Dirac eigenmodes at the QCD Anderson transition

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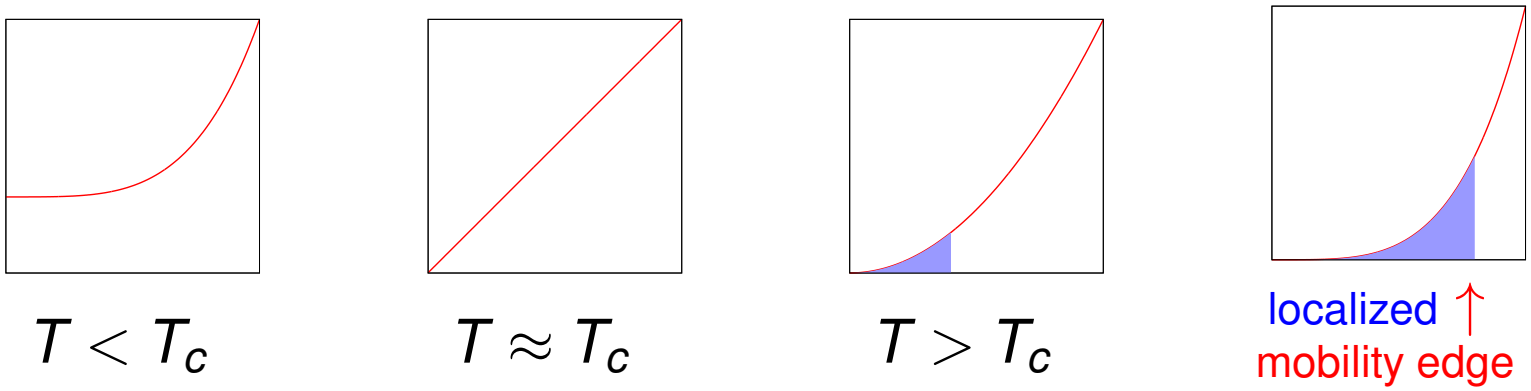
Localized modes in the QCD Dirac spectrum above T_c

Spectral density of the QCD Dirac operator (schematic)



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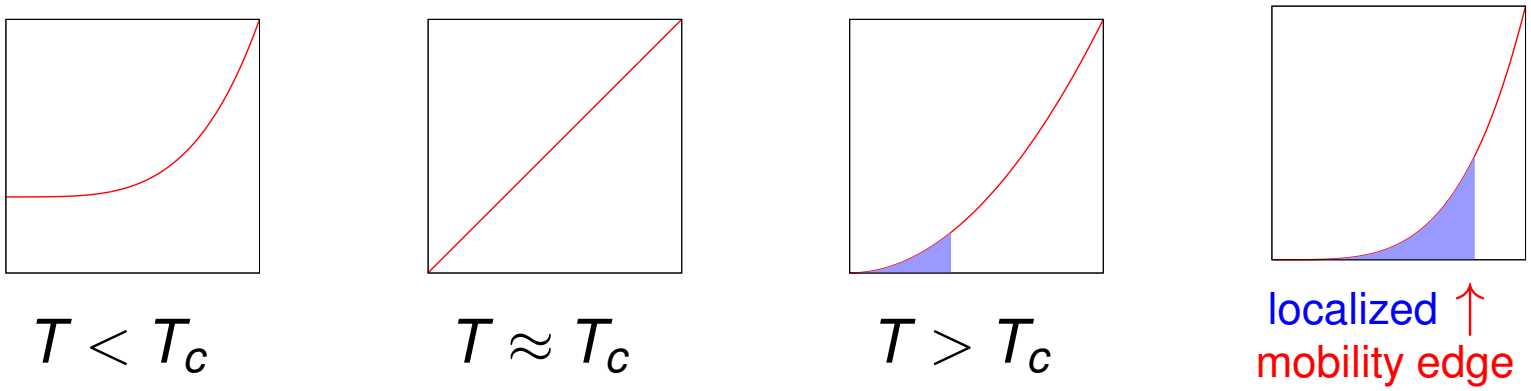


