Shear Viscosity from Pure Yang-Mills Lattice QCD

Simon Mages

Szabolcs Borsányi, Zoltán Fodor,
Andreas Schäfer, Kálmán K. Szabó

June 27, Lattice 2014, NYC

Universität Regensburg

SFB TRR55

RQCD

Outline

Outline

Motivation

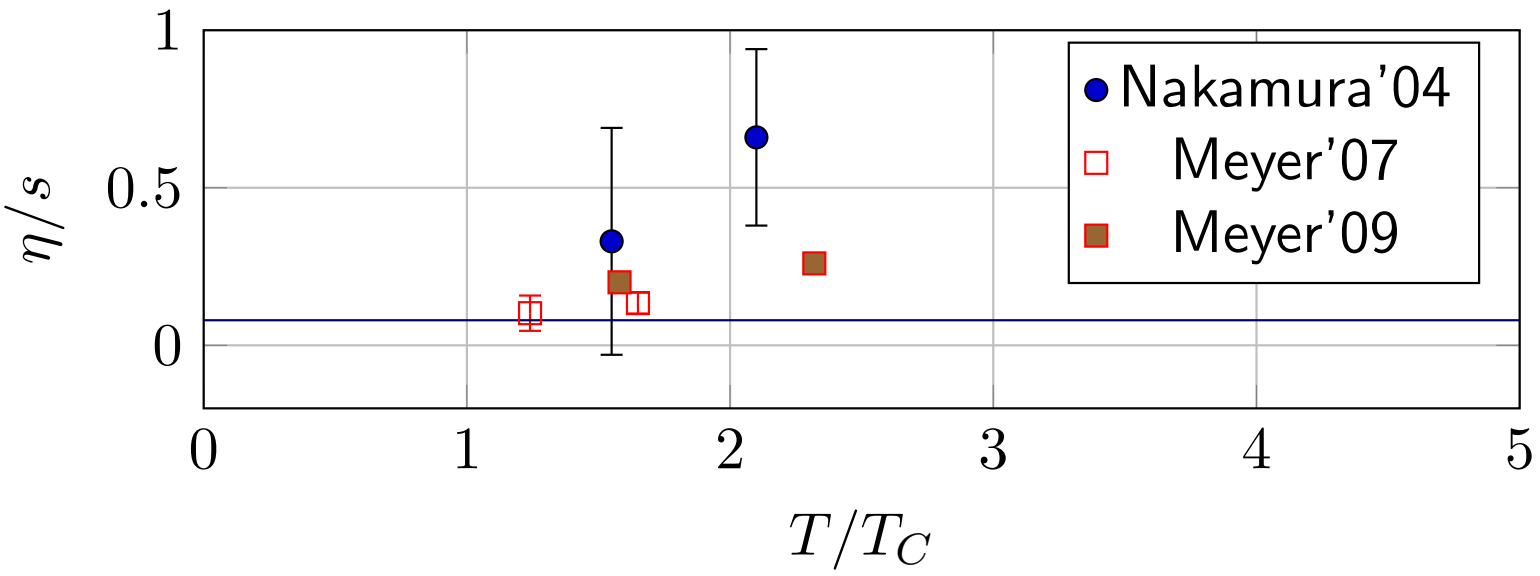
- Medium properties of QGP in high-energy heavy ion collisions [Muller:2013dea]
- shear viscosity over entropy density $\eta/s \propto 0.12$(RHIC)$\propto 0.2$(LHC) [Gale:2012rq]
- η/s close to lower bound of $1/4\pi \sim 0.08$ predicted by AdS/CFT duality [Kovtun:2004de]

$\Rightarrow$ Direct determination from first principles in LQCD?

Motivation

Previous lattice studies:

- [Karsch:1986cq,Nakamura:2004sy,Meyer:2009jp]
- only quenched
- [Meyer:2009jp] has the best errors with multi-level algorithm



Multi-level not available for simulations with dynamical fermions
$\Rightarrow$ now: use conventional algorithm and smearing/Wilson Flow

Transport from Spectral Functions

Traceless part $\Theta_{\mu\nu}$ of $T_{\mu\nu}$ in pure gauge theory

$$\Theta_{\mu\nu}(x) = F_{\mu\sigma}(x)F_{\nu\sigma}(x) - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}(x)F_{\rho\sigma}(x)$$

Euclidean correlators

$$C_{\mu\nu,\rho\sigma}(\tau,\mathbf{q}) = T^{-5}\int d^3\mathbf{x}e^{i\mathbf{q}\cdot\mathbf{x}}\left(\langle\Theta_{\mu\nu}(x)\Theta_{\rho\sigma}(0)\rangle - \langle\Theta_{\mu\nu}(x)\rangle\langle\Theta_{\rho\sigma}(0)\rangle\right)$$

with spectral representation

$$C_{\mu\nu,\rho\sigma}(\tau,\mathbf{q}) = \int_0^\infty \mathrm{d}\omega K(\tau,\omega)\rho_{\mu\nu,\rho\sigma}(\omega,\mathbf{q})$$

$$K(\tau,\omega) = \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))},$$

Transport from Kubo formulae [Teaney:2006nc,Meyer:2009jp]

$$\pi \lim_{\omega\to 0}\lim_{q\to 0}\frac{\rho_{13,13}(\omega,q)}{\omega} = \eta \qquad \pi \lim_{\omega\to 0}\lim_{q\to 0}\frac{\rho_{33,33}(\omega,q)}{\omega} = \frac{4}{3}\eta + \zeta,$$

Finite Temperature Kernel

Systematic problem: Inversion of spectral representation

- MEM $\rightarrow$ problems: low frequency, resolution of more than the spectral weight
- We use an ansatz like [Meyer:2009jp]

High background: $\cosh$-like kernel

- at tree level $6/7$ of $C(1/(2T))$ due to high energy, only $1/7$ due to transport [Meyer:2007ic]
- suppress large energies

Low signal: insensitive kernel

- only higher order sensitivity to low energy features [Aarts:2002cc]

$$K(\tau,\omega) = \frac{2T}{\omega} + \left(\frac{1}{6T} - \tau + T\tau^2\right)\omega + O(\omega^3)$$

- include independent low frequency information: finite momentum

Lattice Setup

Preparatory study for dynamical simulations: pure SU(3)

- Symanzik improved gauge action: $O(a^2)$ errors
- energy momentum tensor $\Theta_{\mu\nu}$ from the clover lattice field strength tensor $F_{\mu\nu}$: $O(a^2)$ errors

Parameters

- $0.75\ T_c \leq T \leq 4.0\ T_c$
- $8 \leq n_t \leq 16$
- $1 \leq r_a \leq 8$ aspect ratio

Algorithm:

- No multilevel: not available for dynamical simulations
- update sweep: 1 heatbath + 4 overrelaxation

Simulations on the QPACE machine in Wuppertal

Outline

Renormalization Structure of $\Theta_{\mu\nu}$

Isotropic case:
Determined by hypercubic symmetry group of the lattice

$$\begin{aligned}\Theta_{\mu\nu} =& Z_{\mu\nu}\theta_{\mu\nu} \\ \theta_{\mu\nu} =& F_{\mu\sigma}F_{\nu\sigma} - 1/4\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma},\end{aligned}$$

where $Z_{\mu\nu} = Z^{diag}$ for $\mu = \nu$, $Z_{\mu\nu} = Z^{rest}$ for $\mu \neq \nu$

Anisotropic case:
Determined by remaining cubic symmetry group of the lattice:
7 Z factors instead of 2

Diagonal Renormalization Factors

Isotropic diagonal factor from thermodynamics literature
[Borsanyi:2012ve]

$$-\frac{sT}{4} = \langle \Theta_{11} \rangle = Z^{diag} \langle \theta_{11} \rangle$$

