$B \to \pi \ell \nu$ semileptonic form factors from unquenched lattice QCD and $|V_{ub}|$

Daping Du
On the behalf of Fermilab lattice and MILC collaborations

Syracuse University, NY

The 32nd International Symposium on Lattice Field Theory

Columbia University, New York, June 23-28, 2014

Author list

Jon A. Bailey
A. Bazavov
C. Bernard
C. M. Bouchard
C. DeTar
Daping Du *
A. X. El-Khadra
J. Foley
E. D. Freeland
E. Gámiz
Steven Gottlieb
U. M. Heller
A. S. Kronfeld
J. Laiho
L. Levkova
Yuzhi Liu
P. B. Mackenzie
Y. Meurice
E. T. Neil
Si-Wei Qiu
J. Simone
R. Sugar
D. Toussaint
R. S. Van de Water
Ran Zhou

(Fermilab lattice and MILC collaborations)

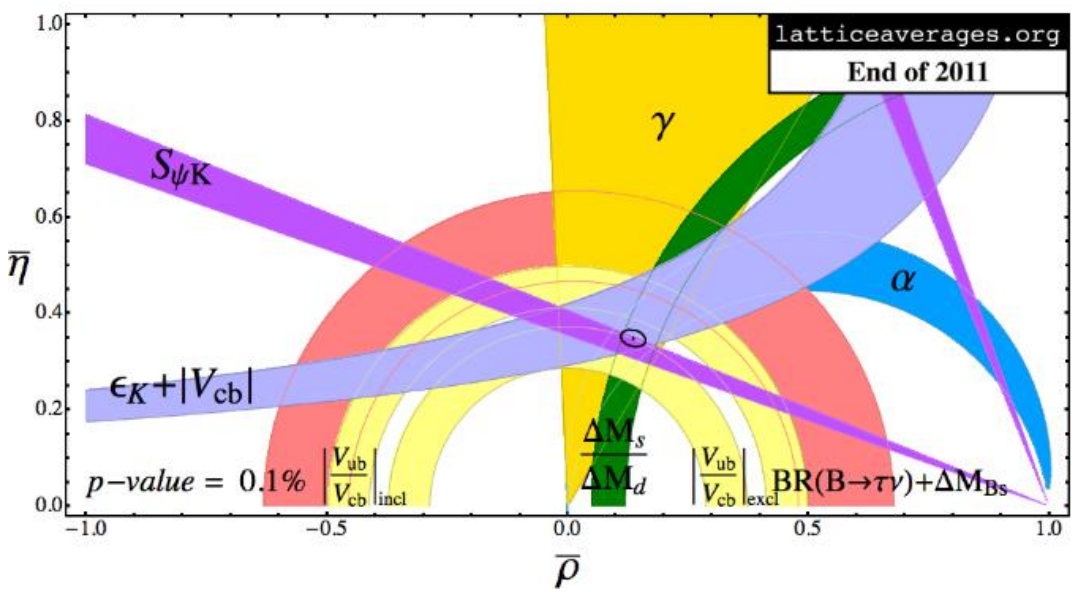
Motivation

- A precise $|V_{ub}|$ will improve understanding on unitary triangle, CP violation and weak decay.
- **Exclusive** semileptonic decay $B \to \pi \ell \nu$ to determine $|V_{ub}|$:

  Theory(Lattice or LCSR) + Experiment

- Rare decay $B^{\pm} \to \pi^{\pm} \ell^{+} \ell^{-}$

  GIM, Cabibbo, loop suppressed



Exclusive decays:

BF % from **experiments**:

- BaBar 2010: 6.1% **1005.3288v2**
- Belle 2011: 5.3% **1012.0090**
- BaBar 2012: 5% **1208.1253**
- Belle 2013 tagged: 7.7% **1306.2781**

HFAG PDG 2013: 3.1%

Unquenched **Lattice** : at $q^2 \sim 20\ GeV^2$

- HPQCD 2006: ~10%
- FNAL/MILC 2008: ~9.4%

Form factors

- The partial decay rate is related to $|V_{ub}|$ and form factor f_+ (zero lepton mass limit) by

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^+ \ell^- \nu) = \text{Phase space} \times |V_{ub}|^2 |f_+(q^2)|^2$$

- The heavy-to-light hadronic matrix element

$$\begin{aligned}\langle \pi | \mathcal{V}^\mu | B \rangle &= f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu \\ &= \sqrt{2M_B}[v^\mu f_{\|}(E_\pi) + p_\perp^\mu f_\perp(E_\pi)]\end{aligned}$$

- Form factor f_0 is also important in some BSM models (τ lepton).
- We also calculate the tensor form factor f_T

$$\langle \pi | \mathcal{T}^{\mu\nu} | B \rangle = -\frac{q^2(p_B + p_\pi)^\mu - (M_B^2 - M_\pi^2) q^\mu}{M_B + M_\pi} f_T(q^2)$$

for the rare $B^\pm \to \pi^\pm \ell^+ \ell^-$ decay **(1303.6010, 1312.2523)**

Simulation data

- **Twelve** ($N_f = 2 + 1$) asqtad MILC ensembles; **Four** lattice spacings
- Full-QCD asqtad staggered light quarks; Fermilab b quark.
- Improvements with respect to FNAL/MILC 2008 **0811.3640**
  - **Increased statistics**: 2X ensembles, ~3X number of configurations
  - **Finer lattice spacings**: a_{min} = 0.09 fm → 0.045 fm
  - **Smaller light quark masses**: (M_π=177~450 MeV)

