Hadronic interactions

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Lattice2014

The 32nd International Symposium on Lattice Field Theory
@ Columbia University, June 23-28, 2014

Hadronic interactions

One of ultimate goals of Lattice QCD

quantitatively understand properties of hadrons

Current status of lattice QCD

very close to reproduce mass for stable hadrons

Experiment

Many hadrons decay through hadronic interaction,

originating from strong interaction

Next task: hadronic interactions

Decay and scattering

Cannot be treated separately to understand properties of unstable hadrons

finial states of unstable particle = scattering states

More difficult to calculate, but important for the ultimate goal

Hadronic interactions

One of ultimate goals of Lattice QCD

quantitatively understand properties of hadrons

Famous hadronic interaction

nuclear force : bind nucleons into nucleus

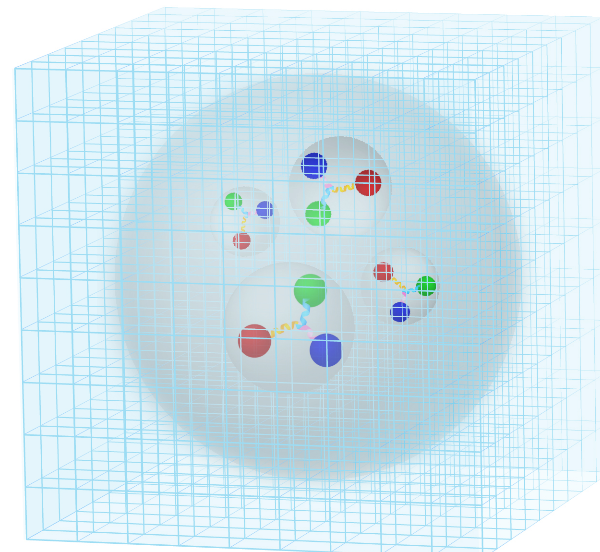
originate from strong interaction

$\leftarrow$ well known in experiment

Another ultimate goal of lattice QCD

quantitatively understand formation of nuclei

from first principle of strong interaction



http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/

Review recent results related to scatterings, decays, and light nuclei

I will not cover

- Scattering lengths
  $I = 0$ $\pi\pi$, $I = 3/2$ $K\pi$, $I = 1$ KK, $I = 1$ $K\overline{K}$, $\cdots$
- Scattering phase shifts
  $I = 2$ $\pi\pi$, $I = 3/2$ $K\pi$, $I = 1/2$ $K\pi$, $\cdots$
- Charmed scattering and bound states [Plenary:Prelovsek Fri 9:00]
- Decays
  $\pi\pi \to \sigma$
  [Talk:Howarth Tue 4B 17:30], [Talk:Wakayama Wed 6B 11:10]
  $\eta\pi \to a_0$
  [Talk:Berlin Wed 6B 11:30], [Talk:Abedel-rehim Wed 6B 11:50]
  $N\pi \to N^*$ [Poster:Verduci Tue]
  $K\pi \to \kappa$, $\rho\pi \to a_1$, $\omega\pi \to b_1$, $\cdots$
- Amplitude (transfer matrix) method for resonances
  [Talk:Petschlies Thu 7B 14:35]
- Multi-hadron correlation function [Poster:Vachaspati Tue]
- Theoretical development [Plenary:Briceño Tue 11:15]

Contents

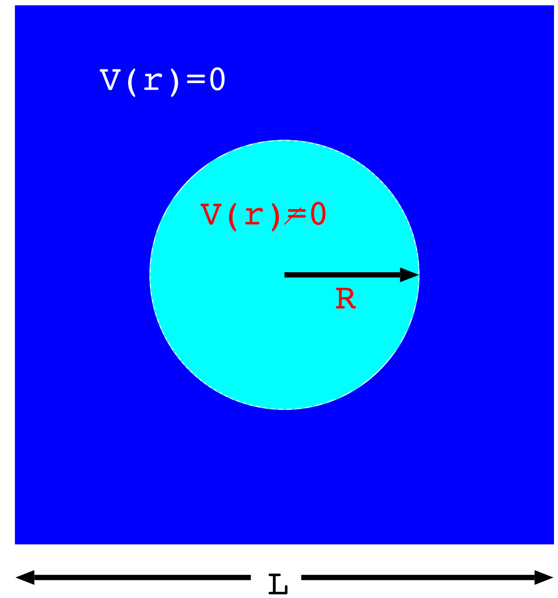
- Introduction
  - Lüscher's finite volume method
- Scattering lengths
  - $I = 2$ $\pi\pi$, $I = 1/2$ $K\pi$
- Scattering phase shifts (resonances)
  - $\pi\pi \to \rho$, $K\pi \to K^*$
- Comparison with HALQCD method
  - $I = 2$ $\pi\pi$, H dibaryon, two-nucleon channels
- Light nuclei
- Summary

Lüscher's finite volume method

Lüscher, CMP105:153(1986),NPB354;531(1991)

spinless two-particle elastic scattering in center of mass (CM) frame

Important assumption



1. Two-particle interaction is localized.

   $\rightarrow$ Interaction range R exists.

$$V(r) \begin{cases} \neq 0 & (r \leq R) \\ = 0\ (\sim e^{-cr}) & (r > R) \end{cases}$$

2. $V(r)$ is not affected by boundary. $\rightarrow R < L/2$

Two-particle wave function $\phi_p(\vec{r})$ satisfies Helmholtz equation

$$\left(\nabla^2 + p^2\right)\phi_p(\vec{r}) = 0 \text{ in } r > R\ (R < L/2)$$

$\leftarrow$ Klein-Gordon eq. of free two particles

$$E = 2\sqrt{m^2 + p^2},\ p^2 \neq \left(\frac{2\pi}{L}\cdot\vec{n}\right)^2 \text{ in general}$$

Lüscher's finite volume method (cont'd)

Lüscher, CMP105:153(1986),NPB354;531(1991)

Helmholtz equation on L^3

1. Solution of $(\nabla^2 + p^2)\phi_p(\vec{r}) = 0$ in $r > R$

$$\phi_p(\vec{r}) = C \cdot \sum_{\vec{n}\in Z^3} \frac{e^{i\vec{r}\cdot\vec{n}(2\pi/L)}}{\vec{n}^2 - q^2}, \quad q^2 = \left(\frac{Lp}{2\pi}\right)^2 \neq \text{integer}$$

2. Expansion by spherical Bessel $j_l(pr)$ and Noeman $n_l(pr)$ functions

$$\phi_p(\vec{r}) = \beta_0(p) n_0(pr) + \alpha_0(p) j_0(pr) + (l \geq 4)$$

3. S-wave Scattering phase shift $\delta_0(p)$ <u>in infinite volume</u>

$$\frac{\beta_0(p)}{\alpha_0(p)} = \boxed{\tan\delta_0(p) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)}}$$

$$Z_{00}(s;q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in Z^3} \frac{1}{\left(\vec{n}^2 - q^2\right)^s}, \quad q = \frac{2\pi}{L} p$$