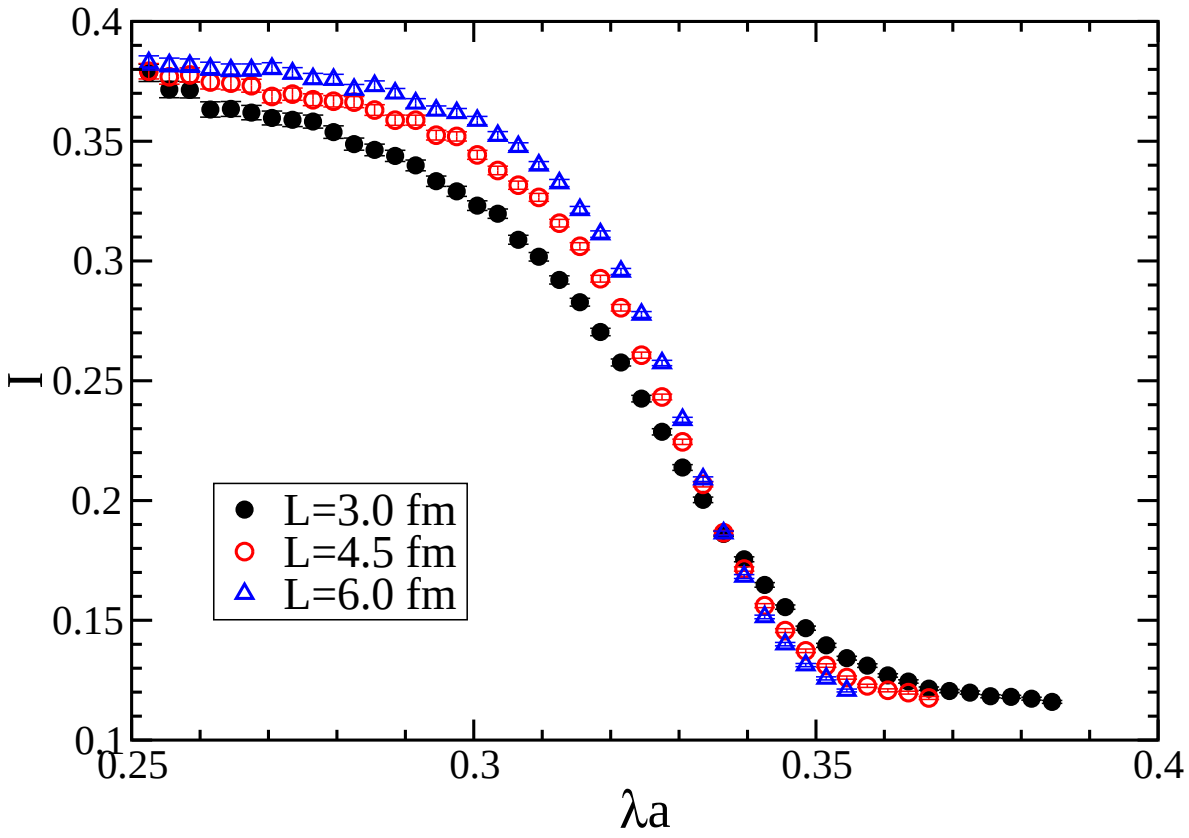
- Mobility edge controlled by the temperature
- Goes to zero around T_c

  $\rightarrow$ localized modes disappear
- Does this have anything to do with the chiral transition?

