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# Magnetic monopole and confinement/deconfinement phase transition in SU(3) Yang-Mills theory

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# Outline of talk

- **Introduction**
  - dual superconductivity
  - Lattice new formulation (quick review)
- Non-Abelain dual-superconductivity in SU(3)-YM theory
- Monopoles and confinement/deconfinemet phase transition at finite temperature
- Summary and outlook

# dual superconductivity

- Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]

**superconductor**

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

**dual superconductor**

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

# The evidence for dual superconductivity

**By using Abelian projection**

**String tension (Linear potential)**

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]

**Chromo-flux tube (dual Meissner effect)**

- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
- ◆ Type the super conductor is of order between Type I and Type II [Y.Matsubara, et.al. 1994]

- ✓ **only obtained in the case of special gauge such as MA gauge**
- ✓ **gauge fixing breaks the gauge symmetry as well as color symmetry**

# The evidence for dual superconductivity

**Gauge decomposition method (a new lattice formulation)**

- **Extracting the relevant mode *V* for quark confinement** by solving the defining equation in the gauge independent way (gauge-invariant way)

➢ For SU(2) case, the decomposition is a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.

➢ For SU(N) case, the formulation is the extension of the SU(2) case.

**➔ we have showed in the series of lattice conferences that**

➢ V-field dominance, magnetic monopole dominance in string tension,

➢ chromo-flux tube and dual Meissner effect.

➢ The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect

# A new formulation of Yang-Mills theory (on a lattice)

## Decomposition of SU(N) gauge links

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

☐ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

☐ SU(3) Yang-Mills link variables: Two options

**maximal option :** $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

**minimal option** : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

# The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[ P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

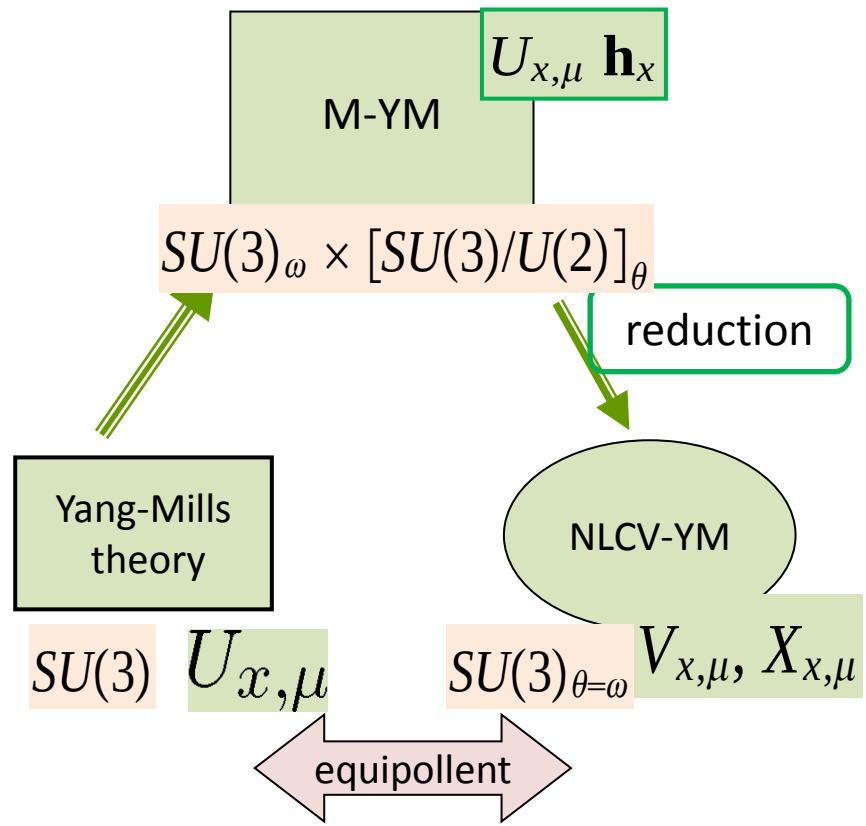
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[ P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$




$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

# Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D^\epsilon_\mu[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}^\dagger_{x,\mu}(\det\hat{L}_{x,\mu})^{1/N}g_x^{-1} \quad V_{x,\mu} = X^\dagger_{x,\mu}U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L^\dagger_{x,\mu}}\right)^{-1}L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

# Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{h}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$

$$F_{\mathrm{red}}[\mathbf{h}_x; U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left\{ (D_\mu^\epsilon[U]\mathbf{h}_x)^\dagger (D_\mu^\epsilon[U]\mathbf{h}_x) \right\}$$

$$SU(3)_\omega \times [SU(3)/U(2)]_\theta \rightarrow SU(3)_{\omega=\theta}$$

- This is invariant under the gauge transformation θ=ω
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- We choose a reduction condition of the same type as the SU(2) case

# Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig\int_{S:C=\partial\Sigma} dS^{\mu\nu}\sqrt{\frac{N-1}{2N}}\,\mathrm{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x))\right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig\sqrt{\frac{N-1}{2N}}(k,\Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j,N_\Sigma)\right)$$

magnetic current $k := \delta^* F = {}^*dF$, $\quad \Xi_\Sigma := \delta^*\Theta_\Sigma\Delta^{-1}$

electric current $j := \delta F$, $\quad N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$

$\Delta = d\delta + \delta d$, $\quad \Theta_\Sigma := \int_\Sigma d^2S^{\mu\nu}(\sigma(x))\delta^D(x - x(\sigma))$

$k$ and $j$ are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

**K.-I. Kondo PRD77 085929(2008)**

The lattice version is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg\,\mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x\right)V_{x,\mu}V_{x+\mu,\mu}V^\dagger_{x+\nu,\mu}V^\dagger_{x,\nu}\right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta^8_{\alpha\beta},$$

# Outline of talk

- Introduction
- **Non-Abelain dual-superconductivity in SU(3)-YM theory**

  We investigate the minimal option for fundamental fermion

  – V-field dominance, non-Abelian magnetic monopole dominance

  – Non-Abelian chromo-flux and chromo-electronic flux tube

  – Non-Abelian dual-meissner effect and induced magnetic monopole
- Monopoles and confinement/deconfinemet phase transition at finite temperature
- Summary and outlook

## ■ SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable $V$, and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

***gauge independent "Abelian" dominance***

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

***Gauge independent non-Abalian monople dominance***

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

**$U^*$ is from the table** in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).

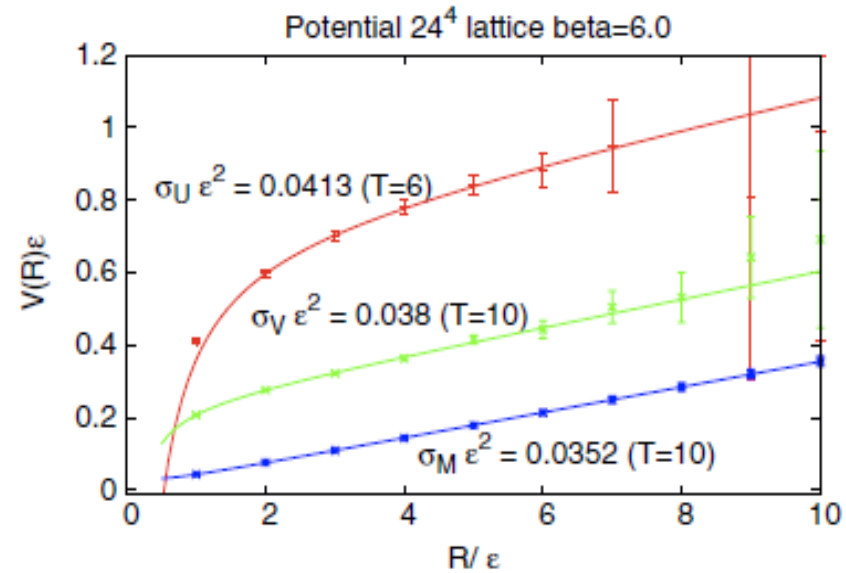



FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance $R$: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on $24^4$ using 500 configurations where $\epsilon$ is the lattice spacing.