Relation between $\boxed{\delta(p) \text{ and } p}$ $\left(E = 2\sqrt{m^2 + p^2}\right)$

Wave function: CP-PACS, PRD70:094504(2005), Sasaki and Ishizuka, PRD78:014511(2008)

Potential: Ishii, Aoki, and Hatsuda, PRL99:022001(2007), $\cdots$

Calculation of phase shift in function of p

Lüscher's method $\delta(p)$ from $E = 2\sqrt{m^2 + p^2}$ in CM frame

Extended to Moving frames $|P| \neq 0$ with $m_1 = m_2$

$\delta(p_{\rm cm})$ from $E_P^2 = 4\left(m^2 + p_{\rm cm}^2\right) + P^2$ Rummukainen and Gottlieb, NPB450:397(1995)

$\pi(P)\pi(0)$ on lattice — Kim, Sachrajda, and Sharpe, NPB727:218(2005)

$\rightarrow \pi(p_{\rm cm})\pi(-p_{\rm cm})$ in CM frame — Christ, Kim, and TY, PRD72:114506(2005)

Feng, Jansen, and Renner, PoS(Lattice 2010):104(2010)

Dudek, Edwards, Thomas, PRD86:034031(2012)

Extension of moving frames $|P| \neq 0$ to $m_1 \neq m_2$

$\delta(p_{\rm cm})$ from $E_P^2 = \left(\sqrt{m_1^2 + p_{\rm cm}^2} + \sqrt{m_2^2 + p_{\rm cm}^2}\right)^2 + P^2$

mixing of even and odd l — Fu, PRD85:014506(2012)

Leskovec and Prelovsek, PRD85:114507(2012), Döring *et al.*, EPJA48:114(2012)

Göckeler *et al.*, PRD86:094513(2012) Li and Liu, PRD87:014502(2013)

Scattering length a_0^I

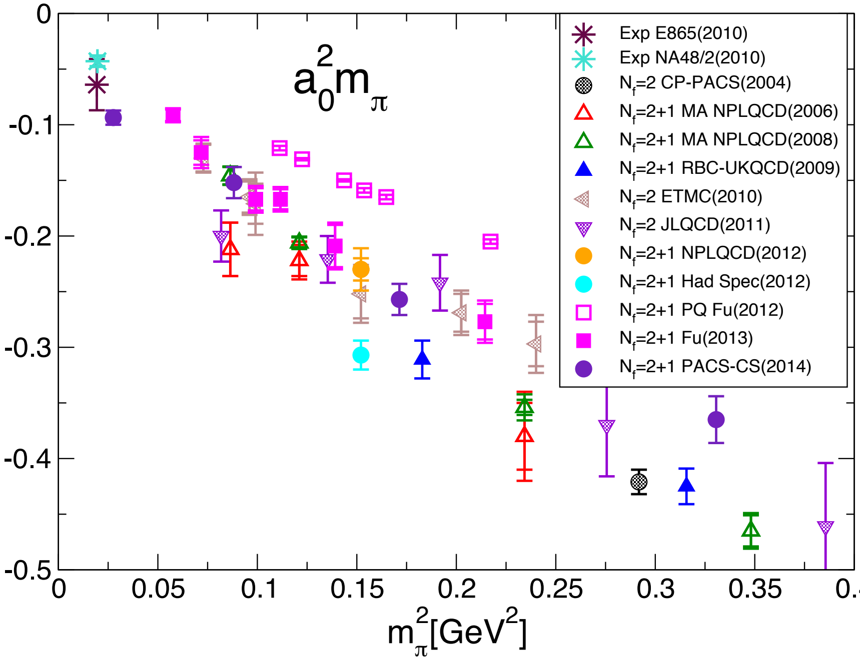
$$a_0 = \lim_{p \to 0} \frac{\tan \delta(p)}{p}$$

$I = 2$ $\pi\pi$ a_0^2 and $I = 1/2$ $K\pi$ $a_0^{1/2}$

Scattering length I

$I = 2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations



Fu PQ(2012)

ASQTAD: same to Fu(2013)

$m_{\rm sea} = m_{\rm val}$ at PQ lightest data

not only PQ effect

$\rightarrow$ other systematic error

$\bigcirc$ Wilson type; $\square$ ASQTAD; $\triangle$ DWF; $\triangleleft$ Twisted; $\triangledown$ overlap

MA:DWF on ASQTAD; PQ:partial quenched

Scattering length I

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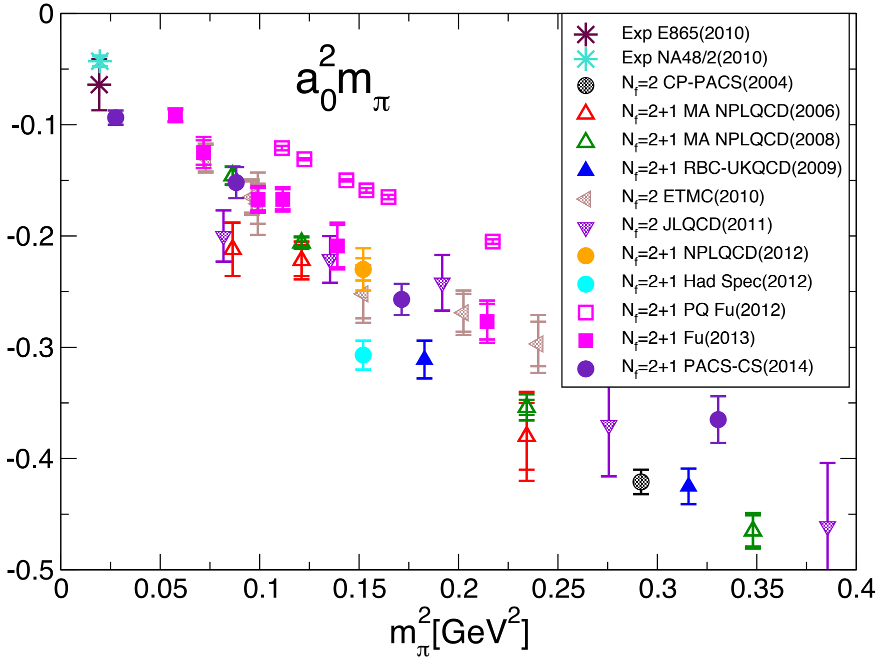
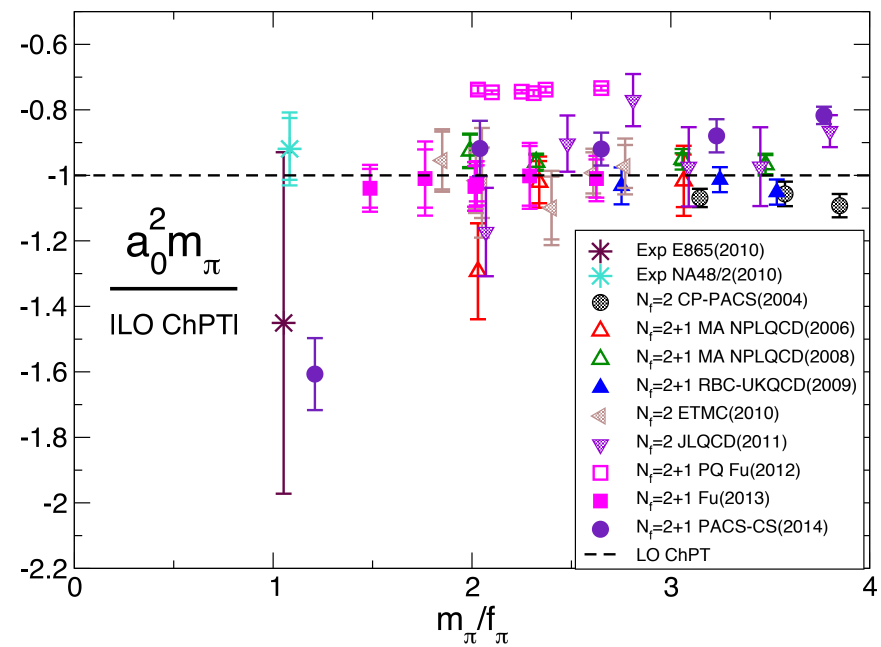




