Evidence of BRST-Symmetry Breaking in Lattice Minimal Landau Gauge

Attilio Cucchieri*, David Dudal+, Tereza Mendes* & Nele Vandersickel+

* IFSC – Universidade de São Paulo, Brazil & + Department of Physics and Astronomy, Ghent University, Belgium

BRST-Symmetry Breaking

The study of color confinement in Yang-Mills theories in minimal Landau gauge is an active area of research [1]. Let us recall that, in this case, the gauge condition is implemented by restricting the functional integral over gauge-field configurations to the so-called Gribov region Ω. This restriction can be achieved by adding a nonlocal term S_h, the horizon function, to the usual Landau gauge-fixed Yang-Mills action $S_{YM}+S_{gf}$. One thus obtains the Gribov-Zwanziger (GZ) action $S_{GZ}=S_{YM}+S_{gf}+\gamma^4 S_h$, where the massive parameter γ, known as the Gribov parameter, is dynamically determined (in a self-consistent way) through the so-called horizon condition. In order to localize the GZ action [2] one introduces a pair of complex-conjugate bosonic fields $(\bar{\phi}^{ac}_\mu, \phi^{ac}_\mu)$ and a pair of Grassmann complex-conjugate fields $(\bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$. Then, the GZ action can be written as $S_{GZ}=S_{YM}+S_{gf}+S_{aux}+S_\gamma$, where

$$
\begin{aligned}
S_{aux} &= \int d^4x \left[\bar{\phi}^{ac}_\mu \partial_\nu \left(D^{ab}_\nu \phi^{bc}_\mu\right) - \bar{\omega}^{ac}_\mu \partial_\nu \left(D^{ab}_\nu \omega^{bc}_\mu\right)\right. \\
&\quad \left. - g_0 \left(\partial_\nu \bar{\omega}^{ac}_\mu\right) f^{abd} D^{be}_\nu \eta^e \phi^{dc}_\mu\right] \qquad (1) \\
S_\gamma &= \int d^4x \left[\gamma^2 D^{ba}_\nu \left(\phi^{ab}_\nu + \bar{\phi}^{ab}_\nu\right) - 4\left(N_c^2-1\right)\gamma^4\right]. \qquad (2)
\end{aligned}
$$

Under the nilpotent BRST variation s, the four auxiliary fields form two BRST doublets, i.e. $s\phi^{ac}_\mu = \omega^{ac}_\mu$, $s\omega^{ac}_\mu = 0$, $s\bar{\omega}^{ac}_\mu = \bar{\phi}^{ac}_\mu$ and $s\bar{\phi}^{ac}_\mu = 0$, giving rise to a BRST quartet. At the same time, one can check that the localized GZ theory is not BRST-invariant. Indeed, while $s(S_{YM}+S_{gf}+S_{aux})=0$, one finds that $sS_\gamma \propto \gamma^2 \neq 0$. Since a nonzero value for the Gribov parameter γ is related to the restriction of the functional integration to the Gribov region Ω, it is clear that BRST-symmetry breaking is a direct consequence of this restriction.

The Bose-Ghost Propagator

In order to study numerically the effect of the BRST-breaking term S_γ, one can consider the expectation value of a BRST-exact quantity. One such possibility is the correlation function

$$Q^{abcd}_{\mu\nu}(x,y) = \langle s(\phi^{ab}_\mu(x)\bar{\omega}^{cd}_\nu(y))\rangle = \langle \omega^{ab}_\mu(x)\bar{\omega}^{cd}_\nu(y) + \phi^{ab}_\mu(x)\bar{\phi}^{cd}_\nu(y)\rangle. \qquad (3)$$

While the above expectation value should be zero for a BRST-invariant theory, it does not necessarily vanish if BRST symmetry is broken. Indeed, at tree level (and in momentum space) one finds [2, 3]

$$Q^{abcd}_{\mu\nu}(p,p') = \frac{(2\pi)^4\,\delta^{(4)}(p+p')\,g_0^2\gamma^4 f^{abe} f^{cde} P_{\mu\nu}(p)}{p^2\left(p^4+2g_0^2 N_c\gamma^4\right)}, \qquad (4)$$

where $P_{\mu\nu}(p)$ is the usual transverse projector. Thus, this propagator is proportional to the Gribov parameter γ, i.e. its nonzero value is clearly related to the breaking of the BRST symmetry in the GZ theory. One should also recall that this Bose-ghost propagator has been proposed as a carrier of long-range confining force in minimal Landau gauge [4, 5, 6].

