Matrix elements for D-meson mixing from 2+1 lattice QCD

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Lattice 2014, Columbia University
D-meson mixing phenomenology

**Short distance**

- Strange and down are **GIM suppressed**.
- Bottom is $|V_{ub}V_{cb}^*|^2$ suppressed.
- Described by two flavors in the SM. This implies that D mixing has no CP violation in the SM.

**HFAG 2013:** CLEO, Belle, BaBar

\[
\begin{align*}
x &= (0.419 \pm 0.211)\% \\
y &= (0.465 \pm 0.186)\%
\end{align*}
\]

**SM short distance estimate:**

\[
x \sim y \sim 10^{-4}\%
\]


**Long distance**

- Effects not well understood
- Possibly the dominant contribution in the Standard Model.

**Lattice vs. long distance diagrams:**

- Disconnected diagrams
- Many contributions from different intermediate states
- Multi-particle intermediate states

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Motivation

• **New physics** enhancements enter with short distance contributions.

• Charm community interested in unquenched matrix elements for **model discrimination**. [Golowich et al., 0705.3650]

• Strong effort from many **experiments**, with work coming from LHCb, Belle II, BES III.

• **Gold plated** process.
D-meson mixing overview

**Effective mixing Hamiltonian**

\[
M_{12} - \frac{i}{2} \Gamma_{12} = \sum_i C_i^{(2)} \left< \bar{D}^0 \left| O_i^{\Delta c = 2} \right| D^0 \right>
\]

\[
+ \sum_{f; jk} \frac{C_j^{(1)} C_k^{(1)} \left< \bar{D}^0 \left| O_j^{\Delta c = 1} \right| f \right> \left< f \left| O_k^{\Delta c = 1} \right| D^0 \right>}{E_f - M_{D^0} + i\epsilon}
\]

**4-quark operators**

\(O_1 = \bar{c}^\alpha \gamma^\mu Lu^\alpha \bar{c}^\beta \gamma^\mu L u^\beta\)

\(O_2 = \bar{c}^\alpha Lu^\alpha \bar{c}^\beta L u^\beta\)

\(O_3 = \bar{c}^\alpha Lu^\beta \bar{c}^\beta L u^\alpha\)

\(O_4 = \bar{c}^\alpha Lu^\alpha \bar{c}^\beta Ru^\beta\)

\(O_5 = \bar{c}^\alpha Lu^\beta \bar{c}^\beta Ru^\alpha\)

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# MILC 2+1 asqtad ensembles

Lattice actions: Staggered light sea & valence, Fermilab clover charm

<table>
<thead>
<tr>
<th>$a(fm)$</th>
<th>$(\frac{L}{a})^3 \times \frac{T}{a}$</th>
<th>$m_l/m_s$</th>
<th>$m_\pi$(MeV)</th>
<th>$am_q$</th>
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</thead>
<tbody>
<tr>
<td>0.12</td>
<td>$24^3 \times 64$</td>
<td>0.1</td>
<td>269</td>
<td>0.0050, 0.0070, 0.0100, 0.0200, 0.0300, 0.0349, 0.0415, 0.0500</td>
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<td>0.12</td>
<td>$20^3 \times 64$</td>
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<td>0.09</td>
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<td>508</td>
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<tr>
<td>0.06</td>
<td>$64^3 \times 144$</td>
<td>0.1</td>
<td>224</td>
<td>0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188</td>
</tr>
<tr>
<td>0.06</td>
<td>$56^3 \times 144$</td>
<td>0.14</td>
<td>265</td>
<td>0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188</td>
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<tr>
<td>0.06</td>
<td>$48^3 \times 144$</td>
<td>0.2</td>
<td>318</td>
<td>0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188</td>
</tr>
<tr>
<td>0.06</td>
<td>$48^3 \times 144$</td>
<td>0.4</td>
<td>452</td>
<td>0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188</td>
</tr>
<tr>
<td>0.045</td>
<td>$64^3 \times 192$</td>
<td>0.2</td>
<td>324</td>
<td>0.0018, 0.0028, 0.0040, 0.0056, 0.0084, 0.0130, 0.0160</td>
</tr>
</tbody>
</table>