Part of FNAL/MILC 2008

a(fm)	am'_l/am'_s	Volume	$N_{src} \times N_{\rm cfg}$	$M_\pi L$	M_π(MeV)
~0.12	0.01/0.05	$20^3 \times 64$	4×2259 (592)	4.5	389
	0.007/0.05	$20^3 \times 64$	4×2110 (836)	3.8	327
	0.005/0.05	$24^3 \times 64$	4×2099 (529)	3.8	277
~0.09	0.0062/0.031	$28^3 \times 96$	4×1931 (557)	4.1	354
	0.00465/0.031	$32^3 \times 96$	4×984	4.1	307
	0.0031/0.031	$40^3 \times 96$	4×1015	4.2	249
	0.00155/0.031	$64^3 \times 96$	4×791	4.8	177
~0.06	0.0072/0.018	$48^3 \times 144$	4×593	6.3	450
	0.0036/0.018	$48^3 \times 144$	4×673	4.5	316
	0.0025/0.018	$56^3 \times 144$	4×801	4.4	264
	0.0018/0.018	$64^3 \times 144$	4×827	4.3	224
~0.045	0.0028/0.014	$64^3 \times 192$	4×801	4.6	324

Correlator fits

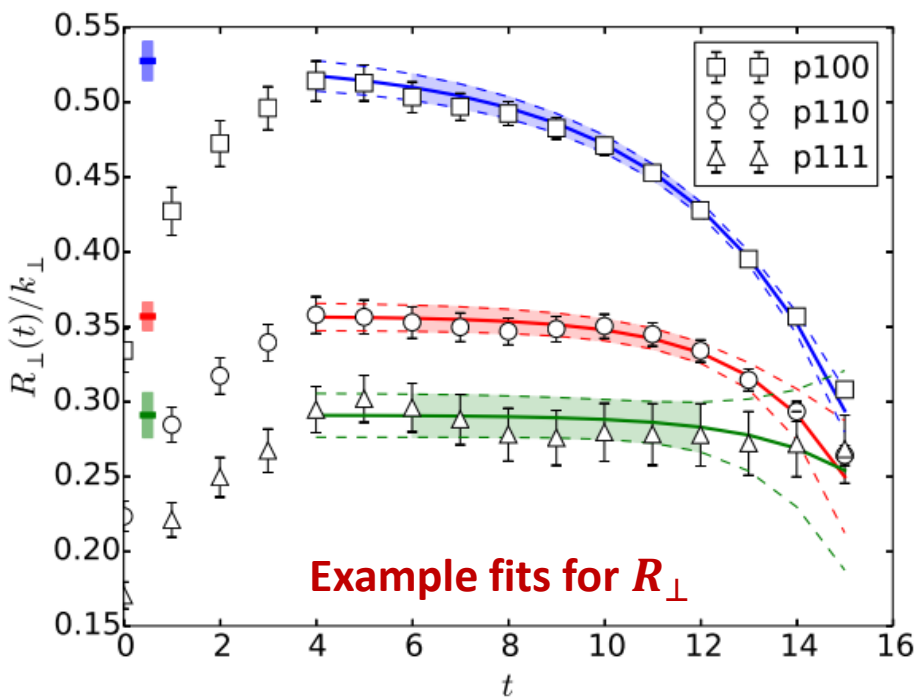
- Use ratios constructed from two- and three-point functions **0811.3640**

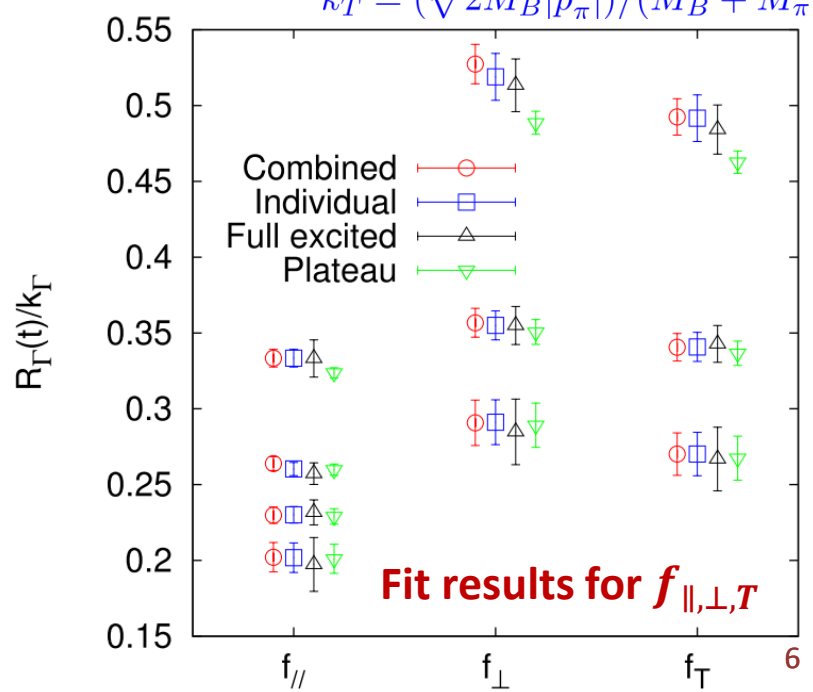
$$R_\Gamma(t,T_a;\mathbf{p}) = \frac{\overline{C^\Gamma_{3pt}}(t,T_a;\mathbf{p})}{\sqrt{\overline{C^\pi_{2pt}}(t;\mathbf{p})\overline{C^B_{2pt}}(T_a-t;\mathbf{0})}}\sqrt{\frac{2E^{(0)}_\pi(\mathbf{p})}{e^{-E^{(0)}_\pi(\mathbf{p})t-M^{(0)}_B(T_a-t)}}}, \qquad \Gamma = \parallel, \perp, T$$

- Fitting with B-meson excited state to reduce systematic effect

$$R_\Gamma(t)/k_\Gamma = f^{lat}_\Gamma\left[1+\mathcal{A}_\Gamma e^{-\Delta M_B(T_a-t)}\right]$$

$k_\parallel = 1,$
$k_\perp = |p^i_\pi|,$
$k_T = (\sqrt{2M_B}|p^i_\pi|)/(M_B+M_\pi)$





Chiral and continuum extrapolation

- We use NLO staggered heavy-light meson χPT + NNLO analytics terms

$$f = f^{(0)}\Big(c_0(1+\text{logs}) + c_1 m_{q,val} + c_3 E_\pi + c_4 E_\pi^2 + c_5 a^2 + \text{NNLO. analyt.}\Big)$$

