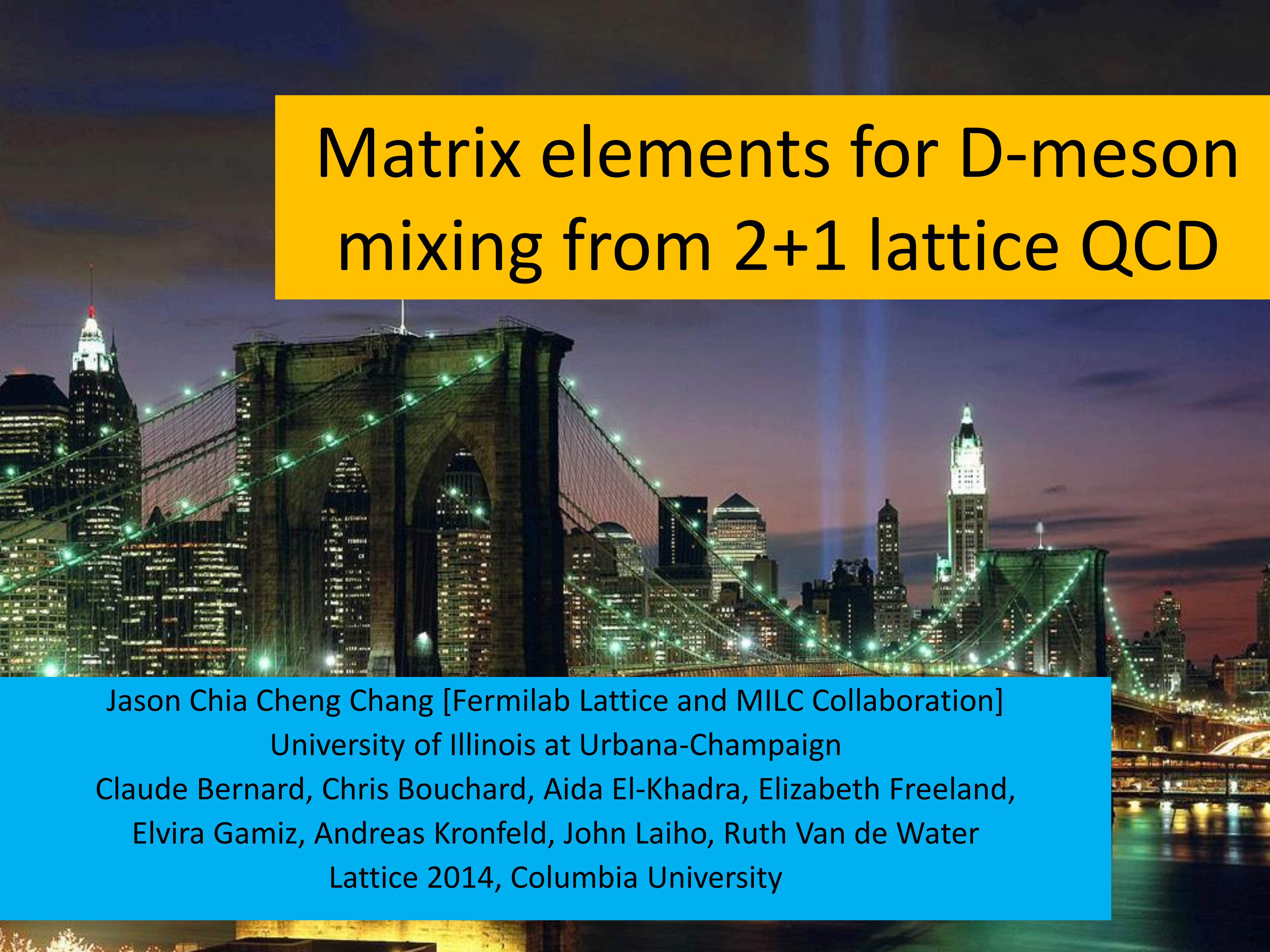

Matrix elements for D-meson mixing from 2+1 lattice QCD

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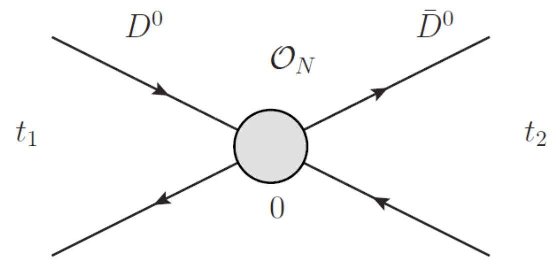
Lattice 2014, Columbia University

D-meson mixing phenomenology

Short distance



- Strange and down are GIM suppressed.
- Bottom is $|V_{ub}V_{cb}^*|^2$ suppressed.
- Described by two flavors in the SM. This implies that D mixing has no CP violation in the SM.
- HFAG 2013: CLEO, Belle, BaBar

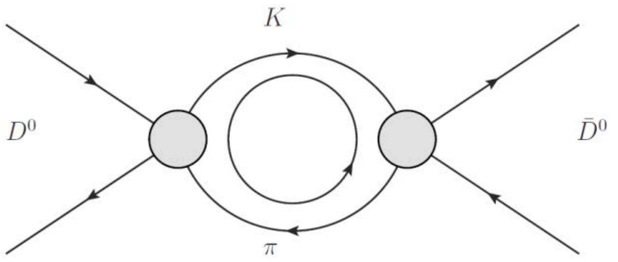
  $$x = (0.419 \pm 0.211)\%$$
  $$y = (0.465 \pm 0.186)\%$$

- SM short distance estimate:

  $$x \sim y \sim 10^{-4}\%$$

  [Golowich & Petrov, hep-ph/0506185v1] w/ quenched lattice

Long distance



- Effects not well understood
- Possibly the dominant contribution in the Standard Model.
- Lattice vs. long distance diagrams:
  - Disconnected diagrams
  - Many contributions from different intermediate states
  - Multi-particle intermediate states

Motivation

- **New physics** enhancements enter with short distance contributions.
- Charm community interested in unquenched matrix elements for **model discrimination.** [Golowich et al., 0705.3650]
- Strong effort from many **experiments**, with work coming from LHCb, Belle II, BES III.

- **Gold plated** process.