○ Wilson type; □ ASQTAD; △ DWF; ◁ Twisted; ▽ overlap

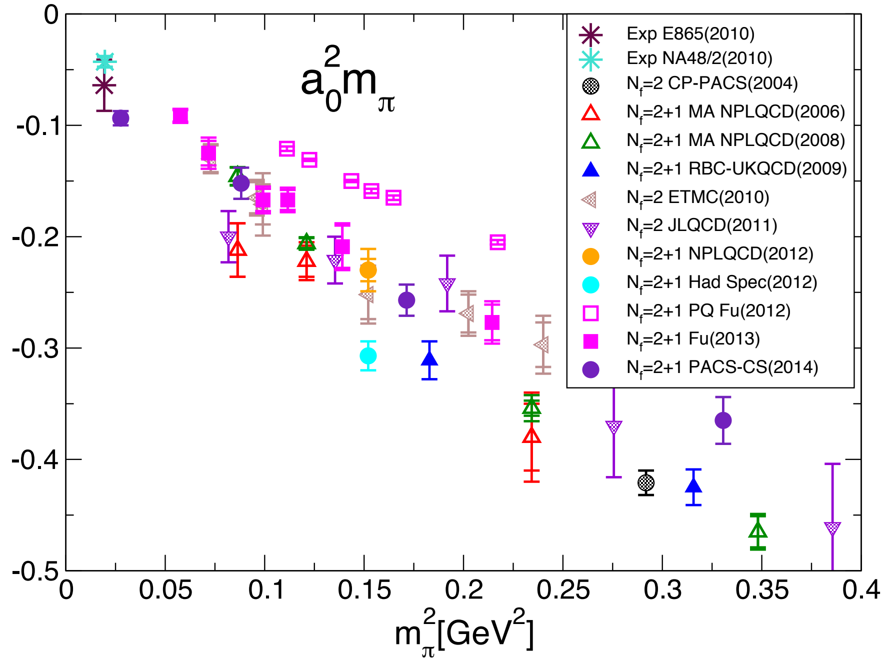
MA:DWF on ASQTAD; PQ:partial quenched

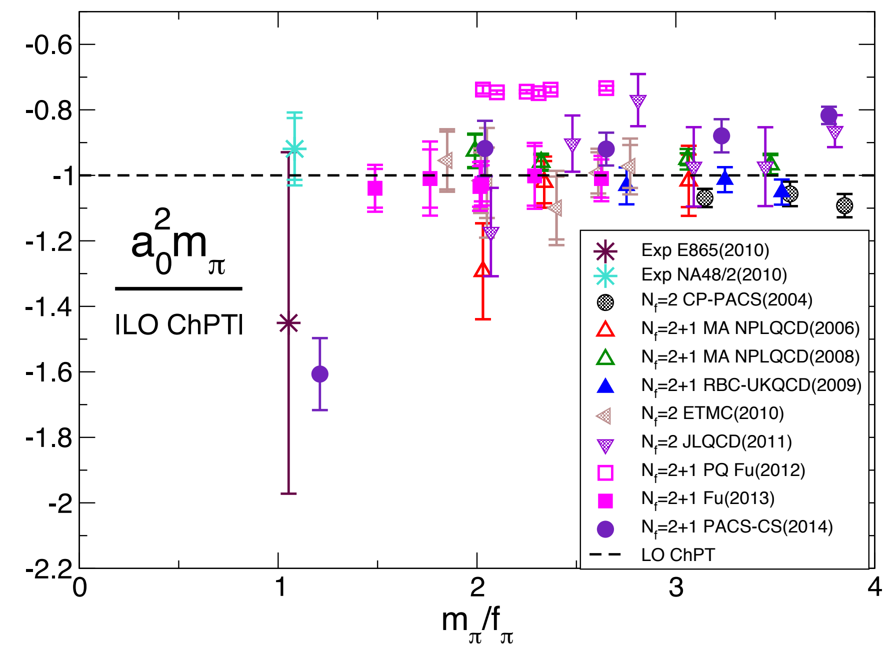
$$\text{NLO ChPT: } a_0^2 m_\pi = \frac{m_\pi^2}{8\pi f_\pi^2}\left[-1 + \frac{32}{f_\pi^2}\left[m_\pi^2 L_{\pi\pi} + \text{analytic} + \text{log}\right]\right]$$

