Beyond-the-Standard-Model Neutral Kaon Mixing from a Mixed-Action Lattice Calculation

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Supported in part by the Fermilab Fellowship in Theoretical Physics

June 24th, 2014

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work in progress

Background

Integrating out the **Standard-Model** weak bosons gives a low-energy hamiltonian that contains the operator

$$\mathcal{O}_1^{\Delta S} = [\overline{s}^{\alpha}\gamma_\mu(1-\gamma_5)d^{\alpha}][\overline{s}^{\beta}\gamma_\mu(1-\gamma_5)d^{\beta}]$$

Parametrize nonperturbative hadronic contribution via

$$B_K = -\frac{\langle \bar{K}^0|\mathcal{O}_1^{\Delta S=2}|K^0\rangle}{N_1\langle \bar{K}^0|L_\mu|0\rangle\langle 0|L_\mu|K^0\rangle}$$

where $L_\mu = \bar{s}\gamma_\mu(1-\gamma_5)d$

Determining B_K gives the Standard-Model predictions for ΔM_K, ϵ_K

Background

In theories **beyond-the-Standard-Model (BSM)** the hamiltonian receives contributions from additional operators

$$\mathcal{O}_2^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\alpha}][\overline{s}^{\beta}(1-\gamma_5)d^{\beta}]$$

$$\mathcal{O}_3^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\beta}][\overline{s}^{\beta}(1-\gamma_5)d^{\alpha}]$$

$$\mathcal{O}_4^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\alpha}][\overline{s}^{\beta}(1+\gamma_5)d^{\beta}]$$

$$\mathcal{O}_5^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\beta}][\overline{s}^{\beta}(1+\gamma_5)d^{\alpha}]$$

Parametrize hadronic contributions from these operators via

$$B_i^{\mathrm{BSM}} = -\frac{\langle \bar{K}^0 | \mathcal{O}_i^{\Delta S=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}$$

Background

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$$\mathcal{O}_2^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\alpha}][\overline{s}^{\beta}(1-\gamma_5)d^{\beta}]$$

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$$\mathcal{O}_4^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\alpha}][\overline{s}^{\beta}(1+\gamma_5)d^{\beta}]$$

$$\mathcal{O}_5^{\Delta S} = [\overline{s}^{\alpha}(1-\gamma_5)d^{\beta}][\overline{s}^{\beta}(1+\gamma_5)d^{\alpha}]$$

B_i^{BSM}

$+$

Experimental

$\Delta M_K \quad \epsilon_K$

Constraints on new physics

Background

Numerical lattice QCD calculations of B_i have been performed by **ETM, RBC/UKQCD** and **SWME** collaborations.

	N_f	B_K	B_2	B_3	B_4	B_5
ETM	2	0.51(2)	0.47(2)	0.78(4)	0.75(3)	0.60(3)
RBC/UKQCD	2+1	0.53(2)	0.43(5)	0.75(9)	0.69(7)	0.47(6)
SWME	2+1	0.518(04)(23)	0.532(05)(23)	0.785(07)(34)	0.913(32)(40)	0.660(22)(29)

in $\overline{\mathrm{MS}}$ at $\mu = 3\,\mathrm{GeV}$

Because results are in tension, additional simulations are especially useful

Mixed action lattice set-up

We present progress in a **mixed-action** calculation of B_i,

We use MILC asqtad-improved **staggered sea quarks** and **domain-wall valence quarks**

light quarks → light dynamical pions, large volumes, fine lattice spacings

reduce mixing between wrong-chirality operators

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		sea sector		valence sector		
$\approx a(\mathrm{fm})$	$\left(\frac{L}{a}\right)^3 \times \frac{T}{a}$	am_l/am_h	am_π	am_x	am_π	$N_{\mathrm{conf.}}$
0.06	$64^3 \times 144$	0.0018/0.018	0.06678(03)	0.0026, 0.0108, 0.033		96
0.06	$48^3 \times 144$	0.0036/0.018	0.09353(07)	0.0036, 0.0072, 0.0108, 0.033		129
0.09	$40^3 \times 96$	0.0031/0.0031		0.004, 0.0124, 0.0186, 0.046		103
0.09	$40^3 \times 96$	0.0031/0.031	0.10538(06)	0.004, 0.0124, 0.0186, 0.046	0.0999(12)	151
0.09	$28^3 \times 96$	0.0093/0.031		0.0062, 0.0124, 0.0186, 0.046		199
0.09	$28^3 \times 96$	0.0062/0.0186	0.14619(14)	0.0062, 0.0124, 0.0186, 0.046	0.1212(17)	169
0.09	$28^3 \times 96$	0.0062/0.031	0.14789(18)	0.0062, 0.0124, 0.0186, 0.046	0.1222(12)	374
0.09	$28^3 \times 96$	0.0124/0.031	0.20635(18)	0.0062, 0.0124, 0.0186, 0.046	0.1216(11)	199
0.12	$32^3 \times 64$	0.005/0.005	0.16081(09)	0.007, 0.02, 0.03, 0.05		175
0.12	$28^3 \times 64$	0.01/0.05	0.22421(12)	0.01, 0.03		116
0.12	$24^3 \times 64$	0.005/0.05	0.15971(20)	0.007, 0.02, 0.03, 0.05, 0.065	0.1718(11)	217
0.12	$20^3 \times 64$	0.007/0.05	0.18891(20)	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	0.1968(08)	279
0.12	$20^3 \times 64$	0.01/0.03	0.22357(19)	0.01, 0.02, 0.03, 0.05, 0.065	0.1946(18)	162
0.12	$20^3 \times 64$	0.01/0.05	0.22447(17)	0.01, 0.02, 0.03, 0.05, 0.065	0.1989(08)	227
0.12	$20^3 \times 64$	0.02/0.05	0.31125(16)	0.01, 0.03, 0.05, 0.065	0.1949(13)	117