D-meson mixing overview

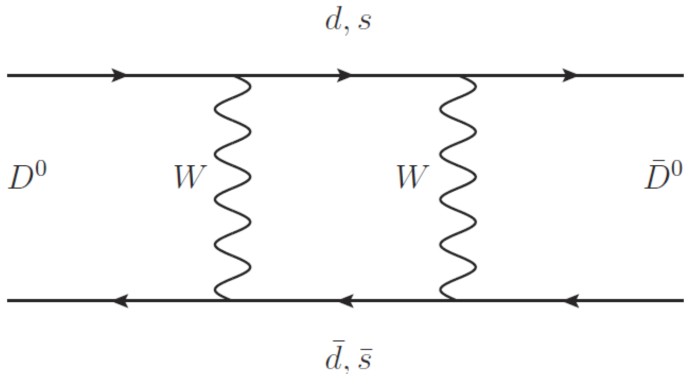
Effective mixing Hamiltonian

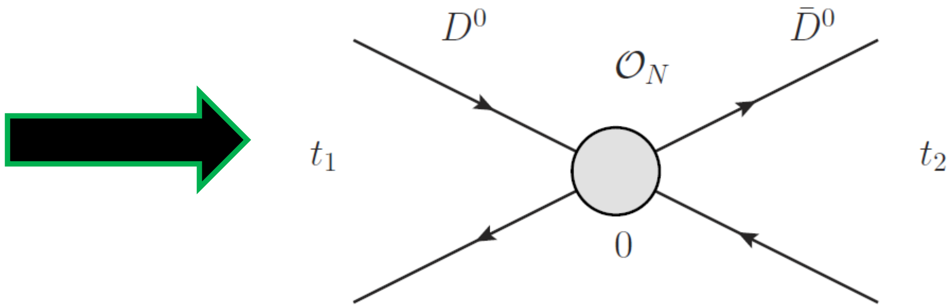
$$M_{12} - \frac{i}{2}\Gamma_{12} = \sum_i C_i^{(2)} \left\langle \bar{D}^0 \left| \mathcal{O}_i^{\Delta_c=2} \right| D^0 \right\rangle$$

$$+ \sum_{f;jk} \frac{C_j^{(1)} C_k^{(1)} \left\langle \bar{D}^0 \left| \mathcal{O}_j^{\Delta_c=1} \right| f \right\rangle \left\langle f \left| \mathcal{O}_k^{\Delta_c=1} \right| D^0 \right\rangle}{E_f - M_{D^0} + i\epsilon}$$

4-quark operators

$$\begin{aligned}
\mathcal{O}_1 &= \bar{c}^\alpha \gamma^\mu L u^\alpha \bar{c}^\beta \gamma^\mu L u^\beta \\
\mathcal{O}_2 &= \bar{c}^\alpha L u^\alpha \bar{c}^\beta L u^\beta \\
\mathcal{O}_3 &= \bar{c}^\alpha L u^\beta \bar{c}^\beta L u^\alpha \\
\mathcal{O}_4 &= \bar{c}^\alpha L u^\alpha \bar{c}^\beta R u^\beta \\
\mathcal{O}_5 &= \bar{c}^\alpha L u^\beta \bar{c}^\beta R u^\alpha
\end{aligned}$$





MILC 2+1 asqtad ensembles

Lattice actions: Staggered light sea & valence, Fermilab clover charm

$a(fm)$	$\left(\frac{L}{a}\right)^3 \times \frac{T}{a}$	m_l/m_s	m_π(MeV)	am_q
0.12	$24^3 \times 64$	0.1	269	0.0050, 0.0070, 0.0100, 0.0200, 0.0300, 0.0349, 0.0415, 0.0500
0.12	$20^3 \times 64$	0.14	326	0.0050, 0.0070, 0.0100, 0.0200, 0.0300, 0.0349, 0.0415, 0.0500
0.12	$20^3 \times 64$	0.2	390	0.0050, 0.0070, 0.0100, 0.0200, 0.0300, 0.0349, 0.0415, 0.0500
0.12	$20^3 \times 64$	0.4	559	0.0050, 0.0070, 0.0100, 0.0200, 0.0300, 0.0349, 0.0415, 0.0500
0.09	$64^3 \times 96$	0.05	177	0.00155, 0.0031, 0.0062, 0.0093, 0.0124, 0.0261, 0.0310
0.09	$40^3 \times 96$	0.1	246	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.0310
0.09	$32^3 \times 96$	0.14	307	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.0310
0.09	$28^3 \times 96$	0.2	356	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.0310
0.09	$28^3 \times 96$	0.4	508	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.0310
0.06	$64^3 \times 144$	0.1	224	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188
0.06	$56^3 \times 144$	0.14	265	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188
0.06	$48^3 \times 144$	0.2	318	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188
0.06	$48^3 \times 144$	0.4	452	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.0160, 0.0188
0.045	$64^3 \times 192$	0.2	324	0.0018, 0.0028, 0.0040, 0.0056, 0.0084, 0.0130, 0.0160

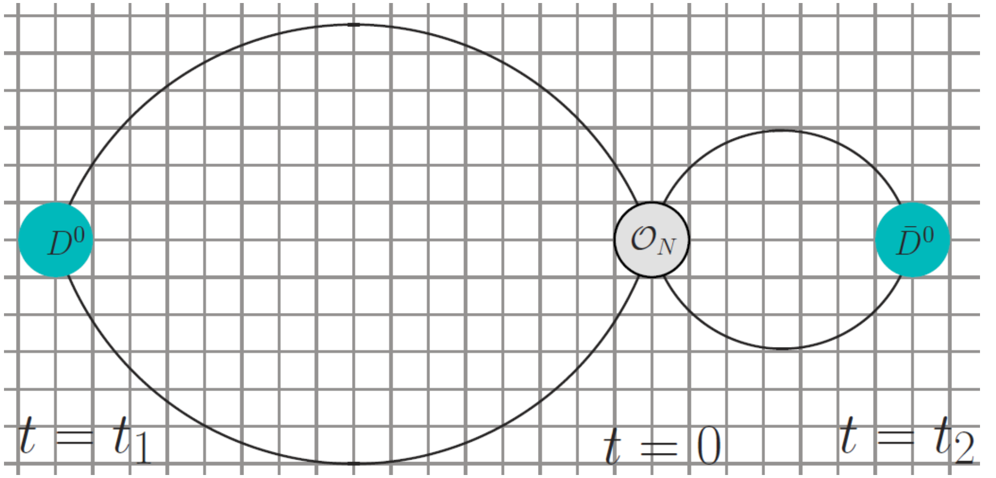
- Four lattice spacings a ← continuum extrapolation
- Lattice size $m_\pi L \sim 4$ ← negligible finite volume effects
- Several light sea-quarks am_l ← sea quark chiral extrapolation
- Multiple light valence-quarks am_q ← valence chiral extrapolation