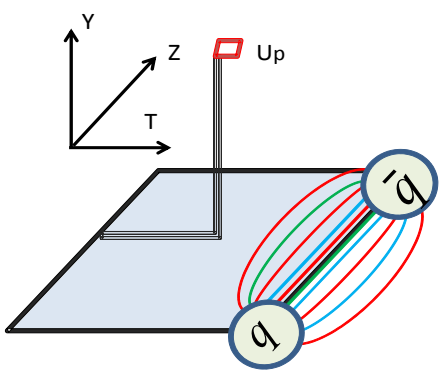
PRD 83, 114016 (2011)

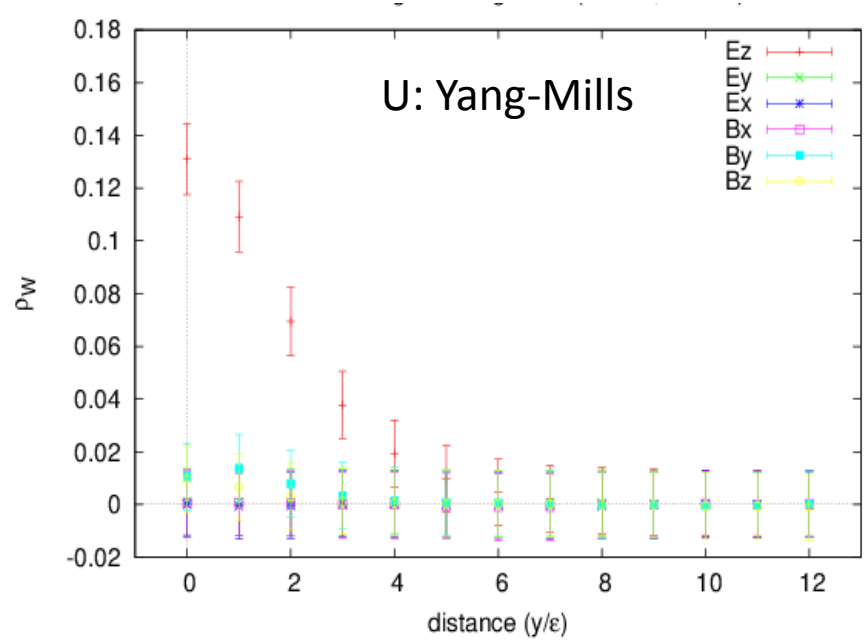
# Chromo flux

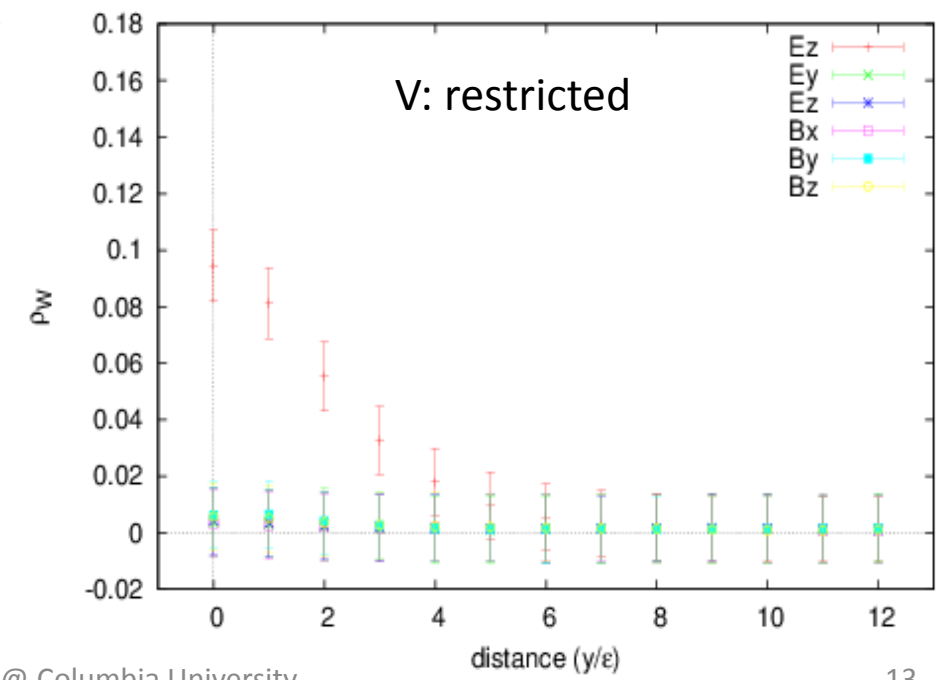
$$\rho_W = \frac{\langle \mathrm{tr}(WLU_pL^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

**Gauge invariant correlation function:** This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et.al. PLB236:199,1990 NPBB347:441-460,1990]

$\mathrm{tr}(U_pLWL^\dagger)$

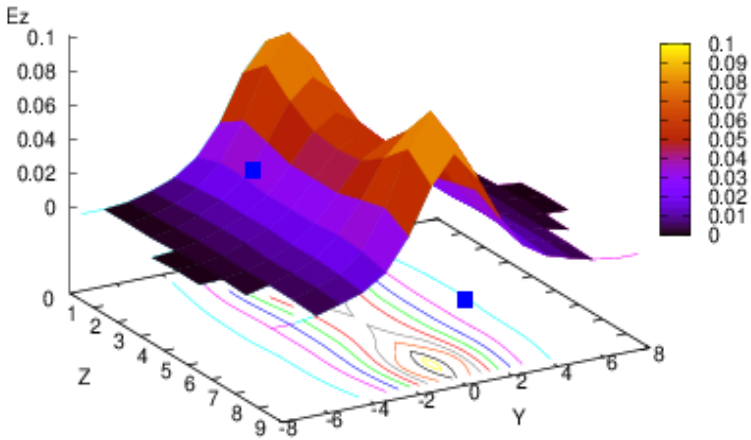


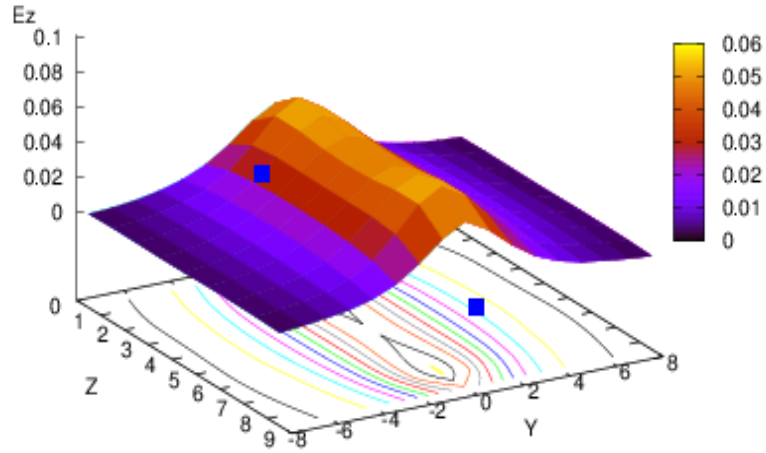



# Chromo-electric (color flux) Flux Tube

Original YM filed




Restricted field




A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

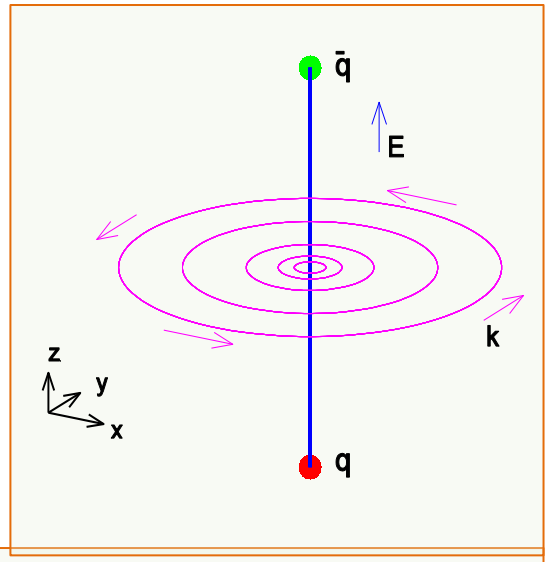
**Flux tube is observed for V-field case. :: dual Meissner effect**

# Magnetic current induced by quark and antiquark pair

Yang-Mills equation (Maxwell equation) for $\mathbf{V}_\mu$ field, the magnetic monopole (current) can be calculated as

$$\mathbf{k} = {}^*dF[\mathbf{V}] \ ,$$

$F[\mathbf{V}]$ is the field strength 2-form of $V_\mu$ field
$d$ the exterior derivative and $*$ denotes the Hodge dual.




