Electric polarizability of neutral hadrons from lattice QCD

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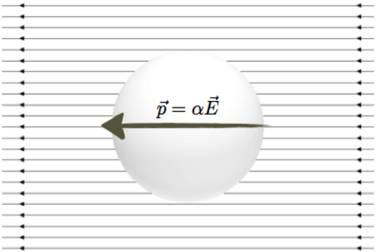
Lattice 2014

Review of Polarizability

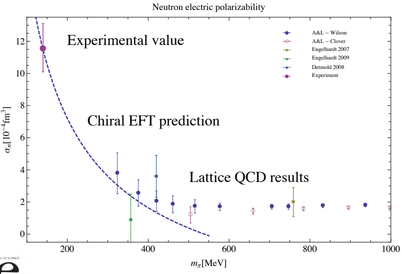
$$
\begin{aligned}
\mathcal{H}_{em} &= -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha\mathcal{E}^2 - \frac{1}{2}\beta B^2 \\
&+ -\frac{1}{2}\gamma_{E1}\vec{\sigma}\cdot\vec{\mathcal{E}}\times\dot{\vec{\mathcal{E}}} - \frac{1}{2}\gamma_{M1}\,\vec{\sigma}\cdot\vec{B}\times\dot{\vec{B}} \\
&+ \gamma_{E2}\sigma_i\mathcal{E}_{ij}B_j - \gamma_{M2}\sigma_i B_{ij}\mathcal{E}_j \\
&- \frac{1}{12}\alpha_{E2}\mathcal{E}_{ij}^2 - \frac{1}{12}\beta_{M2}B_{ij}^2 + \ldots
\end{aligned}
$$

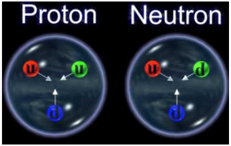


$$\Delta E = \frac{1}{2}\alpha\mathcal{E}^2$$

Motivation

- proton radius puzzle: largest uncertainty is from the proton polarizability. Pohl et al. *Nature 466 (07, 2010) 213-216*
- proton-neutron mass splitting. Largest uncertainty comes from $\beta^p - \beta^n = -0.87(85) \times 10^{-4}\,\mathrm{fm}^3$. Walker-Loud et al. *Phys.Rev.Lett. 108 (2012) 232301*
- First principle calculation.





Background Field Method

- Minimal coupling: $D_\mu = \partial_\mu - igG_\mu - iqA_\mu$

  Amounts to an overall phase factor to the original links

$$U_\mu \rightarrow e^{-iqaA_\mu} U_\mu$$

- Euclidean vector potential: $x_4 = ix_0$ and $A_4 = -iA_0$.

  For electric field in the x-direction:

$$A_M = \langle 0, \mathcal{E}t, 0, 0 \rangle \Rightarrow A_E = \langle -i\mathcal{E}x_4, 0, 0, 0 \rangle$$

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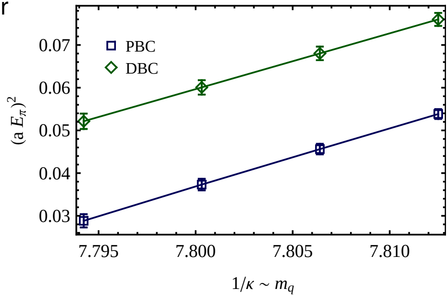
Dirichlet Boundary Conditions

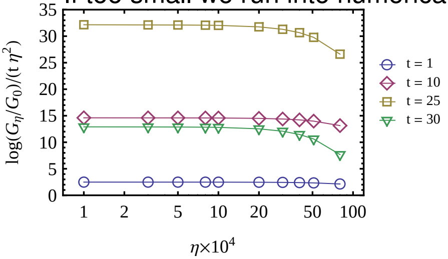
- PBC is not easily implemented.
- Vacuum is stable i.e. no Schwinger pair production.
- Limits the maximal separation between charges.
- Can use arbitrarily small values of the field.
- Introduces boundary effects
- non-zero momentum $\Rightarrow E = \sqrt{m^2 + (\pi/L)^2}$.
- Mass shift $\delta m = (E/m)\delta E$.



We can quantify the volume effects by using larger lattices.

Tuning the Electric Field

- Dirichlet boundary conditions allows us to use arbitrarily small values of the electric field
- We introduce a dimensionless parameter $\eta \equiv q_d a^2 \mathcal{E}$ to gauge the size of the field
- If η is too large we can't neglect higher order terms in the field expansion. If too small we run into numerical instabilities in fitting the data.