0704.0795

hard-pion SU(2) limit

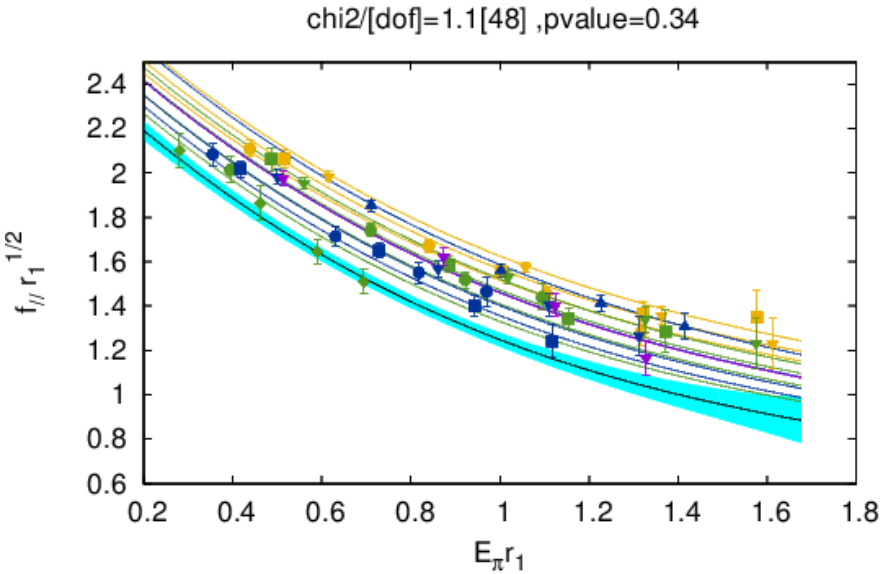
Leading order form: $f_{\parallel}^{(0)} = 1/f_\pi,\ f_{\perp,T}^{(0)} = (1/f_\pi)(g/(E_\pi + \Delta_{BB^*}))$

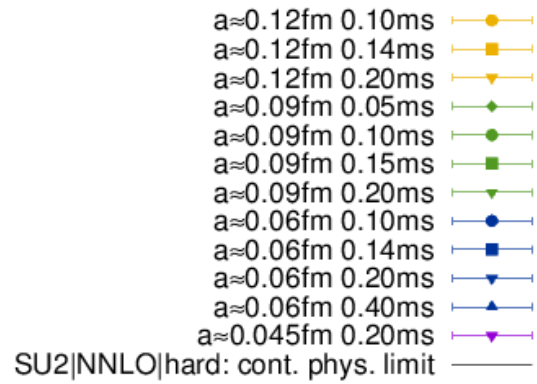
- Use hard-pion approximation because the external pions are too energetic to satisfy $E_\pi \sim M_\pi$. 1011.6531 ,1006.1197
- Use SU(2) χPT in which the strange sea quark is integrated out.

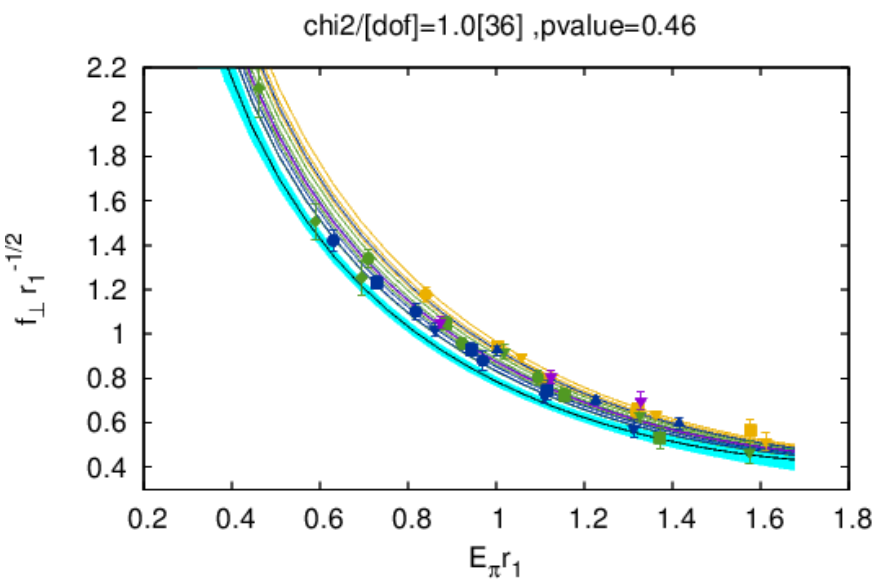
  The NNLO SU(3) extrapolation is not stable with the data used: almost identical $m_{s,sea}$ → not able to fit reliably with $m_{s,sea}$ dependence. A new ensemble with $a \approx 0.12$fm, m_l/m_s=0.005/0.005 is included to stablize the fit.
- The B-B^*-π coupling $g_\pi = 0.45 \pm 0.08$ is from lattice determinations. 1109.2480
- Incorporate HQ discretization effects in the chiral fit 1112.3051

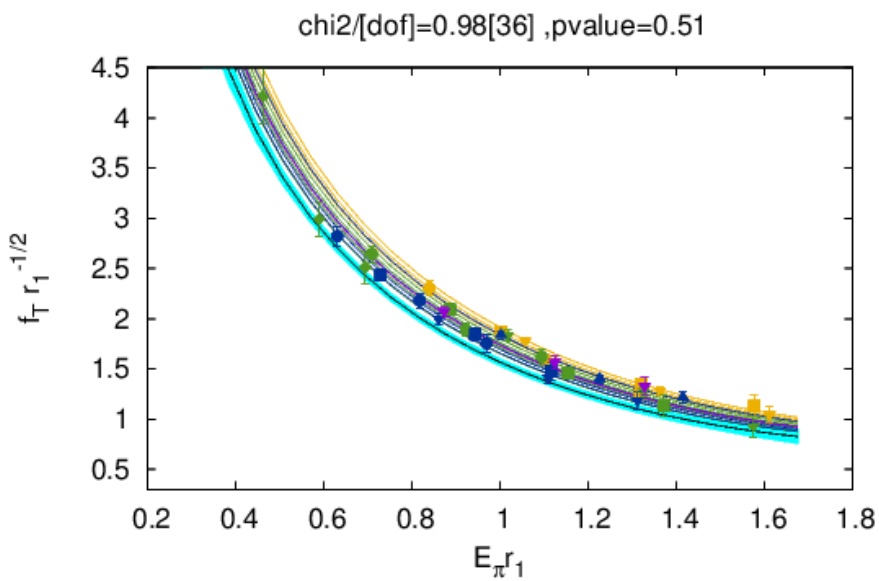
$$f = f^{(0)}\left[c_0(1+\text{logs}_{hard}^{NLO}) + \delta f_{analytic}^{NLO} + \delta f_{analytic}^{NNLO}\right](1+\delta f^{\text{HQ disc.}}).$$