Scattering length I

$I=2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations





○ Wilson type; □ ASQTAD; △ DWF; ◁ Twisted; ▽ overlap

MA:DWF on ASQTAD; PQ:partial quenched

vending of PACS-CS(2014) due to chiral symmetry breaking effect

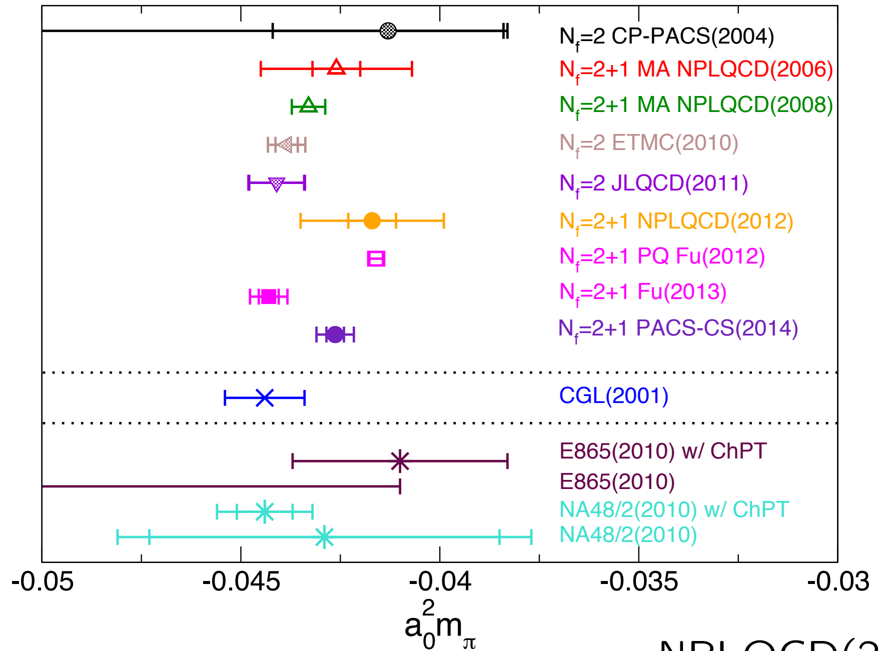
$N_f=3$ NLO WChPT: $a_0^2 m_\pi = \dfrac{m_\pi^2}{8\pi f_\pi^2}\left[-1+\dfrac{32}{f_\pi^2}\left[m_\pi^2(L'-\dfrac{L_5}{2})+\text{analytic}+\log\right]\right]-\dfrac{c_2 a^2}{8\pi f_\pi^2}$

Simultaneous fit with a_0^I for $I=2$ $\pi\pi$, $I=1$ KK, $I=3/2,1/2$ $K\pi$

Scattering length I

$I = 2\ \pi\pi$ Simplest scattering system

Comparison of dynamical calculations at physical m_π



NPLQCD(2012): m_π/f_π from MA calc.

$\bigcirc$ Wilson type; $\square$ ASQTAD; $\triangle$ DWF; $\triangleleft$ Twisted; $\triangledown$ overlap

MA:DWF on ASQTAD; PQ:partial quenched

Sources of systematic error: finite volume effects, Δ_{MA}, Δ_{Wilson}, $\cdots$

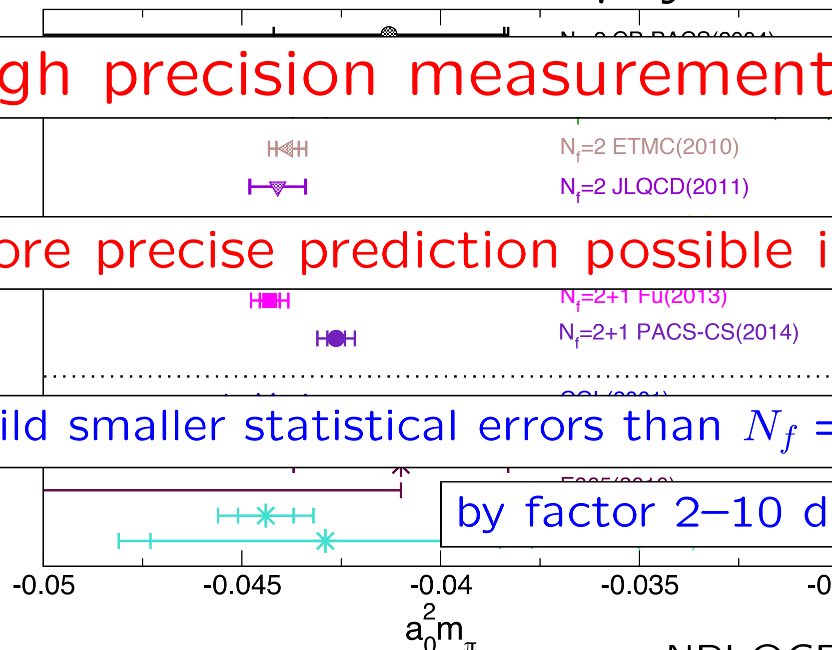
might able to take weighted average for lattice prediction

Scattering length I

$I = 2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations at physical m_π

high precision measurements era



more precise prediction possible in near future

Talk:Knippschild smaller statistical errors than $N_f = 2$ case

by factor 2–10 depending on m_π

NPLQCD(2012): m_π/f_π from MA calc.

○ Wilson type; □ ASQTAD; △ DWF; ◁ Twisted; ▽ overlap

MA:DWF on ASQTAD; PQ:partial quenched

$N_f = 2 + 1$ Twisted $m_\pi = 0.32$–0.40[GeV][Talk:Knippschild Mon 1B 14:35]

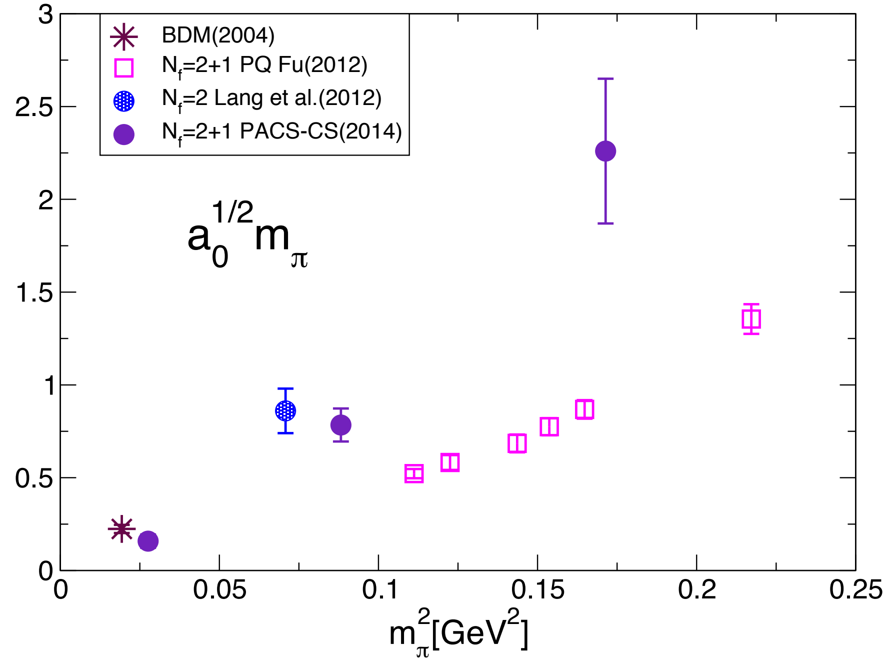
Sources of systematic error: finite volume effects, Δ_{MA}, Δ_{Wilson}, $\cdots$

might able to take weighted average for lattice prediction

Scattering length II

$I = 1/2$ $K\pi$ rectangle diagram

c.f. stochastic LapH method:
Morningstar *et al.*, PRD83:114505(2011)



