion
- Idea is to decompose lattice observable $\mathcal{O}(S_F)$ into
  - Cheap part, calculated with inexact fermion propagators (S'_F):

$$\mathcal{O}' = \mathcal{O}(S'_F)$$

  - Expensive part, calculated with exact fermion propagators (S_F):

$$\Delta\mathcal{O} = \mathcal{O}(S_F) - \mathcal{O}(S'_F)$$

- AMA estimate is

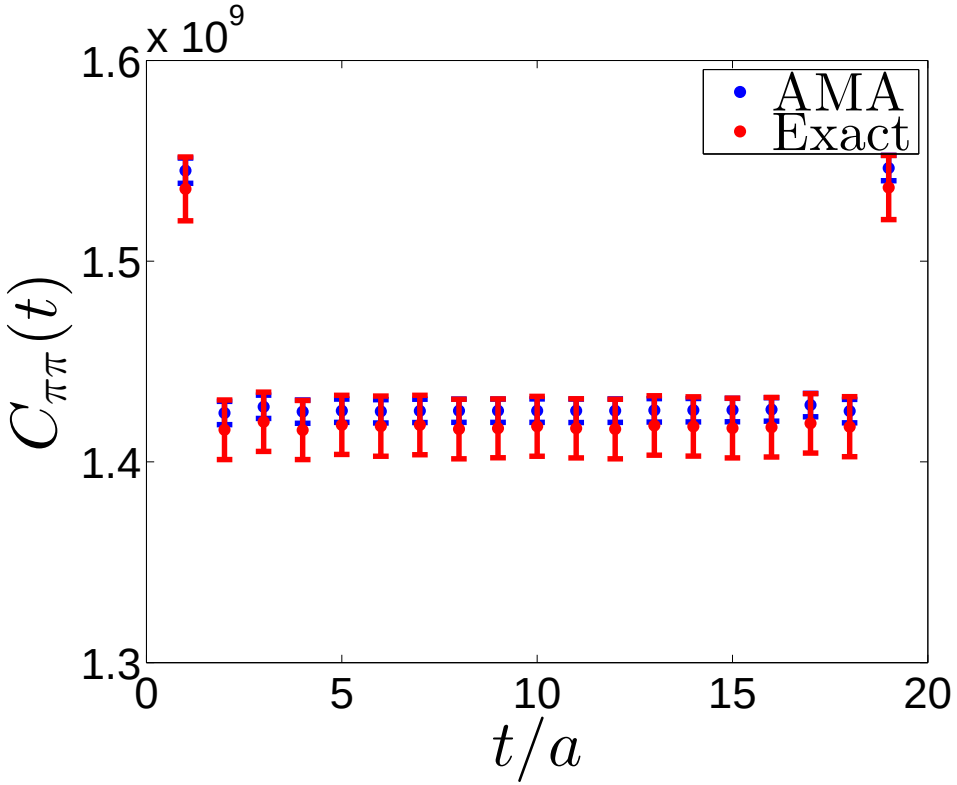
$$\mathcal{O}_{\mathrm{AMA}} = \frac{1}{N_g} \sum_{g \in G} \mathcal{O}' \circ g + \Delta\mathcal{O},$$

  where G is the group of lattice translations
- For K_{l3} analysis:
  - Compute S'_F for all source/sink separations
  - Compute S_F for 7(8) source/sink separations on 48I(64I) ensemble
  - Form AMA observables, averaging $\mathcal{O}'$ over all time translations

All-Mode Averaging [Blum et al., Phys.Rev. D88 (2013)]

- Exact and AMA correlators with a $\pi - \pi$ separation of 20 on 48I ensemble:

	m_π	m_K	E_π	Z_V^π
Exact	0.08046(16)	0.28859(33)	0.15632(59)	0.71172(555)
AMA	0.08049(11)	0.28862(17)	0.15683(33)	0.71080(38)
$\sigma_{exact}/\sigma_{AMA}$	1.5	1.9	1.8	14.6



Outline

Measurement Strategy

- Form factor and vector current renormalization constants extracted using ratio method [P.A. Boyle et al., JHEP 0705 (2007)]
  - K_{l3} form factors:

$$R_1^{(\mu)}(\vec{p}_K, \vec{p}_\pi) = 4\sqrt{E_\pi E_K}\sqrt{\frac{\mathcal{C}_{K\pi}^{(\mu)}(t, t_{\rm snk}; \vec{p}_K, \vec{p}_\pi)\,\mathcal{C}_{\pi K}^{(\mu)}(t, t_{\rm snk}; \vec{p}_\pi, \vec{p}_K)}{\tilde{\mathcal{C}}_\pi(t_{\rm snk}; \vec{p}_\pi)\,\tilde{\mathcal{C}}_K(t_{\rm snk}; \vec{p}_K)}} \tag{4}$$

