Gluonic Correlations around Deconfinement

Tereza Mendes

in collaboration with Attilio Cucchieri

Instituto de Física de São Carlos
Universidade de São Paulo
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We address these issues analyzing data from finite-temperature simulations of the gluon propagator in SU(2) Landau gauge on large lattices.
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On the other hand, studies of the gluon propagator at $T = 0$ have shown a (dynamical) mass, so we can try to use this knowledge to define temperature-dependent masses for the region $T \approx T_c$
Gluon at Criticality

First (small lattice) studies of SU(2) theory around $T_c$ found:

- $D_T(p^2)$ is IR-suppressed and decreases as $T$ increases
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Strong response of $D_L$ to the transition implies that it contains information about the location of $T_c$. If this info is unrelated to the center symmetry restoration, one could define an alternative order parameter for the deconfinement transition.
This Work (Finite $T$): Parameters

- pure SU(2) case, with a standard Wilson action
- cold start, projection on positive Polyakov loop configurations
- Landau-gauge fixing using stochastic overrelaxation
- lattice sizes ranging from $48^3 \times 4$ to $192^3 \times 16$
- several $\beta$ values, allowing several values of the temperature $T = 1/N_t a$ around $T_c$
- gluon dressing functions normalized to 1 at 2 GeV
- masses extracted from Gribov-Stingl behavior (fits shown in plots below)
Results: Low Temperatures

As $T$ is turned on, magnetic propagator gets more strongly suppressed (3d-like), electric one increases.
Results: Low Temperatures

At larger $T$, magnetic propagator slightly more suppressed, electric one increases (showing IR plateau?)
Real-Space Propagator at $T \neq 0$

Another qualitative response of the propagator to temperature: $D_L$ ceases to show violation of reflection positivity as $T$ is turned on, while such violation is still observed in the magnetic sector.

Plots of transverse and longitudinal real-space propagator at $T = 0.25T_c$:
Longitudinal and transverse gluon at $T_c$

Electric (left) and magnetic (right) propagators at $T_c$
Results: Propagators at $0.98 \, T_c$

Just below $T_c$, systematic errors for $D_L(p)$ are already present.
Results: Propagators at $1.01 \, T_c$

Just above $T_c$, systematic errors for $D_L(p)$ seem much less severe, IR plateau for $D_L(p)$ drops significantly for $N_t \leq 8$
Results: Propagators at $1.02\, T_c$

Just above $T_c$, systematic errors for $D_L(p)$ seem much less severe, IR plateau for $D_L(p)$ drops somewhat for $N_t \leq 8$
Discussion

Clearly, the thing that stands out more about $T_c$ is the presence of very large finite-size corrections, but the (large-volume) behavior of $D_L$ itself seems to be smooth around the critical region.
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$\Rightarrow$ To get an idea let us consider $D_L(0)$ as a function of the temperature
Infrared Plateau for $D_L(p)$ vs. $T$

IR plateau [from $D_L(0)$]:

![Graph showing the infrared plateau for $D_L(0)$ vs. $T/T_c$]
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IR plateau [from $D_L(0)$]:

Peak at $T_c$ for $N_t = 4$ $\Rightarrow$ finite maximum at $\lesssim 0.9 \, T_c$ for $N_t = 16$
Infrared Plateau for $D_L(p)$ vs. $T$

IR plateau value [estimated as $D_L(0)$] for all $T$ values (left) and smaller range (right).

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So?

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It is still interesting to characterize the behavior of the gluon propagator at these temperatures in terms of its analytic structure, performing fits to extract mass scales; can make a comparison with $T = 0$ case.
Fitting forms

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Might try interpolation (inspired by dimensional reduction in transverse case) of more elaborated fits used for the $T = 0$ 4d and 3d cases:

$$D_{4d}(p^2) = C \frac{p^2 + d}{p^4 + u^2 p^2 + t^2}$$

$$D_{3d}(p^2) = C \frac{(p^2 + d)(p^2 + 1)}{(p^4 + u^2 p^2 + t^2)(p^2 + v)}$$
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These (polynomial) Gribov-Stingl forms allow for complex-conjugate poles. At nonzero $T$ they do not work well...
Consider generalized versions of Gribov-Stingl forms above, e.g.

\[ D_{L,T}(p^2) = C \frac{1 + dp^{2\eta}}{(p^2 + a)^2 + b^2} \quad \text{or} \quad C \left[ \frac{p^2 + d}{(p^2 + a)^2 + b^2} \right]^\eta \]
Our Proposal

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Both fits correspond to poles at masses

\[ m^2 = a \pm i b \Rightarrow m = m_R + i m_I \]

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These fits (shown in above plots) work quite well. The masses obtained have comparable real and imaginary parts and are smooth around the transition. At higher \( T \): imaginary part gets smaller in longitudinal case.
## Electric and Magnetic Masses vs. $T$

<table>
<thead>
<tr>
<th>$T/T_c$</th>
<th>$N_s^3 \times N_t$</th>
<th>$m_R^{(E)}$</th>
<th>$m_I^{(E)}$</th>
<th>$m_R^{(M)}$</th>
<th>$m_I^{(M)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>($E$)</td>
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<td>($M$)</td>
<td>($M$)</td>
</tr>
<tr>
<td>0</td>
<td>$64^3 \times 64$</td>
<td>0.83 GeV</td>
<td>0.43 GeV</td>
<td>0.86 GeV</td>
<td>0.51 GeV</td>
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<tr>
<td>0.25</td>
<td>$96^3 \times 16$</td>
<td>0.61 GeV</td>
<td>0.28 GeV</td>
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</tr>
<tr>
<td>0.5</td>
<td>$48^3 \times 8$</td>
<td>0.51 GeV</td>
<td>0.13 GeV</td>
<td>0.59 GeV</td>
<td>0.36 GeV</td>
</tr>
<tr>
<td>0.7</td>
<td>$96^3 \times 8$</td>
<td>0.31 GeV</td>
<td>0.13 GeV</td>
<td>0.37 GeV</td>
<td>0.24 GeV</td>
</tr>
<tr>
<td>0.9</td>
<td>$96^3 \times 16$</td>
<td>0.10 GeV</td>
<td>0.06 GeV</td>
<td>0.15 GeV</td>
<td>0.10 GeV</td>
</tr>
<tr>
<td>0.98</td>
<td>$96^3 \times 8$</td>
<td>0.19 GeV</td>
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<td>0.28 GeV</td>
<td>0.20 GeV</td>
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<td>1.0</td>
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<td>2.0</td>
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<td>0.07 GeV</td>
<td>0.19 GeV</td>
<td>0.14 GeV</td>
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- Good fits (for transverse and longitudinal cases) to several generalized Gribov-Stingl forms, including an exponentiated form, suggesting the presence of branch cuts in addition to simple poles

- Main qualitative feature of gluonic correlations in the deconfined phase seems to be lack of violation of reflection positivity for $D_L(x)$ (observed however for all $T \neq 0$)