Variational analysis $\leftarrow$ κ expected

non negligible κ effect

in large m_π in PACS-CS

Fu: possibly systematic errors

PACS-CS: large Δ_{Wilson}

◯ Wilson type; □ ASQTAD PQ:partial quenched

Fu, PRD85:074501(2012), Lang *et al.*, PRD86:054508(2012), PACS-CS, PRD89:054502(2014)
other works: NPLQCD, PRD74:114503(2006)(indirect), Nagata *et al.*, PRC80:045203(2009)

$$\text{NLO ChPT: } a_0^{1/2}\mu_{\pi K} = \frac{\mu_{\pi K}^2}{4\pi f_\pi^2}\left[2 + \frac{32}{f_\pi^2}\left[m_\pi m_K L' + \frac{m_\pi^2 + m_K^2}{2} L_5 + \text{analytic} + \text{log}\right]\right]$$

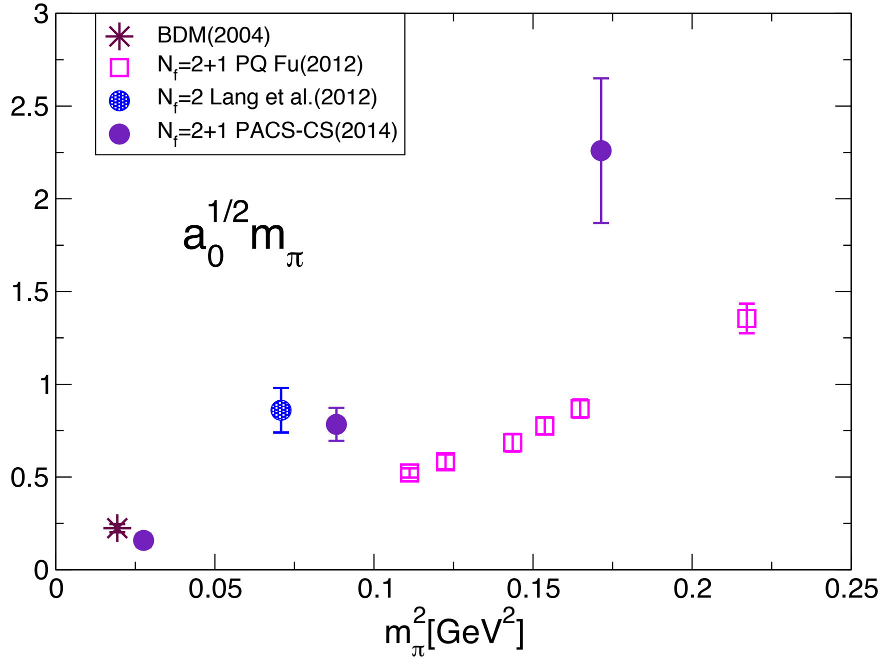
$$\mu_{\pi K} = m_\pi m_K/(m_\pi + m_K)$$

Scattering length II

$I = 1/2\ K\pi$ rectangle diagram

c.f. stochastic LapH method:

Morningstar *et al.*, PRD83:114505(2011)



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NLO WChPT: $a_0^{1/2}\mu_{\pi K} = \dfrac{\mu_{\pi K}^2}{4\pi f_\pi f_K}\left[2 + \dfrac{32}{f_\pi f_K}\left[m_\pi m_K L' + \dfrac{m_\pi^2 + m_K^2}{2} L_5 + \text{analytic} + \text{log}\right]\right]$

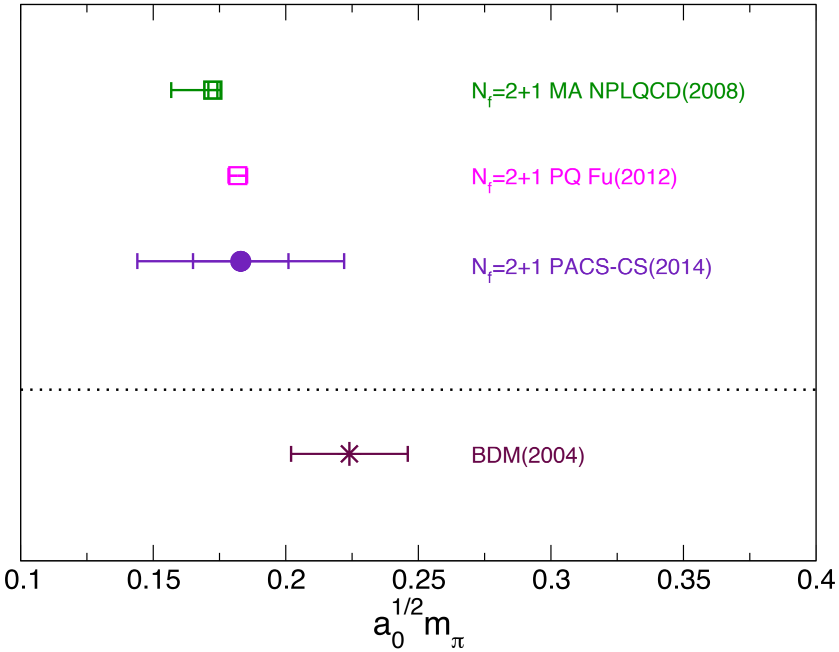
PACS-CS due to chiral symmetry breaking effect $\quad - \dfrac{c_2 a^2}{4\pi f_\pi f_K}\dfrac{\mu_{\pi K}^2}{m_\pi m_K}$

Simultaneous fit with a_0^I for $I = 2\ \pi\pi$, $I = 1\ KK$, $I = 3/2, 1/2\ K\pi$

Scattering length II

$I = 1/2\ K\pi$

Comparison at physical m_π



○ Wilson type; □ ASQTAD; △ DWF; MA:DWF on ASQTAD; PQ:partial quenched

NPLQCD, PRD74:114503(2006)(indirect; LEC from $a_0^{3/2}$ of $K\pi$), Fu, PRD85:074501(2012), PACS-CS, PRD89:054502(2014)

other works: Nagata *et al.*, PRC80:045203(2009)

more accurate direct calculation is desired

$N_f = 2+1$ DWF $m_\pi^{\rm phys}$, $m_K^{\rm phys}$ on $L = 5.5$fm[Talk:Janowski Mon 1B 15:15]