Works also below T_c where s is very small:
calculate Z at the same β above T_c

Non-Diagonal Renormalization Factors

No scalar thermodynamic expectation value available:

$$\langle \Theta_{\mu \neq \nu} \rangle = 0.$$

$\Rightarrow$ use correlators at finite τ and general tensor decomposition/rotational invariance [Karsch:1986cq,Meyer:2007ic]

$$\frac{\left(Z^{rest}\right)^2}{\left(Z^{diag}\right)^2} = \frac{1}{4} \frac{\langle (\theta_{11} - \theta_{22})(\tau)(\theta_{11} - \theta_{22})(0) \rangle}{\langle \theta_{12}(\tau)\theta_{12}(0) \rangle}.$$

Renormalization Summary

- Determination of all relative renormalization factors of the traceless energy momentum tensor
- Absolute factor can be determined from thermodynamics (literature)
- isotropic and anisotropic
- non-perturbative

Outline

Choice of the Ansatz for the Spectral Function

Different ansätze in literature

- Breit-Wigner [Karsch:1986cq,Nakamura:2004sy,Sakai:2005fa] $\rightarrow$ no continuum contribution
- Modified tree-level [Meyer:2007ic] $\rightarrow$ no momentum dependence
- Hydro+Continuum [Meyer:2009jp] $\rightarrow$ also our choice

Choice of the Ansatz for the Spectral Function

Hydrodynamics (valid up to $\omega, q < \pi T$) [Teaney:2006nc]

$$\frac{\rho_{13,13}(\omega,q)}{\omega} \xrightarrow{\omega,q\to 0} \frac{\eta}{\pi}\frac{\omega^2}{\omega^2 + (\eta q^2/(\epsilon+p))^2}$$
$$\frac{\rho_{33,33}(\omega,q)}{\omega} \xrightarrow{\omega,q\to 0} \frac{\Gamma_s}{\pi}\frac{(\epsilon+p)\omega^4}{(\omega^2 - v_s(q)^2 q^2)^2 + (\Gamma_s \omega q^2)^2},$$

with sound attenuation length $\Gamma_s = (\frac{4}{3}\eta + \zeta)/(\epsilon + p)$, speed of sound $v_s(q)$

$v_s(q)$ in a conformal theory [Baier:2007ix]

$$v_s(q) = v_s\left(1 + \frac{\Gamma_s}{2}q^2\left(\tau_\Pi - \frac{\Gamma_s}{4v_s^2}\right) + O(q^4)\right),$$

with the relaxation time for shear stress τ_Π.

Choice of the Ansatz for the Spectral Function

$$
\begin{aligned}
\rho_{33,33}(\omega,q) =& \rho_{low}(\omega,q) + \rho_{high}(\omega,q) \\
\frac{\rho_{low}(\omega,q)}{\tanh(\omega/2T)} =& \frac{2\Gamma_s}{\pi} \frac{(\epsilon+p)\omega^4}{(\omega^2 - v_s(q)^2 q^2)^2 + (\Gamma_s \omega q^2)^2} \frac{1+\sigma_1\omega^2}{1+\sigma_2\omega^2} \\
\frac{\rho_{high}(\omega,q)}{\tanh(\omega/2T)} =& \omega^4 \tanh^2\left(\frac{\omega}{2T}\right) \frac{2d_A}{15(4\pi)^2}
\end{aligned}
$$

No Breit-Wigner $\rho_{medium}(\omega,q)$ like in [Meyer:2009jp], because it did not improve the fit.

Usage of Smearing

N_{Stout} steps of stout smearing:

$$r_{smear} = a\sqrt{8\rho_{Stout}N_{Stout}}$$

Stout smearing is used to

1. suppress the statistical errors,
2. damp cutoff effects,
3. suppress the high energy parts of the spectrum down to the scale $\pi/r_{smear} =: \Omega$.

Smearing is done at constant scale in physical units: equivalent to a discretized Wilson Flow

$$r_{smear} = \sqrt{8t}$$

Usage of Smearing

Model the suppression directly in the spectral function

$$W_\Omega^{\tanh,\Sigma}(\omega) = \frac{1}{2}\left(1 + \tanh\left(\frac{\Omega - \omega}{\Sigma\sqrt{6}/\pi}\right)\right),$$
$$W_\Omega^{\mathrm{erf},\Sigma}(\omega) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\Omega - \omega}{\Sigma}\right)\right)$$

with smearing scale $\Omega = \pi/r_{smear}$
Alltogether:

$$C_\Omega(\tau, q) = \int_0^\infty \mathrm{d}\omega K(\tau, \omega) W_\Omega(\omega)\rho(\omega, q).$$

Also: more datapoints by another effective direction in the dataset, the smearing scale Ω

Systematic Error Assessment

Selection of datapoints for the fit

- Cut in the smearing radius to suppress discretization errors: $r_{smear}T > \lambda$
- Choose fit range in Euclidean time depending on the smearing: $\tau > r_{smear}x$
- stronger condition for finite momentum: use $x_p \cdot x$ instead of x

$\Rightarrow$ $O(20)$ - $O(80)$ datapoints for the 6 fit parameters (5 in ρ and Ω in the parametrization)