We plot $\log \frac{G_\eta(t)}{G_0(t)} / (t\eta^2)$.

$G_\eta(t) = (G_{+\eta}(t) + G_{-\eta}(t))/2.$

We chose $\eta = 10^{-4}$.

Extracting the energy shift

- Polarizability calculations require 5 times more quark propagators than hadron mass calculations.
- Due to the fact that u and d quarks have different charges.
- And because we compute both $+\eta$ and $-\eta$ correlators for more statistics.

$$\Rightarrow G_0, \;\; G_{+\eta}(t), \;\; G_{-\eta}(t)$$

Highly correlated because they are computed from the same Monte Carlo samples.

$$C_{ij} = \begin{bmatrix} G_0\,G_0 & G_0\,G_+ & G_0\,G_- \\ G_+\,G_0 & G_+\,G_+ & G_+\,G_- \\ G_-\,G_0 & G_-\,G_+ & G_-\,G_- \end{bmatrix}$$

$$\chi^2 = \sum_{ij} \left[f(t_i) - G(i)\right] C_{ij}^{-1} \left[f(t_j) - G(j)\right]$$

$$f(t) = (A + \delta A)\, e^{-(E+\delta E)t}$$

- with full correlation matrix: $a\delta E = 4.3 \pm 1.2 \times 10^{-7}$.
- excluding off-diagonal blocks: $a\delta E = 8.2 \pm 150000 \times 10^{-7}$.

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Ensemble Details

2-flavor n-HYP clover ensembles

$\eta = 10^{-4}$: Field size that an electron would create 0.5 fm away

- $m_\pi = 306$ MeV
- $a = 0.1255$ fm
- 4 volumes

1. $16 \times 16^2 \times 32$
2. $24 \times 24^2 \times 48$
3. $30 \times 24^2 \times 48$
4. $48 \times 24^2 \times 48$

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300+ configs/ensemble

100 quark propagators/config.

Done on multi-GPU systems: GWU Colonial One, USQCD, IMPACT clusters

370,000 GPU hours ⇒ 6 million CPU hours.

+10 million CPU hours for gauge generation

Larger lattices are elongated only in the field direction

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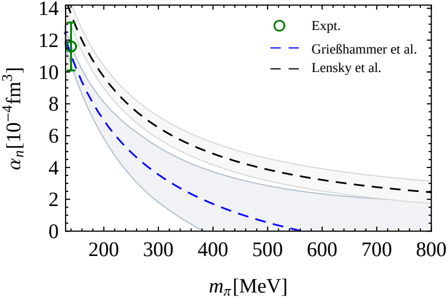
Neutron χPT prediction



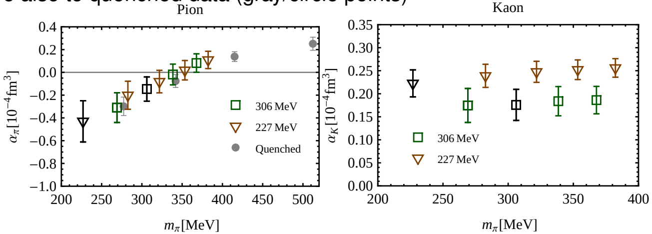
- H. Grießhammer et al. *Prog.Part.Nucl.Phys. 67 (2012) 841-897*
- J. McGovern et al. *Eur.Phys.J. A49 (2013) 12*
- V. Lensky and V. Pascalutsa, *Eur.Phys.J. C65 (2010) 195-209*

Leading order: $\alpha_n \propto 1/m_\pi$

More accurate as m_π decreases.

Sea and valence quark dependence: $L \simeq 3\,\text{fm}$

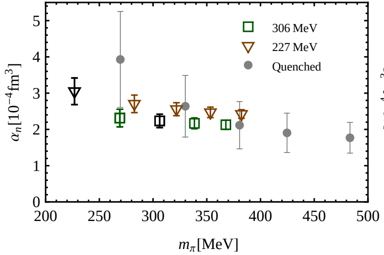
- Two sea unitary points (black points)
- Use multiple valence quark masses to gauge the quark dependence: Partial quenching
- Chose valence quark masses around the dynamical points
- $m_\pi = 227$ MeV (orange/triangle points), $m_\pi = 306$ MeV (green/square points).
- Compare also to quenched data (gray/circle points)