$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^*dF[\mathbf{V}] = {}^*ddF[\mathbf{V}] = 0$
(Bianchi identity)

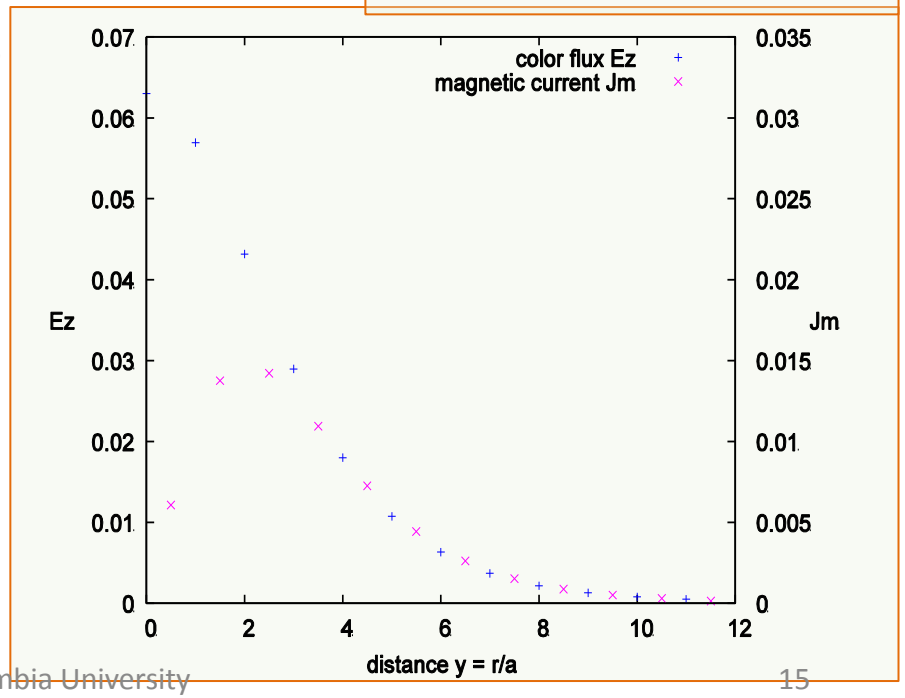



Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

# Outline of talk

- Introduction
- Non-Abelain dual-superconductivity in SU(3)-YM theory
- **Monopoles and confinement/deconfinemet phase transition at finite temperature**

  We investigate chromo-flux by using Polyakov loop source

  – Polyakov loops and Polyakov loop average (convesional order parameter)

  – Chromo-flux at fineite temperature

  – Induced magnetic monopole by quark and antiquark source
- Summary and outlook

# Lattice set up

- Standard Wilson action
- $24^3 \times 6$ lattice
- Temperature is controlled by using $\beta$ ($=6/g^2$);

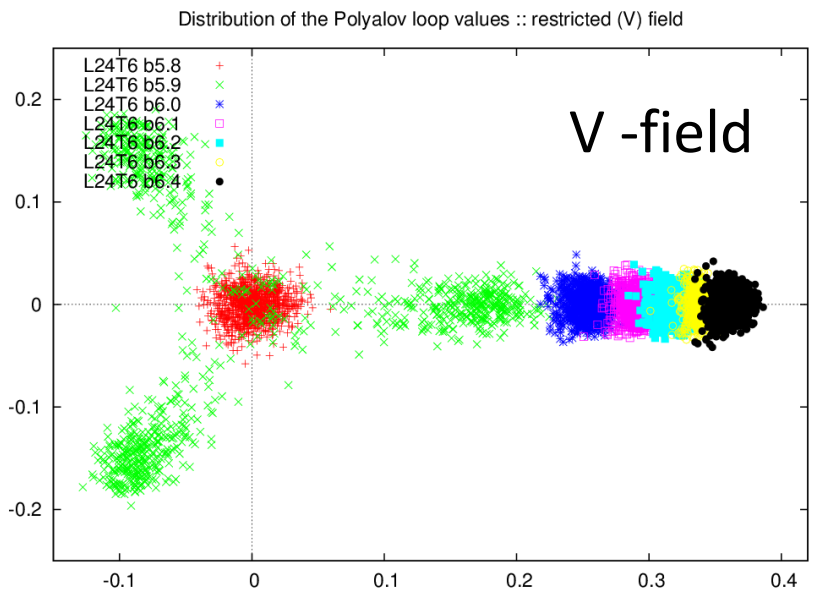
  $\beta$=5.8, 5.9, 6.0, 6.1, 6.2, 6.3
- Measurement by 1000 configurations

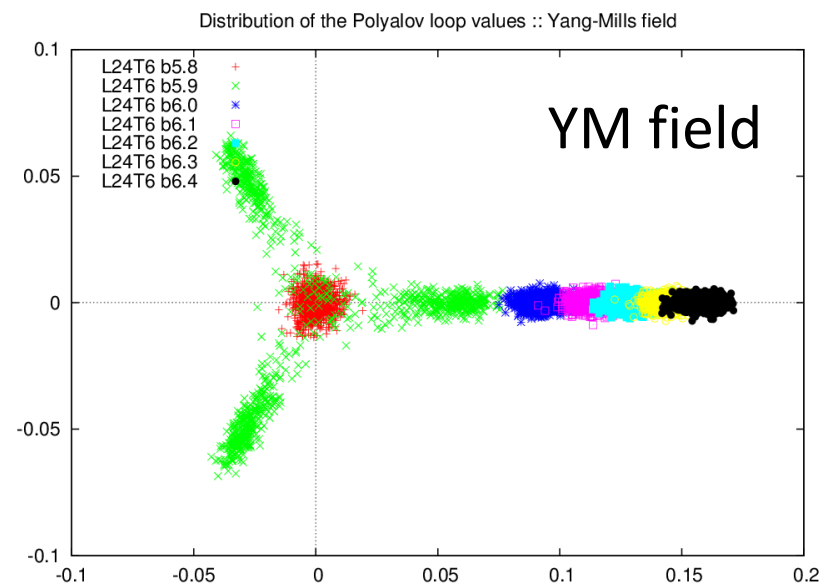
- We investigate chromo-flux by using Polyakov loop source in place of the Wilson loop.
- V-filed dominance in Polyakov loops both in confinement and deconfinement phase (reported in the last conference)

# Distribution of Polyakov loop

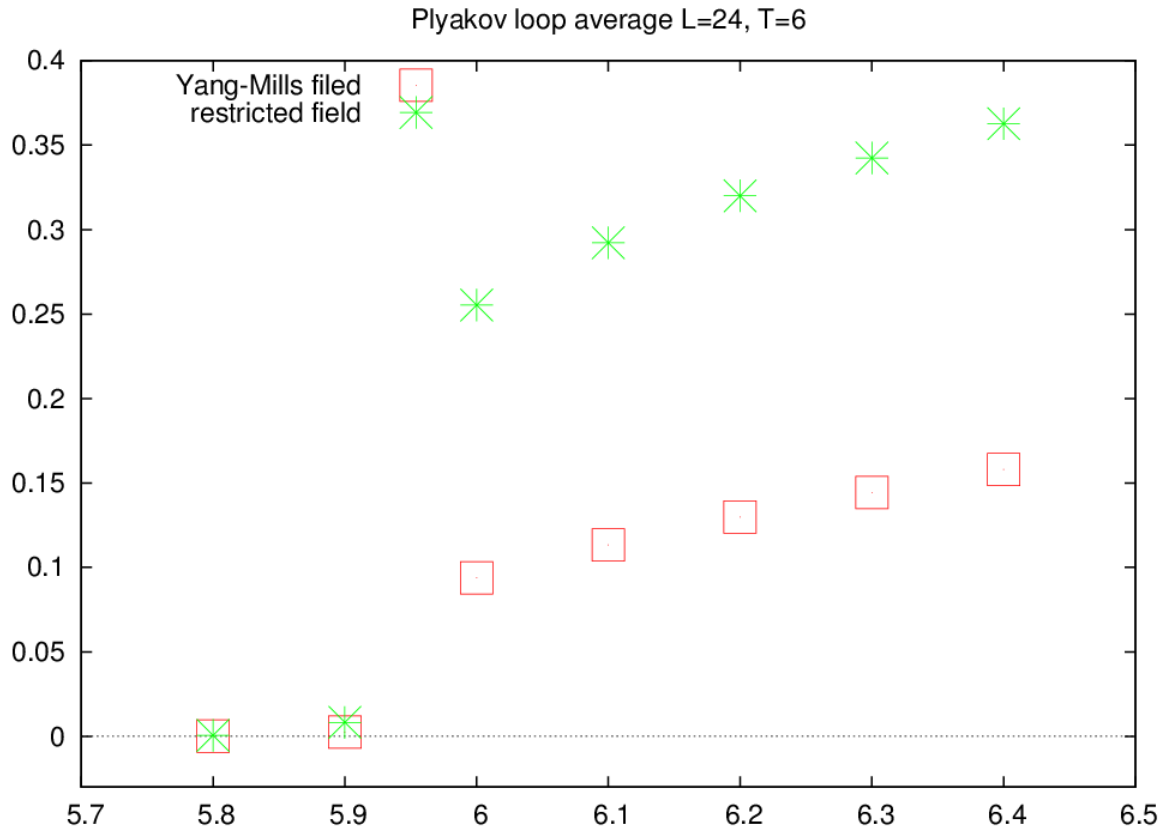
$$P_U(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} U_{(x,t),4}\right) \text{ for original Yang–Mills filed}$$

$$P_V(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} V_{(x,t),4}\right) \text{ for restricted field}$$