On the lattice one does not have direct access to the auxiliary fields $(\bar{\phi}^{ac}_\mu, \phi^{ac}_\mu)$ and $(\bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$. On the other hand, by 1) adding suitable sources to the GZ action, 2) explicitly integrating over the four auxiliary fields and 3) taking the usual functional derivatives with respect to the sources, one can verify that [4]

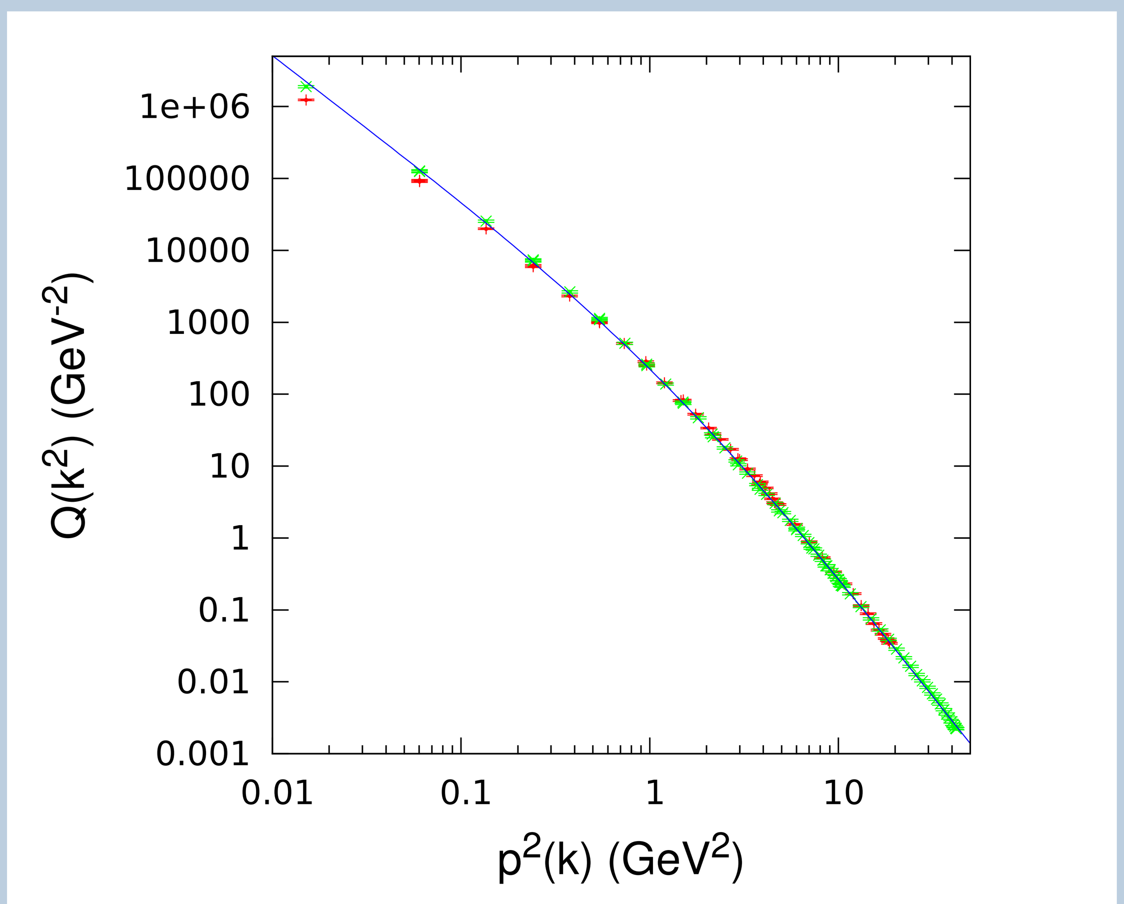
$$Q^{abcd}_{\mu\nu}(x-y) = \gamma^4 \left\langle R^{ab}_\mu(x) R^{cd}_\nu(y)\right\rangle, \qquad (5)$$

where $R^{ac}_\mu(x) = \int d^4z\,(\mathcal{M}^{-1})^{ae}(x,z)\,B^{ec}_\mu(z)$ and $B^{bc}_\nu(x) = g_0 f^{bec} A^e_\nu(x)$.