Chiral-continuum fit results



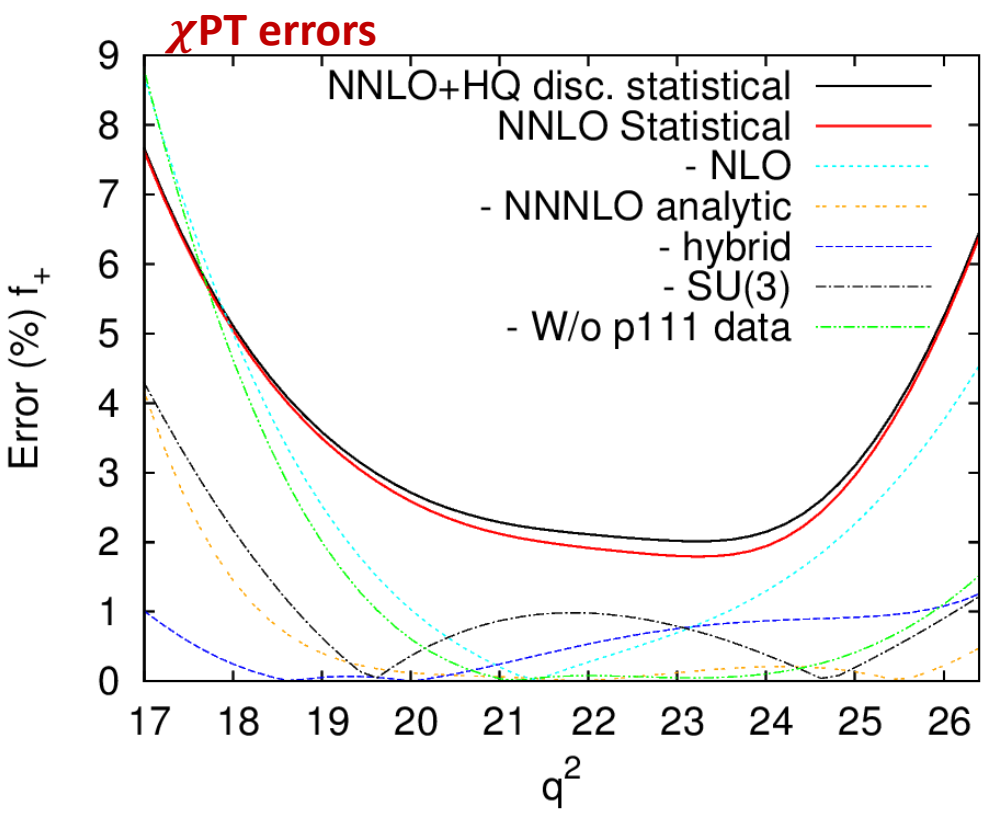


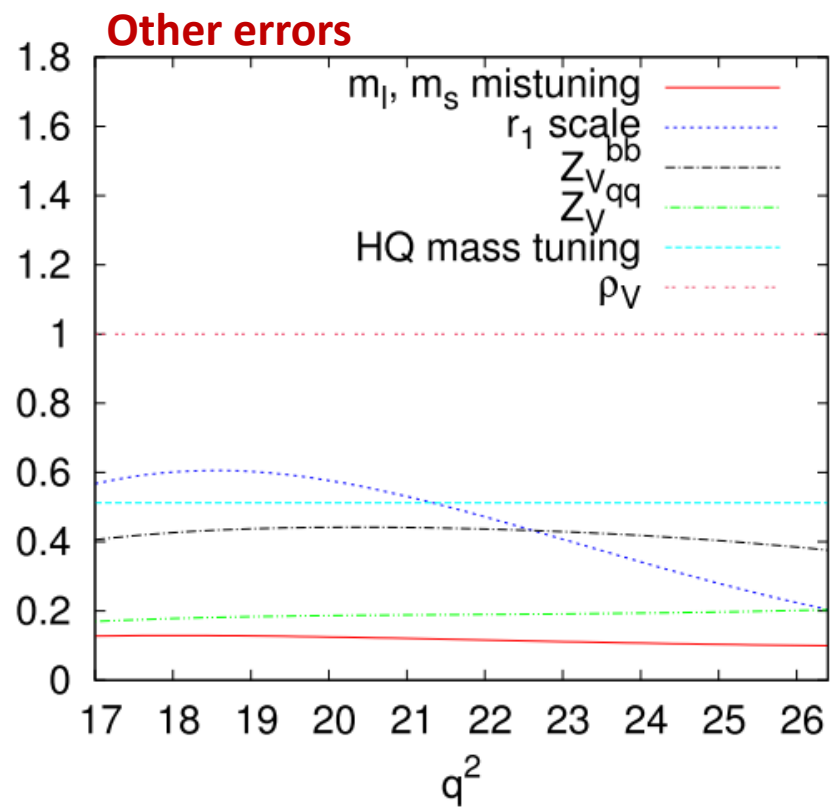




Systematic errors

- The NNLO statistical error covers different variations in the chiral and continuum extrapolation. Quote no additional systematic for χPT.
- All other errors are sub percent and are largely independent of q^2.