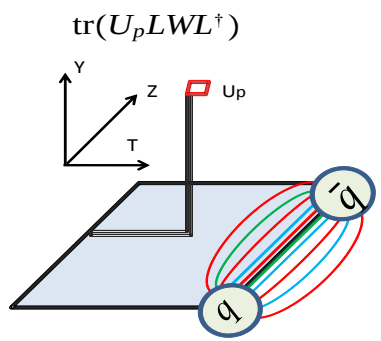


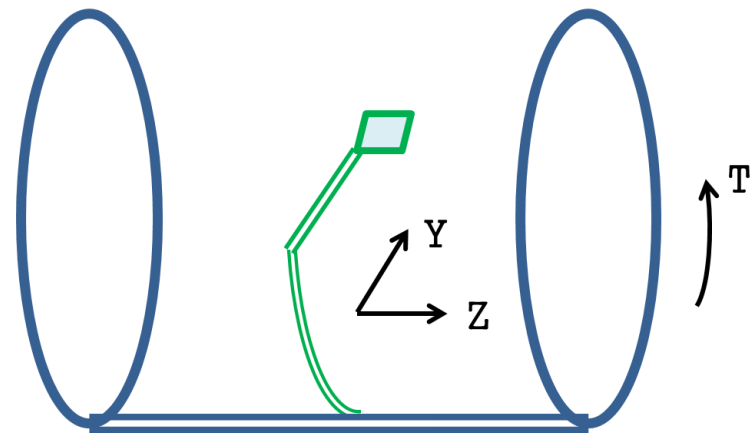



# Polyakov loop average YM-field v.s. V - field



# Chromo-electric flux at finite temperature

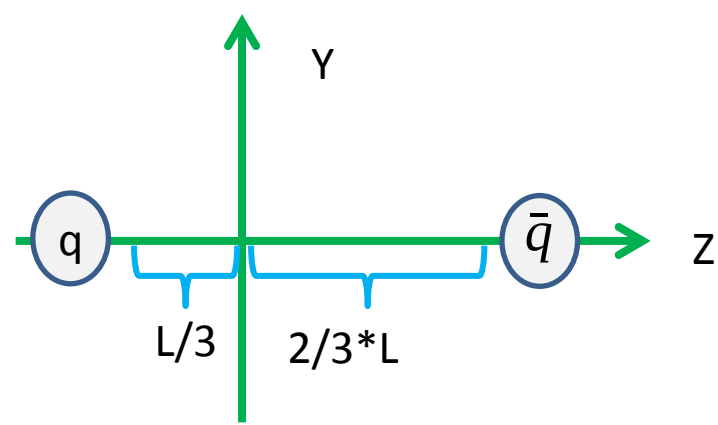


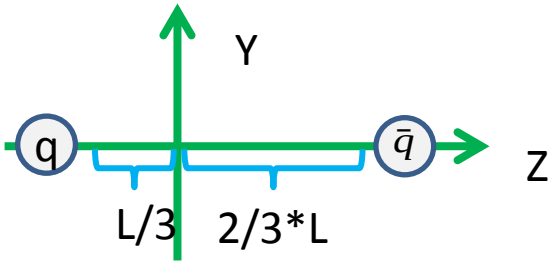

Size of Wilson loop T-direction = Nt
➔The quark and antiquark sources are given by **Plyakov loops**.

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

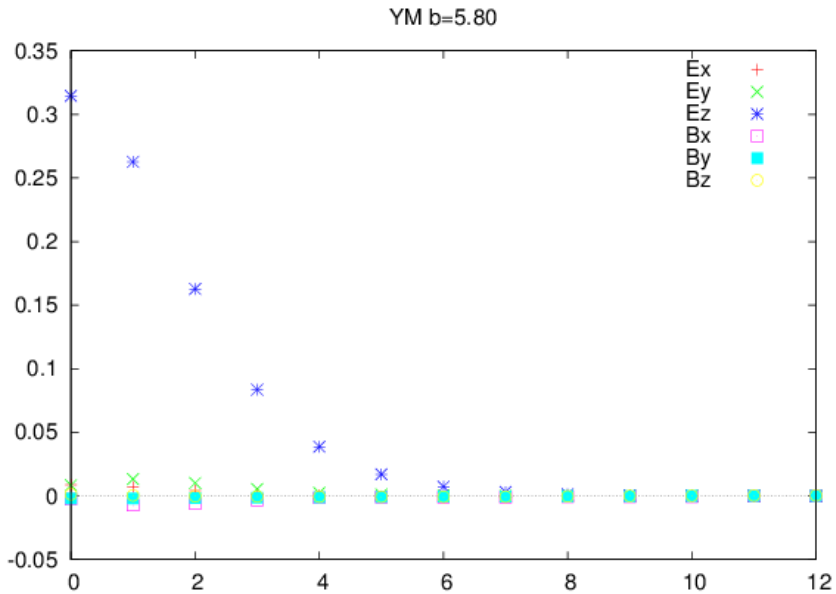
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$