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		sea sector		valence sector	
$\approx a$(fm)	$\left(\frac{L}{a}\right)^3 \times \frac{T}{a}$	am_l/am_h	$\approx m_\pi$ (MeV)	am_x	$\approx m_\pi$ (MeV)
0.06	$64^3 \times 144$	0.0018/0.018	220	0.0026, 0.0108, 0.033	
0.06	$48^3 \times 144$	0.0036/0.018	310	0.0036, 0.0072, 0.0108, 0.033	
0.09	$40^3 \times 96$	0.0031/0.0031		0.004, 0.0124, 0.0186, 0.046	
0.09	$40^3 \times 96$	0.0031/0.031	250	0.004, 0.0124, 0.0186, 0.046	240
0.09	$28^3 \times 96$	0.0093/0.031		0.0062, 0.0124, 0.0186, 0.046	
0.09	$28^3 \times 96$	0.0062/0.0186	350	0.0062, 0.0124, 0.0186, 0.046	290
0.09	$28^3 \times 96$	0.0062/0.031	350	0.0062, 0.0124, 0.0186, 0.046	290
0.09	$28^3 \times 96$	0.0124/0.031	500	0.0062, 0.0124, 0.0186, 0.046	290
0.12	$32^3 \times 64$	0.005/0.005	260	0.007, 0.02, 0.03, 0.05	
0.12	$28^3 \times 64$	0.01/0.05	390	0.01, 0.03	340
0.12	$24^3 \times 64$	0.005/0.05	270	0.007, 0.02, 0.03, 0.05, 0.065	290
0.12	$20^3 \times 64$	0.007/0.05	320	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	340
0.12	$20^3 \times 64$	0.01/0.03	380	0.01, 0.02, 0.03, 0.05, 0.065	330
0.12	$20^3 \times 64$	0.01/0.05	390	0.01, 0.02, 0.03, 0.05, 0.065	340
0.12	$20^3 \times 64$	0.02/0.05	550	0.01, 0.03, 0.05, 0.065	340

Extraction of lattice B parameters

$$B_i^{\mathrm{BSM}} = -\frac{\langle \bar{K}^0 | \mathcal{O}_i^{\Delta S=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}$$

We extract lattice B parameters by fitting the ratio of Euclidean-time-dependent lattice correlators to a plateau in the region far away from the source and sink

