Investigation of the tetra-quark candidate $a_0(980)$: preliminary results

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Contents

- Interpolating Operators and Correlation Matrix.
- Gauge Configurations.
- Quark Propagators.
- Results.
- Conclusions & outlook.
Interpolating Operators

\[ \mathcal{O}^{q\bar{q}} = \sum_x \left( \bar{d}_x u_x \right) \]

\[ \mathcal{O}^{K\bar{K}, \text{point}} = \sum_x \left( \bar{s}_x \gamma_5 u_x \right) \left( \bar{d}_x \gamma_5 s_x \right) \]

\[ \mathcal{O}^{\eta_s \pi, \text{point}} = \sum_x \left( \bar{s}_x \gamma_5 s_x \right) \left( \bar{d}_x \gamma_5 u_x \right) \]

\[ \mathcal{O}^{Q\bar{Q}} = \sum_x \epsilon_{abc} \left( \bar{s}_{x,b} \left( C \gamma_5 \bar{d}^T_{x,c} \right) \right) \epsilon_{ade} \left( u^T_{x,d} \left( C \gamma_5 \right) s_{x,e} \right) \]

\[ \mathcal{O}^{K\bar{K}, \text{2-part}} = \sum_{x,y} \left( \bar{s}_x \gamma_5 u_x \right) \left( \bar{d}_y \gamma_5 s_y \right) \]

\[ \mathcal{O}^{\eta_s \pi, \text{2-part}} = \sum_{x,y} \left( \bar{s}_x \gamma_5 s_x \right) \left( \bar{d}_y \gamma_5 u_y \right) \]
The Correlation Matrix

\[ C_{jk}(t) = \langle \mathcal{O}_j(t) \mathcal{O}^\dagger_k(0) \rangle \]

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<thead>
<tr>
<th>( \mathcal{O}_{qq} )</th>
<th>( \mathcal{O}_{KK} ) ( \text{point} )</th>
<th>( \mathcal{O}_{\eta_s \pi} ) ( \text{point} )</th>
<th>( \mathcal{O}Q\bar{Q} )</th>
<th>( \mathcal{O}_{K\bar{K}} ) ( \text{2part} )</th>
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For more details, see previous talk by Joshua Berlin (Wed. 11:30-11:50)

A. Abdel-Rehim (The Cyprus Institute) Tetra-quark \( a_0(980) \): Preliminary Results
Gauge configurations with 2+1 dynamical clover fermions and the Iwasaki gauge action.

Configurations available through the PACS-CS collaboration.


Lattice: $32^3 \times 64$, Lattice spacing $\approx 0.09$ fm.

500 configurations at $M_\pi \approx 300$ MeV.

198 configurations at $M_\pi \approx 150$ MeV.
Propagators are smeared at the source and sink with APE smeared links and invariant Gaussian quark smearing.

Forward propagators for $u, d, s$ quarks computed for 5 random source locations on each configuration.

Strange quark loops computed with $Z_4$ noises diluted in time.

5 noises per time slice and only 15 time steps from the source.
### Results I: no 2-particle operators

<table>
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<tr>
<th>$\mathcal{O}^{q\bar{q}}$</th>
<th>$\mathcal{O}_{\text{point}}^{K\bar{K}}$</th>
<th>$\mathcal{O}_{\text{point}}^{\eta_s\pi}$</th>
<th>$\mathcal{O}_{\text{2part}}^{Q\bar{Q}}$</th>
<th>$\mathcal{O}_{\text{2part}}^{K\bar{K}}$</th>
<th>$\mathcal{O}_{\text{2part}}^{\eta_s\pi}$</th>
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Results for $m_\pi \approx 300$ MeV, 2500 measurements
Note: no mixing between $q\bar{q}$ and tetra-quarks (as expected)
Eigenvector Components, 3x3 connected

$M_{\pi}=300\text{ MeV}, 3x3\text{ GEVP},\text{ connected only ground state}$

$M_{\pi}=300\text{ MeV}, 3x3\text{ GEVP},\text{ connected only first excited state}$

$M_{\pi}=300\text{ MeV}, 3x3\text{ GEVP},\text{ connected only second excited state}$
Eigenvector Components, 4x4 connected

$M_{\pi}=300$ MeV, 4x4 GEVP, connected only

ground state

1st excited state

2nd excited state

3rd excited state

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Effective Mass, Connected+disconnected

$M_{\pi}=300$ MeV, $3x3$ GEVP, connected+disconnected

$M_{\pi}=300$ MeV, $4x4$ GEVP, connected+disconnected

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Tetra-quark $a_0(980)$: Preliminary Results
Eigenvector Components, 3x3 connected+disconnected

M_{\pi}=300 \text{ MeV}, 3x3 GEVP, connected+disconnected

- ground state
- first excited state
- second excited state

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Tetra-quark a_0(980): Preliminary Results
Eigenvector Components, 4x4 connected+disconnected

$M_{pi}=300$ MeV, 4x4 GEVP, connected+disconnected

ground state

1st excited state

2nd excited state

3rd excited state

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Tetra-quark $a_0(980)$: Preliminary Results
Comments on the eigenvector components