$\Rightarrow$ Histogram method for systematic error

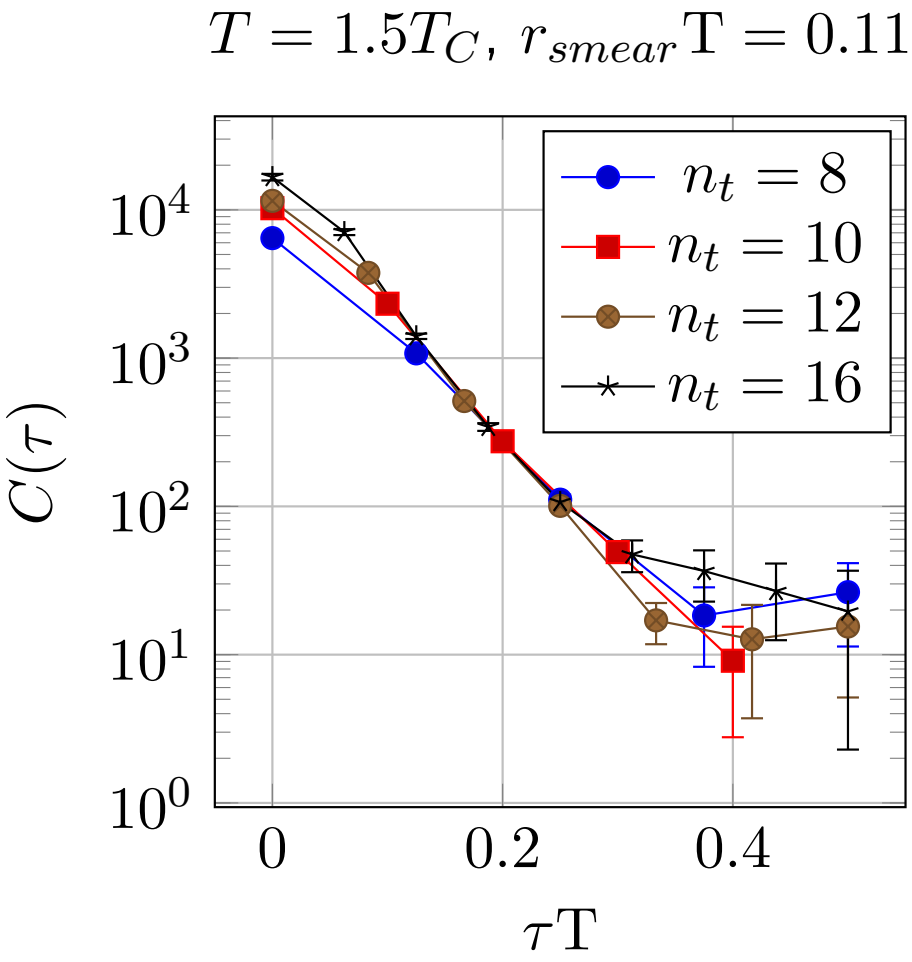
Strategy Summary

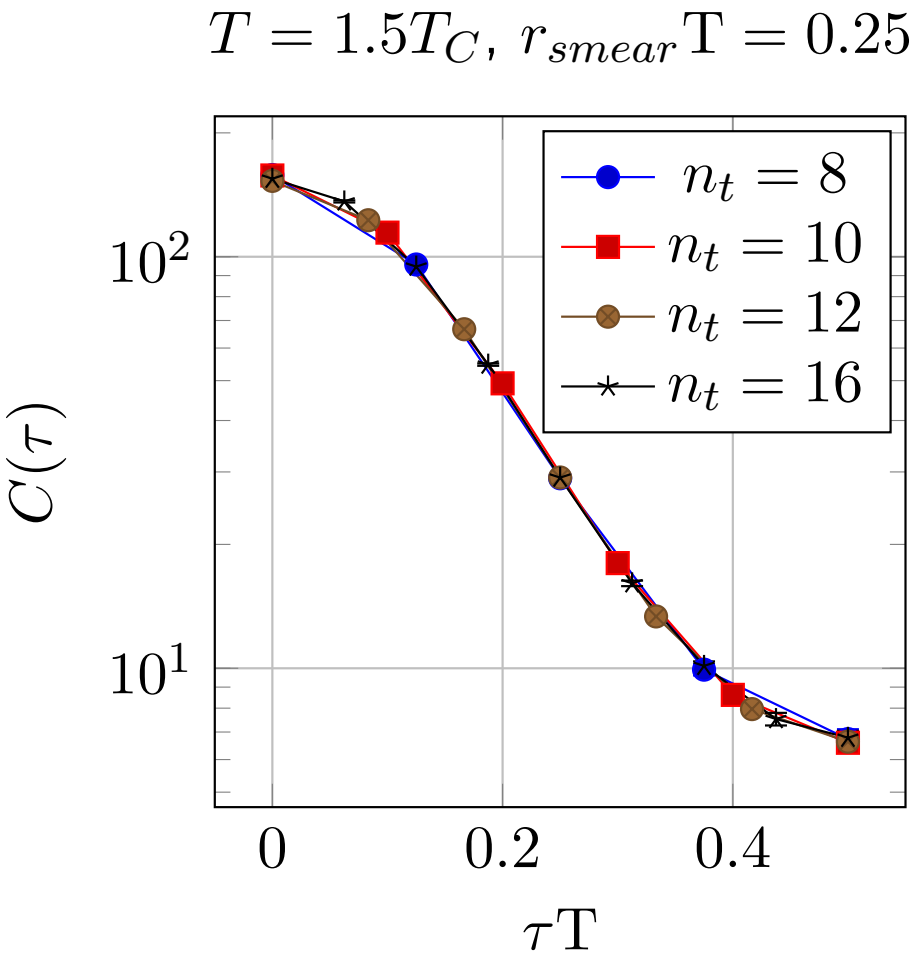
- hydro+continuum ansatz for low energy sensitivity
- use different r_{smear}
- include smearing dependent weightfunction to parameterize the effect of smearing
- systematic error by histogram method

Outline

Statistical and Discretization Errors

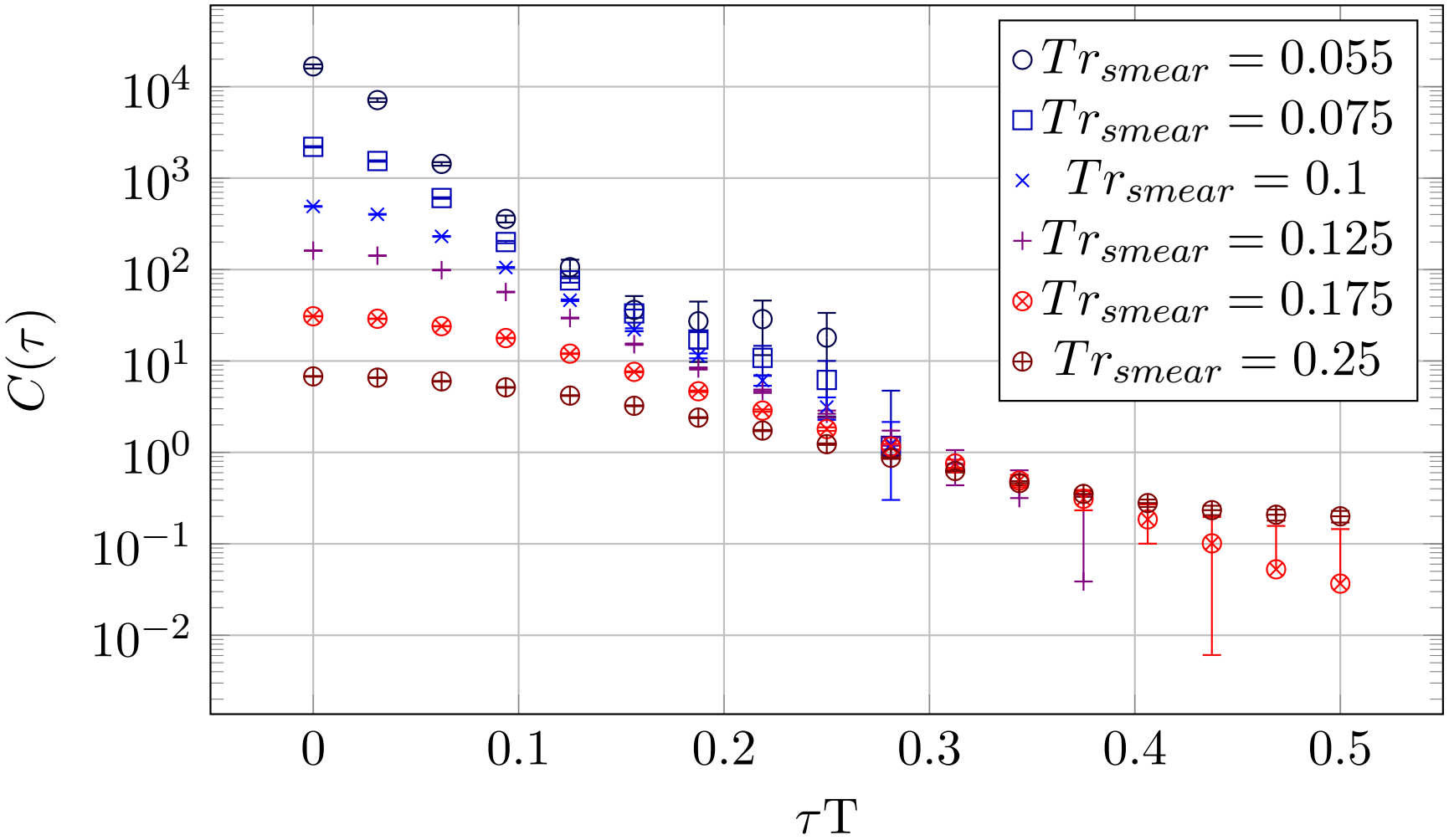
Both errors decrease with the smearing radius:





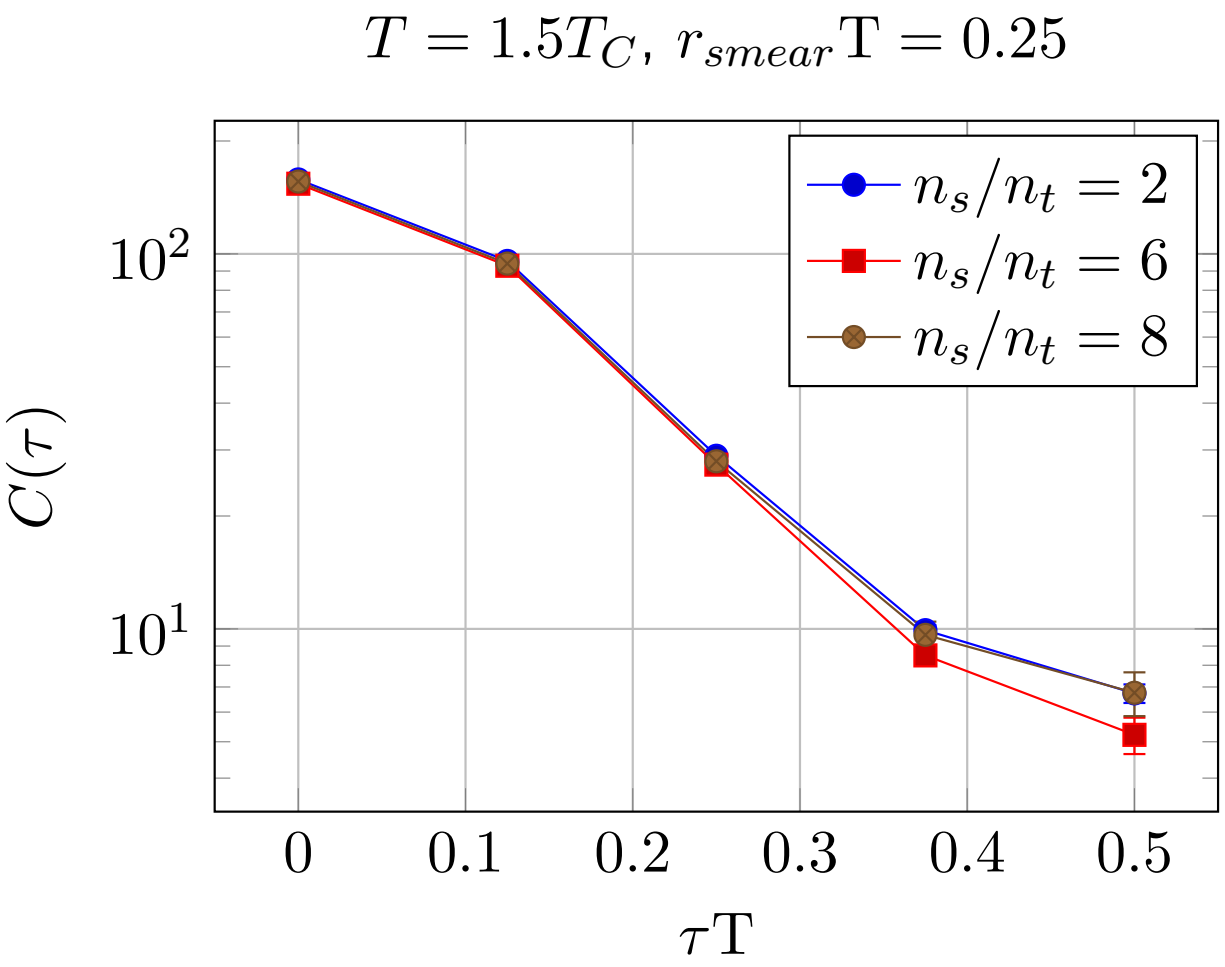
Suppression of high frequency contributions

$T = 0.75T_c$ with different aspect ratios
shear channel $C(\tau) = \frac{1}{T^5}\int \mathrm{d}\mathbf{x}\langle\Theta_{12}(0,0)\Theta_{12}(\tau,\mathbf{x})\rangle$



Finite Volume Errors

$T = 1.5T_c$ with different aspect ratios
shear channel $C(\tau) = \frac{1}{T^5}\int \mathrm{d}\mathbf{x}\langle\Theta_{12}(0,0)\Theta_{12}(\tau,\mathbf{x})\rangle$

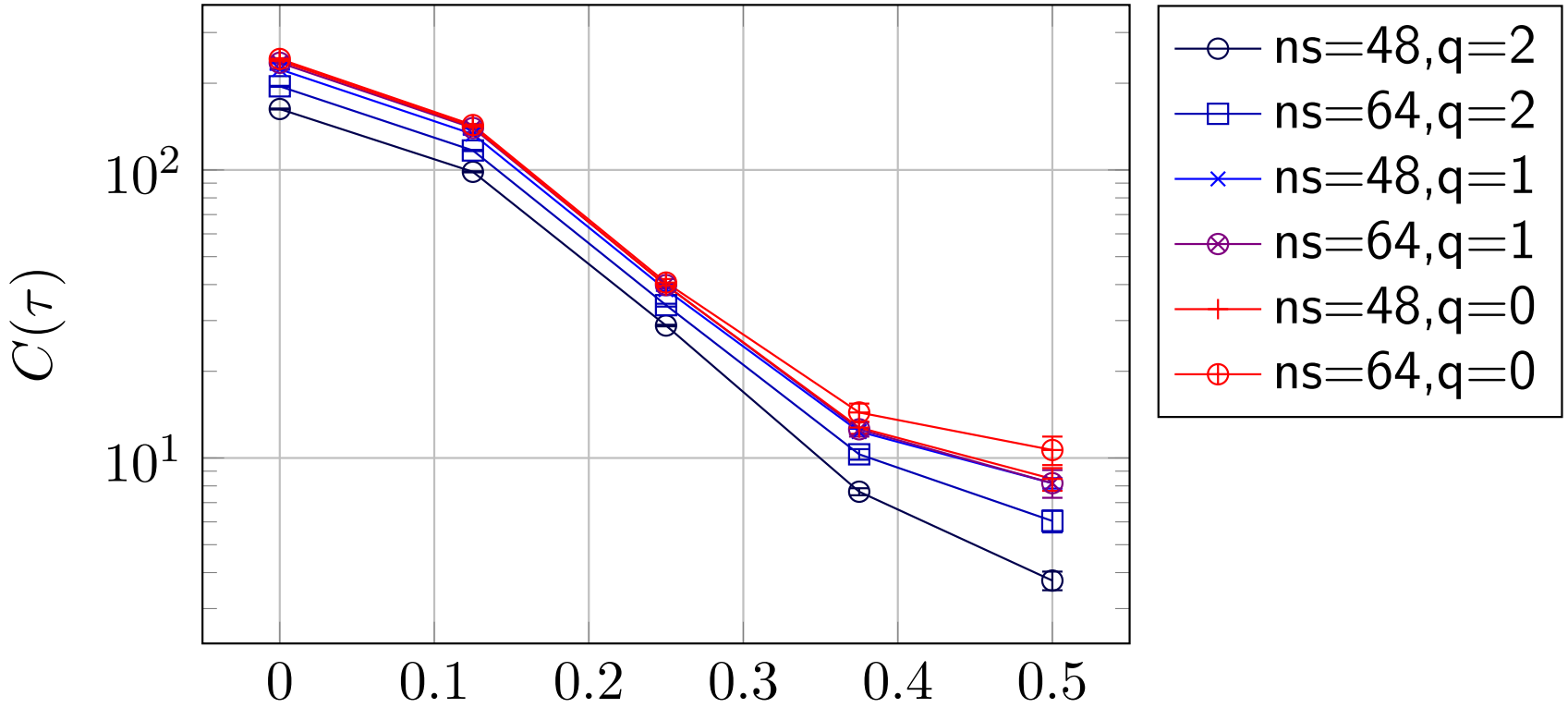


Momentum Dependence

momenta $q_n = 2\pi T n / r_a$ with $n \in \{0, 1, 2\}$, $r_a \in \{6, 8\}$
all well in the hydro regime $|\mathbf{q}| \leq \pi T$