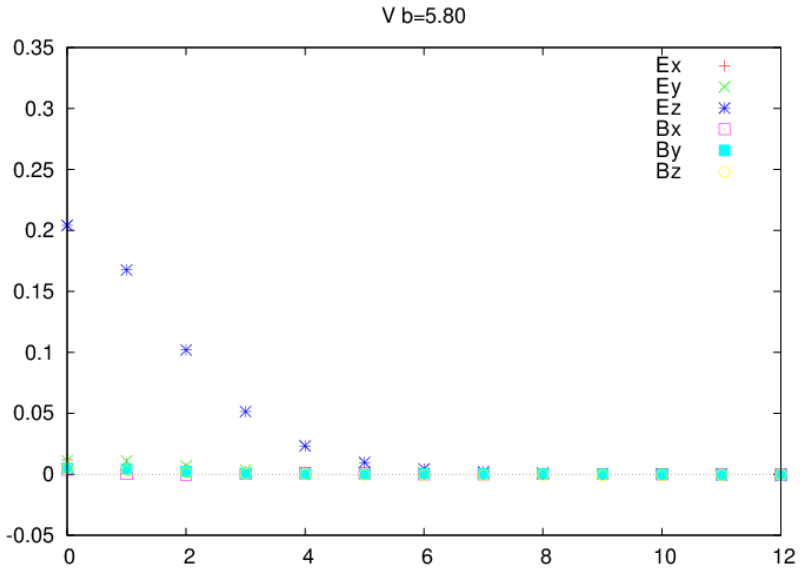
# Chromo-flux β=5.8




YM field

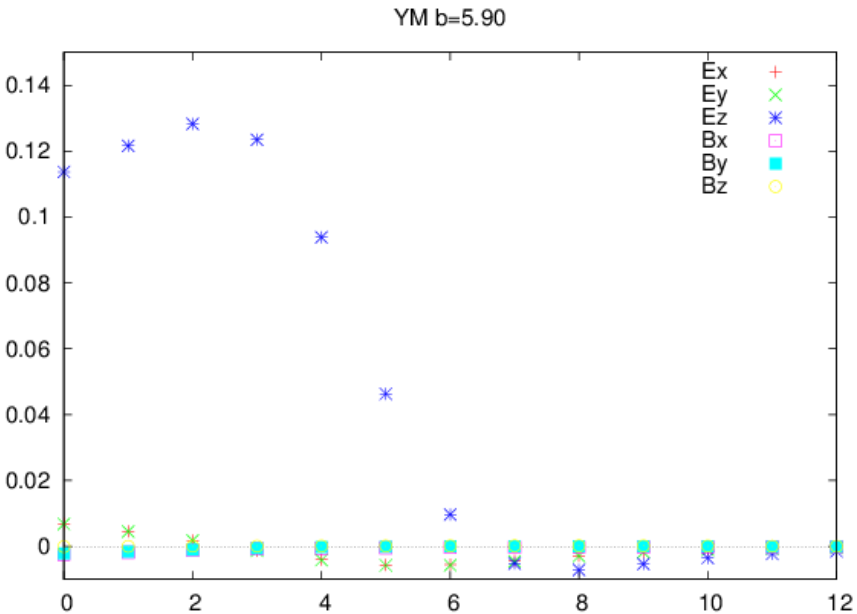



V field

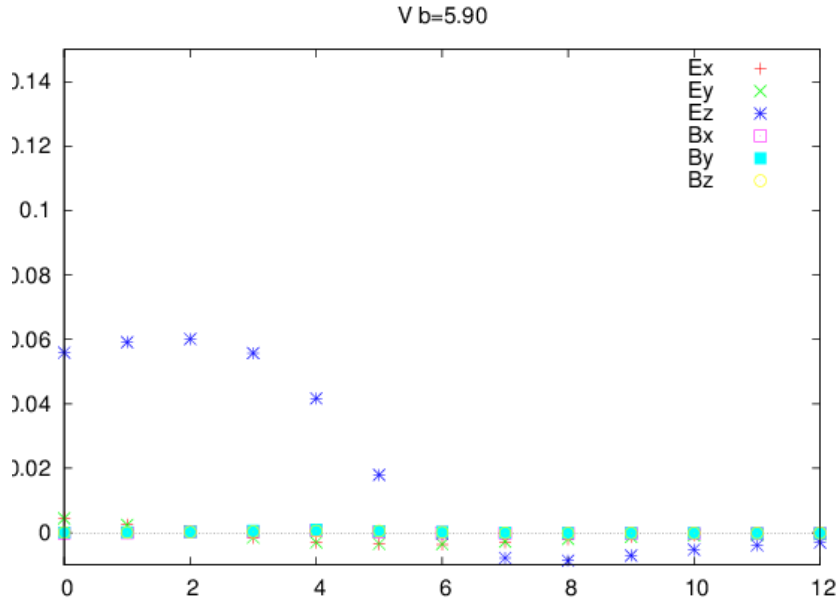


# Chromo-flux β=5.9

YM field

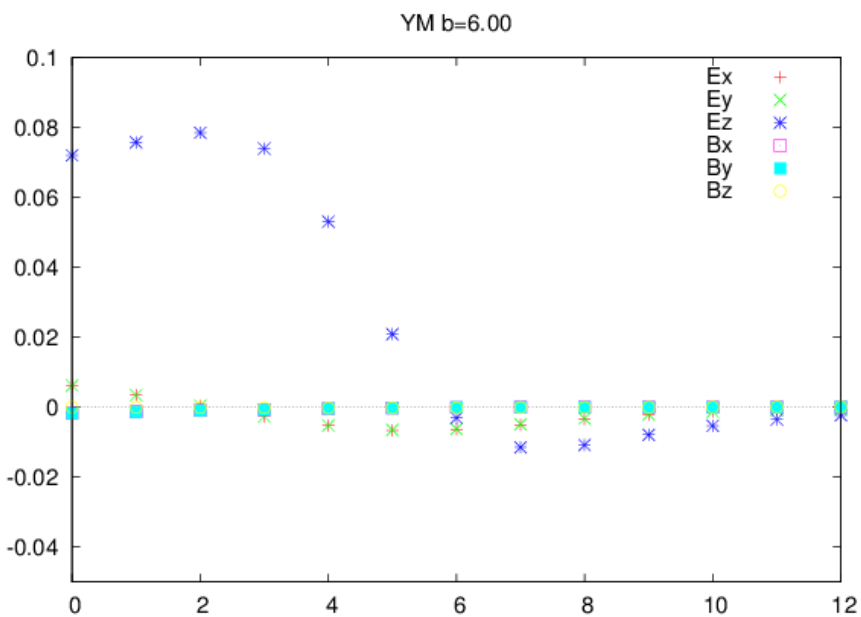



V field

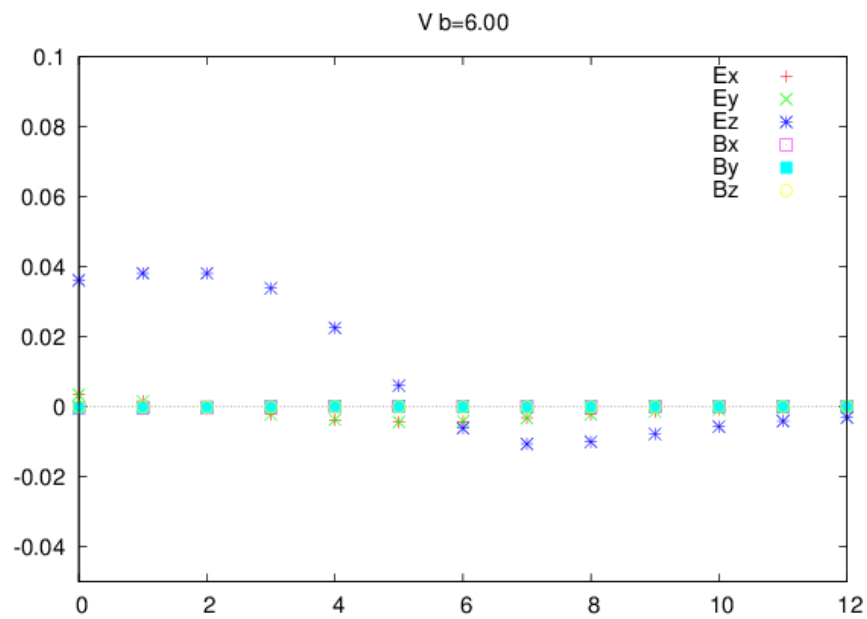


# Chromo-flux β=6.0

YM field

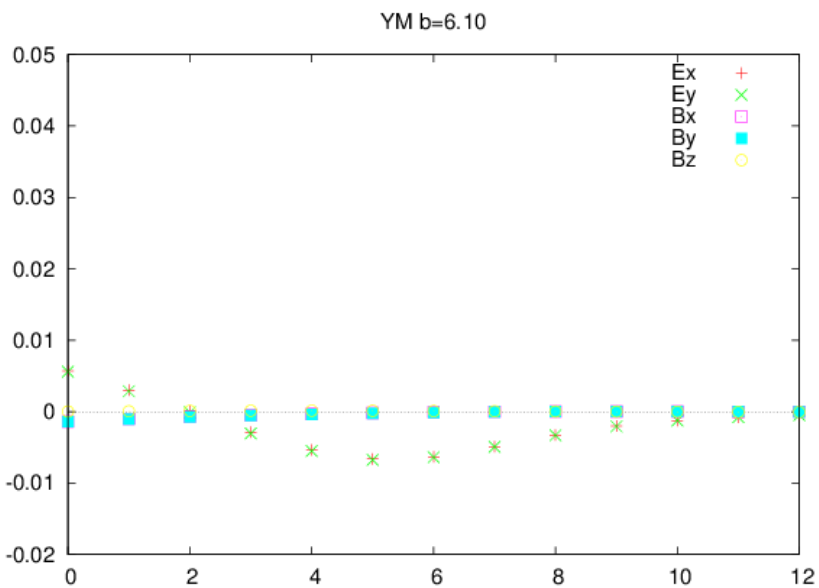



V field

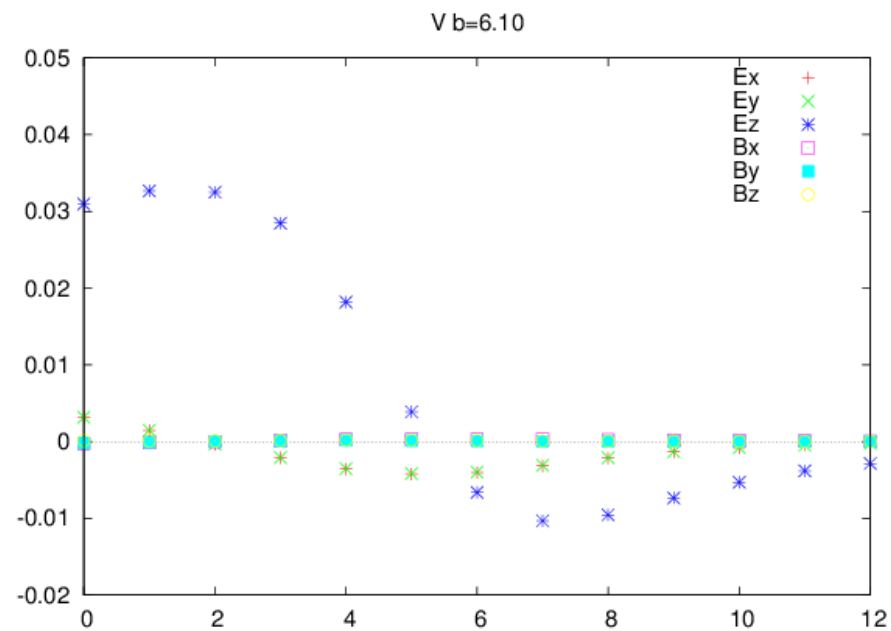


# Chromo-flux β=6.1

YM field