Two- and three-point correlators

D-meson lattice operators

$$\chi_{D^0}(x) = \bar{u}\gamma_5 c(x)$$

$$\chi_{\bar{D}^0}(x) = \bar{c}\gamma_5 u(x)$$



Correlators

$$C^{2pt}(x,0) = \langle T\{\bar{\chi}^0(x)\,\chi^0(0)\}\rangle$$

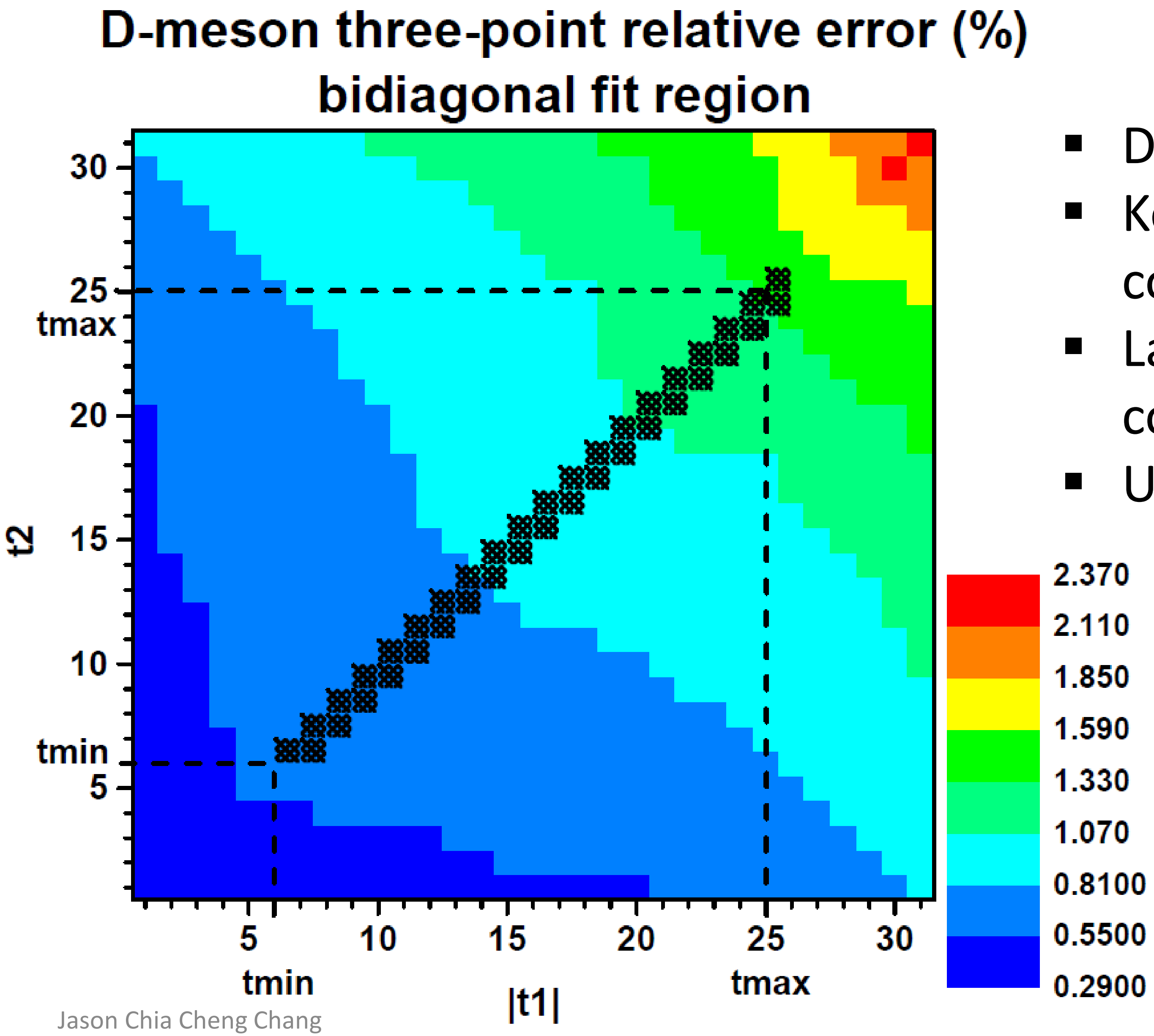
$$C^{3pt}_N(x_1,x_2,0) = \langle T\{\chi^0(x_2)\,\mathcal{O}_N(0)\,\chi^0(x_1)\}\rangle$$

Fit functions

$$C^{2pt}(t) = \sum_n (-1)^{n(t+1)} \frac{|Z_n|^2}{2E_n}\left(e^{-E_n t} + e^{-E_n(T-t)}\right)$$

