The QCD Equation of State to $\mathcal{O}\left(\mu_B^4\right)$ from the Lattice

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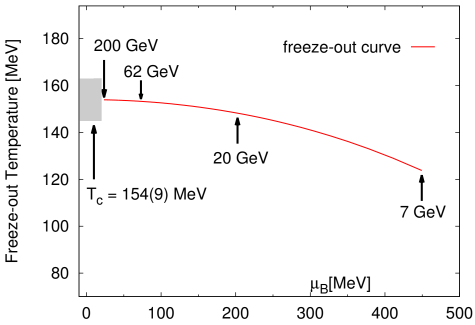
Beam Energy Scan



Sketch of the freezeout curve in the (T, μ_B)-plane, where a knowledge of the EoS is needed. Calculated according to the HRG parametrization of Cleymans *et al.* (2006).

Equation of State

- Accurate determination one of the major goals of finite-temperature lattice QCD.
- Necessary input in hydrodynamic modeling of heavy-ion collisions.
- Continuum results at physical and nearly physical pion masses now available with staggered fermions at $\mathcal{O}(\mu_B^0)$ [Wupp.-Bud. (2010), HotQCD (see talk by A. Bazavov)] as well as for $\mathcal{O}(\mu_B^2)$ [Wupp.-Bud. (2012)].
- Here we will present our results for the $\mathcal{O}(\mu_B^4)$ corrections to the Equation of State, calculated on $N_\tau = 6$ and 8 lattices.

Quark Number Susceptibilities

- Sign problem at $\mu_B > 0$. One approach [R. Gavai and S. Gupta (2001), Swansea-Bielefeld (2001)] is to expand the pressure in a Taylor series:

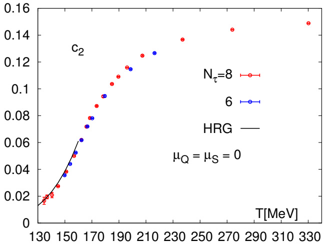
$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i!\,j!\,k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k .$$

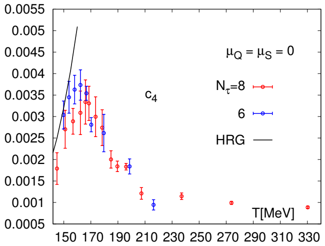
- QNS also yield information about the quantum numbers of the QGP degrees of freedom [V. Koch and S. Jeon (1999)]. They also satisfy scaling relations near the chiral phase transition [Y. Hatta and T. Ikeda (2002)].

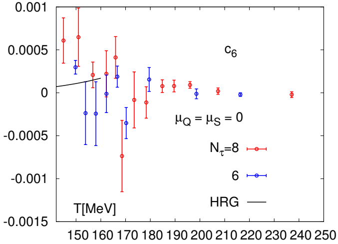
Calculating the Equation of State

- Our calculations were performed with the HISQ action [C. Davies, E. Follana *et al.*(2006)]. The ensembles were the same as the ones used to calculate the $\mu_B = 0$ EoS (see the talk by A. Bazavov at this conference).
- We calculated all the susceptibilities upto sixth order in the temperature range 140 MeV $\lesssim T \lesssim$ 330 MeV.
- $m_\pi \approx 160$ MeV while the strange quark mass was set to its physical value.
- $\mathcal{O}(5,000 - 15,000)$ statistics at two lattice spacings $a = 1/6T$ and $a = 1/8T$.
- Operator traces needed for the susceptibilities evaluated stochastically using ∼1,500 random vectors per configuration.

Pressure Corrections







$$\frac{P}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_B}{T}\right)^n .$$

Sixth-order corrections noisy, but around 1-5% of second-order corrections.

Other Observables

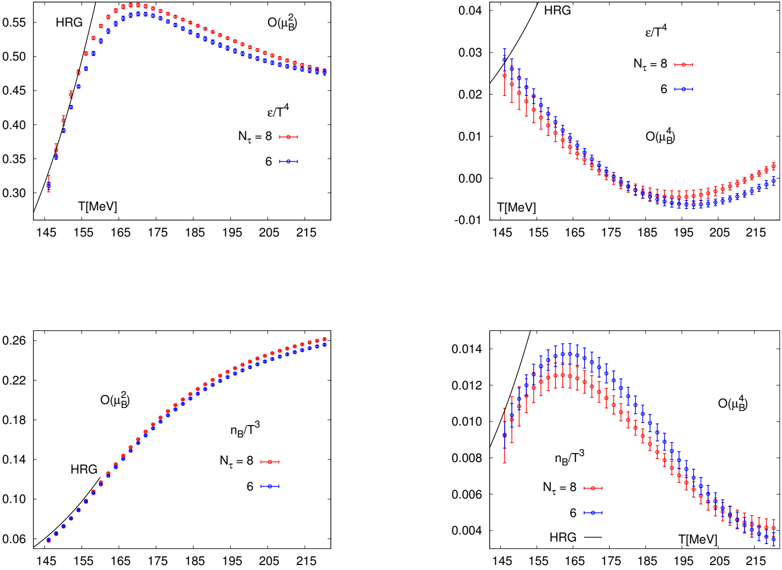
The energy density (ε), entropy density (s) and baryon density (n_B) were obtained from spline fits to the susceptibilities:

$$\frac{\varepsilon}{T^4} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + 3c_n\right\},$$