Error budget at $q^2 = 20\ \ GeV^2$

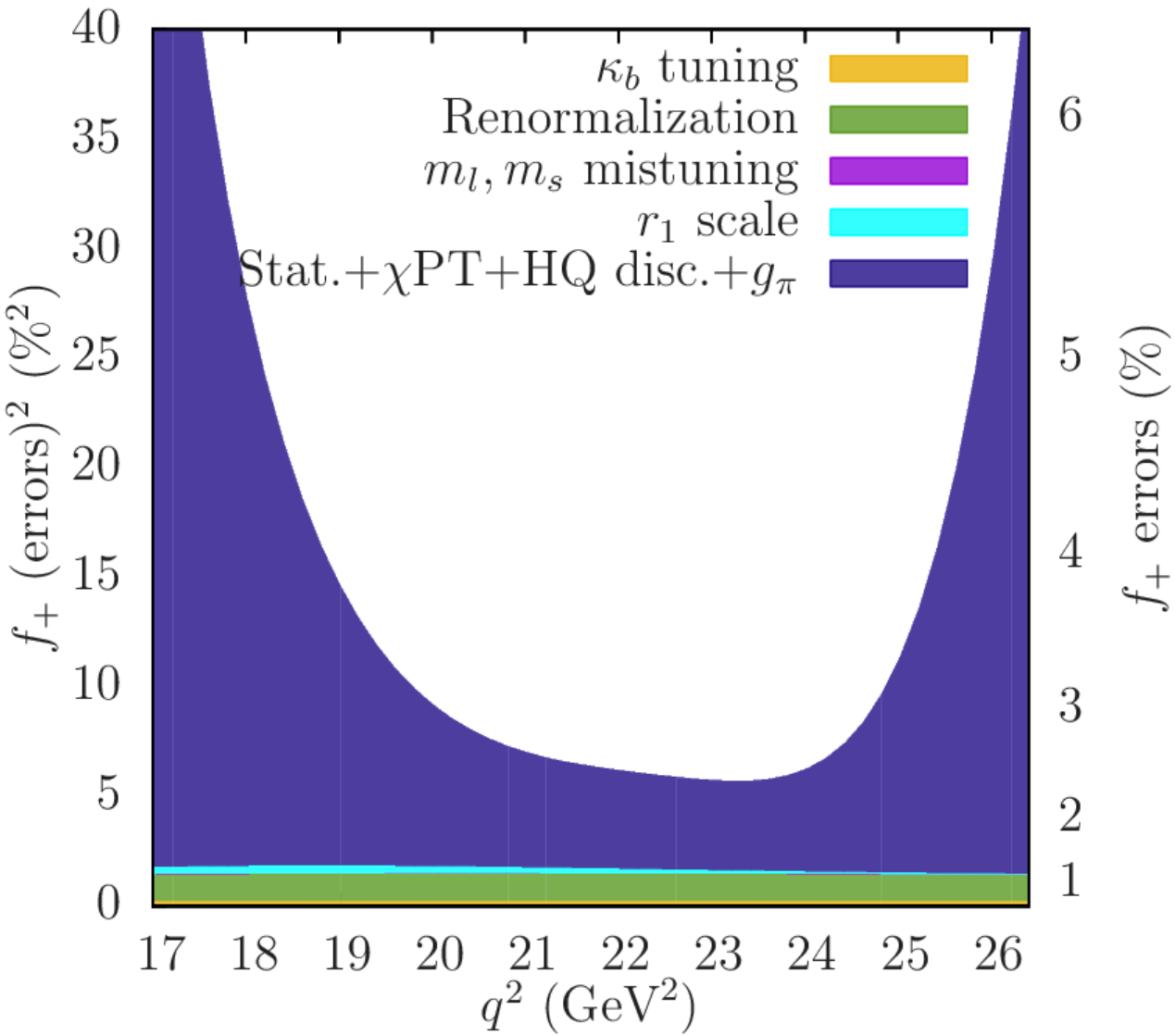
- The kinematic point $q^2 = 20\ GeV^2$ represents the region where the lattice uncertainty and experimental uncertainty are comparable.

  Decisive to determine $|V_{ub}|$.
- The errors in "()" are already included in "Statistical+χPT+HQ+g_π"

Uncertainty (%)	$\delta f_+(q^2=20)$	$\delta f_0(q^2=20)$	$\delta f_T(q^2=20)$
Statistical+χPT+HQ+g_π	2.7	3.2	3.2
(NLO χPT)	(1.1)	(0.6)	(0.8)
(Heavy quark discretization)	(<1.5)	(<1.5)	(<1.5)
(Coupling $g_{BB^*\pi}$)	(0.5)	(0.7)	(0.5)
Scale r_1	0.6	0.7	0.7
Non-perturbative Z_V^{bb}	0.4	0.5	0.5
Non-perturbative Z_V^{qq}	0.2	0.2	0.2
Perturbative ρ	1.0	1.0	1.0
Heavy b quark mass mistuning	0.5	0.7	0.6
Light quark mass tuning	0.1	0.2	0.2
Total	3.0	3.4	3.5

Preliminary

Error budget for f_+



Extrapolation in q^2: z-expansion

- Use model-independent expansion in parameter z to extrapolate to full kinematic range. **hep-ph/9702300**
- Use Bourrely-Caprini-Lellouch (BCL) formulae, which have the right asymptotic at high q^2 and threshold) **0807.2722**

$$f_{+,T}(z) = \frac{1}{1-t(z)/t_*}\sum_{n=0}^{N_z-1} b_n\left[z^n - (-1)^{n-N_z}\frac{n}{N_z}z^{N_z}\right],$$

$$f_0(z) = \sum_{n=0}^{N_z} b_n z^n,$$

$$t = q^2, \quad t_* = M_{B^*}^2$$

- Functional z-expansion:

Advantage over <u>synthetic data point method</u>: d.o.f $\approx$ # of independent functions in $f^{\chi PT}$ instead of arbitrary # of synthetic points $\rightarrow$ correct correlation.