Preliminary result $a_0^{1/2} m_\pi = 0.174(60)$

Scattering phase shift $\delta(p)$

$I = 1$ $\pi\pi \to \rho$, $I = 1/2$ $K\pi \to K^*$

Phase shift I

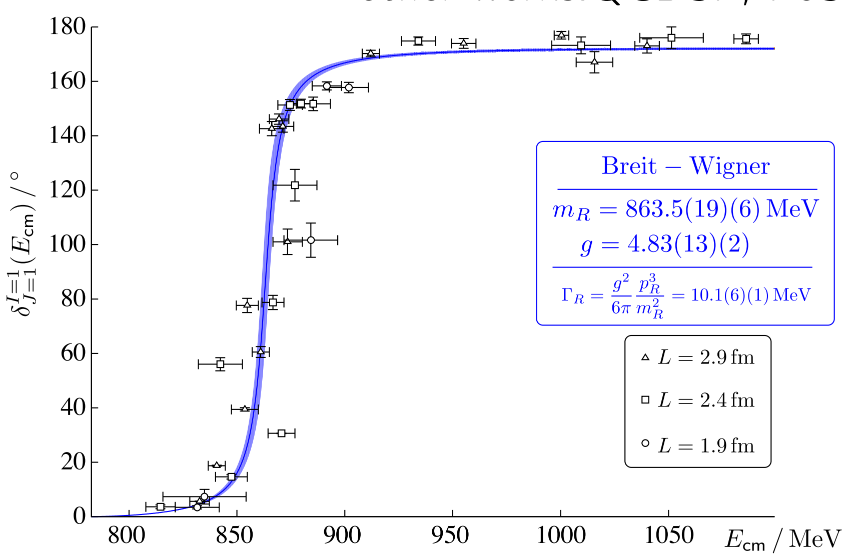
$I = 1$ P-wave $\pi\pi \to \rho$

	1.	2.	3.	4.	5.	6.
$L\|P\|/2\pi$	1	0,1,$\sqrt{2}$	0,1,$\sqrt{2}$	0,1^2,$\sqrt{2}$	0*	0,1^2,$\sqrt{2}^3$,$\sqrt{3}^2$,2
N_{mom}	2	5–6	5	6	6	29
m_π[MeV]	320	290-480	270	410, 300	300	390
$m_\pi L$	4.2	$\geq$ 3.7	2.7	6.0, 4.4	$\geq$ 4.6	$\geq$ 3.8

* asymmetric lattice $L^2 \times \eta L$, $\eta = 1, 1.25, 2$

1. CP-PACS, PRD76:094506(2007), 2. ETMC, PRD83:094505(2011),
3. Lang *et al.*, PRD84:054503(2011), 4. PACS-CS, PRD84:094505(2011),
5. Pelissier *et al.*, PRD87:014503(2013), 6. Hadron Spectrum, PRD87:034505(2013)

other works:QCDSF, PoS(LATTICE 2008)136, BMW, PoS(Lattice 2010)139



Hadron Spectrum, PRD87:034505(2013)

Breit-Wigner form fit

$$\frac{p^3}{\sqrt{s}} \cot \delta(p) = \frac{6\pi}{g^2_{\rho\pi\pi}}(m_\rho^2 - s)$$

$$s = E^2_{\text{cm}}$$

$$\Gamma_\rho = \frac{p_\rho^3}{m_\rho^2}\frac{g^2_{\rho\pi\pi}}{6\pi}, \quad p_\rho^2 = \frac{m_\rho^2}{4} - m_\pi^2$$

Phase shift I
$I = 1$ P-wave $\pi\pi \to \rho$

	1.	2.	3.	4.	5.	6.
$L\|P\|/2\pi$	1	0,1,$\sqrt{2}$	0,1,$\sqrt{2}$	0,1^2,$\sqrt{2}$	0^*	0,1^2,$\sqrt{2}^3$, $\sqrt{3}^2$,2
$N_{\rm mom}$	2	5–6	5	6	6	29
m_π[MeV]	320	290-480	270	410, 300	300	390
$m_\pi L$	4.2	≥ 3.7	2.7	6.0, 4.4	≥ 4.6	≥ 3.8

* asymmetric lattice $L^2 \times \eta L$, $\eta = 1, 1.25, 2$

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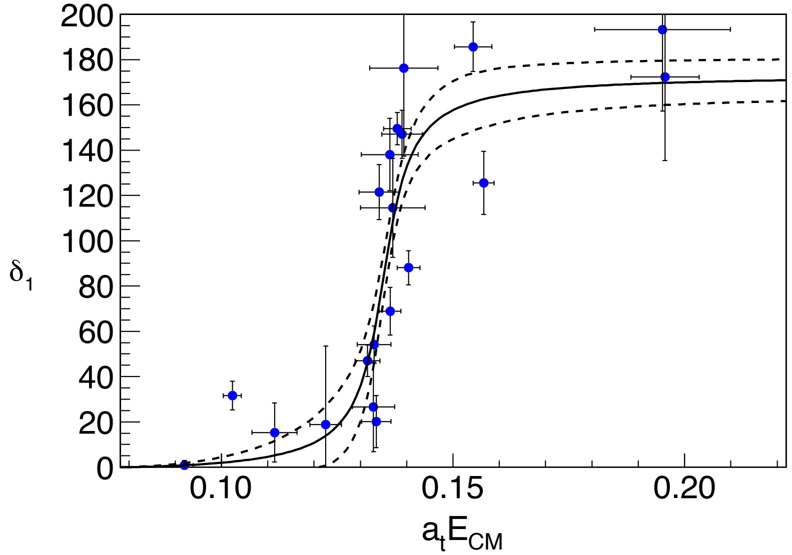


[Talk:Fahy Mon 1B 14:15]

Preliminary results

1. [Talk:Fahy Mon 1B 14:15]

$m_\pi \sim 0.24$ GeV at $m_\pi L = 4.4$

$g_{\rho\pi\pi} = 4.3(1.6)$

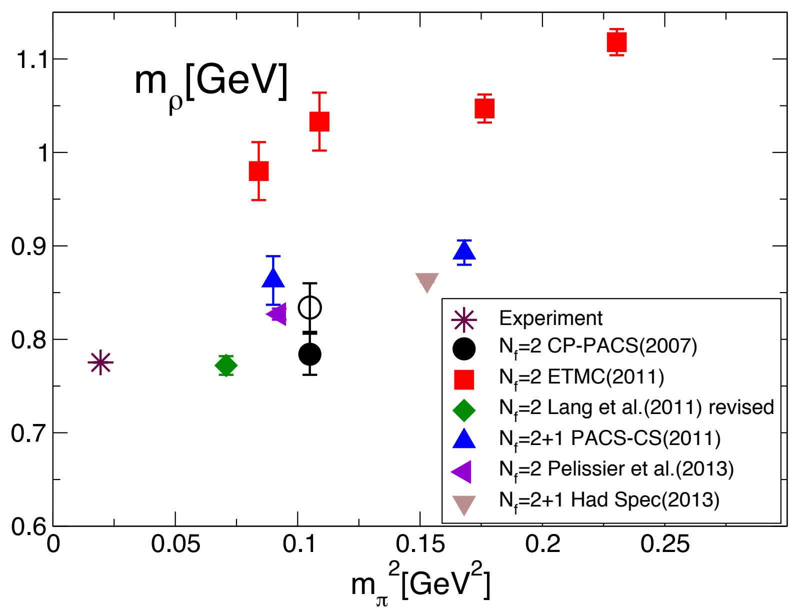
$a_t m_\rho = 0.1355(19) \to m_\rho \sim 0.79$ GeV