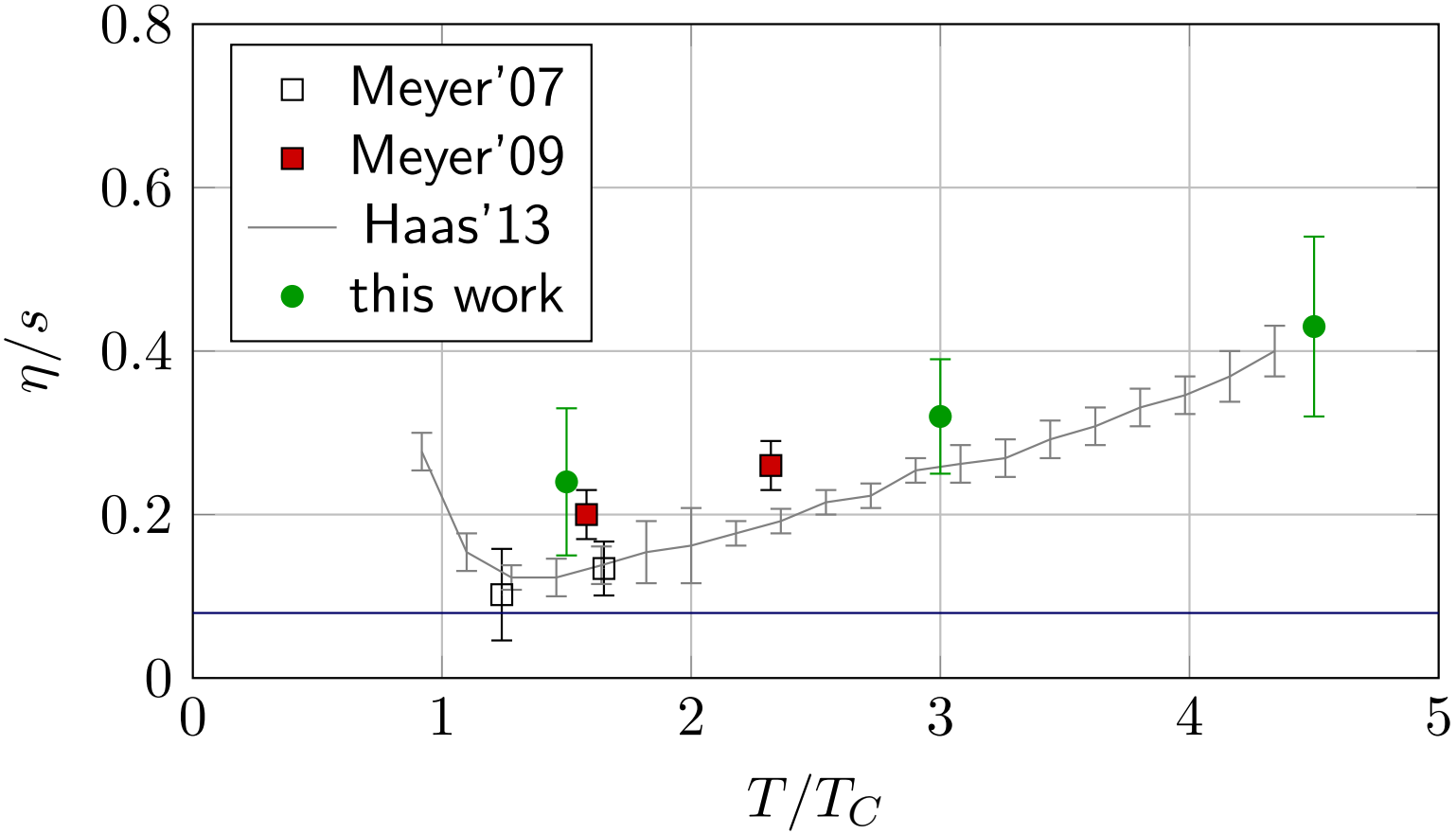
$$C_{33,33}(\tau, q) = \frac{1}{T^5} \int \mathrm{d}\mathbf{x} e^{i2\pi \mathbf{q}\mathbf{x}/n_s} \langle \Theta_{33}(0,0) \Theta_{33}(\tau, \mathbf{x}) \rangle$$

$$T = 1.5 T_C,\ r_{smear}\mathrm{T} = 0.25$$



Fit Result

T/T_c	1.5	3.0	4.5
χ^2/DOF	1.2(2)(6)	2.2(3)(5)	2.1(3)(8)
η/s	0.24(7)(6)	0.32(5)(5)	0.43(9)(7)



Lattice Results Summary

- Smearing strongly decreases statistical and cutoff effects
- Negligible finite volume errors
- η/s without multilevel

Outline

Discussion and Outlook

Technical:

- non-perturbative renormalization strategy for the (an)isotropic traceless energy momentum tensor
- Smearing to increase sensitivity to low energies and to remedy cutoff effects and to reduce statistical errors

Phenomenological:

- η/s consistent with expectations

Outlook:

- method can be extended for dynamical fermions
- replace smearing by Wilson Flow and get analytical understanding of the effect of smearing

Stout Smearing

damp UV fluctuations i.e. high energy part of the spectrum
generate "fat" links by

$$
\begin{aligned}
U'_\mu =& \mathrm{e}^{\mathrm{i}Q_\mu} U_\mu, \\
Q_\mu =& \frac{\mathrm{i}}{2}\left(\Omega_\mu^\dagger - \Omega_\mu - \frac{1}{3}\mathrm{tr}[\Omega_\mu^\dagger - \Omega_\mu]\right), \\
\Omega_\mu =& \left(\rho_{Stout} \sum_{\nu\neq\mu} C_{\mu\nu}\right) U_\mu^\dagger,
\end{aligned}
$$

with staples $C_{\mu\nu}$
effective smearing radius

$$r_{smear} = a\sqrt{8\rho_{Stout} N_{Stout}}.$$

Wilson Flow

flow $B_\mu(t,x)$ defined by

$$\frac{d}{dt}B_\mu = D_\nu G_{\nu\mu},$$
$$B_\mu|_{t=0} = A_\mu,$$

with field strength tensor G at finite t
Generated by stout smearing steps

$$\frac{d}{d\hat{t}}V_\mu = \mathrm{i}Q_\mu V_\mu,$$
$$V_\mu|_{t=0} = U_\mu,$$

$\rightarrow$ approximation by small stout smearing steps with
$r^2_{smear} = 8t = 8N_{stout}\rho_{stout}$

Smoothing Wilson Flow

The Wilson flow is a smoothing operation on gauge configurations. This is directly seen by its action on the gauge fields in leading order of perturbation theory in the bare coupling g_0 [Luscher:2010iy]

$$B_\mu(t,x) = g_0 \int d^4y K_t(x-y) A_\mu(y) + O(g_0^2),$$
$$K_t(z) = \frac{e^{-z^2/4t}}{(4\pi t)^2}\mu.$$

This makes the smoothing effect explicit with a radius of $\sqrt{8t}$. It has also been shown, that all observables measured at finite Wilson flow time are finite renormalized quantities and in particular with removed discretization errors.[Luscher:2010iy]

Renormalization Strategy

O_i: set of multiplicatively renormalizable observables
$\hat{O}_i$: set of discretized operators
Z_i: set of renormalization factors

$$\langle O_i \rangle = Z_i \langle \hat{O}_i \rangle.$$

Z_i, $\hat{O}_i$ depend on discretization, O_i is independent.

$$\begin{aligned} Z_i =& Z_i(a,\xi); \qquad \langle \hat{O}_i \rangle = \langle \hat{O}_i \rangle (a,\xi); \qquad O_i = O_i. \\ \Rightarrow & \langle O_i \rangle = Z_i(\xi_0) \langle \hat{O}_i \rangle (\xi_0) = Z_i(\xi_1) \langle \hat{O}_i \rangle (\xi_1). \end{aligned}$$

If $\langle \hat{O}_i \rangle \neq 0$ then measure $Z_i(\xi_0)/Z_i(\xi_1)$ by two simulations.