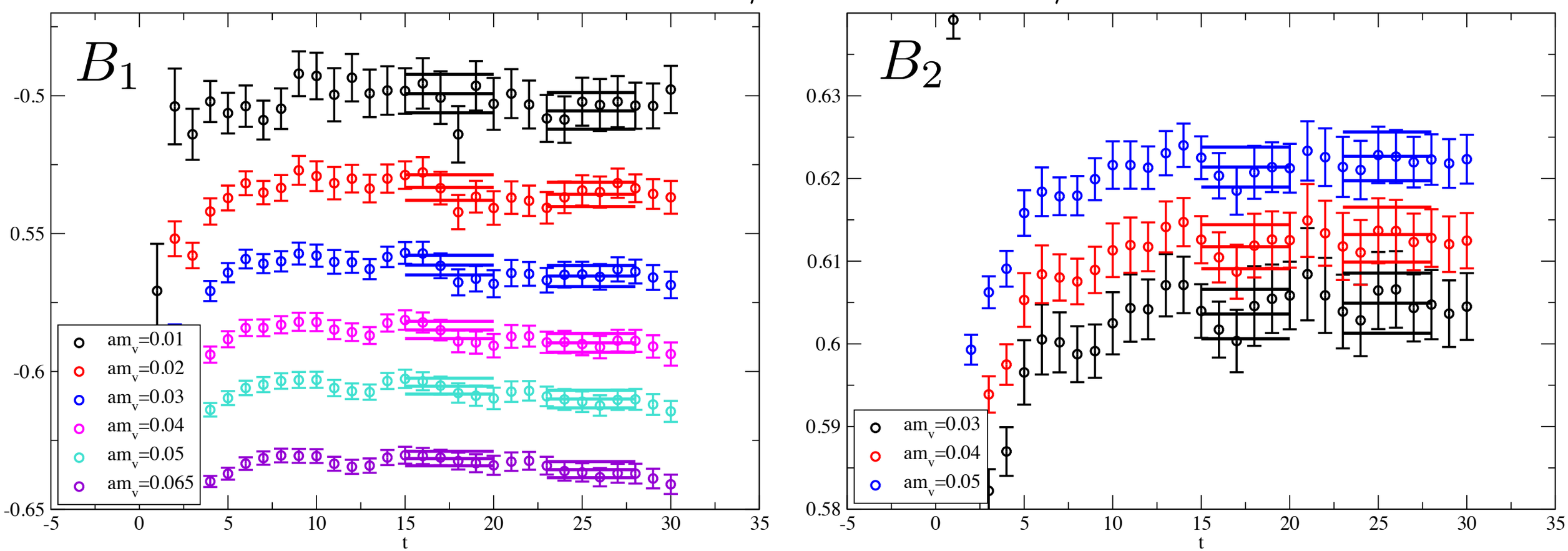
- fit uncertainties determined via jackknifing
- autocorrelations investigated via blocking & found to be negligible
- dependence on fit range thoroughly investigated

Extraction of lattice B parameters

$$B_i^{\rm BSM} = -\frac{\langle \bar{K}^0 | \mathcal{O}_i^{\Delta S=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}$$

We extract lattice B parameters by fitting the ratio of Euclidean-time-dependent lattice correlators to a plateau in the region far away from the source and sink

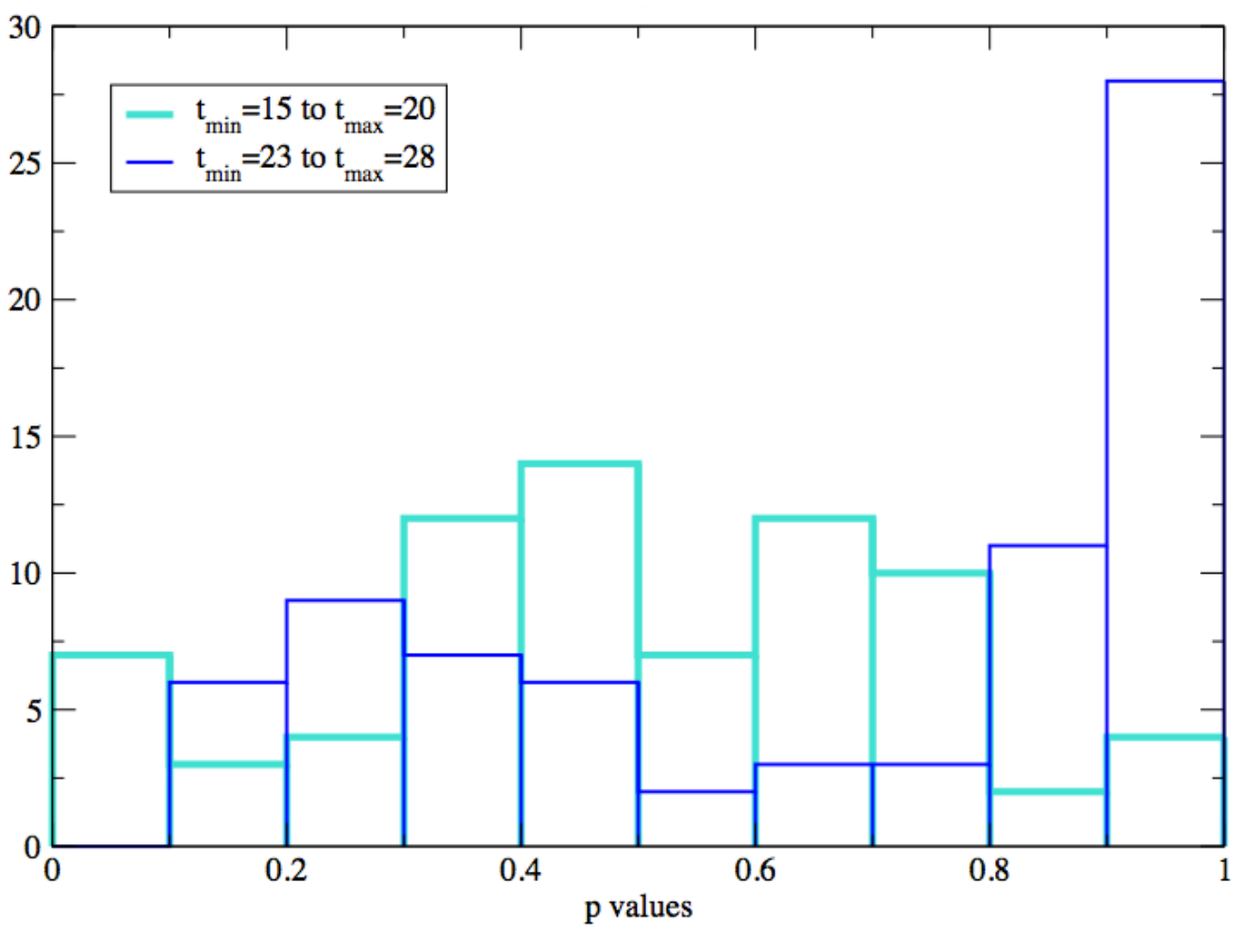
$$a \approx 0.12\text{fm} \quad am_l/am_h = 0.007/0.05$$