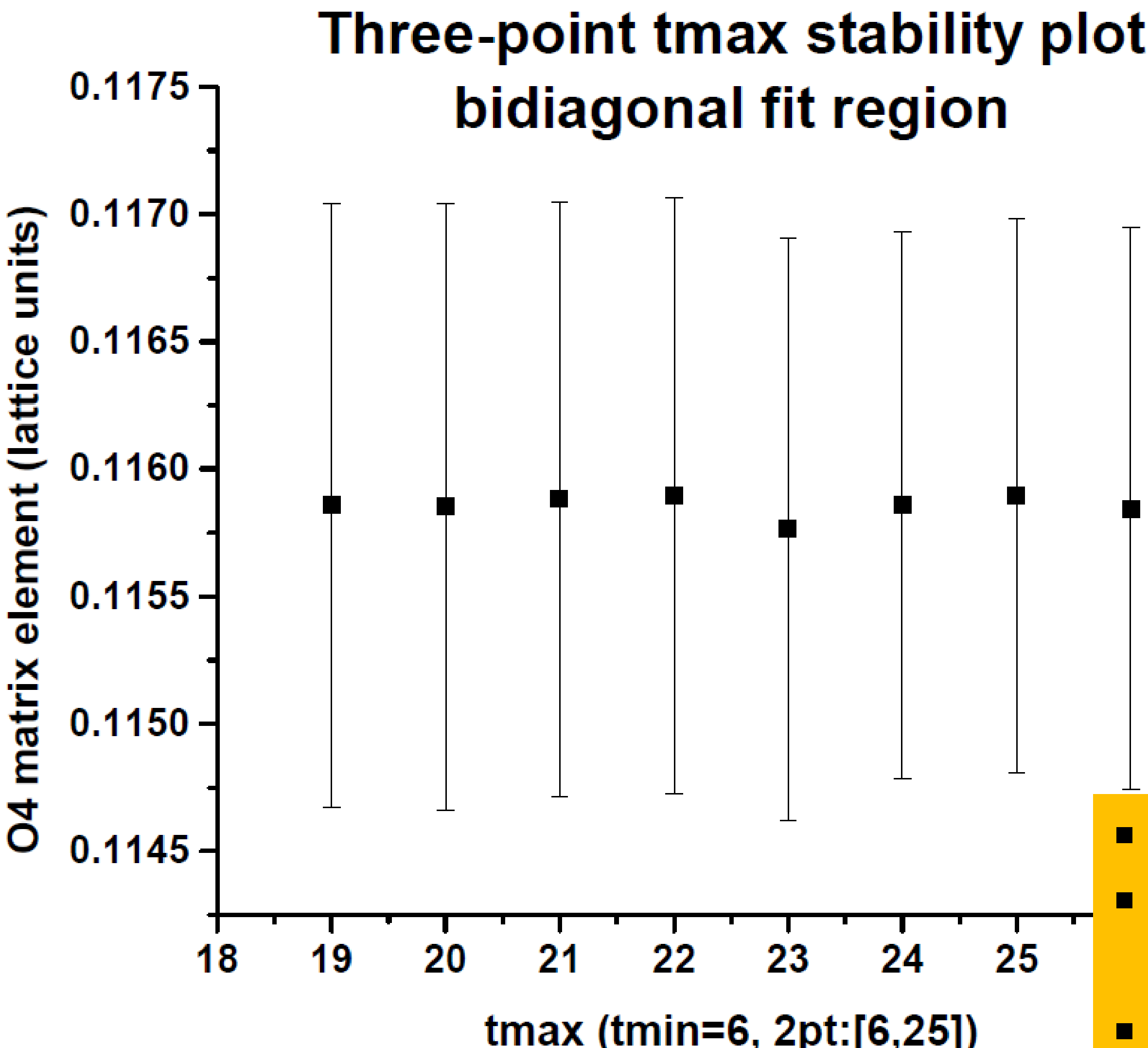
$$C^{3pt}(t_2,t_1) = \sum_{m,n} (-1)^{n(t_2+1)} (-1)^{m(|t_1|+1)} \frac{\langle n|\mathcal{O}_i|m\rangle Z_n^\dagger Z_m}{4E_n E_m} e^{-E_n t_2} e^{-E_m|t_1|} + \mathcal{O}\left(e^{-ET}\right)$$

Fit regions



- Data reduction
- Keep important correlations
- Large ground state contribution
- Use $|t_1| = t_2(\pm 1)$

Bidiagonal correlator fits



- Stable.
- Data reduction minimizes systematic errors.
- $\sim$ 1% precision

Renormalization and matching

- One-loop matching from lattice to continuum with 1-loop tadpole-improved action. [Lepage & Mackenzie, hep-lat/9209022]
- Match lattice regularization to dimensional regularization with the $\overline{MS}$ scheme at the charm quark scale.

Example for O1:

$$\langle \mathcal{O}_1 \rangle^{\overline{\mathrm{MS}}} = (1 + \alpha_s \rho_{11}) \langle \mathcal{O}_1 \rangle^{\mathrm{lat}} + \alpha_s \rho_{12} \langle \mathcal{O}_2 \rangle^{\mathrm{lat}} + \mathcal{O}\left(\alpha_s^2, \alpha_s \frac{\Lambda_{\mathrm{QCD}}}{M} \right)$$

- Match lattice charm-quark action to continuum HQET through $O(a)$.

Wilson action improved by Clover term.

Operator improved by heavy quark rotation.

Heavy quark discretization errors

Heavy quark effective theory describes

$$\mathcal{L}_{\rm QCD} \doteq \mathcal{L}_{\rm HQET} = \sum_i C_i^{\rm cont}(m_Q)\mathcal{O}_i$$

$$\mathcal{L}_{\rm lat} \doteq \mathcal{L}_{{\rm HQET}(m_0 a)} = \sum_i C_i^{\rm lat}(m_Q, m_0 a)\mathcal{O}_i$$

$$\text{error}_i = \left| \left[C_i^{\rm lat}(m_Q, m_0 a) - C_i^{\rm cont}(m_Q) \right] \mathcal{O}_i \right|$$

HQ discretization errors start at $\mathcal{O}(a^2, \alpha_s a)$.

Include $\mathcal{O}(a^2, \alpha_s a)$ errors with functions as given above with unknown coefficients that are determined in the chiral fit.

SU(3) PQrSHMχPT simultaneous fits

Chiral and continuum extrapolation achieved through:
SU(3) partially quenched rooted staggered heavy meson χPT

$\langle \overline{D}^0 | \mathcal{O}_N | D^0 \rangle = \beta_N \big(1 + \text{logs} + \text{W.S. logs } (\beta_M / \beta_N) \big)$ +analytic terms