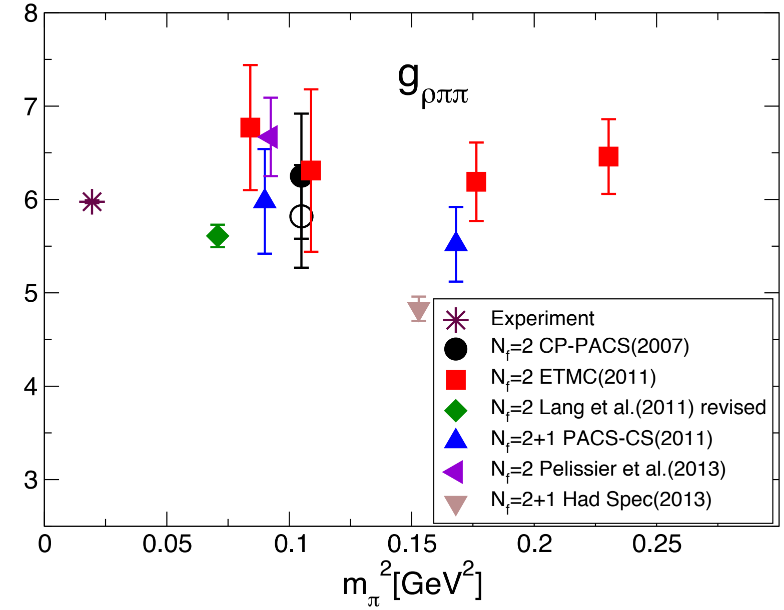
2. [Talk:Metivet Mon 1B 14:55]

5 data: $m_\pi \sim 0.135$–0.3 GeV at $m_\pi L \gtrsim 4$

Phase shift I

$I = 1$ P-wave $\pi\pi \to \rho$





open symbol: lattice dispersion relation

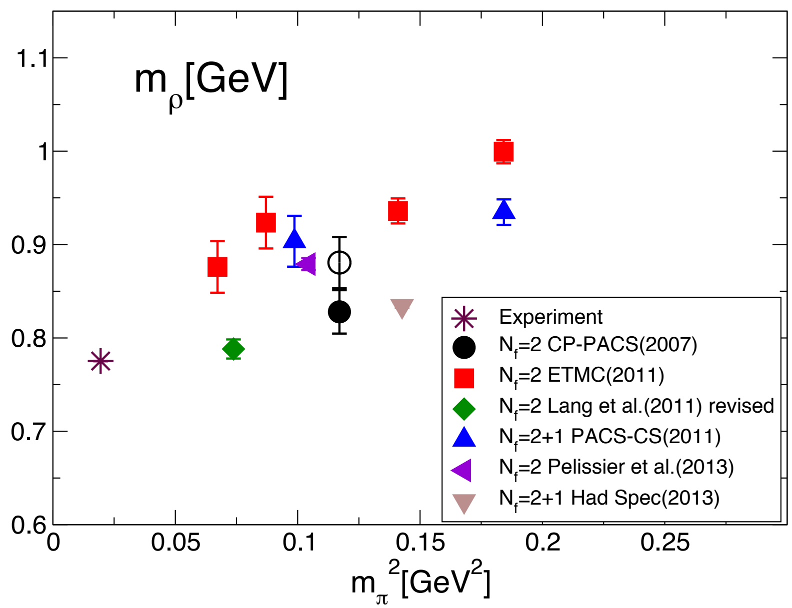
CP-PACS, PRD76:094506(2007), ETMC, PRD83:094505(2011), Lang *et al.*, PRD84:054503(2011), PACS-CS, PRD84:094505(2011), Pelissier *et al.*, PRD87:014503(2013), Hadron Spectrum, PRD87:034505(2013)

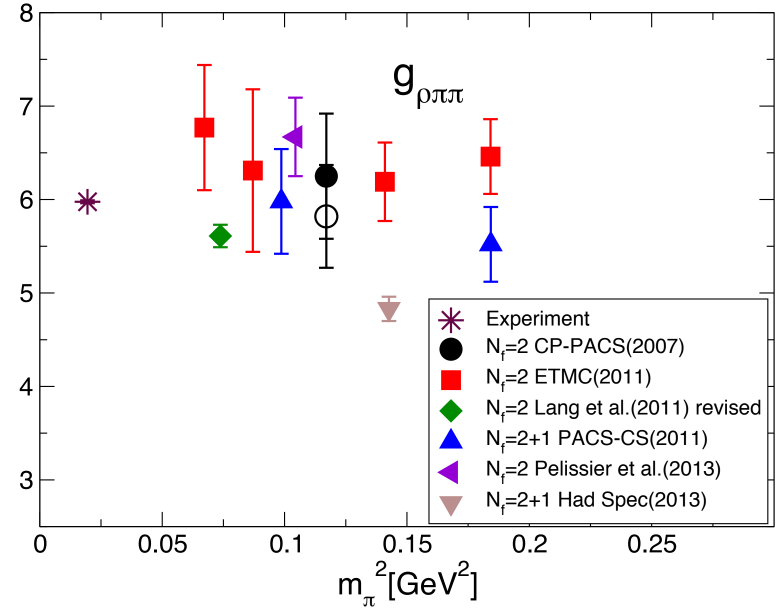
others [Talk:Fahy Mon 1B 14:15] [Talk:Metivet Mon 1B 14:55]

m_ρ scattered, but depends on scale setting quantities

Phase shift I

$I = 1$ P-wave $\pi\pi \to \rho$





open symbol: lattice dispersion relation

scale fixed by $r_0 = 0.47$ fm

CP-PACS, PRD76:094506(2007), ETMC, PRD83:094505(2011), Lang *et al.*, PRD84:054503(2011), PACS-CS, PRD84:094505(2011), Pelissier *et al.*, PRD87:014503(2013), Hadron Spectrum, PRD87:034505(2013)

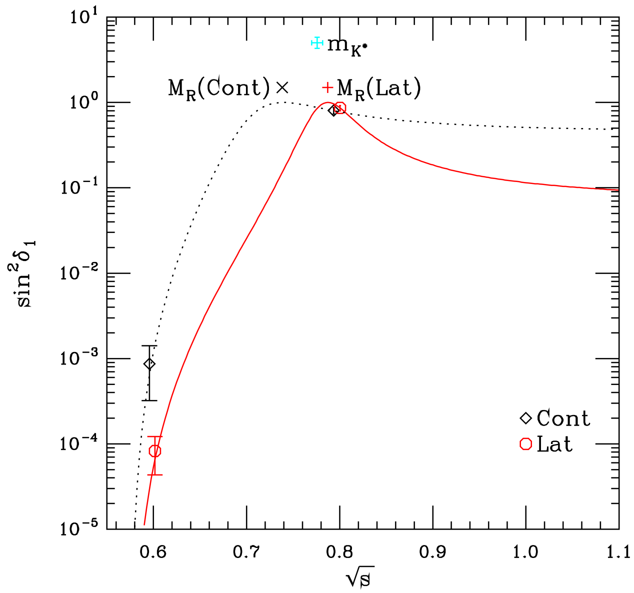
others [Talk:Fahy Mon 1B 14:15] [Talk:Metivet Mon 1B 14:55]