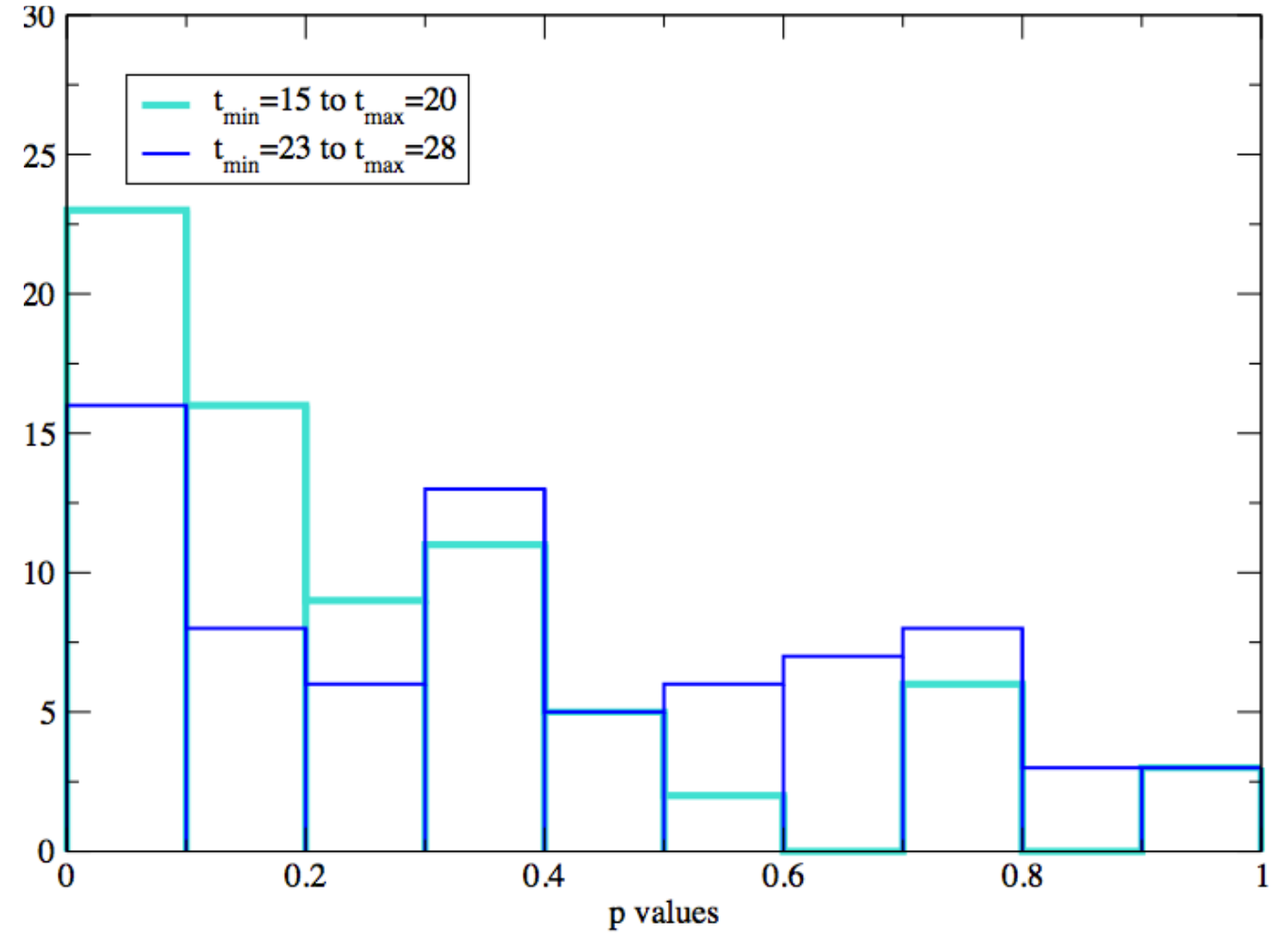
Extraction of lattice B parameters

p-value distributions found to be reasonable across the full set of fits to all valence-quark mass combinations, and for both fitting ranges