$$R_2^{(\mu)}(\vec{p}_K, \vec{p}_\pi) = 2\sqrt{E_\pi E_K}\sqrt{\frac{\mathcal{C}_{K\pi}^{(\mu)}(t, t_{\rm snk}; \vec{p}_K, \vec{p}_\pi)\,\mathcal{C}_{\pi K}^{(\mu)}(t, t_{\rm snk}; \vec{p}_\pi, \vec{p}_K)}{\mathcal{C}_{\pi\pi}^{(0)}(t_{\rm snk}; \vec{0})\,\mathcal{C}_{KK}(t_{\rm snk}; \vec{0})}} \tag{5}$$

$$R_\alpha^{(\mu)} \overset{t, t_{\rm snk} \gg 1}{=} (p_K + p_\pi)^\mu f_+^{K\pi}(q^2) + (p_K - p_\pi)^\mu f_-^{K\pi}(q^2) \tag{6}$$

  - Z_V:

$$Z_V^\pi \overset{t \gg 1}{=} \frac{\tilde{\mathcal{C}}_\pi(t_{\rm snk})}{\mathcal{C}_{\pi\pi}(t_{\rm src}, t, t_{\rm snk})}, \quad Z_V^K \overset{t \gg 1}{=} \frac{\tilde{\mathcal{C}}_K(t_{\rm snk})}{\mathcal{C}_{KK}(t_{\rm src}, t, t_{\rm snk})} \tag{7}$$

- We have also explored global fit approach:
  - Fit directly to correlators
  - Reduce statistical error using maximal set of simultaneous constraints

Summary of Lattice Data

Lattice	m_π (MeV)	$f_+^{K\pi}(0)$	Stat. error
24I	678	0.9992(1)	0.01%
24I	563	0.9956(4)	0.04%
24I	422	0.9870(9)	0.09%
24I	334	0.9760(43)	0.4%
24I	334	0.9858(28)	0.3%
48I (PRELIMINARY)	**139**	**0.9727(25)**	**0.3%**
32ID	248	0.9771(21)	0.2%
32ID	171	0.9710(45)	0.5%
32I	398	0.9904(17)	0.2%
32I	349	0.9845(23)	0.2%
32I	295	0.9826(35)	0.4%
64I (PRELIMINARY)	**139**	**0.9701(22)**	**0.2%**

- I: Iwasaki gauge action, ID: Iwasaki+DSDR gauge action
- $f_+^{K\pi}(0)$ extracted from vector current
- Unphysical ensembles analyzed in [P.A. Boyle et al., JHEP 1308 (2013)]

Physical Point Extrapolation (PRELIMINARY)

- Physical point measurements still contain small errors from mistuning of input quark masses, cutoff effects, etc.
- Can use heavy data to correct to physical point:
  1. $SU(3)$-breaking ansatz [P.A. Boyle et al., JHEP 1308 (2013)] :

$$f_+^{K\pi}(0; m_\pi^2, m_K^2) = A + \frac{(m_K^2 - m_\pi^2)^2}{m_K^2}\left(A_0 + (m_K^2 + m_\pi^2)A_1\right) \tag{8}$$

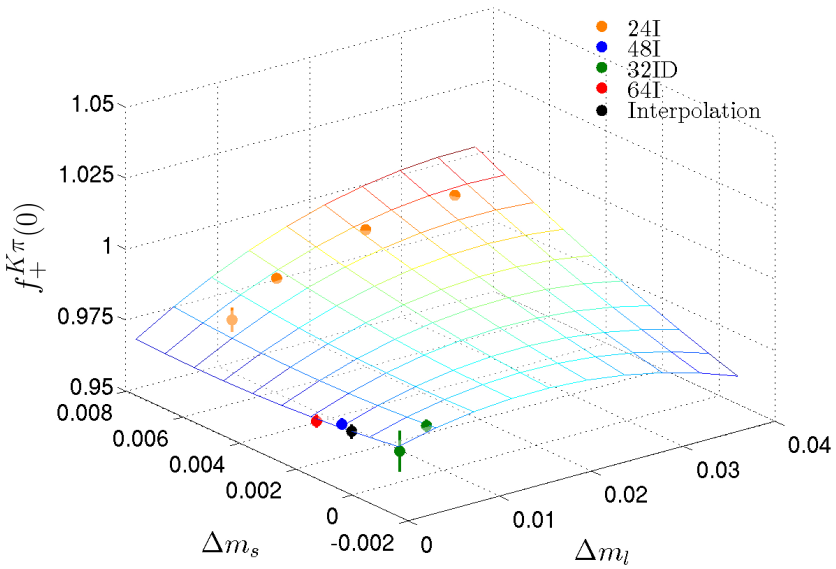
  2. Analytic ansätze:

$$f_+^{K\pi}(0; m_l, m_s) = f_+^{K\pi}(0; m_l^{\text{phys}}, m_s^{\text{phys}}) + \frac{\partial f_+^{K\pi}}{\partial m_l}\Delta m_l + \frac{\partial f_+^{K\pi}}{\partial m_s}\Delta m_s + \frac{\partial^2 f_+^{K\pi}}{\partial m_l^2}\Delta m_l^2 + \frac{\partial^2 f_+^{K\pi}}{\partial m_l \partial m_s}\Delta m_l \Delta m_s + \frac{\partial^2 f_+^{K\pi}}{\partial m_s^2}\Delta m_s^2 \tag{9}$$