Transition at λ_c sharper for larger volume

$N_f = 2+1$ QCD with stout staggered quarks, physical point

Spectral statistics at fixed $T > T_c$, scan through the spectrum

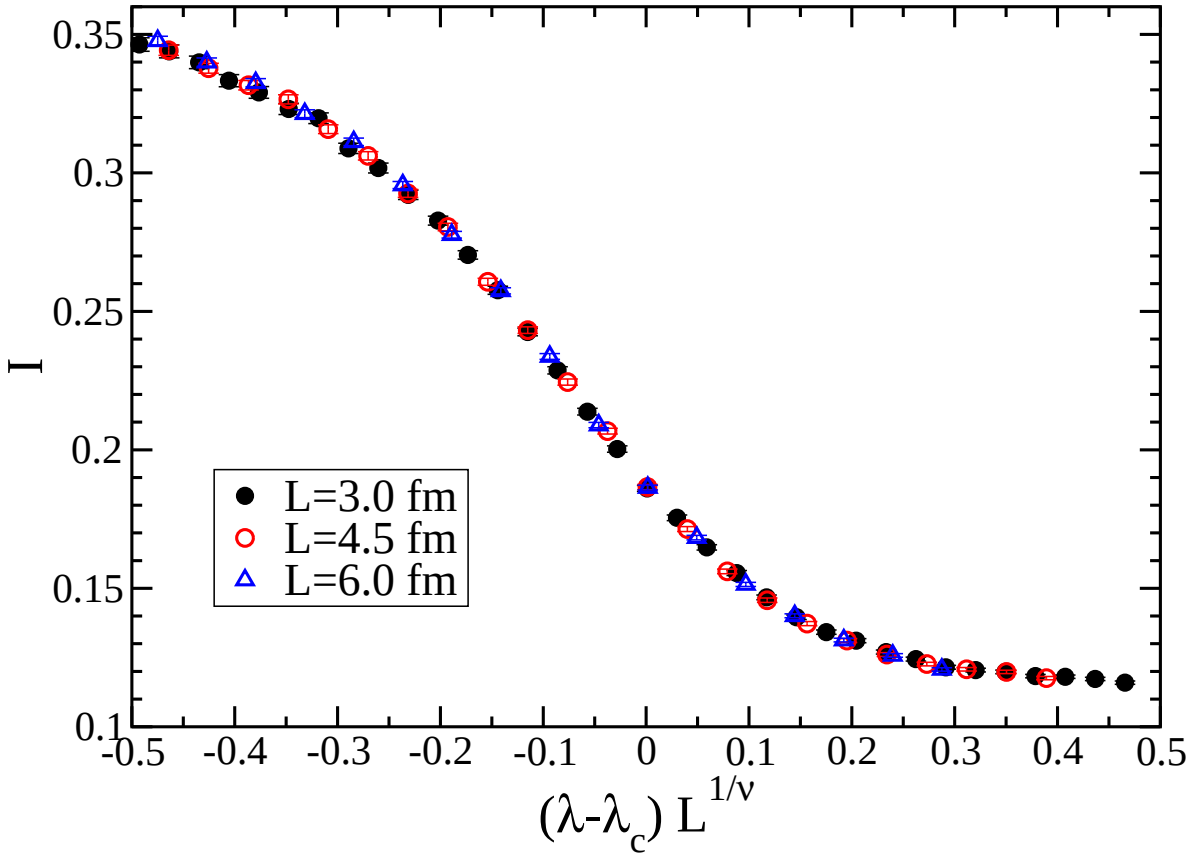


Finite size scaling works

$$I(\lambda, \mu, L) = f\left(L^{1/\nu}(\lambda - \lambda_c)\right)$$

$\nu = 1.43(5)$ compatible with 3d unitary Anderson model

M. Giordano,TGK and F. Pittler, PRL 112 (2014)



Anderson transition in the Dirac spectrum

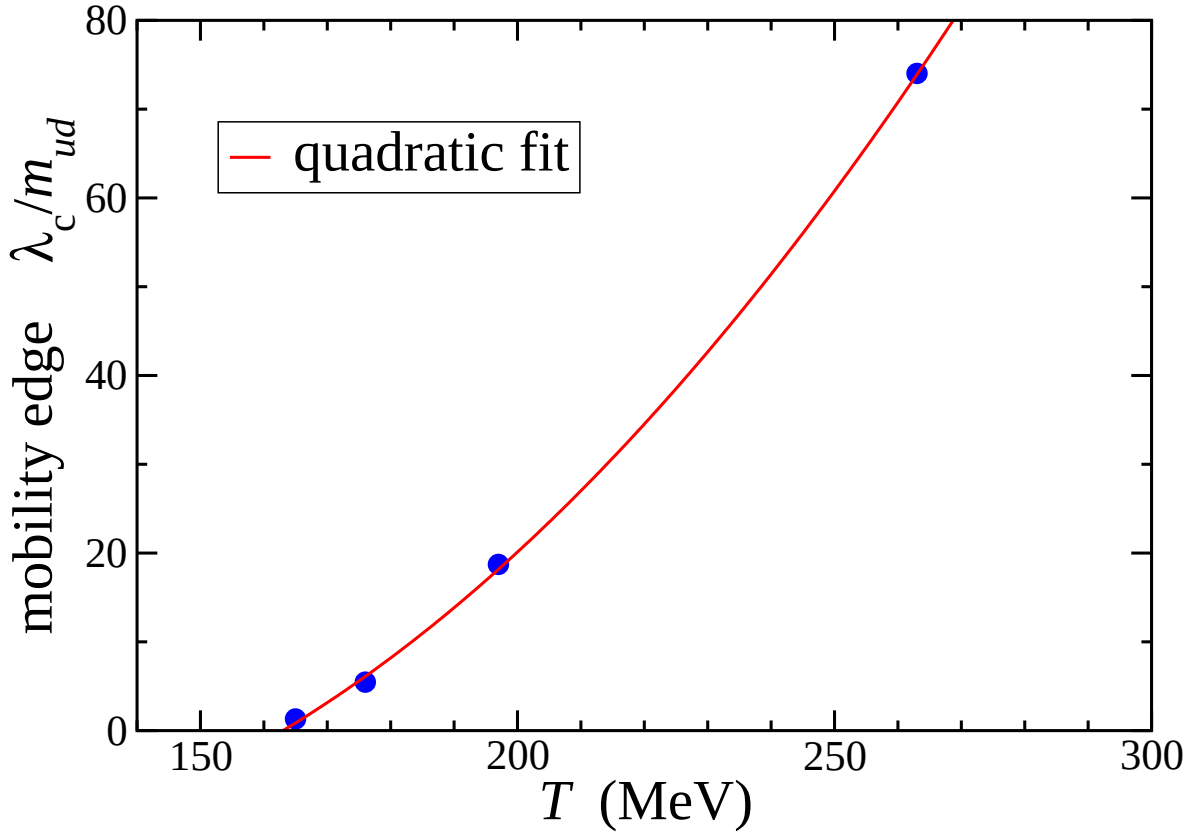
- Singularity in spectral statistics at λ_c
- 2nd order "phase transition"
- Eigenmode correlation length diverges
- Above T_c there is a critical point λ_c in the spectrum
  - Does not imply singularity in thermodynamic quantities like

$$\langle \bar{\psi}\psi \rangle \propto \int d\lambda \, \frac{m\rho(\lambda)}{m^2 + \lambda^2} \quad \text{or} \quad \langle \bar{\psi}\psi(0) \cdot \bar{\psi}\psi(x) \rangle$$

- How can this be connected to the chiral (phase) transition?

Localized modes disappear around T_c

$N_f = 2+1$ flavor stout staggered, physical quark masses



Fit: $\lambda_c \to 0$ at $T = 163(2)$ MeV$\approx T_c$

Why look at critical eigenmodes at λ_c?

- Strategy:
  - understand behavior at λ_c (fixed T)
  - see what happens when $T \searrow T_c$ and $\lambda_c \searrow 0$
- Use knowledge about Anderson transitions
- Possible role of critical eigenmodes when $\lambda_c \searrow 0$:

  $\langle |\psi(0)|^2 \, |\psi(x)|^2 \rangle$ correlator $\leftrightarrow$ disconnected chiral susceptibility

Critical eigenmodes have scale invariant structure

- Expectation from Anderson transition:
  - critical wave functions have multifractal structure
  - scale invariance $\rightarrow$ finite size scaling from eigenmodes
- What is a multifractal? $f : \mathbb{R}^d \rightarrow \mathbb{R}$ function
  - f smooth:
    "level sets" $S(a) = \{x : a < f(x) < a + \Delta a\}$ have dimension d
  - multifractal:
    sets $S(a)$ have fractal dimensions $D(a)$ depending on a
- Random multifractals:

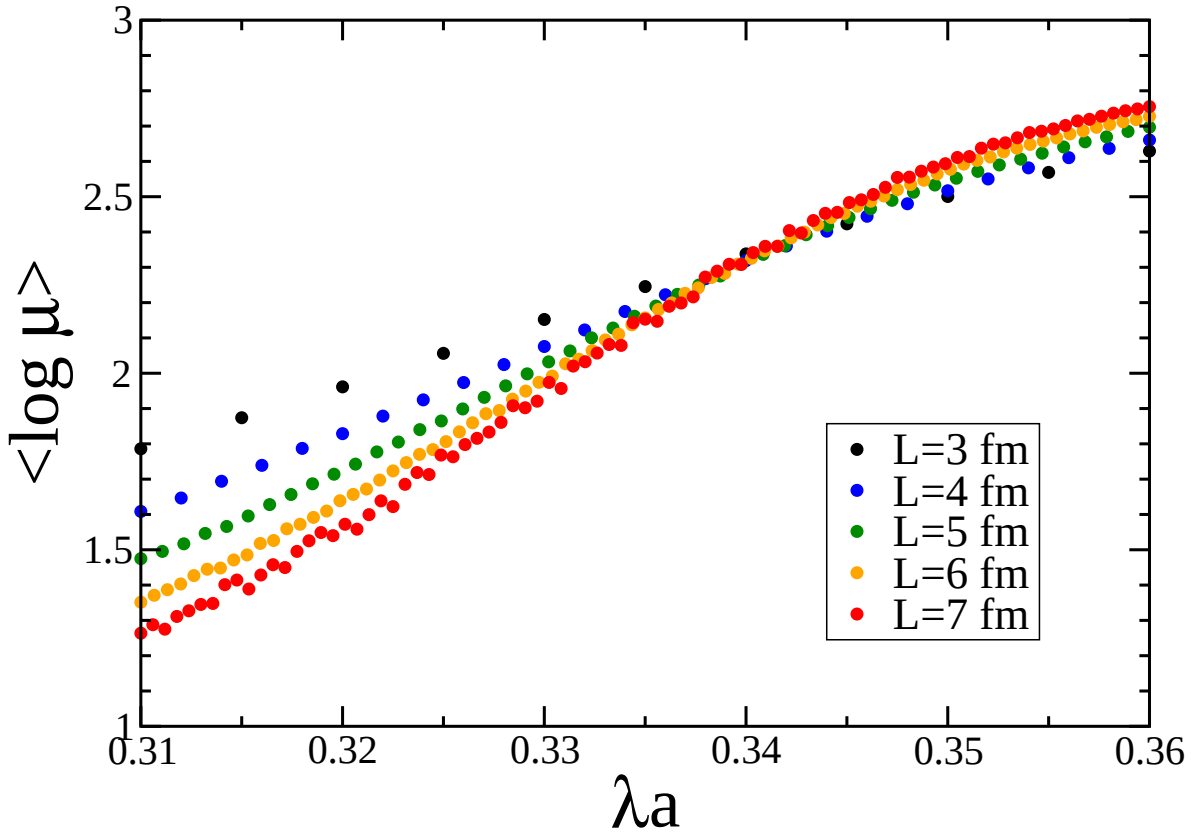
  $\langle f(0) f(x) \rangle$ correlators $\leftrightarrow$ multifractal dimensions $D(a)$

Multifractal analysis

- Subdivide system of size L into equal boxes of size $l \ll L$
- Compute box weights:
$$\mu_k(l) = \sum_{x \in \text{box } k} |\psi(x)|^2$$
- Scale invariance $\rightarrow$
$p(\mu)$, distribution of μ depends only on l/L
- Dependence of $p(\mu)$ on l/L $\rightarrow$ multifractal dimensions

Scale invariance at the mobility edge

Mobility edge: common crossing point
up to scaling violations $\rightarrow$ irrelevant exponents



Conclusions and outlook

- Above T_c there is a localization-delocalization transition in the QCD Dirac spectrum
- Real Anderson transition,
  same universality class as 3d Anderson model

  3+1d QCD $\rightarrow$ 3d Anderson...talk by M. Giordano
- At T_c Anderson critical point $\rightarrow 0$

  study in $N_f = 3$ staggered with real phase transition...talk by F. Pittler
- Final goal: understand connection between chiral and Anderson transition in QCD-like theories

  originally proposed by Garcia-Garcia and Osborn PR D75 (2007)