$a \approx 0.12\text{fm} \quad (am_l)/(am_h) = 0.01/0.03$



$a \approx 0.12\text{fm} \quad (am_l)/(am_h) = 0.005/0.05$



Operator renormalization

We use one-loop tadpole-improved Lattice Perturbation Theory to relate

$$B_n^{\mathrm{latt}} \longrightarrow B_n^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$$

At this order the BSM operators mix in pairs

$$\begin{pmatrix} B_1^{\overline{\mathrm{MS}}} \\ B_2^{\overline{\mathrm{MS}}} \\ B_3^{\overline{\mathrm{MS}}} \\ B_4^{\overline{\mathrm{MS}}} \\ B_5^{\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} 1+\bullet & & & & \\ & 1+\bullet & \bullet & & \\ & \bullet & 1+\bullet & & \\ & & & 1+\bullet & \bullet \\ & & & \bullet & 1+\bullet \end{pmatrix} \begin{pmatrix} B_1^{\mathrm{latt}} \\ B_2^{\mathrm{latt}} \\ B_3^{\mathrm{latt}} \\ B_4^{\mathrm{latt}} \\ B_5^{\mathrm{latt}} \end{pmatrix}$$

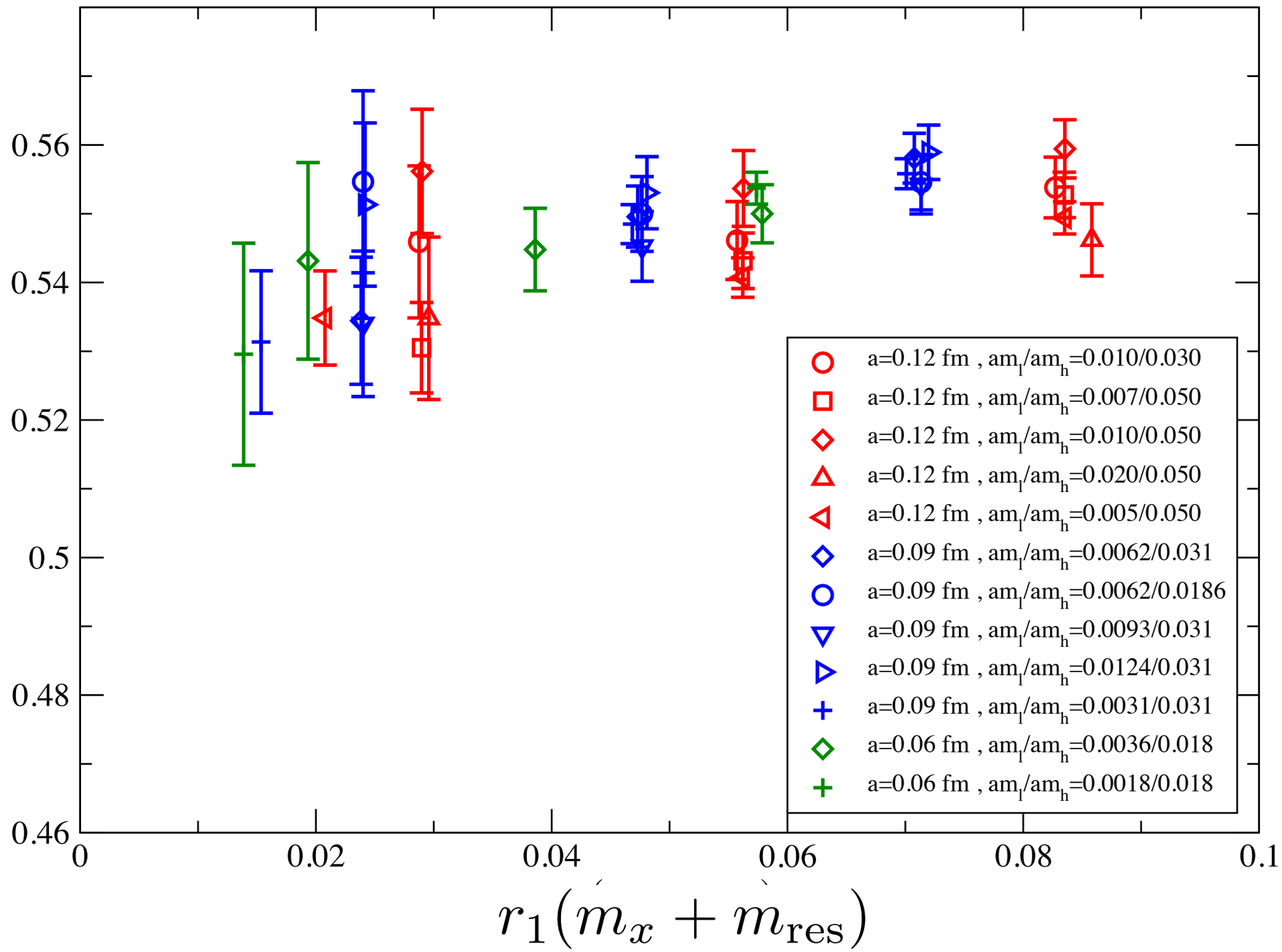
The mixing results in larger uncertainties in B_i^{BSM} than in B_K.