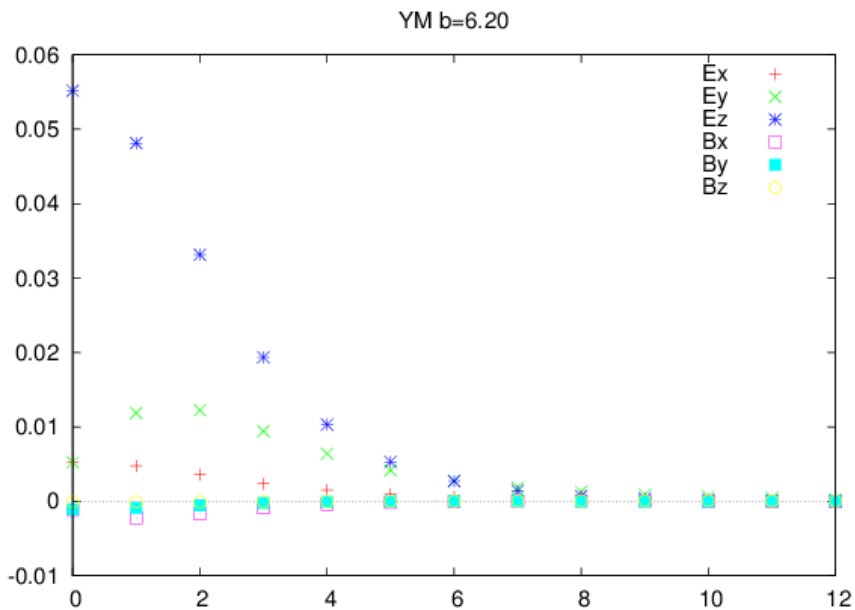



V field

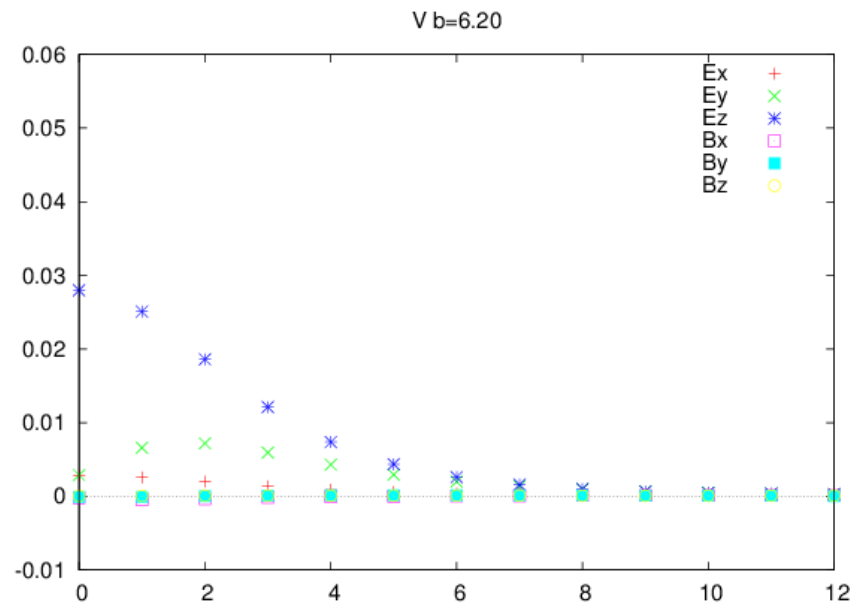


# Chromo-flux β=6.2

YM field

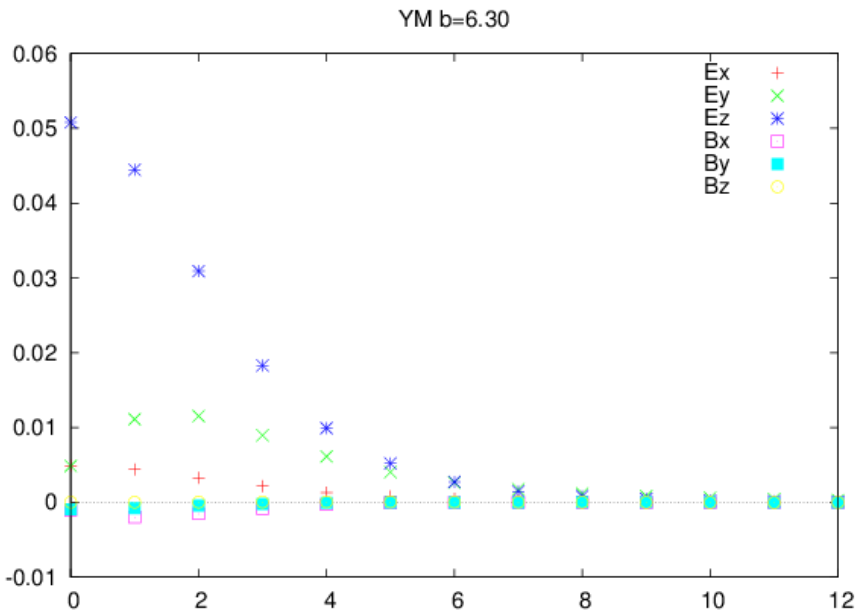



V field

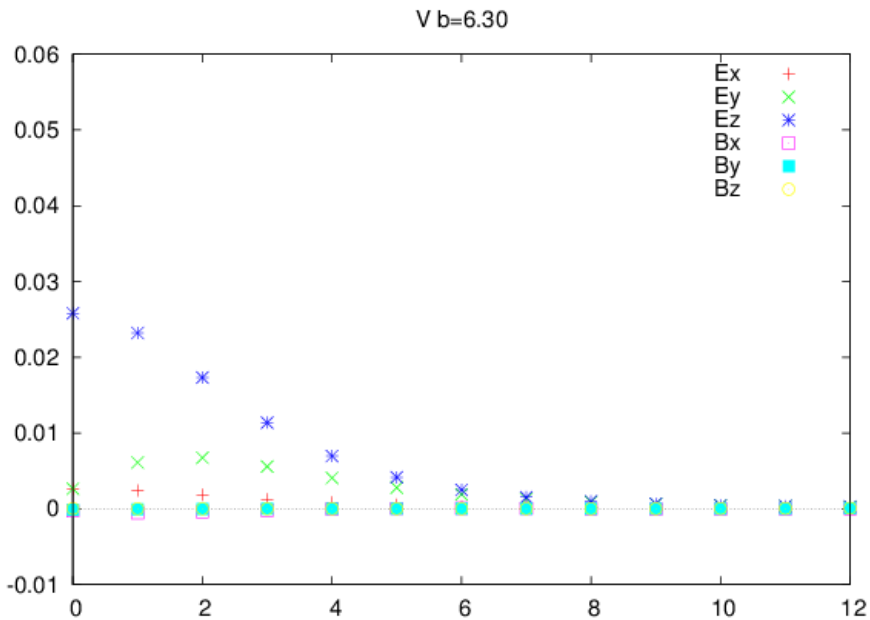


# Chromo-flux β=6.3

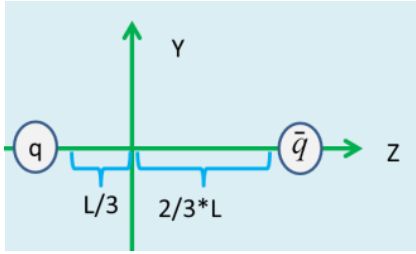
YM field




V field

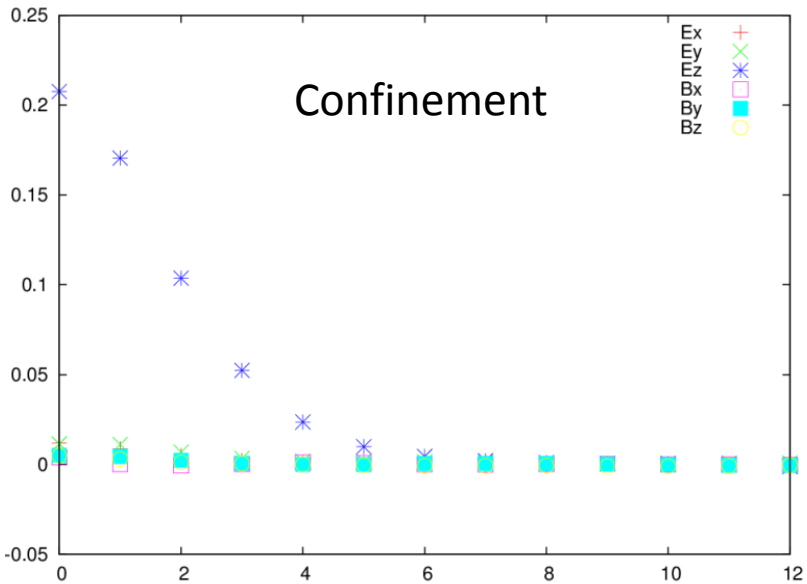


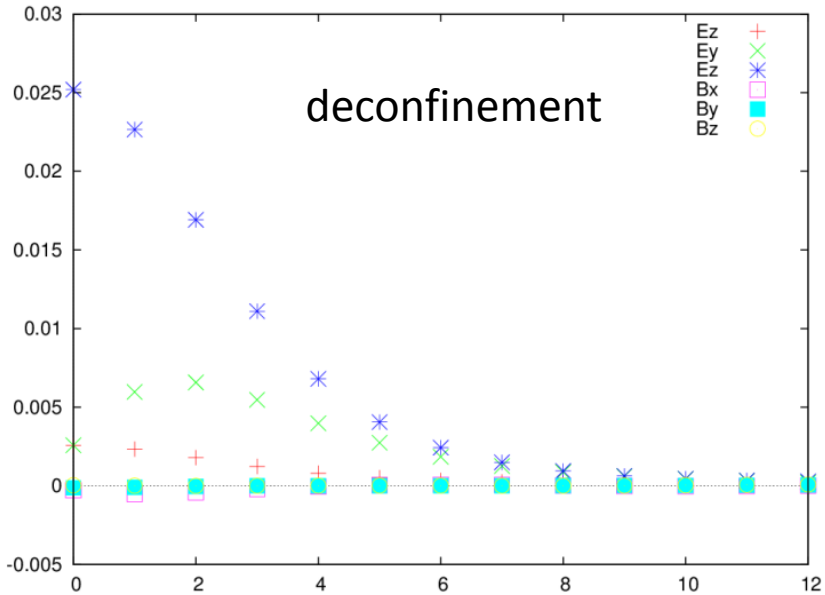
# Chromo-electric flux in deconfinement phase




- $E_y \neq 0$ for deconfinemnte phase i.e., No sharp chromo-flux tube