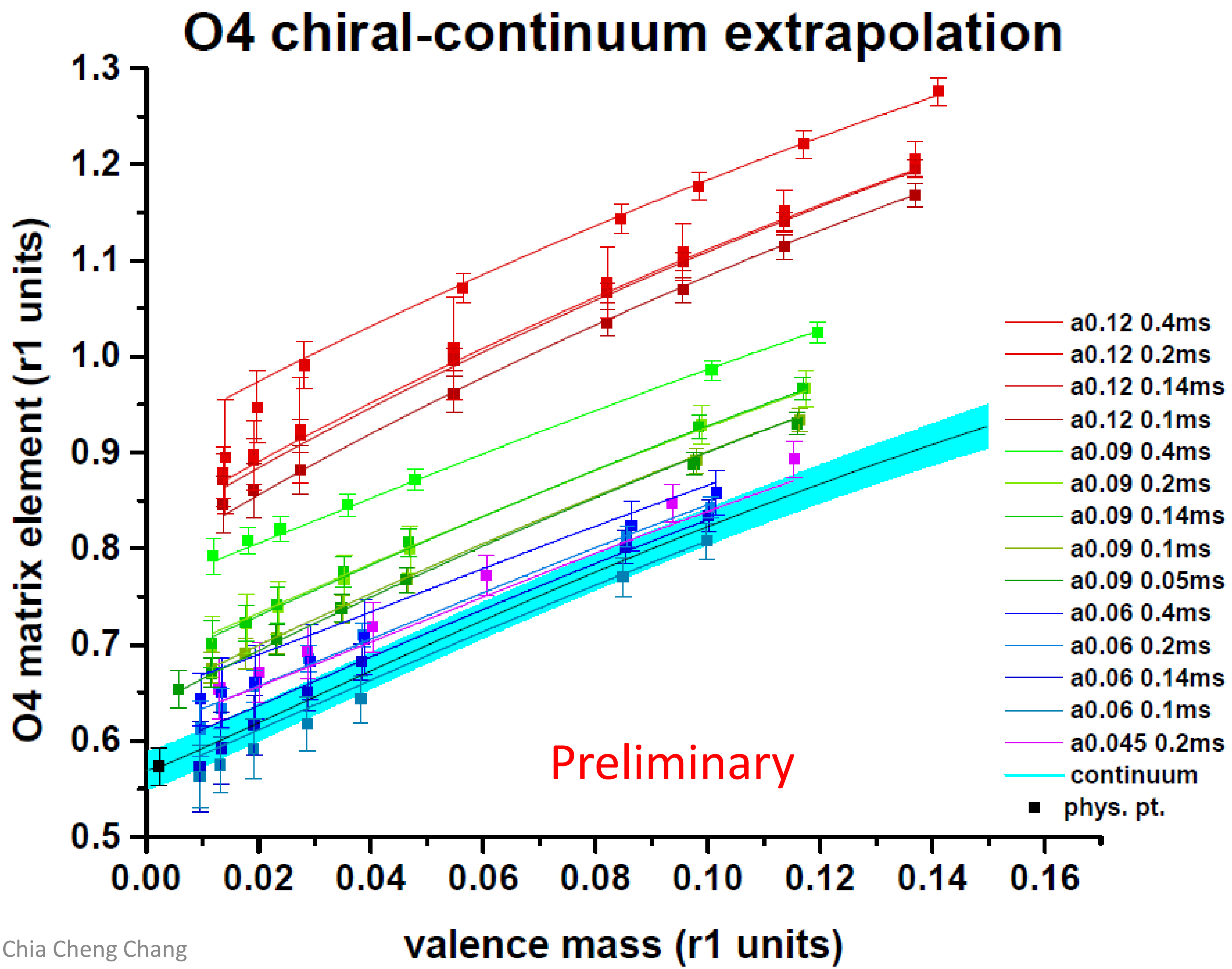
NLO wrong spin, taste mixing effects.
→ Leading order LECs mix at NLO.
→ Simultaneous fits: $\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\}$ and $\{\mathcal{O}_4, \mathcal{O}_5\}$

Fits with NLO chiral logs + (N)NLO analytic terms.

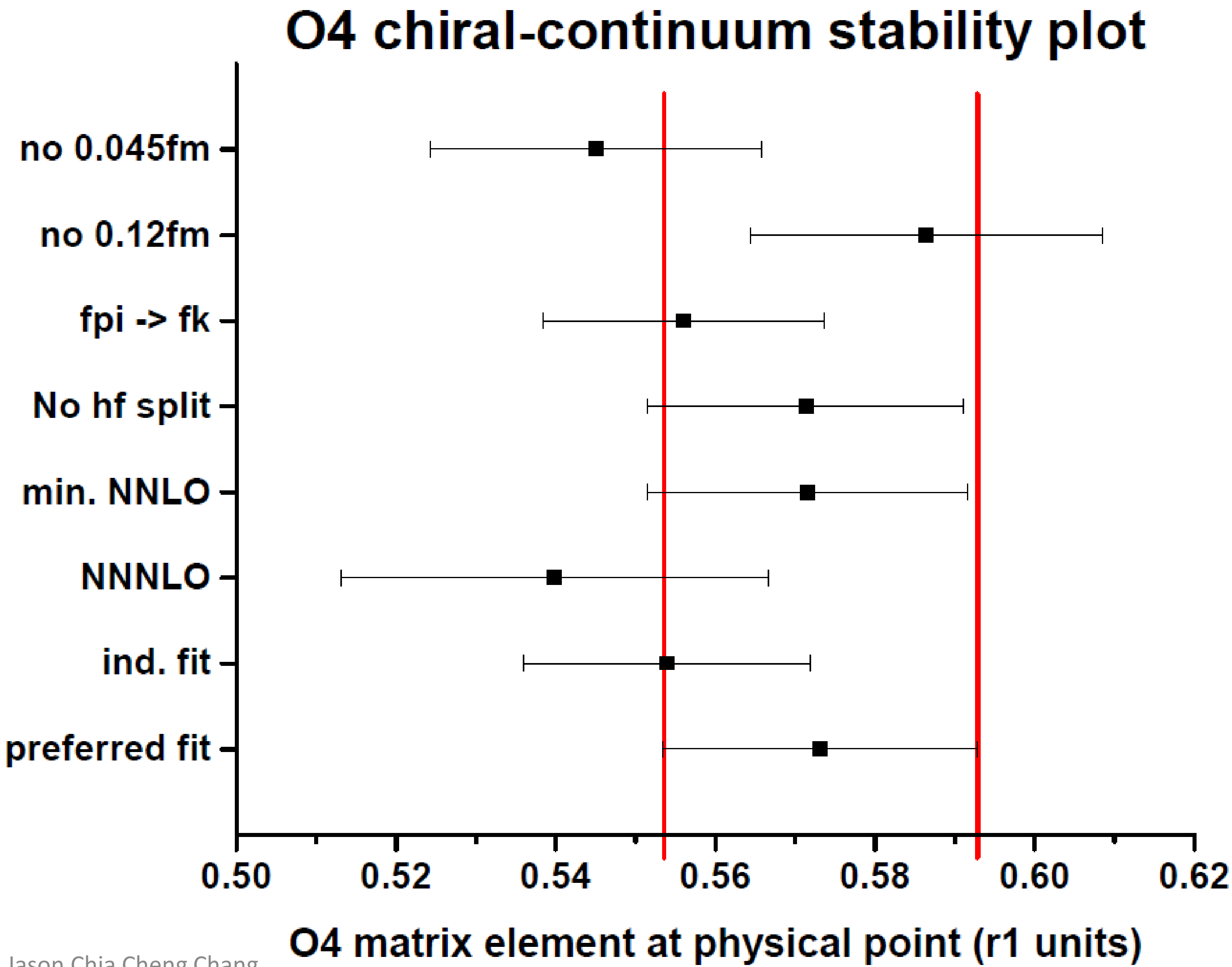
Benefits of simultaneous fits:

- Correlations between data sets preserved.
- Consistent values for all (Bayesian) fit parameters across SM/BSM fits.

Chiral-continuum extrapolation



Chiral-continuum fit variations



Complete error estimation

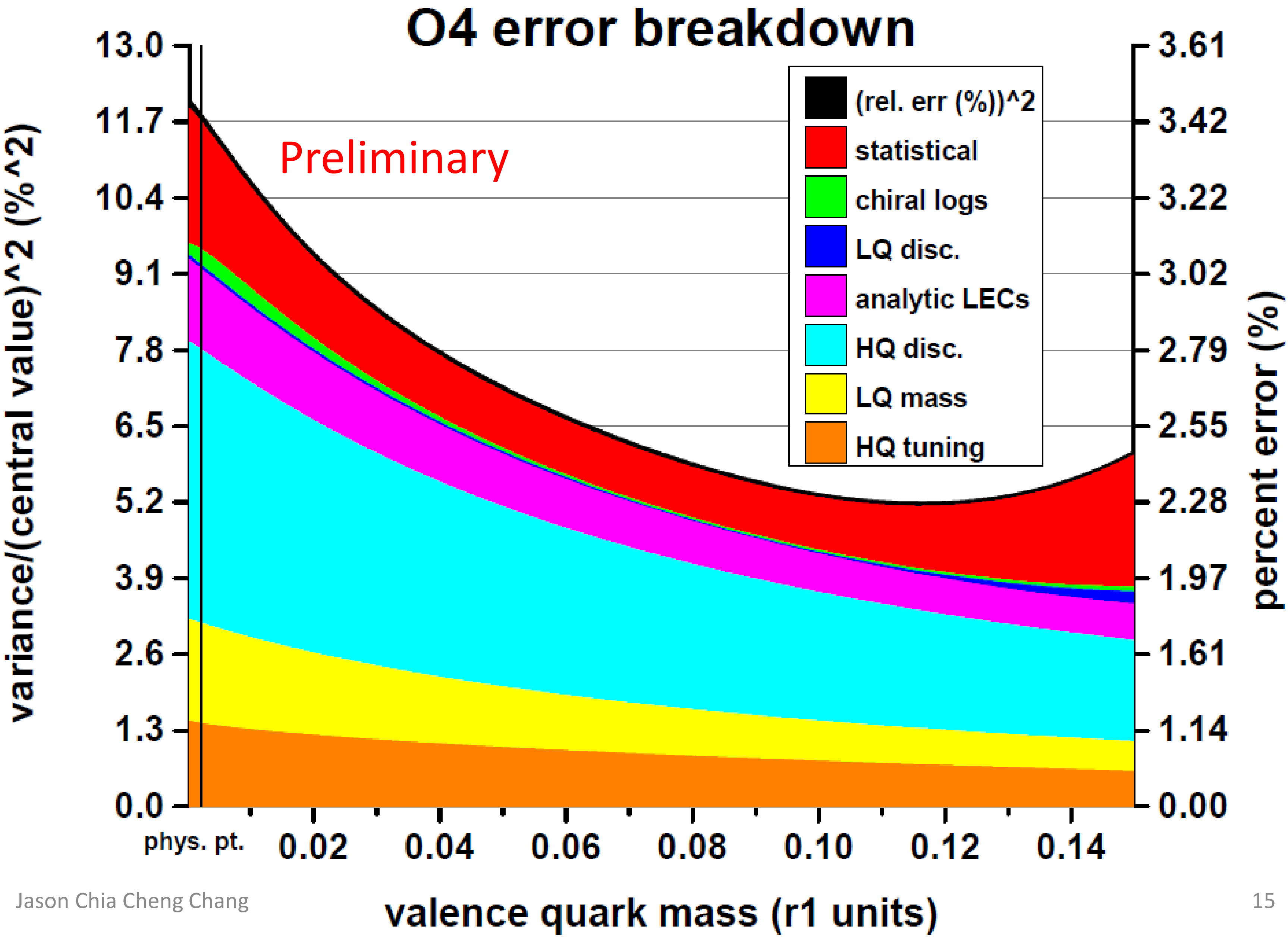
- Statistical
- χPT contribution
  - Chiral logs (LO LECs, input errors)
  - Light quark discretization
  - Analytic LECs
- Heavy quark discretization error
- Charm-quark mass tuning error
- Renormalization and matching error
- Light quark mass uncertainties
- Finite volume effects

Chiral fit function

+ in quadrature

Covariance matrix includes statistical and systematic errors.

Error breakdown



Preliminary results

Preliminary error budget

Percent errors (%)	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$
Statistical	2.1	2.5	2.2	1.5	2.8
Chiral logs	0.6	1.0	0.7	0.5	1.2
Analytic LECs	1.8	1.3	1.0	1.2	1.7
LQ disc.	0.5	0.3	0.1	0.3	0.8
HQ disc.	2.8	1.9	2.0	2.2	2.9
LQ mass	0.5	0.7	0.4	1.3	0.6
HQ tuning	1.2	1.4	0.7	1.2	0.9
Combined (missing renorm & F.V.)	4.2	4.9	3.3	3.4	4.8

Remaining tasks:

Account for renormalization and matching error.