We estimate this by using two alternative values for α_s in our renormalization calculation

α_s at q* from BLM prescription

$$B_K^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$$

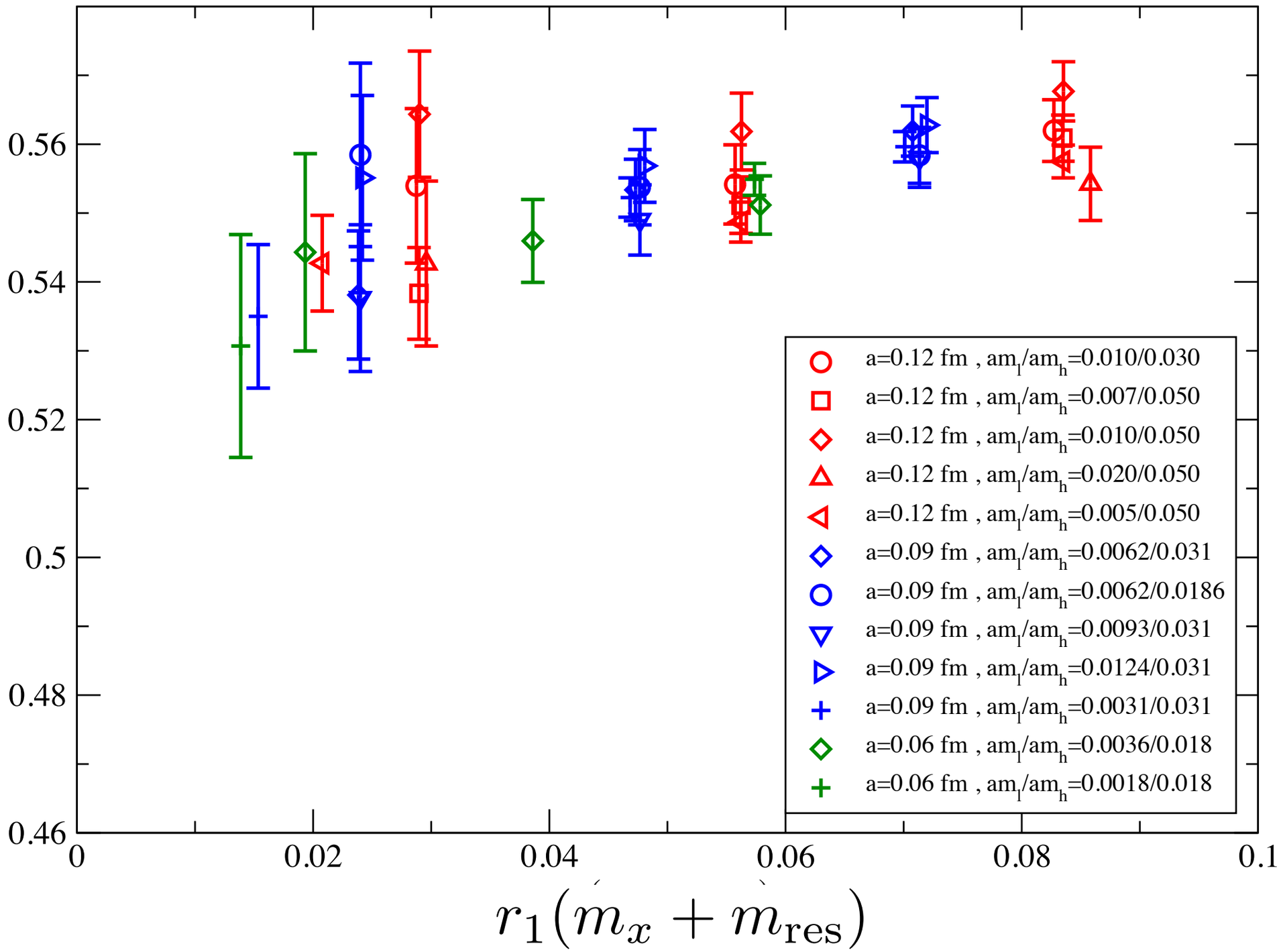
$$m_y \approx m_{\mathrm{strange}}$$



α_s at q* = 2/a

$$B_K^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$$

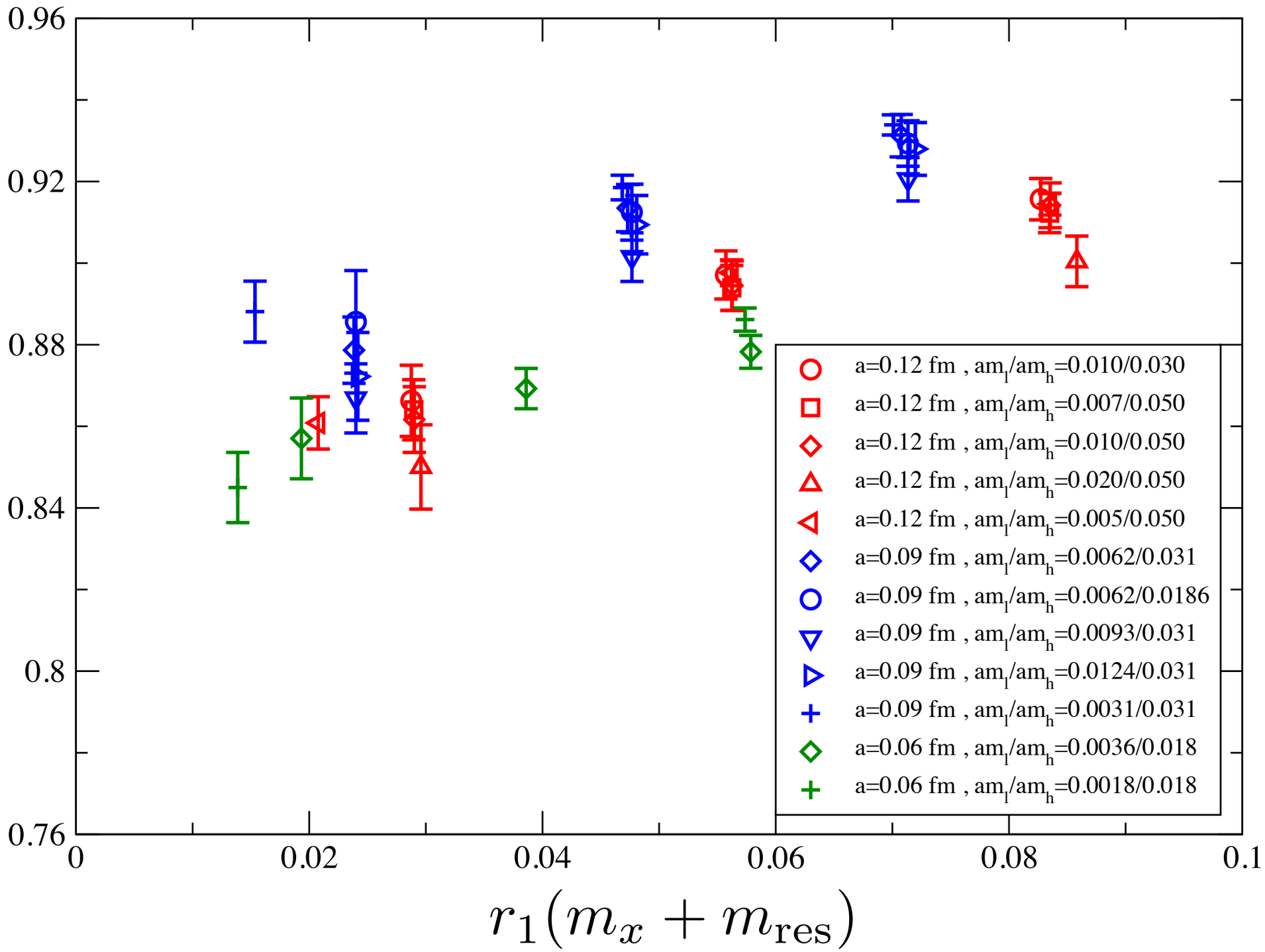
$$m_y \approx m_{\mathrm{strange}}$$



α_s at q* from BLM prescription

$$B_4^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$$

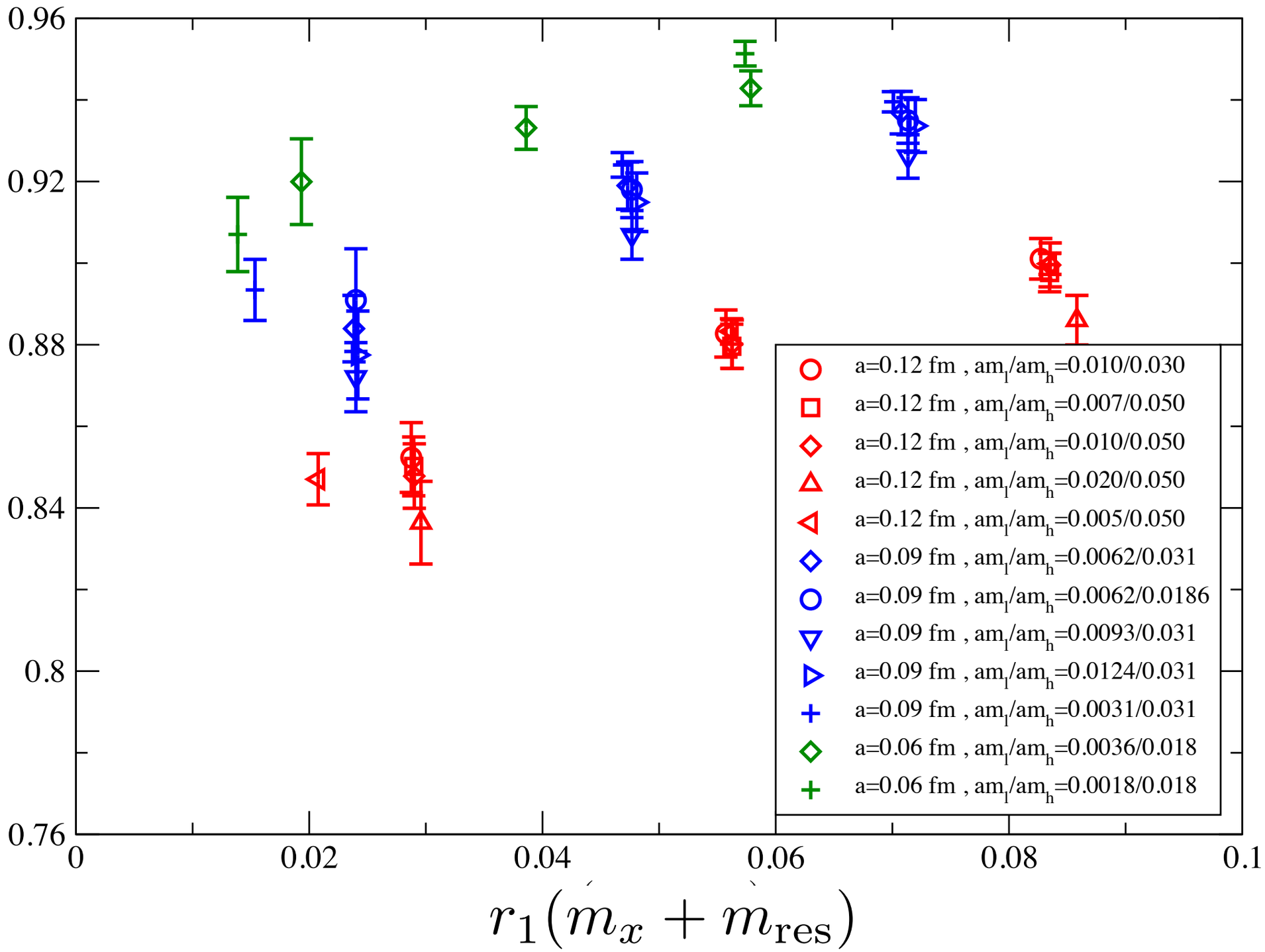
$$m_y \approx m_{\mathrm{strange}}$$



α_s at q* = 2/a

$$B_4^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$$

$$m_y \approx m_{\mathrm{strange}}$$



BSM parameters more difficult

- BSM parameters have large uncertainty from truncation error in renormalization

	renorm. unc.
B_K	$< 1\%$
B_2	$\sim 3 - 5\%$
B_3	$\sim 3 - 5\%$
B_4	$\sim 5 - 10\%$
B_5	$\sim 5 - 15\%$

statistical uncertainty
0.5%-2.5%

Rough estimate of individual data point uncertainties:
from comparing two choices of α_s .

- BSM parameters also have larger variation with valence-quark mass and lattice spacing

Future plans for nonperturbative renormalization

We have data for NPR in RI-SMOM-γ_μ scheme

Unfortunately conversion to $\overline{\mathrm{MS}}$ is not currently available

We will calculate renormalization factors in this nonperturbative scheme

Result can be updated using NPR when continuum conversion factors are calculated

Future plans for chiral-continuum extrapolation

Plan to fit data using both $SU(2)$ and $SU(3)$ ChPT

Preliminary fits to BSM data (with only statistical) uncertainties lead to poor p values

Plan to incorporate renormalization uncertainties before fitting

Plan to consider golden-combinations, which remove leading chiral logs and potentially improve fit