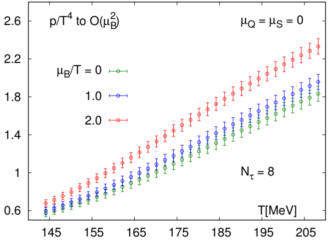
$$\frac{s}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + (4-n)c_n\right\},$$

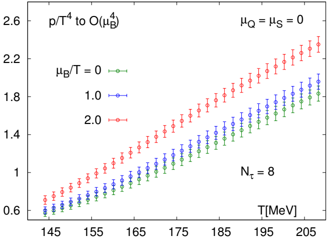
$$\frac{\mu_B n_B}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n n \cdot c_n \; (\mu_Q = \mu_S = 0).$$

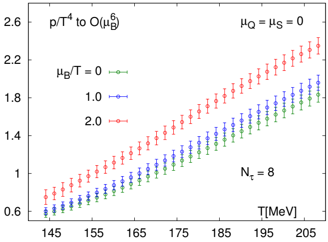
Correction Coefficients



Putting Everything Together

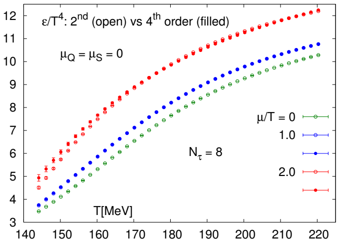


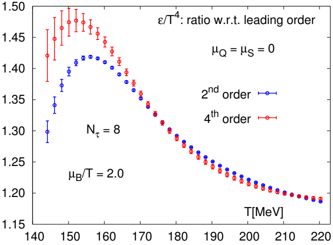




Finite-μ_B corrections under control for $\frac{p}{T^4}$ for $\frac{\mu_B}{T} \lesssim 2.0$.

Energy Density to $\mathcal{O}\left(\mu_B^4\right)$





$\mathcal{O}(\mu_B^4)$-corrections important below $T \sim 170$ MeV and for $\mu_B/T \gtrsim 2.0$.

Constraints in Heavy-Ion Collisions

- Net strangeness zero, charge-to-baryon number ratio fixed (0.4 for Pb, 0.5 for Cu). Thus,

$$N_S = 0 \qquad \text{and} \qquad N_Q = r N_B$$

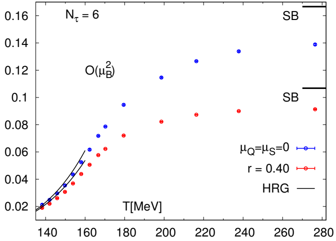
where $r = N_p/(N_p + N_n) = 0.4$ - 0.5 in most cases.

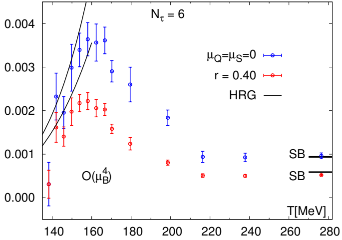
- These determine the electric and strangeness chemical potentials as functions of μ_B and T:

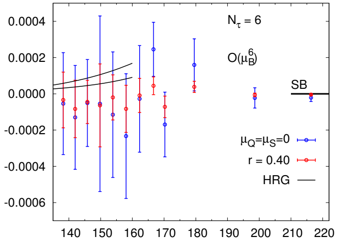
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \frac{\mu_B^3}{T^3} + \ldots, \qquad \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \frac{\mu_B^3}{T^3} + \ldots$$

- Necessary to take these constraints into account for a more accurate phenomenology of heavy-ion collisions.

Pressure Corrections



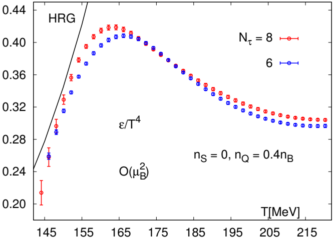


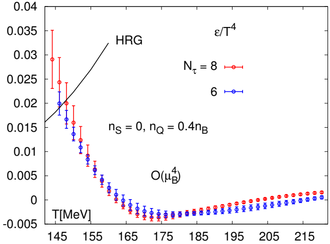


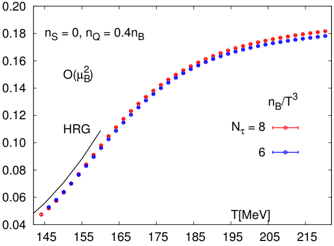
$$\frac{P}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_B}{T}\right)^n .$$

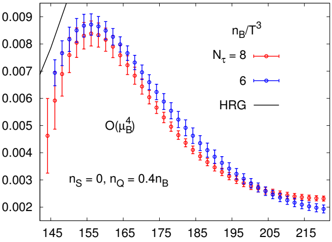
In general, coefficients smaller in magnitude than for the $\mu_Q = \mu_S = 0$ case.