Numerical Simulations

We evaluated [7] the Bose-ghost propagator as defined in Eq. (5) above —modulo the global factor γ^4— using numerical simulations in the SU(2) case. In order to check discretization and finite-volume effects, we considered three different values of the lattice coupling β and five different physical volumes, ranging from about $(3.366\,fm)^4$ to $(10.097\,fm)^4$. Numerical results for the scalar function $Q(k^2)$, defined through the relation $Q^{ac}(k) \equiv Q^{abcb}_{\mu\mu}(k) \equiv \delta^{ac} N_c P_{\mu\mu}(k) Q(k^2)$ [see Eq. (4)], are shown in Figs. 1 and 2. The data scale quite well, even though small deviations are observable in the IR limit (see Fig. 2). We also fit the data using the fitting function

$$f(p^2) = \frac{c}{p^4}\,\frac{p^2+s}{p^4+u^2p^2+t^2}. \qquad (6)$$



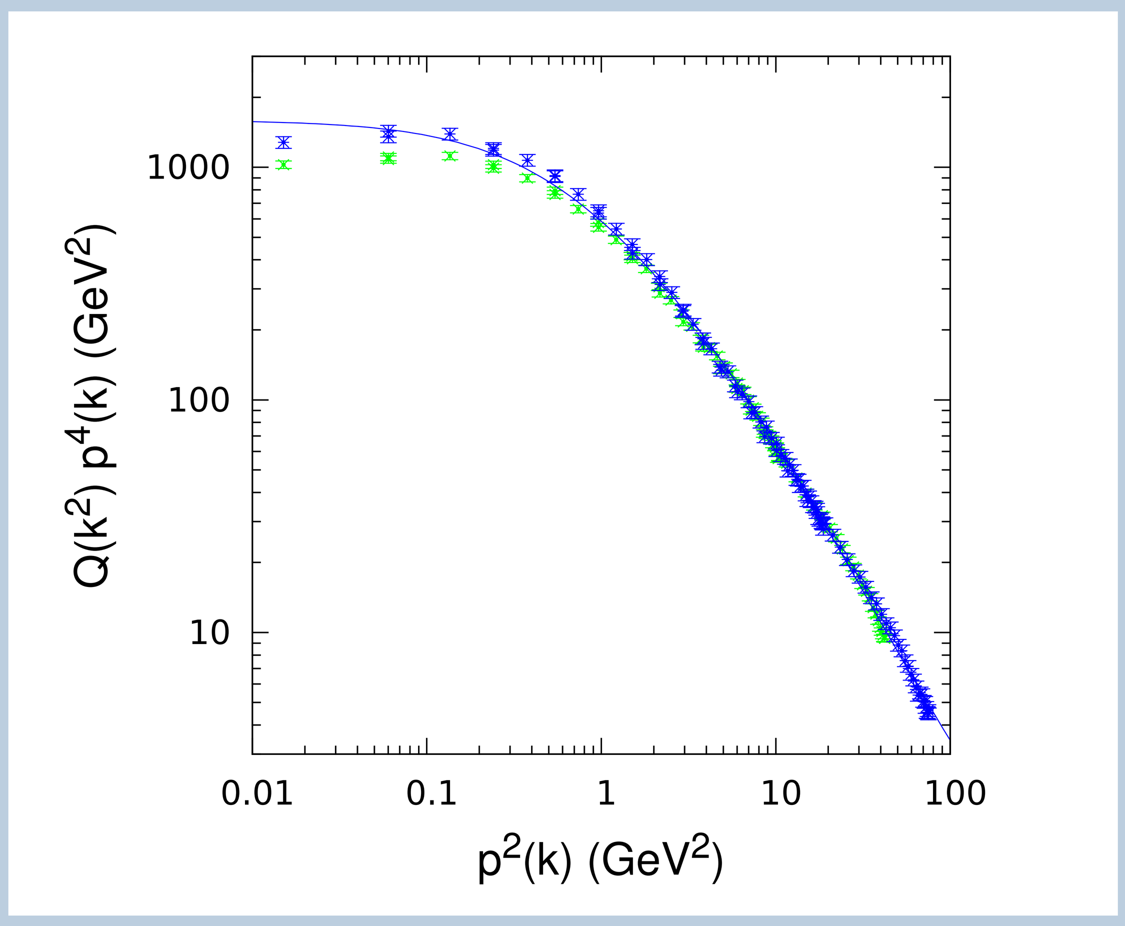


Figure 1 (left): data for $\beta = 2.2$, $V = 48^4$ (+) matched [8] with data for $\beta = 2.34940204$, $V = 72^4$ ($\times$), fitted using Eq. (6) with $t = 3.2(0.3)(GeV^2)$, $u = 3.6(0.4)(GeV)$, $s = 46(13)(GeV^2)$ and $c = 114(13)$. **Figure 2 (right):** data for $\beta = 2.34940204$, $V = 72^4$ ($\times$) matched [8] with data $\beta = 2.43668228$, $V = 96^4$ ($*$), fitted using Eq. (6) with $t = 3.0(0.2)(GeV^2)$, $u = 3.9(0.3)(GeV)$, $s = 58.0(9.8)(GeV^2)$ and $c = 247(16)$.

Following the analysis in [4, 6], this fitting function corresponds to considering an infrared-free ghost propagator $G(p^2)$ and a massive gluon propagator $D(p^2)$. The fit describes the data quite well. As a consequence, the Bose-ghost propagator presents a p^{-4} singularity in the infrared (IR) limit. This result is in agreement with the one-loop analysis carried out in [9]. One should stress that, even though a double-pole singularity is suggestive of a long-range interaction, the above result does not imply a