Renormalization Strategy

If $\langle \hat{O}_i \rangle = 0$ and $O_i = \int \mathrm{d}^4 x o_i(x)$, consider correlators

$$\begin{aligned} C_{ij}(\tau) := &\int \mathrm{d}^3\mathbf{x} \langle o_i(\tau, \mathbf{x}) o_j(0) \rangle \\ =&Z_i(\xi_0) Z_j(\xi_0) \int \mathrm{d}^3 x \langle \hat{o}_i(\tau, \mathbf{x}) \hat{o}_j(0) \rangle (\xi_0) \\ =&Z_i(\xi_1) Z_j(\xi_1) \int \mathrm{d}^3 x \langle \hat{o}_i(\tau, \mathbf{x}) \hat{o}_j(0) \rangle (\xi_1). \end{aligned}$$

$\Rightarrow$ overdetermined system of equations for $Z_i(\xi_0)/Z_i(\xi_1)$

- Choice of τ sets the renormalization scale.
- For absolute scale compare with isotropic (renormalized) data.

Renormalization Structure of $\Theta_{\mu\nu}$

Anisotropic case:
Determined by remaining cubic symmetry group of the lattice

$$\begin{aligned}
\Theta_{00}^{\overline{iso}} &= Z^{EE_0}\theta_{00}^{EE_0} + Z^{BB_0}\theta_{00}^{BB_0} \\
\Theta_{kk}^{\overline{iso}} &= Z^{EE_0}\theta_{kk}^{EE_0} + Z^{BB_0}\theta_{kk}^{BB_0} + Z^{EE_1}\theta_{kk}^{EE_1} + Z^{BB_1}\theta_{kk}^{BB_1} \\
\Theta_{0k}^{\overline{iso}} &= Z^{EB}\theta_{0k}^{EB} \\
\Theta_{kl}^{\overline{iso}} &= Z^{EE_2}\theta_{kl}^{EE_2} + Z^{BB_2}\theta_{kl}^{BB_2}.
\end{aligned}$$

7 Z factors instead of 2
degeneration in the isotropic case

$$\begin{aligned}
Z^{diag} &= Z^{EE_0} = Z^{BB_0} = Z^{EE_1} = Z^{BB_1} \\
Z^{rest} &= Z^{EB} = Z^{EE_2} = Z^{BB_2}
\end{aligned}$$

Operators and Renormalization Factors

operators	E and B	Z factor
$\sum_i \overline{F_{0i}^{iso}}^2$	$\sum_i E_i^2$	Z^{EE_0}
$\overline{F_{0i}^{iso}}^2 - \overline{F_{0j}^{iso}}^2$	$E_i^2 - E_j^2$	Z^{EE_1}
$\overline{F_{0i}^{iso}}\,\overline{F_{0j}^{iso}}$	$E_i E_j$	Z^{EE_2}
$\sum_{i<j} \overline{F_{ij}^{iso}}^2$	$\sum_k B_k^2$	Z^{BB_0}
$\overline{F_{ij}^{iso}}^2 - \overline{F_{jk}^{iso}}^2$	$B_k^2 - B_i^2$	Z^{BB_1}
$\overline{F_{ij}^{iso}}\,\overline{F_{jk}^{iso}}$	$B_k B_i$	Z^{BB_2}
$\overline{F_{0i}^{iso}}\,\overline{F_{ji}^{iso}}$	$E_i B_k$	Z^{EB}

Table: Operators belonging to irreducible representations of the cubic group in terms of the anisotropic field strength tensor $\overline{F_{\mu\nu}^{iso}}$

Anisotropic Energy Momentum Tensor

$$
\begin{aligned}
\Theta_{00}^{\overline{iso}} &= Z^{EE_0}\theta_{00}^{EE_0} + Z^{BB_0}\theta_{00}^{BB_0} \\
\Theta_{kk}^{\overline{iso}} &= Z^{EE_0}\theta_{kk}^{EE_0} + Z^{BB_0}\theta_{kk}^{BB_0} + Z^{EE_1}\theta_{kk}^{EE_1} + Z^{BB_1}\theta_{kk}^{BB_1} \\
\Theta_{0k}^{\overline{iso}} &= Z^{EB}\theta_{0k}^{EB} \\
\Theta_{kl}^{\overline{iso}} &= Z^{EE_2}\theta_{kl}^{EE_2} + Z^{BB_2}\theta_{kl}^{BB_2}.
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_{00}^{EE_0} &= +\frac{1}{2}\sum_i F_{0i}^{\overline{iso}\,2} = +\frac{1}{2}\sum_i E_i^2 \\
\theta_{00}^{BB_0} &= -\frac{1}{2}\sum_{i<j} F_{ij}^{\overline{iso}\,2} = -\frac{1}{2}\sum_l B_l^2
\end{aligned}
$$

Anisotropic Energy Momentum Tensor

$$\theta^{EE_0}_{kk} = -\frac{1}{6}\sum_i \overline{F^{iso}_{0i}}^2 = -\frac{1}{6}\sum_i E_i^2 = -\frac{1}{3}\theta^{EE_0}_{00}$$

$$\theta^{BB_0}_{kk} = +\frac{1}{6}\sum_{i<j} \overline{F^{iso}_{ij}}^2 = +\frac{1}{6}\sum_k B_k^2 = -\frac{1}{3}\theta^{BB_0}_{00}$$

$$\theta^{EE_1}_{kk} = +\frac{1}{3}(2\ \overline{F^{iso}_{0k}}^2 - \sum_{i\neq k} \overline{F^{iso}_{0i}}^2) = +\frac{1}{3}(2E_k^2 - \sum_{i\neq k} E_i^2)$$

$$\theta^{BB_1}_{kk} = -\frac{1}{3}(2 \sum_{i<j, i\neq k\neq j} \overline{F^{iso}_{ij}}^2 - \sum_{i\neq k} \overline{F^{iso}_{ki}}^2) = -\frac{1}{3}(2B_k^2 - \sum_{j\neq k} B_j^2)$$

$$\theta^{EB}_{0k} = \sum_{l\neq k} \overline{F^{iso}_{0l}}\,\overline{F^{iso}_{kl}} = \sum_{l\neq k}\sum_{k\neq i\neq l} E_l B_i$$

$$\theta^{EE_2}_{kl} = \overline{F^{iso}_{k0}}\,\overline{F^{iso}_{l0}} = E_k E_l$$

$$\theta^{BB_2}_{kl} = \sum_{k\neq i\neq l} \overline{F^{iso}_{ki}}\,\overline{F^{iso}_{li}} = B_l B_k.$$

Diagonal Renormalization Factors

- cubic symmetry: $\langle \Theta^{XX_1}_{kk} \rangle = 0 = \langle \Theta^{EB}_{0k} \rangle = \langle \Theta^{XX_2}_{kl} \rangle$
- two diagonal factors Z^{XX_0} remaining

isotropic diagonal factor from from thermodynamics study [Borsanyi:2012ve]

$$\frac{sT}{4} = \langle \Theta^{iso}_{11} \rangle = Z^{diag} \langle \theta^{iso}_{11} \rangle$$

measure the anisotropic counterparts

$$Z^{XX_0} \langle \theta^{XX_0}_{11} \rangle (\xi) = Z^{diag} \langle \theta^{XX_0}_{11} \rangle (0).$$

Works also below T_c where s is very small:
calculate Z at the same β above T_c

Non-Diagonal Renormalization Factors

- Z^{XX_1} by degeneracy of the factors in the isotopic Z^{diag}
- BUT: $\langle \Theta_{kk}^{XX_1} \rangle = 0 \Rightarrow$ use correlators at finite τ

$$\left(Z^{XX_1}\right)^2 \int d\mathbf{x} \langle \theta_{11}^{XX_1}(\tau, \mathbf{x}) \theta_{11}^{XX_1}(0, \mathbf{0}) \rangle(\xi) =$$
$$= \left(Z^{diag}\right)^2 \int d\mathbf{x} \langle \theta_{11}^{XX_1}(\tau, \mathbf{x}) \theta_{11}^{XX_1}(0, \mathbf{0}) \rangle(0),$$