Covariance function $K_f(z,z') = \langle\, \delta f^{\chi PT}(z)\, \delta f^{\chi PT}(z')\,\rangle$ can be uniquely diagonalized in a basis $\{\psi_i(z)\}$.

$$\chi^2 = \int_{z_1}^{z_2} dz \int_{z_1}^{z_2} dz'\, [f^{\chi PT}(z) - f^{BCL}(z)] K_f^{-1}(z,z') [f^{\chi PT}(z') - f^{BCL}(z')],$$

$$= \sum_i (1/\lambda_i)\left[\int_{z_1}^{z_2} [f^{\chi PT}(z) - f^{BCL}(z)]\psi_i(z)\right]^2$$

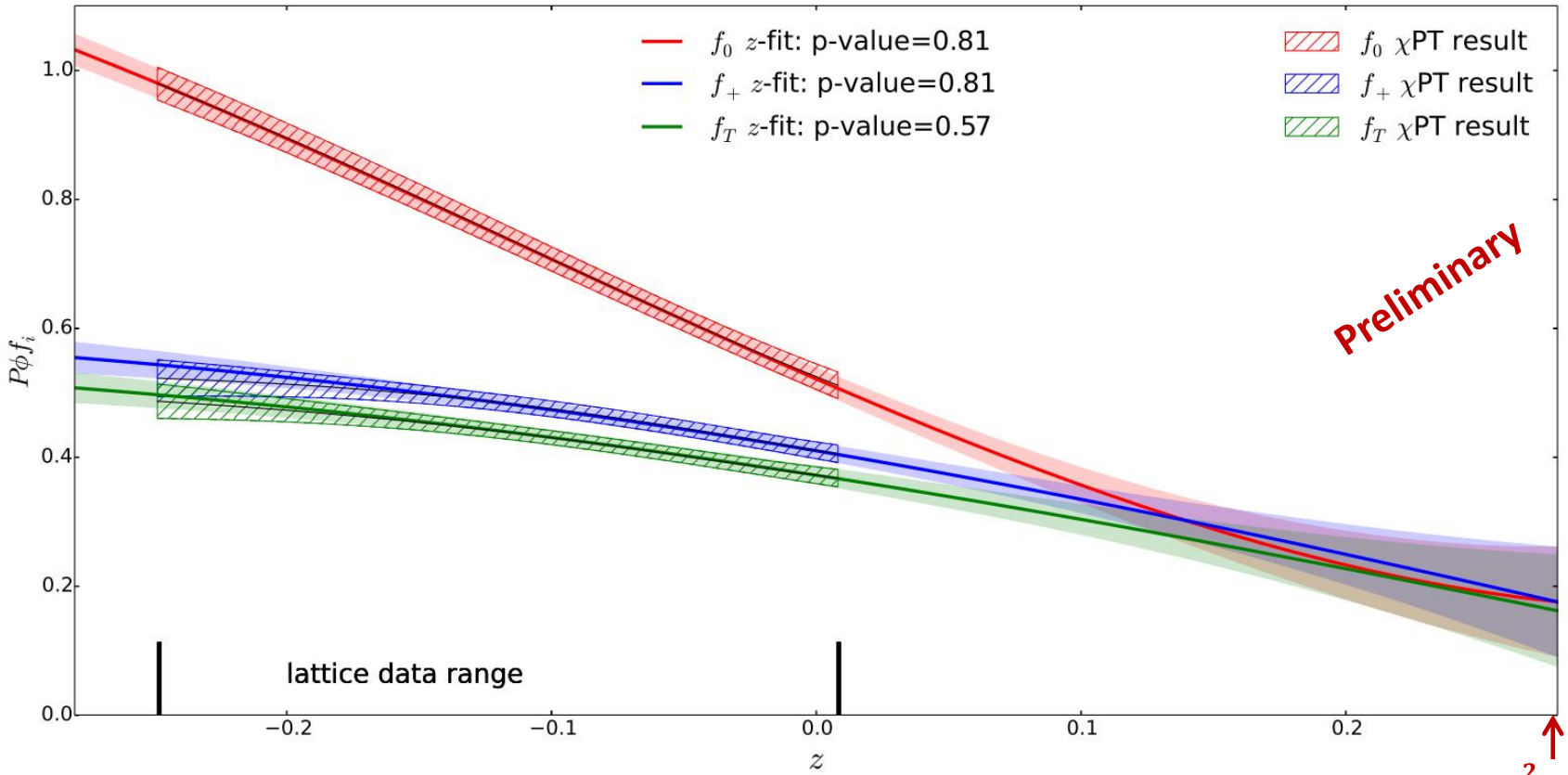
$$K_f(z,z') = \sum_i \lambda_i \psi_i(z)\psi_i(z')$$

Minimizing χ^2 to find expansion coefficients b_n.