➔ Disappearance of dual superconductivity.

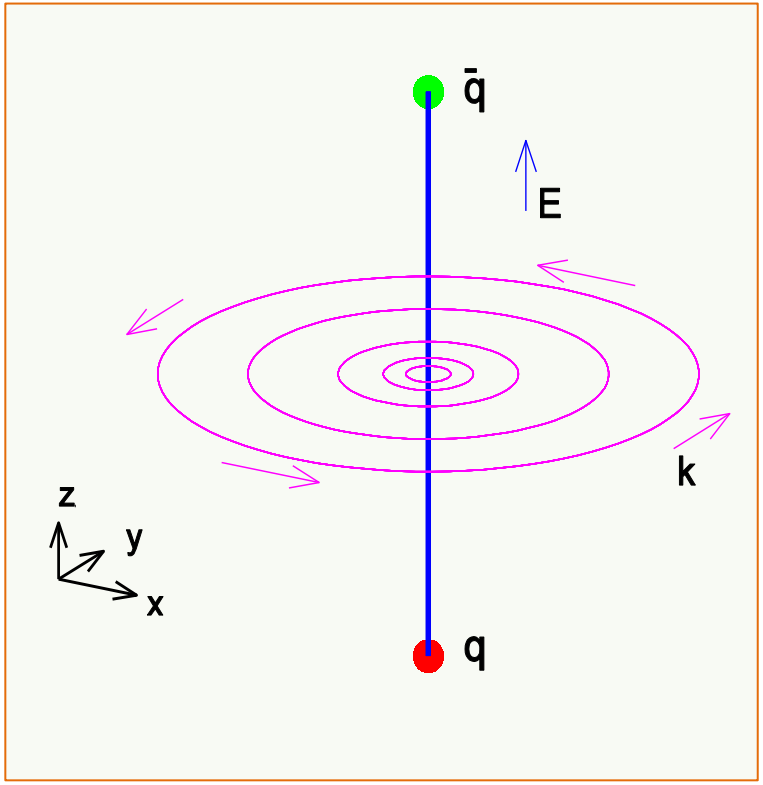


# Chromo-magnetic current (monopole current)

- To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxell equation for V field.

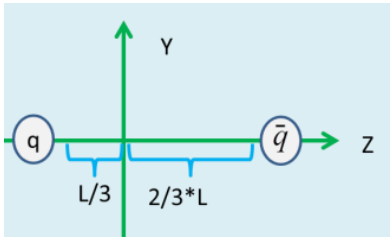
$$k = {}^*dF[V]$$



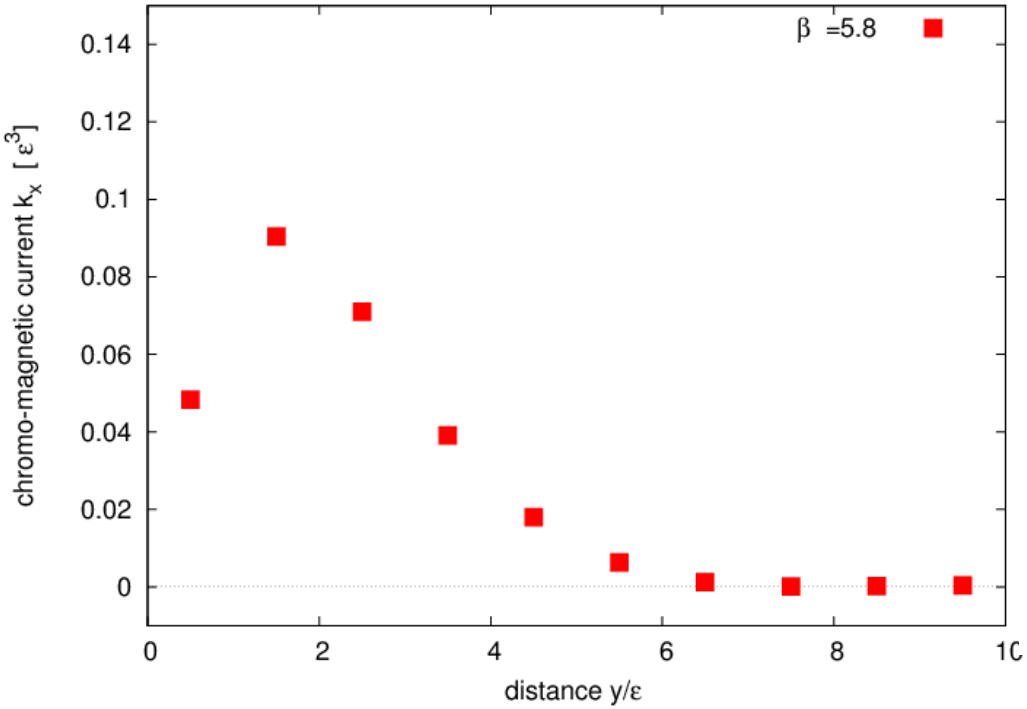

$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^*dF[\mathbf{V}] = {}^*ddF[\mathbf{V}] = 0$
(Bianchi identity)