Analogously relate Z^{XX_2}, Z^{EB} to Z^{rest}.
BUT: $\nexists$ scalar expectation value $\propto Z^{rest}$ for absolute scale
Use general tensor decomposition [Karsch:1986cq,Meyer:2007ic]

$$\left(Z^{rest}\right)^2 \langle \theta_{12}(\tau, \mathbf{x}) \theta_{12}(0, \mathbf{0}) \rangle =$$
$$= \frac{1}{4} \left(Z^{diag}\right)^2 \langle (\theta_{11} - \theta_{22})(\tau, \mathbf{x}) (\theta_{11} - \theta_{22})(0, \mathbf{0}) \rangle.$$

General tensor decomposition

[Karsch:1986cq,Meyer:2007ic]

$$\langle \Theta_{ij}(x)\Theta_{kl}(y)\rangle = A(x-y)(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) + B(x-y)\delta_{ij}\delta_{kl},$$

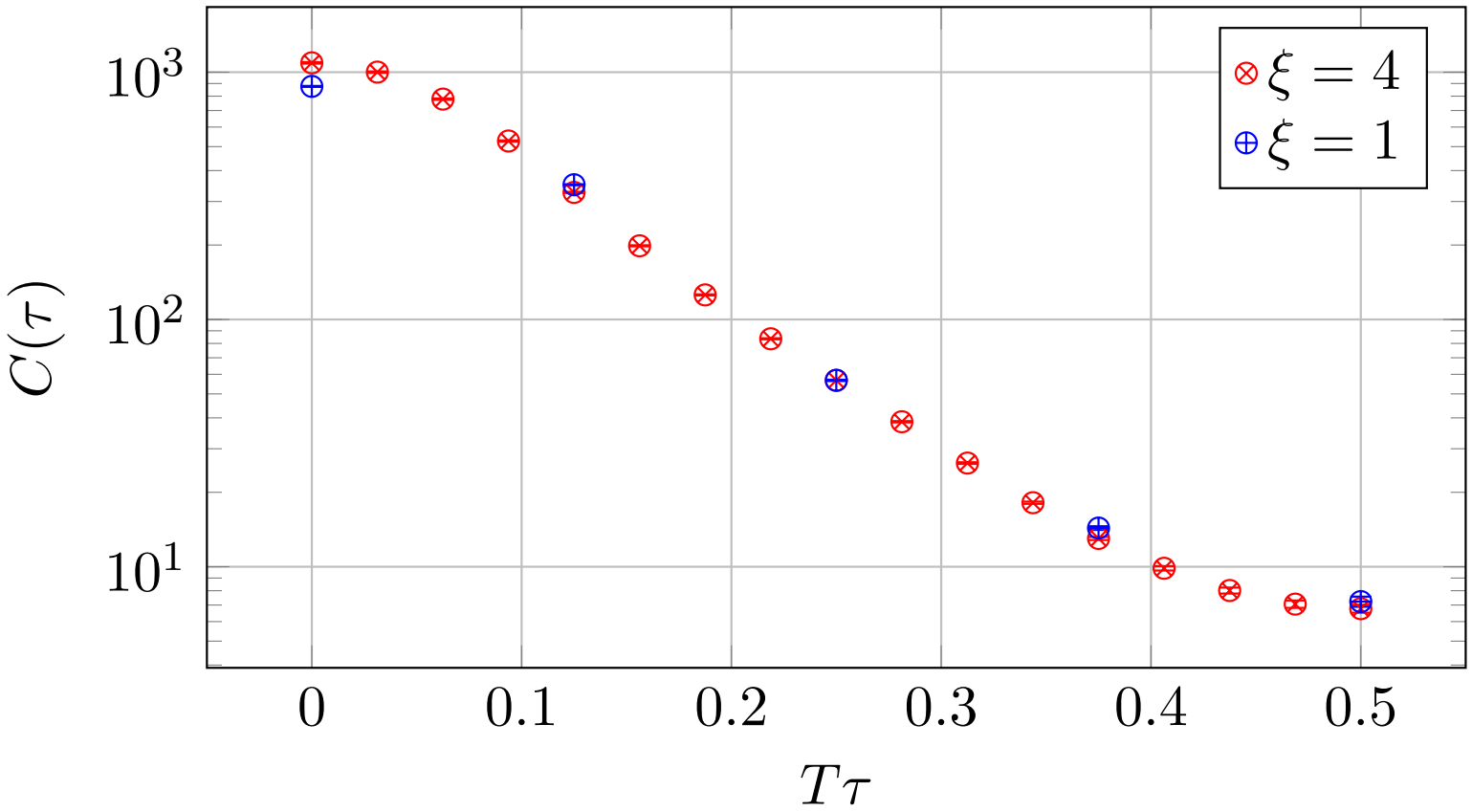
where i,j,k, and l are spatial indices. Valid for the continuum and makes use of periodic boundary conditions and the cubic symmetry in the spatial directions.

$$\begin{aligned}\langle \Theta_{12}(\tau,\mathbf{x})\Theta_{12}(0,\mathbf{0})\rangle =& \frac{1}{4}\langle(\Theta_{11}-\Theta_{22})(\tau,\mathbf{x})(\Theta_{11}-\Theta_{22})(0,\mathbf{0})\rangle \\ \Leftrightarrow \langle \theta_{12}(\tau,\mathbf{x})\theta_{12}(0,\mathbf{0})\rangle =& \frac{\left(Z^{diag}\right)^2}{4\left(Z^{rest}\right)^2}\langle(\theta_{11}-\theta_{22})(\tau,\mathbf{x})(\theta_{11}-\theta_{22})(0,\mathbf{0})\rangle.\end{aligned}$$

Anisotropic Renormalization

- simulations @ $T = 1.5\ T_C$
- $\xi = 1.0, 4.0$
- physically equivalent smearing/flow
- "renormalization scale" is at a Euclidean time $T\tau = 0.25$
- Shear channel buildt from diagonal components
  $C(\tau) = \frac{1}{T^5} \int \mathrm{d}\mathbf{x} \langle \Theta_{12}(0,0) \Theta_{12}(\tau, \mathbf{x}) \rangle$

Anisotropic Renormalization



- Good overall agreement between the (an)isotropic correlator
- resulting factors: $Z^{BB_1} = 0.937(2)$ and $Z^{EE_1} = 13.19(6)$ (tree-level: $Z^{BB_1}_{t.l.} = 1$ and $Z^{EE_1}_{t.l.} = \xi^2 = 16$)

Systematic Parameter Choices

choices for fixed parameters

3x λ, 2x x, 3x x_p, 3x smearing width Σ, 2x smearing form, 3x fit initial conditions

λ	0.1, 0.14, 0.20
x	1.2, 1.4
x_p	1.2, 1.4, 1.6
smearing width Σ	$0.5T$, $1T$, $2T$
smearing form	$W^{\tanh}$, W^{erf}
fit i.con. Γ	$0.2T^3, 0.35T^3, 0.5T^3$

$\Rightarrow$ 324 separate analyses for every temperature, systematic error

Anisotropic Parameters

n_t	n_s	β	ξ_0	N_{Sto}	ρ_{Sto}	ξ_{Sto}	N_{conf}	N_{sep}	N_{str}
8	16	4.81166	1.0	1	0.0791572	1.0	$8.7 \cdot 10^5$	1	5
32	16	5.06027	3.59877	11	0.00719611	4.0	$2.9 \cdot 10^6$	1	23

Table: Parameters of the simulation to demonstrate the anisotropic renormalization strategy.