Correction Coefficients: Constrained Case

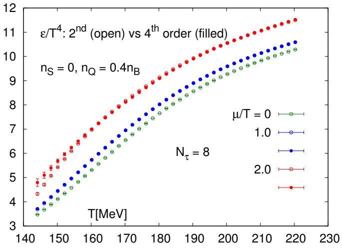


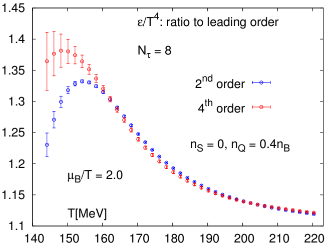






Energy Density to $\mathcal{O}\left(\mu_B^4\right)$





Larger $\mathcal{O}(\mu_B^4)$-corrections when compared to $\mathcal{O}(\mu_B^2)$ corrections in the constrained case than for the $\mu_Q = \mu_S = 0$ case.

Observables on the Freezeout Curve

We will use the values of (T, μ_B) from the parametrization by Cleymans *et al.*:

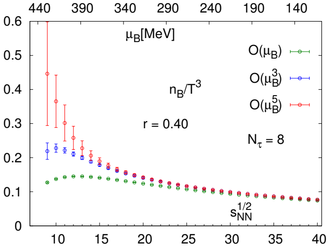
$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \qquad \mu_B = \frac{d}{1 + e\sqrt{s_{NN}}},$$

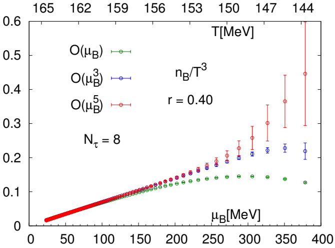
with

a	0.154 ± 0.009	GeV
d	1.308 ± 0.028	GeV
b	0.139 ± 0.016	GeV^{-1}
e	0.273 ± 0.008	GeV^{-1}
c	0.053 ± 0.021	GeV^{-3}

to calculate the values of p, ε, n_B, etc. at freezeout.

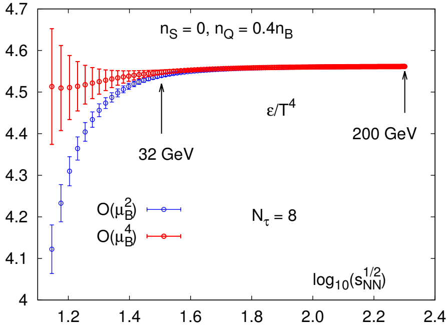
Baryon Density at Freezeout





By comparing different orders, we find that $O(\mu_B^4)$-corrections become important around $\sqrt{s_{NN}} \sim 20$ GeV. Similarly, $O(\mu_B^6)$-corrections are expected to become significant around $\sqrt{s_{NN}} \sim 12$ GeV.

Energy, Entropy and Baryon Density



ε/T^4 nearly constant on the freezeout curve upto $\sqrt{s_{NN}} \sim 32$ GeV and possibly for lower values of $\sqrt{s_{NN}}$ too.

Conclusions

- Calculating the equation of state beyond second order is necessary to extend its applicability for a broader beam energy range. Using our lattice results for the quark number susceptibilities, we calculated the pressure to sixth order and the energy density to fourth order.

- $\mathcal{O}(\mu_B^4)$-corrections contribute $\sim$10% to the total energy density for $T \lesssim 160$ MeV, and this contribution increases when the constraints from heavy-ion collisions are taken into account.

Conclusions

- Baryon number density is especially sensitive to finite-μ_B corrections since its expansion begins at $\mathcal{O}(\mu_B)$. By looking at the contribution of various orders, we estimated that fourth order becomes important around $\sqrt{s_{NN}} \sim 20$ GeV, whereas sixth order starts to become important at $\sqrt{s_{NN}} \sim 12$ GeV.
- It is possible that the energy density ε/T^4 remains constant along the freezeout curve. $\mathcal{O}(\mu_B^4)$ corrections are required for $\sqrt{s_{NN}} \lesssim 32$ GeV to see this, however.
- What remains to be done: second-order quantities *i.e.*specific heat C_v, speed of sound c_s^2 and the compressibility K, as well as calculating P, ε, etc. on isentropic trajectories *i.e.* at constant entropy per baryon.