# Chromo-magnetic (monopole) current β=5.8

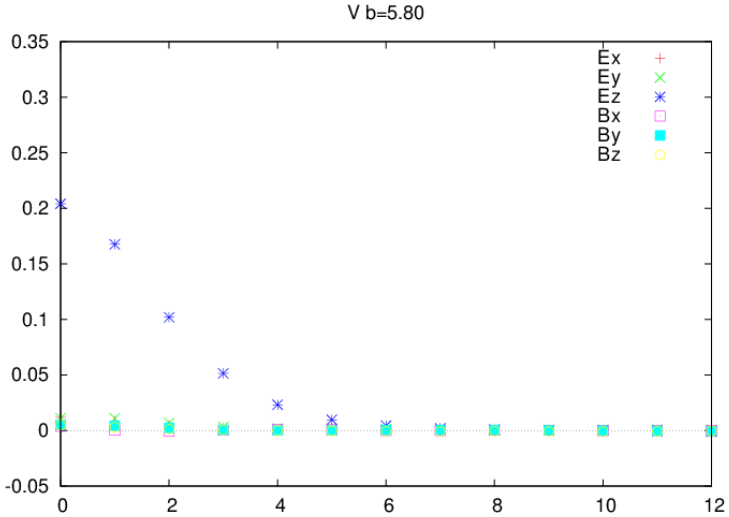
## Confinement phase




chromo-magnetic current $k_x$

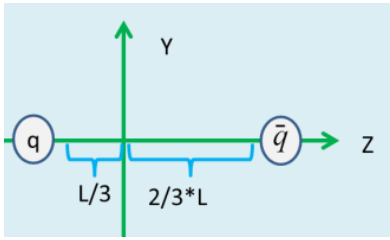



Chromo-flux

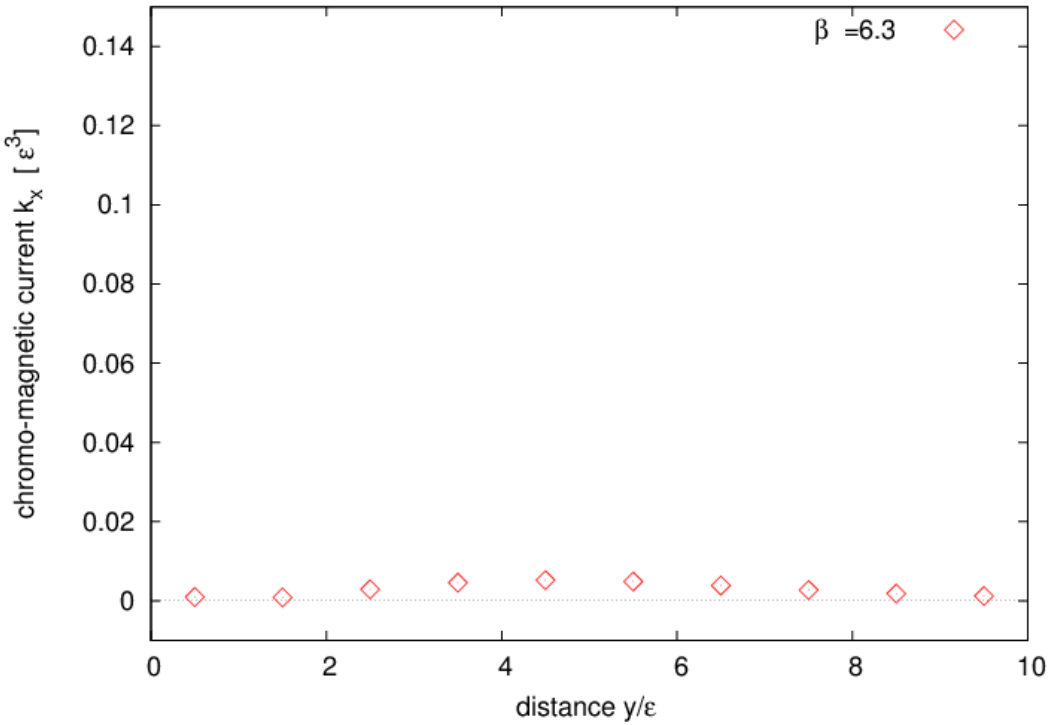


# Chromo-magnetic (monopole) current β=6.3

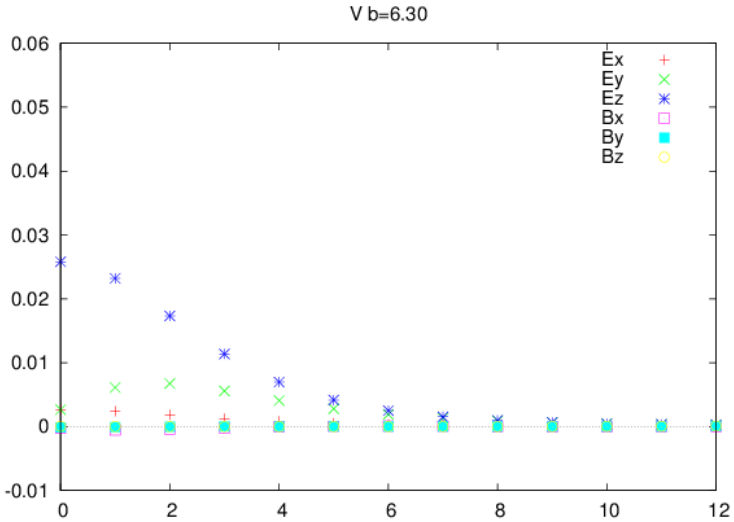
deconfinement phase



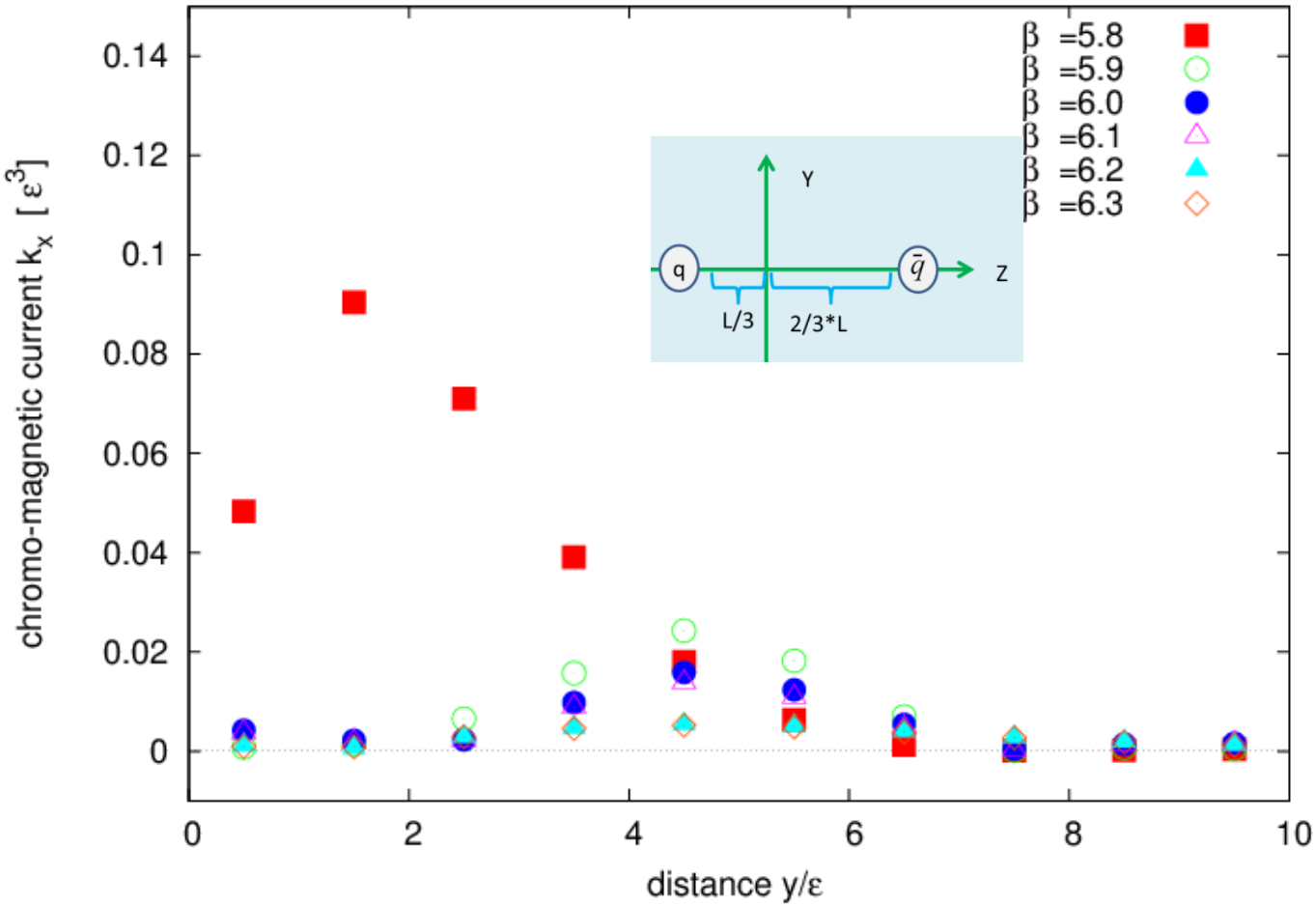

chromo-magnetic current $k_x$




Chromo-flux



# Chromo-magnetic current $k_x$ :: (conbied plot)



# Summary

- **We investigate non-Abelian dual Meissner effects at finite temperature,** applying our new formulation of Yang-Mills theory on the lattice.
- We measure chromo-flux created by a pair of quark and antiquark and the induced chromo-magnetic current (magnetic monopole) due to dua-Meissner effect.

➢ In confinement phase, **observation of the chromo-electric flux tube and induced magnetic monopole**

➢ deconfiment phase, **disappearance of the the chromo-electronic flux tube and vanishing the magnetic monopole**

➔The magnetic monopole plays the dominant role in confinement/ deconfinement phase transition.

**<u>Outlook</u>**

☐ Distribution of chromo-flux and magnetic monopole in 2D (3D) space

☐ Measurment by Magnetic monopole operator $\mathbf{k}_\mu(x) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta_{\alpha\beta}(x)$

# Tank you for your attention.

32nd International Symposium on Lattice Field Theory (Lattice 2014)