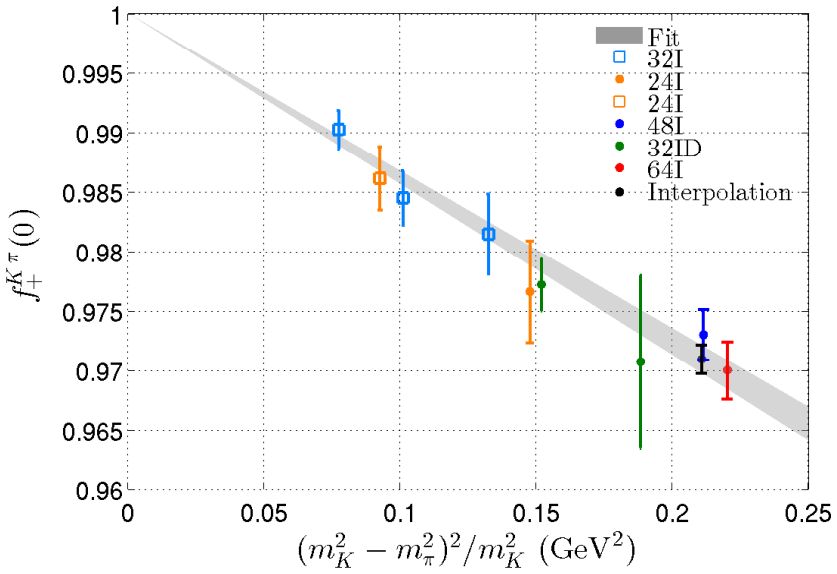
$$f_+^{K\pi}(0; m_l, m_s^{\text{sea}}, m_s^{\text{val}}) = f_+^{K\pi}(0; m_l^{\text{phys}}, m_s^{\text{phys}}, m_s^{\text{phys}}) + \frac{\partial f_+^{K\pi}}{\partial m_l}\Delta m_l + \frac{\partial f_+^{K\pi}}{\partial m_s^{\text{sea}}}\Delta m_s^{\text{sea}} + \frac{\partial f_+^{K\pi}}{\partial m_s^{\text{val}}}\Delta m_s^{\text{val}} \tag{10}$$

- $\Delta m_q \equiv m_q - m_q^{\text{phys}}$
- Physical quark masses (m_q^{phys}) from $SU(2)$ ChPT global fit analysis [R. Arthur et al., Phys.Rev. D87 (2013)]
- Can also add a^2 scaling term

Physical Point Extrapolation (PRELIMINARY)



(a) Quadratic analytic ansatz



(b) NLO $SU(3)$-breaking ansatz

- $SU(3)$ ansatz: $m_\pi^{\text{phys}} = 135.0$ MeV, $m_K^{\text{phys}} = 495.7$ MeV
- Estimate model error from difference in fits
- Preliminary results suggest systematic error from physical point extrapolation is $\lesssim 0.1\%$ effect

Outlook

- We have generated two new physical pion mass lattices, and computed Kl_3 3-point functions on each
- Physical point measurements have reduced extrapolation error to sub-statistical
- Improved precision means deeper understanding of systematics necessary
  - Mass, a^2 dependence of Z_V
- Ongoing work:
  - Finalize fit systematics
  - Incorporate data from scalar current
  - Explicit continuum extrapolation?

Thank you!

Twisted Boundary Conditions

- Momentum states in periodic, cubic lattice with side length L and spacing a are

$$p_i = \frac{2\pi k_i}{L}, \;\; k_i \in \mathbb{Z}$$

- On our physical point ensembles, smallest allowed momentum states are

$$48^3 : \;\; \frac{2\pi}{48a} \approx 226\,\text{MeV}, \quad 64^3 : \;\; \frac{2\pi}{64a} \approx 232\,\text{MeV}$$

- $q^2 = 0$ requires

$$|\vec{p}_\pi| = \frac{m_K^2 - m_\pi^2}{2m_K} \approx 229\,\text{MeV}$$

- Avoid momentum interpolation via twisted boundary conditions

$$\psi(x + L\hat{e}_i) = e^{i\theta_i}\psi(x) \implies p_i = \frac{2\pi k_i}{L} + \frac{\theta_i}{L}$$

- Further reduction in statistical noise from moving pion by twisting in all 3 spatial directions and averaging spatial components of vector correlator