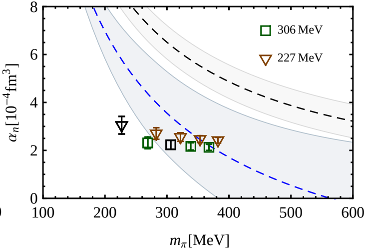


ML *et al.* Phys.Rev. D89 (2014) 074506

Quenched data: Alexandru and Lee PoS LATTICE2010 (2010) 131

continued... Neutron: $L \simeq 3\,\mathrm{fm}$

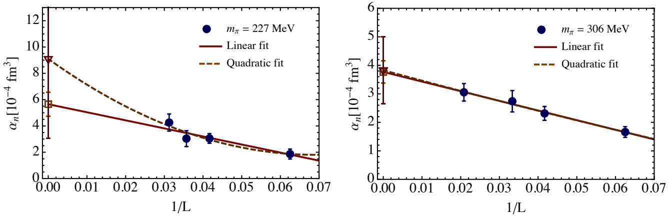




- mild dependence on valence quark mass
- slight shift upwards between sea quark masses
- Not in good agreement with χPT
- Maybe from finite volume corrections

Finite Volume Corrections

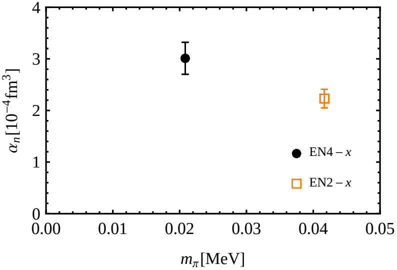
- Consider only the dynamical points.
- For each pion mass: compute α on four different volumes.
- Perform an infinite volume extrapolation:
  - Linear: $a + b/L_x$
  - quadratic: $a + b/L_x + c/L_x^2$



Both are consistent and have good Q.
Algebraic dependence on finite volume corrections. Plot is along the field direction.

Finite Volume Corrections continued...

- Analyze this effect for neutron polarizability: 306 MeV pion.
- Take two volumes: “EN2" = 24×24^2 and “EN4" = 48×24^2.
- Place the electric field along the y-direction on the 48×24^2.
- Expect this to be consistent with our results for the 24×24^2 ensemble.



- Finite volume corrections to α occur along the field direction.
- From now on we show our polarizability results in the infinite volume limit

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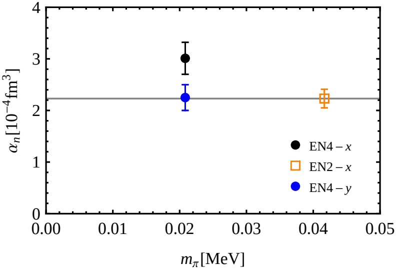

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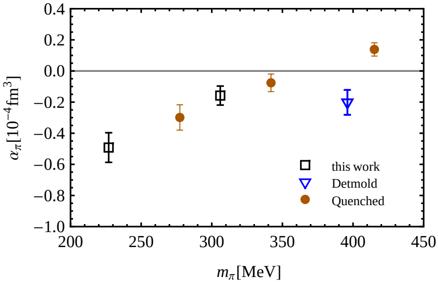
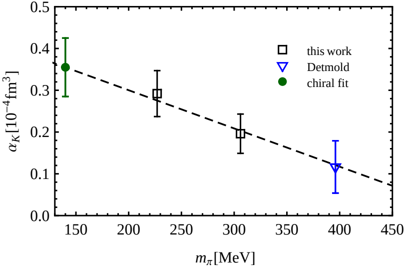
Neutral Pion: Volume Extrapolation



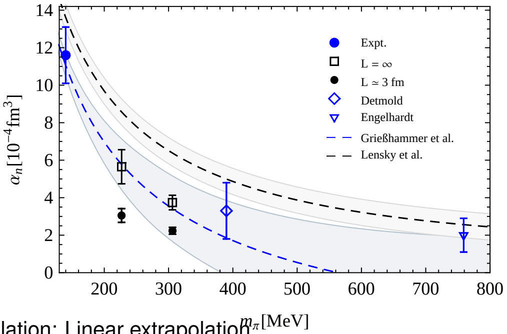
- Infinite volume extrapolation: constant and linear fit. Both have good Q levels
- Still exhibits negative trend. Not due to volume corrections.
- Confirms the quenched study by Alexandru and Lee PoS LATTICE2010 (2010) 131

Neutral Kaon: Volume and Chiral Extrapolation



- Volume extrapolation: Linear extrapolation
- Chiral extrapolation: Linear extrapolation (mild dependence on valence quark mass)
- $\alpha_K^0 = 0.35(7) \times 10^{-4}$ fm^3.
- χPT predicts $\alpha_K^0 = 0$ at $\mathcal{O}(p^4)$.
- No experimental measurements to date.