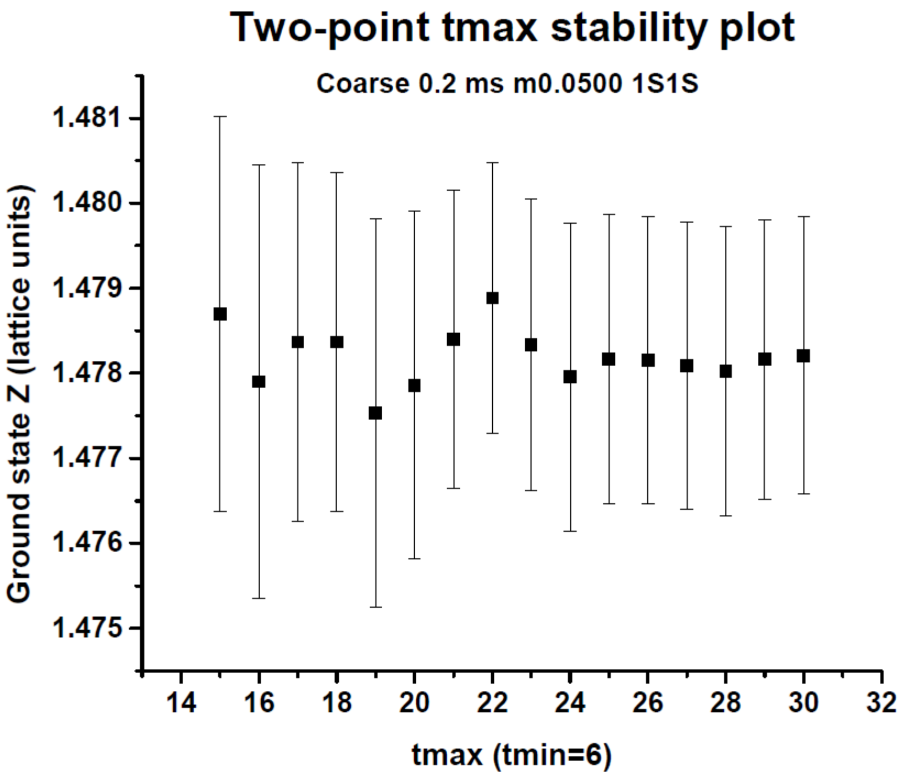
Account for finite-volume corrections.

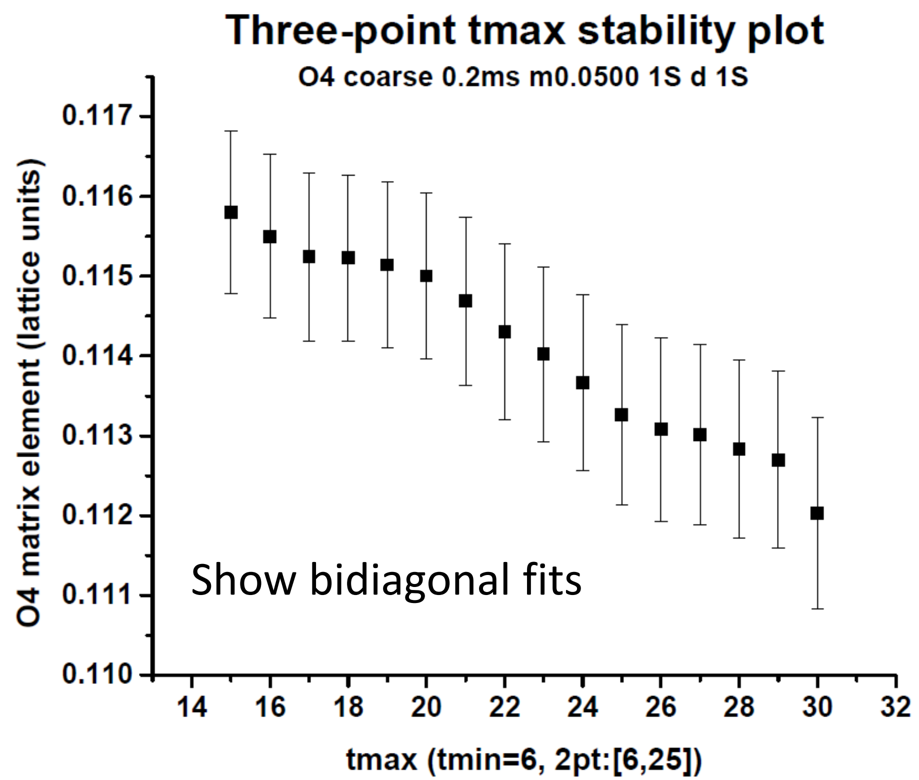
Report bag parameters.