Extrapolation in q^2: z-fit results

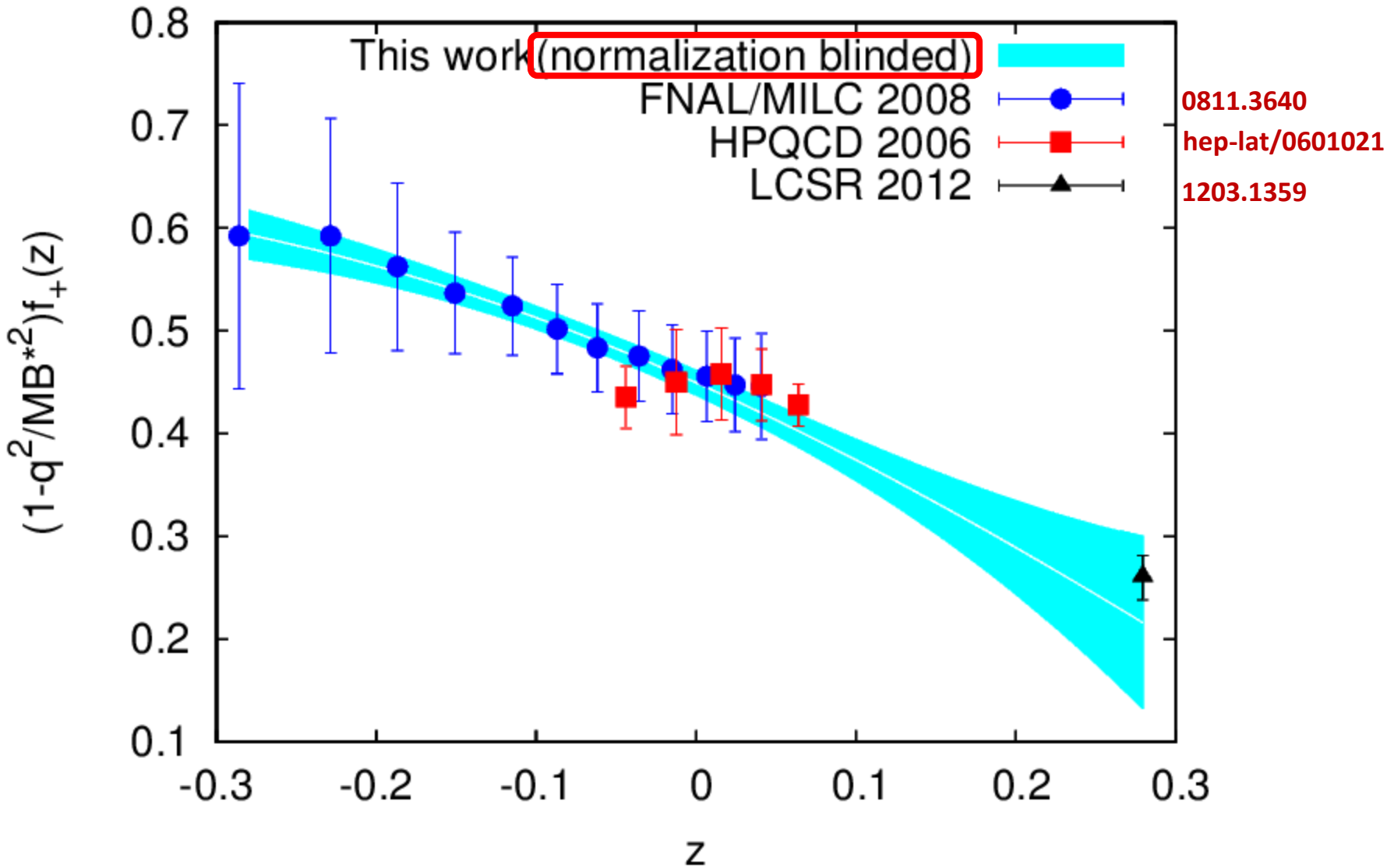
- Fit to N_z=4 order
- Constraints with log normal distribution on $\sum_{ij} B_{ij} b_i b_j$
- Kinematic constraint on $f_{0,+}$:

$$f_0(q^2 = 0) = f_+(q^2 = 0)$$

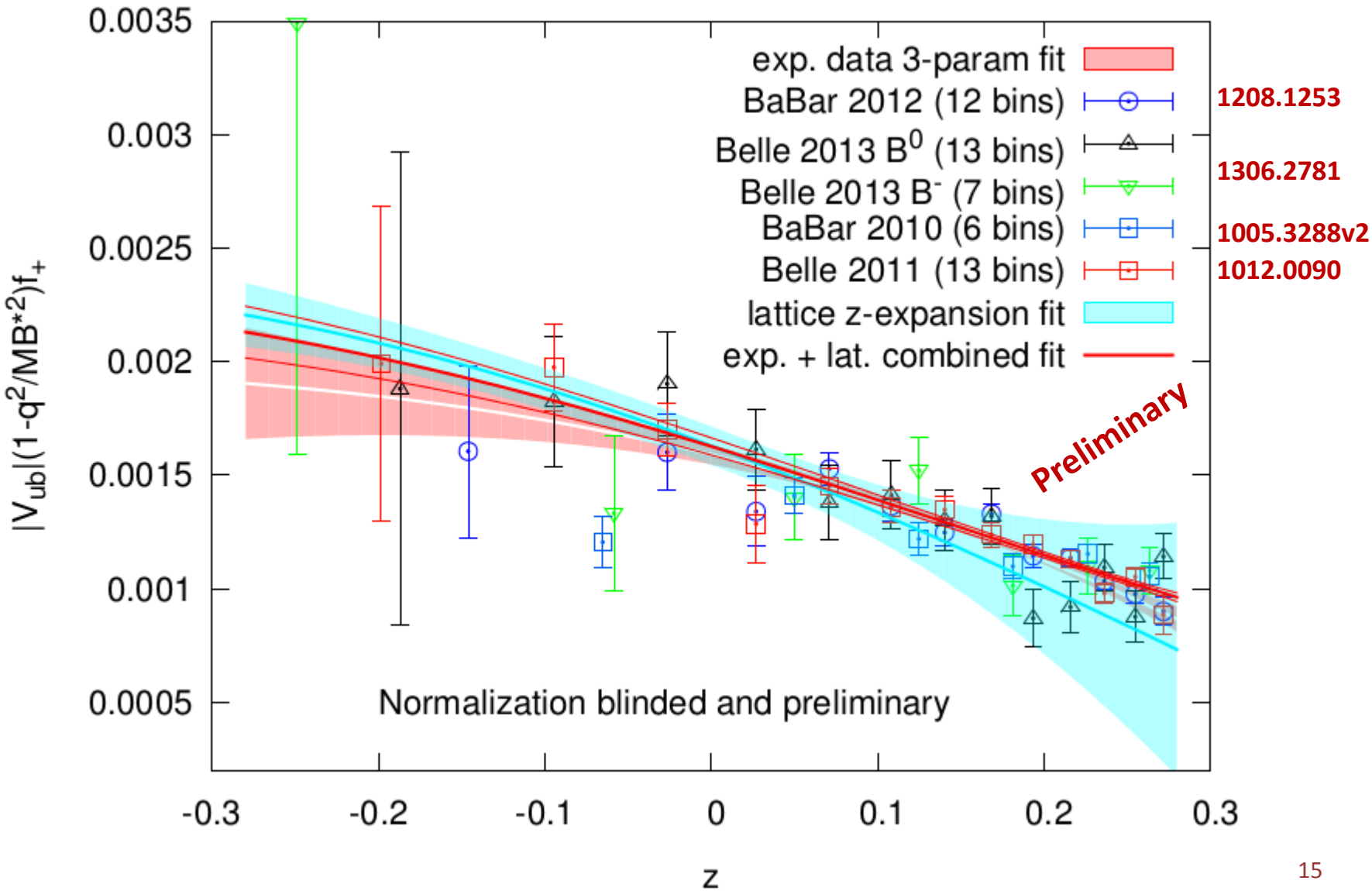


$q^2 = 0$

Extrapolation in q^2: compared to previous results



Determination of $|V_{ub}|$



Summary and outlook

- We are updating our previous result (from 2008) for the $B \to \pi \ell \nu$ semileptonic form factors. We also calculate the tensor form factor which is needed to predict the rare decay $B^{\pm} \to \pi^{\pm} \ell^{+} \ell^{-}$ (in progress).
- We use a new functional method to implement the z-expansion.
- We combine the lattice result (**normalization blinded**) for the form factors with experimental measurements to determine $|V_{ub}|$.

Fit	$\overline{\chi^2}$[dof]	p-value	b_0	b_1	b_2	b_3	$\|V_{ub}\|$ error
Lattice only	0.62 [3]	0.6	0.410(12)	-0.69(12)	-0.54(94)	0.33(150)	-
Exp. only(BaBar 10,12+Belle 11,13)	1.1[51]	0.32	0.397(12)	-0.42(14)	-0.72(52)	-	-
All (exp. + lattice)	1.1[53]	0.27	0.417(11)	-0.580(53)	-0.30(12)	0.58(29)	4.1%

- The error for $|V_{ub}|$ is **4.1%**, compared to the result of using FNAL/MILC 2008 analysis: $|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}$ (8.8%).
- We are also working on the $B_s \to K \ell \nu$ semileptonic form factors (**Y. Liu, 1312.3197**) which provides an another exclusive determination of $|V_{ub}|$. The work is in progress.

Thank you

Backup: Experimental results

- Experimental observable is the differential branching fraction

$$\frac{\Delta\mathcal{B}}{\Delta q^2} \sim \frac{d\mathcal{B}}{dq^2} = \tau_B \frac{d\Gamma}{dq^2} = \boxed{\frac{\tau_B G_F^2}{192\pi^3 M_B^3}} |V_{ub}|^2 |\vec{p}(q^2)|^3 |f_+(q^2)|^2$$

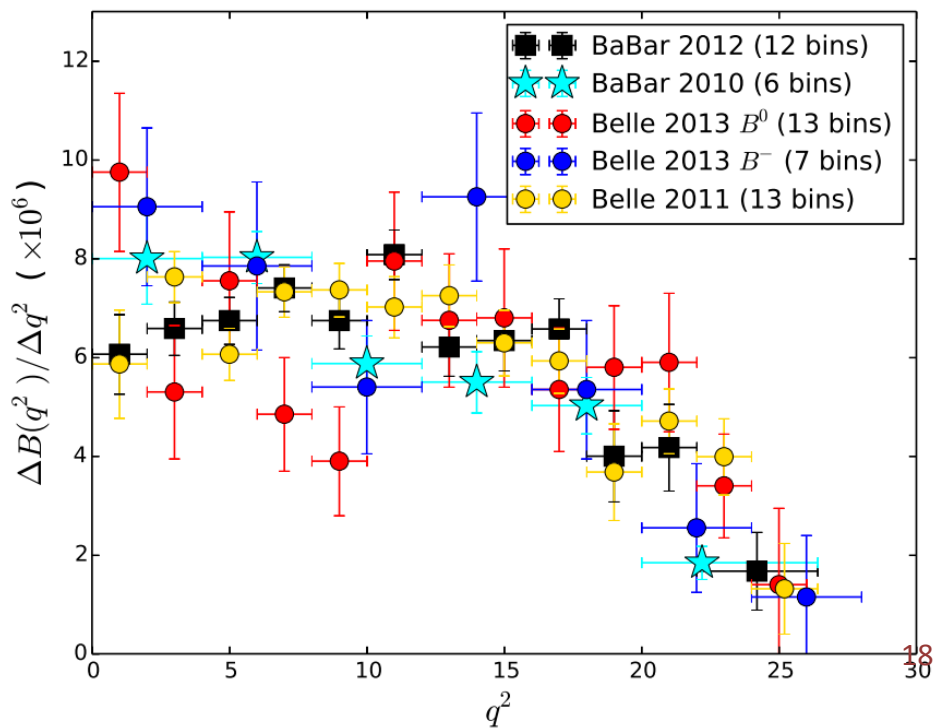
$$\equiv C_{\tau_B}^2$$

- Define an intermediate variable

$$\mathcal{D} = \frac{1}{C_\tau}\sqrt{\frac{\Delta\mathcal{B}}{\Delta q^2}} \sim |V_{ub}||\vec{p}(q^2)|^{3/2} f_+(q^2)$$

BaBar 2010 (1208.1253)
Untagged
Belle 2011 (1208.1253)
Untagged
BaBar 2012 (1208.1253)
untagged
Belle 2013 (1306.2781)
Hadronic tagged
Sys. correlation between B^0 and B^-

final state radiation removed



Backup: Combining lattice and experiments

- Adding the experimental data to the fit

$$\chi^2_{\rm exp} \equiv \sum_{\alpha,\beta} \left[\mathcal{D}_\beta - |V_{ub}| \sum_m b_m Z_{m\beta} \right] \Sigma^{-1}_{\alpha\beta} \left[\mathcal{D}_\alpha - |V_{ub}| \sum_n b_n Z_{n\alpha} \right]$$

Sum over q^2 bins

(Exp. Covariance matrix)$^{-1}$

$$\left[z(q^2_\alpha) - (-1)^n \frac{n}{N_z} z(q^2_\alpha)^{N_z} \right] \frac{|\vec{p}_\pi|^{3/2}}{1 - t(q^2_\alpha)/t_*}$$

- Combine lattice and experiments

$$\chi^2 = \chi^2_{Lat} + \chi^2_{BaBar} + \chi^2_{Belle},$$