Discretization Parameters

T/T_C	n_t	n_s	β	N_{Sto}	ρ_{Sto}	N_{conf}	N_{sep}	N_{str}
0.75	16	16	4.81166	25	0.0837365	$5.4 \cdot 10^4$	16	2
0.75	20	20	4.98659	36	0.0872255	$4.8 \cdot 10^4$	24	4
0.75	24	24	5.13241	49	0.0942036	$3.7 \cdot 10^4$	32	8
0.75	32	32	5.36624	72	0.111649	$1.6 \cdot 10^4$	48	22
1.5	8	16	4.81166	25	0.0837365	$4.8 \cdot 10^4$	8	6
1.5	10	20	4.98659	36	0.0872255	$1.1 \cdot 10^5$	12	3
1.5	12	24	5.13241	49	0.0942036	$1.4 \cdot 10^5$	16	6
1.5	16	32	5.36624	72	0.111649	$6.2 \cdot 10^4$	24	18
3.0	8	16	5.36624	25	0.0837365	$5.9 \cdot 10^4$	8	1
3.0	10	20	5.54986	36	0.0872255	$4.3 \cdot 10^4$	12	2
3.0	12	24	5.70092	49	0.0942036	$5.4 \cdot 10^4$	16	4
3.0	16	32	5.94063	72	0.111649	$4.0 \cdot 10^4$	24	25
4.5	8	16	5.70092	25	0.0837365	$7.6 \cdot 10^4$	8	1
4.5	10	20	5.88676	36	0.0872255	$4.5 \cdot 10^4$	12	3
4.5	12	24	6.03933	49	0.0942036	$3.7 \cdot 10^4$	16	2
4.5	16	32	6.29871	72	0.111649	$3.8 \cdot 10^4$	24	19

Table: Parameters of the simulations used to estimate discretization errors.

Finite Volume Parameters

T/T_C	n_t	n_s	β	N_{Sto}	ρ_{Sto}	N_{conf}	N_{sep}	N_{str}
1.5	8	16	4.81166	25	0.0837365	$4.8 \cdot 10^4$	8	6
1.5	8	48	4.81166	25	0.0837365	$1.8 \cdot 10^4$	32	9
1.5	8	64	4.81166	25	0.0837365	$7.7 \cdot 10^3$	32	9

Table: Parameters of the simulations used to estimate finite volume effects.

Viscosity Parameters

T/T_C	n_t	n_s	β	N_{Sto}	ρ_{Sto}	N_{conf}	N_{sep}	N_{str}
1.5	8	48	4.81166	25	0.0837365	$5.6 \cdot 10^4$	32	27
1.5	8	64	4.81166	25	0.0837365	$2.4 \cdot 10^4$	32	27
3.0	8	48	4.81166	25	0.0837365	$2.2 \cdot 10^4$	32	9
3.0	8	64	4.81166	25	0.0837365	$9.4 \cdot 10^3$	32	9
4.5	8	48	5.70092	25	0.0837365	$2.7 \cdot 10^4$	32	18
4.5	8	64	5.70092	25	0.0837365	$1.1 \cdot 10^4$	32	18

Table: Parameters of the finite momentum runs.

Viscosity Fit Details

- correlated in Euclidean time, flow time, and momentum
- completely inside a Jackknife
- given results are averaged over subset of the analyses with $0.7 \leq \chi^2/DOF \leq 3.0$

Error Details

- The first error is the statistical error. It is the average of the standard error from the jackknife procedure.
- The second error is a systematic error. It is the standard error over all analyses from the histogram method.

Fit Result

$$\frac{\rho_{low}(\omega,q)}{\tanh(\omega/2T)} = \frac{2\Gamma_s}{\pi}\frac{(\epsilon+p)\omega^4}{(\omega^2 - v_s(q)^2q^2)^2 + (\Gamma_s\omega q^2)^2}\frac{1+\sigma_1\omega^2}{1+\sigma_2\omega^2}$$

$$v_s(q) = v_s\left(1 + \frac{\Gamma_s}{2}q^2\left(\tau_\Pi - \frac{\Gamma_s}{4v_s^2}\right) + O(q^4)\right),$$

T/T_c	1.5	3.0	4.5
χ^2/DOF	1.2(2)(6)	2.2(3)(5)	2.1(3)(8)
η/s	0.24(7)(6)	0.32(5)(5)	0.43(9)(7)
σ_1	-0.07(3)(4)	-0.14(2)(3)	-0.12(4)(4)
σ_2	0.03(2)(3)	0.09(4)(4)	0.1(2)(2)
τ	3(11)(5)	7(22)(26)	20(70)(40)
v	4(2)(2)	2(1)(2)	2(2)(1)
Ω_r	0.97(2)(1)	0.93(2)(3)	0.94(2)(3)

- b. müller, arxiv:1309.7616 [nucl-th].
- p. kovtun, d. t. son and a. o. starinets, phys. rev. lett. **94** (2005) 111601 [hep-th/0405231].
- c. gale, s. jeon, b. schenke, p. tribedy and r. venugopalan, phys. rev. lett. **110** (2013) 012302 [arxiv:1209.6330 [nucl-th]].
- l. pang, q. wang and x. -n. wang, nucl. phys. a **904-905** (2013) 811c [arxiv:1211.1570 [nucl-th]].
- g. s. bali, f. bruckmann, g. endrodi and a. schäfer, phys. rev. lett. **112** (2014) 042301 [arxiv:1311.2559 [hep-lat]].
- d. steineder, s. a. stricker and a. vuorinen, jhep **1307** (2013) 014 [arxiv:1304.3404 [hep-ph]].
- f. karsch and h. w. wyld, phys. rev. d **35** (1987) 2518.
- h. b. meyer, phys. rev. d **76** (2007) 101701 [arxiv:0704.1801 [hep-lat]].

G. Aarts and J. M. Martinez Resco, JHEP **0204** (2002) 053 [hep-ph/0203177].

h. b. meyer, nucl. phys. a **830** (2009) 641c [arxiv:0907.4095 [hep-lat]].

m. luscher, jhep **1008** (2010) 071 [arxiv:1006.4518 [hep-lat]].

s. borsanyi, s. durr, z. fodor, c. hoelbling, s. d. katz, s. krieg, t. kurth and l. lellouch *et al.*, jhep **1209** (2012) 010 [arxiv:1203.4469 [hep-lat]].

s. borsanyi, s. durr, z. fodor, s. d. katz, s. krieg, t. kurth, s. mages and a. schäfer *et al.*, arxiv:1205.0781 [hep-lat].

l. del debbio, a. patella and a. rago, jhep **1311** (2013) 212 [arxiv:1306.1173 [hep-th]].

h. suzuki, ptep **2013** (2013) 8, 083b03 [arxiv:1304.0533 [hep-lat]].

d. teaney, phys. rev. d **74** (2006) 045025 [hep-ph/0602044].

m. asakawa, t. hatsuda and y. nakahara, prog. part. nucl. phys. **46** (2001) 459 [hep-lat/0011040].

g. aarts, c. allton, j. foley, s. hands and s. kim, phys. rev. lett. **99** (2007) 022002 [hep-lat/0703008 [hep-lat]].

k. symanzik, nucl. phys. b **226** (1983) 187.

m. lüscher and p. weisz, commun. math. phys. **97**, 59 (1985) [erratum-ibid. **98**, 433 (1985)].

h. baier, h. boettiger, m. drochner, n. eicker, u. fischer, z. fodor, a. frommer and c. gomez *et al.*, pos lat **2009** (2009) 001 [arxiv:0911.2174 [hep-lat]].

c. morningstar and m. j. peardon, phys. rev. d **69** (2004) 054501 [hep-lat/0311018].

s. borsanyi, g. endrodi, z. fodor, s. d. katz and k. k. szabo, jhep **1207** (2012) 056 [arxiv:1204.6184 [hep-lat]].

S. Sakai, A. Nakamura and T. Saito, Nucl. Phys. A **638** (1998) 535 [hep-lat/9810031].

A. Nakamura and S. Sakai, Phys. Rev. Lett. **94** (2005) 072305 [hep-lat/0406009].

S. Sakai and A. Nakamura, PoS LAT **2005** (2006) 186 [hep-lat/0510100].

D. Teaney, Phys. Rev. D **74** (2006) 045025 [hep-ph/0602044].

r. baier, p. romatschke, d. t. son, a. o. starinets and m. a. stephanov, jhep **0804** (2008) 100 [arxiv:0712.2451 [hep-th]].

m. haas, l. fister and j. m. pawlowski, arxiv:1308.4960 [hep-ph].