Thank you

3pt exploratory correlator fits

Correlator fits



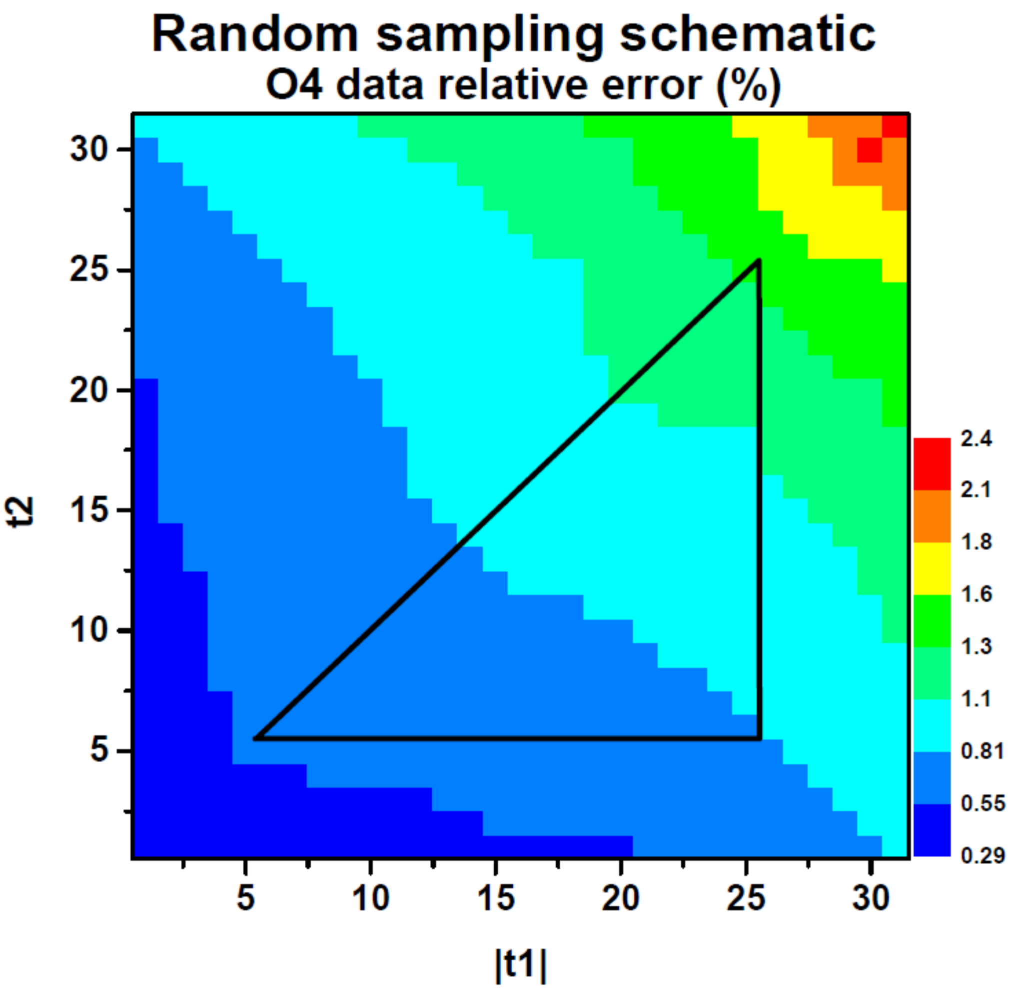


Expected tmax stability plot

Surprising systematic trend

Large time slices → dominantly ground state signal
Ground state parameters expected to be insensitive to tmax
*tmax fits vary both data size and fit region

Random sampling procedure



Procedure

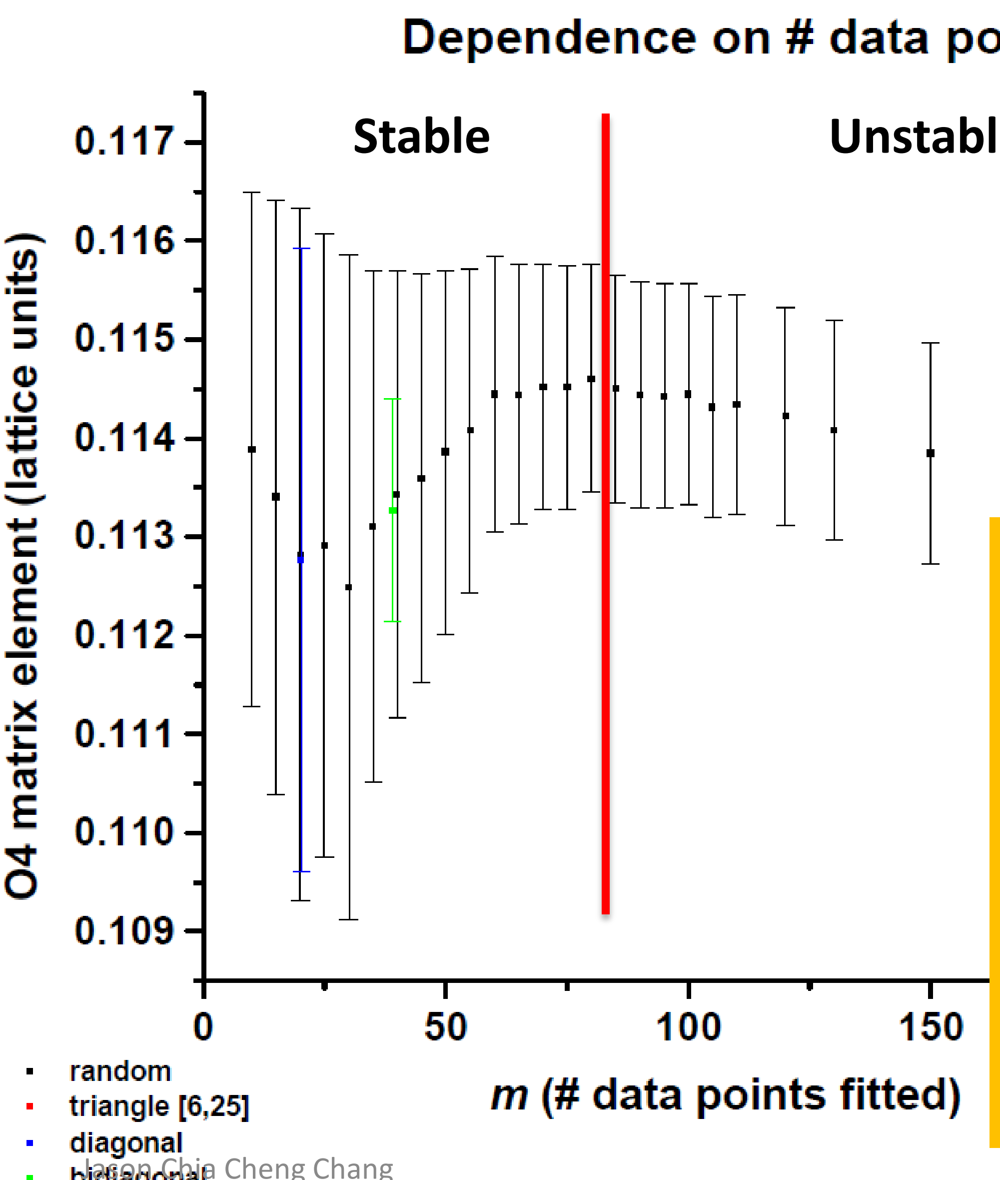
1) Take fixed triangle region
2) Randomly sample *m* points
3) Repeat O(100) times to average out statistical variations.*
4) Plot stability plot vs *m*

*Observed that the standard deviation of repeated samples are much smaller than statistical error.

Goal

Produce fits with varying data size but fixed fit region.

Random sampling result



Insights

Clear trend at large values of *m*.

Suggests data reduction.

→ bidiagonal fits

Advantages of bidiagonal fits:

- Data reduction into stable region.
- Keeps important correlations. (Signal from both parity states).