linearly-rising potential between quarks [4, 6, 9]. Indeed, when coupled to quarks via the $A-\phi$ propagator —which is nonzero due to the vertex term $\bar{\phi}^{ac}_\mu\, g f^{acb} A^c_\nu \partial_\nu \phi^{bc}_\mu$—, the Bose-ghost propagator gets a momentum factor at each vertex [4, 6], i.e. the effective propagator is given by p^{-2} in the IR limit.

Conclusions

We presented the first numerical evaluation of the Bose-ghost propagator in minimal Landau gauge. We find that our data are well described by a simple fitting function, which can be related to a massive gluon propagator in combination with an IR-free (Faddeev-Popov) ghost propagator, implying a p^{-4} singularity in the IR limit. Our results constitute the first numerical manifestation of BRST-symmetry breaking due to the restriction of the functional integration to the Gribov region Ω in the GZ approach. This directly affects continuum functional studies in Landau gauge, which usually employ lattice results as an input and/or as a comparison. At the same time, several questions are still open for a clear understanding of the GZ approach. In particular, one should understand how a physical positive-definite Hilbert space could be defined in this case.

References

[1] J. Greensite, Lect. Notes Phys. **821**, 1 (2011).
[2] N. Vandersickel and D. Zwanziger, Phys. Rept. **520**, 175 (2012).
[3] J. A. Gracey, Eur. Phys. J. **C70**, 451 (2010).
[4] D. Zwanziger, arXiv:0904.2380 [hep-th].
[5] S. Furui, PoS **LAT 2009**, 227 (2009).
[6] D. Zwanziger, Phys. Rev. **D81**, 125027 (2010).
[7] A. Cucchieri, D. Dudal, T. Mendes and N. Vandersickel, arXiv:1405.1547 [hep-lat].
[8] D. B. Leinweber *et al.*, Phys. Rev. **D60**, 094507 (1999) [Erratum-ibid. **D61**, 079901 (2000)]; A. Cucchieri, T. Mendes and A. R. Taurines, Phys. Rev. **D67**, 091502 (2003).
[9] J. A. Gracey, JHEP **1002**, 009 (2010).