- **Four lattice spacings** $a \leftarrow$ continuum extrapolation
- **Lattice size** $m_\pi L \sim 4 \leftarrow$ negligible finite volume effects
- **Several light sea-quarks** $am_l \leftarrow$ sea quark chiral extrapolation
- **Multiple light valence-quarks** $am_q \leftarrow$ valence chiral extrapolation
Two- and three-point correlators

D-meson lattice operators

\[ \chi_{D^0}(x) = \bar{u}\gamma_5 c(x) \]
\[ \chi_{\bar{D}^0}(x) = \bar{c}\gamma_5 u(x) \]

Correlators

\[ C_{2pt}^{2pt}(x, 0) = \left\langle T \left\{ \chi^0(x) \chi^0(0) \right\} \right\rangle \]
\[ C_{3pt}^{3pt}(x_1, x_2, 0) = \left\langle T \left\{ \chi^0(x_2) \mathcal{O}_N(0) \chi^0(x_1) \right\} \right\rangle \]

Fit functions

\[ C_{2pt}^{2pt}(t) = \sum_n (-1)^{n(t+1)} \frac{|Z_n|^2}{2E_n} \left( e^{-E_n t} + e^{-E_n (T-t)} \right) \]
\[ C_{3pt}^{3pt}(t_2, t_1) = \sum_{m,n} (-1)^{n(t_2+1)} (-1)^{m(|t_1|+1)} \langle n | \mathcal{O}_i | m \rangle \frac{Z_n^\dagger Z_m}{4E_n E_m} e^{-E_n t_2} e^{-E_m |t_1|} + \mathcal{O}(e^{-ET}) \]
Fit regions

- Data reduction
- Keep important correlations
- Large ground state contribution
- Use $|t_1| = t_2(\pm 1)$
Bidiagonal correlator fits

- Stable.
- Data reduction minimizes systematic errors.
- \( \sim 1\% \) precision
Renormalization and matching

- One-loop matching from lattice to continuum with 1-loop tadpole-improved action. [Lepage & Mackenzie, hep-lat/9209022]
- Match lattice regularization to dimensional regularization with the $\overline{MS}$ scheme at the charm quark scale.

Example for O1:

$$\langle O_1 \rangle_{\overline{MS}} = (1 + \alpha_s \rho_{11}) \langle O_1 \rangle_{\text{lat}} + \alpha_s \rho_{12} \langle O_2 \rangle_{\text{lat}} + \mathcal{O} \left( \alpha_s^2, \alpha_s \frac{\Lambda_{\text{QCD}}}{M} \right)$$

- Match lattice charm-quark action to continuum HQET through $O(\alpha)$.
  Wilson action improved by Clover term.
  Operator improved by heavy quark rotation.
Heavy quark discretization errors

Heavy quark effective theory describes

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{HQET}} = \sum_i C_i^{\text{cont}}(m_Q) \mathcal{O}_i$$

$$\mathcal{L}_{\text{lat}} = \mathcal{L}_{\text{HQET}(m_0 a)} = \sum_i C_i^{\text{lat}}(m_Q, m_0 a) \mathcal{O}_i$$

$$\text{error}_i = \left| \left[ C_i^{\text{lat}}(m_Q, m_0 a) - C_i^{\text{cont}}(m_Q) \right] \mathcal{O}_i \right|$$

HQ discretization errors start at $O(a^2, \alpha_s a)$.