Neutron Polarizability



- Volume extrapolation: Linear extrapolation
- Agrees very well with χPT (Greißhammer et al.)
- Small errorbars

Summary

This Work

- Analyzed the neutron, pion, and kaon electric polarizability
- Closest pion masses to the physical point: $m_\pi = 220$ MeV and 306 MeV
- Volume extrapolation, chiral extrapolation for the kaon
- Neutron polarizability agrees well with χPT predictions, *after* infinite volume extrapolation

Ongoing Work

- Charging the sea quarks (next talk)
- Chiral extrapolation for neutron with the help of χPT
- Charged particles

Spin 1/2 particles: Compton Polarizability

$$\mathcal{L} = \bar{N}(i\gamma_\mu D^\mu - M)N + \left(\frac{e\kappa}{4M}F_{\mu\nu}\right)\bar{N}\sigma^{\mu\nu}N + \left(\frac{e\kappa}{4M} - \frac{er_E^2}{6}\right)\partial^\nu F_{\mu\nu}\bar{N}\gamma_\mu N$$
$$+ \left(\frac{\bar{\beta}}{4}F_{\mu\nu}F^{\mu\nu}\bar{N}N - \frac{\bar{\alpha}+\bar{\beta}}{4M}F_{\mu\alpha}F^{\nu\alpha}\bar{N}\gamma^\mu iD_\nu N\right) + \ldots .$$

A. L'vov, Int.J.Mod.Phys. A8 (1993) 5267-5305

Without polarizability terms (i.e. $\bar{\alpha} = \bar{\beta} = 0$) there is still a contribution from the particles magnetic moment.

$$i\frac{\partial\Phi}{\partial t} = \left[\frac{p^2}{2m} - \vec{\mu}\cdot\left(\vec{\mathcal{E}}\times\frac{\vec{p}}{m}\right) + \frac{\mu^2}{2m}\mathcal{E}^2\right]\Phi.$$

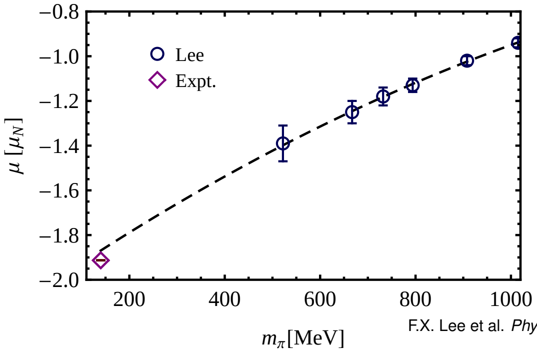
Detmold et al. Phys. Rev. D81 054502

2nd term is zero for Dirichlet boundary conditions

$$\alpha_s = \bar{\alpha} - \mu^2/2m$$

α_s is the static polarizability which we compute on the lattice.

Neutron magnetic moment

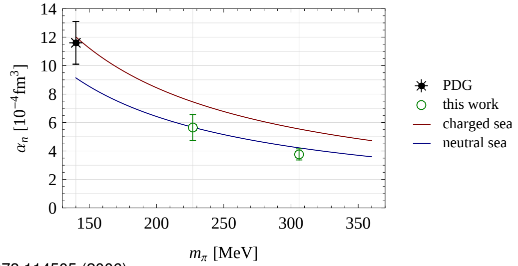


F.X. Lee et al. *Phys.Lett. B627 (2005) 71-76*

At the physical point this correction is $< 5\%$.

Charging the Sea Quarks

- Predictions from χPT



Detmold et al. Phys. Rev. D73 114505 (2006)

- From the lattice: Work pioneered by Engelhardt Phys. Rev. D76 114502. $m_\pi \simeq 800, 320$ MeV.
- Large computational effort to include the effects into our calculation. Freeman et al. PoS LATTICE2012 and PoS LATTICE2013 (2013)

δE and δm