- $q\bar{q}$ mixing with tetraquark operators seem to be small but seems to affect the eigenvalues.
- Including disconnected diagrams but ignoring the $q\bar{q}$ seems to increase the diquark-antidiquark component of the lowest two states.
- Including $q\bar{q}$ seems to give a more consistent picture with or without disconnected diagrams where:
  - Ground state mainly $q\bar{q}$.
  - First and second excited states mainly a mix of $K\bar{K}$ and $\eta - \pi$.
  - Diquark-anti-diquark is the heaviest.
Two Exponential Fits
Two-Exponential Fits, 3x3 connected

M_{pi}=300 MeV, 3x3 correlation matrix, connected only

- State 1, connected
- State 2, connected
- State 3, connected

A. Abdel-Rehim (The Cyprus Institute)  Tetra-quark a_0(980): Preliminary Results
Two-Exponential Fits, 4x4 connected

\[ M_{\pi} = 300 \text{ MeV}, 4 \times 4 \text{ correlation matrix, connected only} \]

State 0, connected

State 1, connected

State 2, connected

State 3, connected

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Tetra-quark \(a_0(980)\): Preliminary Results
Two-Exponential Fits, 3x3 connected + disconnected

$M_{\pi}=300$ MeV, 3x3 correlation matrix, total

State 1, total

State 2, total

State 3, total

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Tetra-quark $a_0(980)$: Preliminary Results
Two-Exponential Fits, 4x4 connected + disconnected

$M_{\pi}=300$ MeV, 4x4 correlation matrix, total

Two exponential fit
State 0, total

Two exponential fit
State 1, total

Two exponential fit
State 2, total

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Extracted Energy Levels at $m_\pi \approx 300$ MeV

- States not shown were either undetermined by the fit or the data was too noisy.
- Only scattering states can be resolved unambiguously at this level of statistics.
- Currently adding the 2-particle operators to the correlation matrix and also using one-end trick to improve the $q\bar{q}$ correlator. This will hopefully allow us to resolve the $a_0(980)$.
Results for $m_\pi \approx 150$ MeV, 198 measurements
Effective Mass, Connected Only

$M_{\pi}=150$ MeV, $3 \times 3$ GEVP, connected only

State 1
State 2
State 3

$M_{\pi}=150$ MeV, $4 \times 4$ GEVP, connected only

State 0
State 1
State 2
State 3

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Eigenvector Components, 3x3 connected

$M_{\pi}=150$ MeV, 3x3 GEVP, connected only

- **Ground State**
  - KKbar
  - $\eta_{ss}\pi$
  - QQbar

- **First Excited State**
  - KKbar
  - $\eta_{ss}\pi$
  - QQbar

- **Second Excited State**
  - KKbar
  - $\eta_{ss}\pi$
  - QQbar

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Tetra-quark $a_0(980)$: Preliminary Results
Eigenvector Components, 4x4 connected

$M_{pi}=150$ MeV, 4x4 GEVP, connected only

ground state

$M_{pi}=150$ MeV, 4x4 GEVP, connected only
1st excited state

$M_{pi}=150$ MeV, 4x4 GEVP, connected only
2nd excited state

$M_{pi}=150$ MeV, 4x4 GEVP, connected only
3rd excited state

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Effective Mass, Connected+disconnected

$M_{\pi}=150$ MeV, 3x3 GEVP, connected+disconnected

State 1
State 2
State 3

$M_{\pi}=150$ MeV, 4x4 GEVP, connected+disconnected

State 0
State 1
State 2
State 3

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Tetra-quark $a_0(980)$: Preliminary Results
Eigenvector Components, 3x3 connected+disconnected

M_{\pi}=150 \text{ MeV}, 3x3 GEVP, connected+disconnected

|vj|² vs. t/a

Ground state:
- KK\bar{b}ar
- \eta_{\Delta\tau}\pi
- QQ\bar{b}ar

First excited state:
- KK\bar{b}ar
- \eta_{\Delta\tau}\pi
- QQ\bar{b}ar

Second excited state:
- KK\bar{b}ar
- \eta_{\Delta\tau}\pi
- QQ\bar{b}ar

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Tetra-quark a₀(980): Preliminary Results
Eigenvector Components, 4x4 connected+disconnected

$M_{pi}=150$ MeV, 4x4 GEVP, connected+disconnected

- **Ground state**
  - qqbar
  - KKbar
  - etass-pi
  - QQbar

- **1st excited state**
  - qqbar
  - KKbar
  - etass-pi
  - QQbar

- **2nd excited state**
  - qqbar
  - KKbar
  - etass-pi
  - QQbar

- **3rd excited state**
  - qqbar
  - KKbar
  - etass-pi
  - QQbar

A. Abdel-Rehim (The Cyprus Institute)  Tetra-quark $a_0(980)$: Preliminary Results
We reported on ongoing study of the scalar $a_0(980)$ using interpolating operators with two and four quarks. Both connected and disconnected contributions are included. The goal is to isolate this state and understand its quark sub-structure. Initial results show that including quark-antiquark operators as well as disconnected diagram will have an important effect on the spectrum. quark-antiquark operator seems to be noisy.

Outlook: currently using the one-end trick to compute the 2-particle operators as well as improve the statistics of the $q\bar{q}$ correlation functions.

Outlook: perform more elaborate fit including 2-particle states with opposite momenta.