Include $O(a^2, \alpha_s a)$ errors with functions as given above with unknown coefficients that are determined in the chiral fit.
SU(3) PQrSHMχPT simultaneous fits

Chiral and continuum extrapolation achieved through:
SU(3) partially quenched rooted staggered heavy meson χPT

\[ \langle \bar{D}^0 | \mathcal{O}_N | D^0 \rangle = \beta_N \left( 1 + \text{logs} + \text{W.S. logs} \left( \beta_M / \beta_N \right) \right) + \text{analytic terms} \]

NLO wrong spin, taste mixing effects.

\[ \rightarrow \text{Leading order LECs mix at NLO.} \]

\[ \rightarrow \text{Simultaneous fits: } \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\} \text{ and } \{\mathcal{O}_4, \mathcal{O}_5\} \]

Fits with NLO chiral logs + (N)NLO analytic terms.

Benefits of simultaneous fits:
- Correlations between data sets preserved.
- Consistent values for all (Bayesian) fit parameters across SM/BSM fits.
Chiral-continuum extrapolation

O4 chiral-continuum extrapolation

O4 matrix element (r1 units)

valence mass (r1 units)

Preliminary

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Chiral-continuum fit variations

O4 chiral-continuum stability plot

- no 0.045fm
- no 0.12fm
- fpi -> fk
- No hf split
- min. NNLO
- NNNLO
- ind. fit
- preferred fit

O4 matrix element at physical point (r1 units)
Complete error estimation

- Statistical
- $\chi$PT contribution
  - Chiral logs (LO LECs, input errors)
  - Light quark discretization
  - Analytic LECs
- Heavy quark discretization error
- Charm-quark mass tuning error
- Renormalization and matching error
- Light quark mass uncertainties
- Finite volume effects

Covariance matrix includes statistical and systematic errors.

Chiral fit function + in quadrature
Error breakdown

O4 error breakdown

Preliminary
Preliminary results

**Preliminary error budget**

<table>
<thead>
<tr>
<th>Percent errors (%)</th>
<th>$O_1$</th>
<th>$O_2$</th>
<th>$O_3$</th>
<th>$O_4$</th>
<th>$O_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>2.1</td>
<td>2.5</td>
<td>2.2</td>
<td>1.5</td>
<td>2.8</td>
</tr>
<tr>
<td>Chiral logs</td>
<td>0.6</td>
<td>1.0</td>
<td>0.7</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Analytic LECs</td>
<td>1.8</td>
<td>1.3</td>
<td>1.0</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>LQ disc.</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>HQ disc.</td>
<td>2.8</td>
<td>1.9</td>
<td>2.0</td>
<td>2.2</td>
<td>2.9</td>
</tr>
<tr>
<td>LQ mass</td>
<td>0.5</td>
<td>0.7</td>
<td>0.4</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>HQ tuning</td>
<td>1.2</td>
<td>1.4</td>
<td>0.7</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Combined (missing renorm &amp; F.V.)</td>
<td>4.2</td>
<td>4.9</td>
<td>3.3</td>
<td>3.4</td>
<td>4.8</td>
</tr>
</tbody>
</table>

**Remaining tasks:**

- Account for renormalization and matching error.
- Account for finite-volume corrections.
- Report bag parameters.
Thank you
3pt exploratory correlator fits
Correlator fits

Large time slices → dominantly ground state signal
Ground state parameters expected to be insensitive to tmax
*tmax fits vary both data size and fit region

Expected tmax stability plot

Surprising systematic trend

Show bidiagonal fits
Random sampling procedure

**Procedure**

1) Take fixed triangle region
2) Randomly sample $m$ points
3) Repeat $O(100)$ times to average out statistical variations.*
4) Plot stability plot vs $m$

*Observed that the standard deviation of repeated samples are much smaller than statistical error.

**Goal**

Produce fits with varying data size but fixed fit region.
Random sampling result

**Insights**
Clear trend at large values of $m$. Suggests data reduction.

→ bidiagonal fits

**Advantages** of bidiagonal fits:
- Data reduction into stable region.
- Keeps important correlations.
  (Signal from both parity states).