Lang *et al.*: small $m_\rho \to$ finite volume effect

Hadron Spectrum: small $m_\rho \to$ uncertainty of scale determination

small $g_{\rho\pi\pi} \to$ need to check systematic error

Phase shift II
$I = 1/2$ P-wave $K\pi \to K^*$



Breit-Wigner form fit of $\sin^2\delta(p)$

$N_f = 2+1$ ASQTAD

$L|P|/2\pi = 1$ (irrep A_1)

assume $\delta_0 = 0$

$L = 3$fm @ $m_\pi = 0.24$GeV

Fu and Fu, PRD86:094507(2012)

mixing of even and odd l in Moving frames — Fu, PRD85:014506(2012)

Leskovec and Prelovsek, PRD85:114507(2012), Döring *et al.*, EPJA48:114(2012)

Göckeler *et al.*, PRD86:094513(2012) Li and Liu, PRD87:014502(2013)

$L|P|/2\pi = 1$ (irrep A_1, $\delta_{l\geq 2} = 0$): $E(p) \to \delta_{l=0}(p)$ and $\delta_{l=1}(p)$

$\delta_0(p)$ is not negligible in experiment

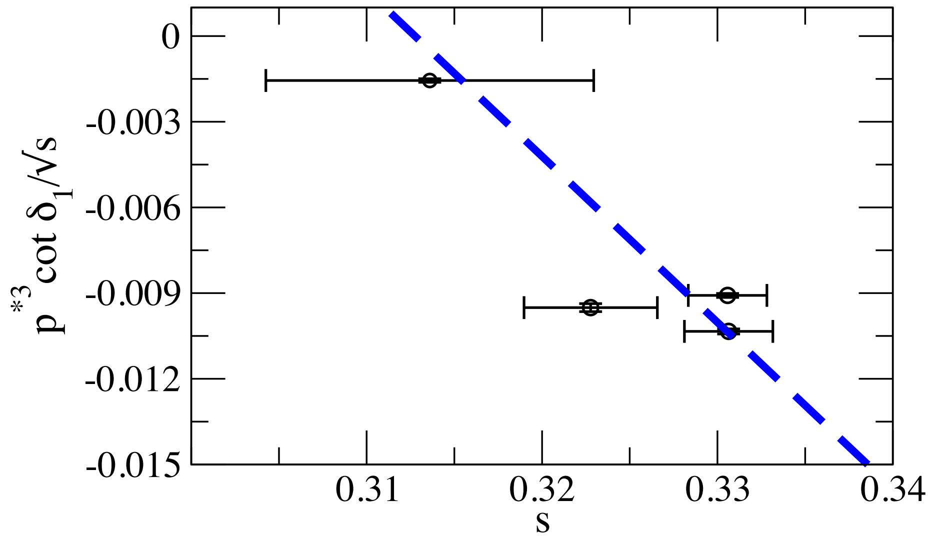
Systematic errors should be estimated.

- $\delta_0 \neq 0$ neglected
- other taste scatterings, $\pi_{SC}K_{SC}, \cdots,$ than $\pi_5 K_5$

Phase shift II

$I = 1/2$ P-wave $K\pi \to K^*$

Breit-Wigner form fit of $\dfrac{p^3}{\sqrt{s}} \cot\delta(p)$



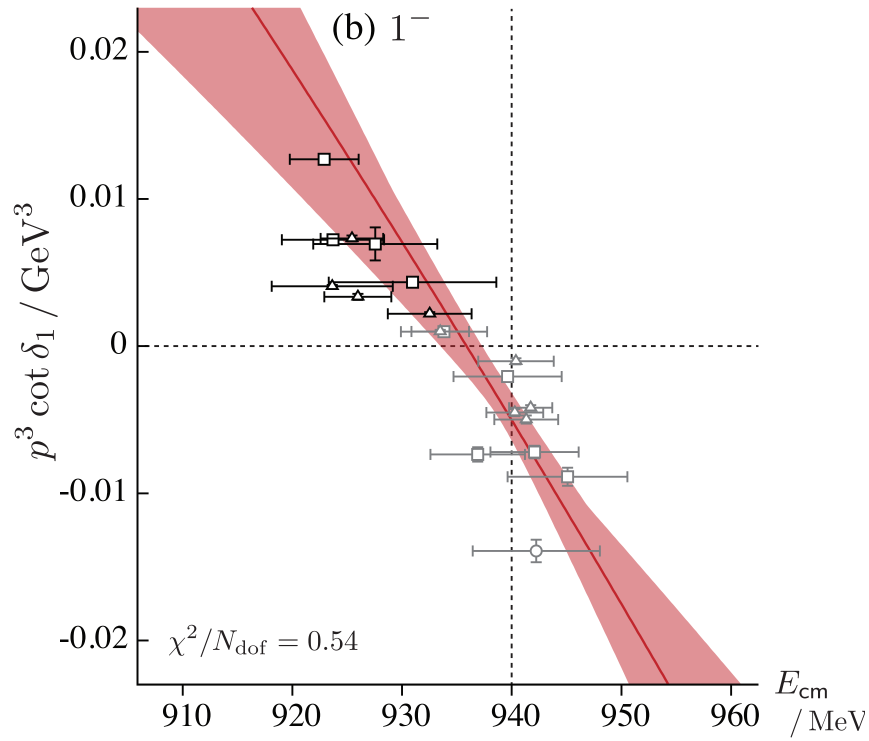
$N_f = 2$ clover: $L|P|/2\pi = 0, 1, \sqrt{2}$, choose irreps where $l \geq 1$

- $L = 1.9$ fm @ $m_\pi = 0.27$ GeV Prelovsek *et al.*, PRD88:054508(2013)
- 4 data in resonance region
- $g_{K^*\pi K} = 5.7(1.6) \leftrightarrow g^{\text{exp}}_{K^*\pi K} = 5.65(5)$, $m_{K^*} = 0.891(14)$GeV
- no data in $m_{K^*} < E_\pi(p_{\text{cm}}) + E_K(p_{\text{cm}})$

Phase shift II

$I = 1/2$ P-wave $K\pi \to K^*$

Breit-Wigner form fit



$N_f = 2+1$ aniso. clover: $L|P|/2\pi = 0, 1, \sqrt{2}, \sqrt{3}, 2$

take care of mixing of $l = 0, 1, 2$

– 19 data near threshold $m_\pi + m_K$

[Talk:Wilson Mon 1B 15:35]

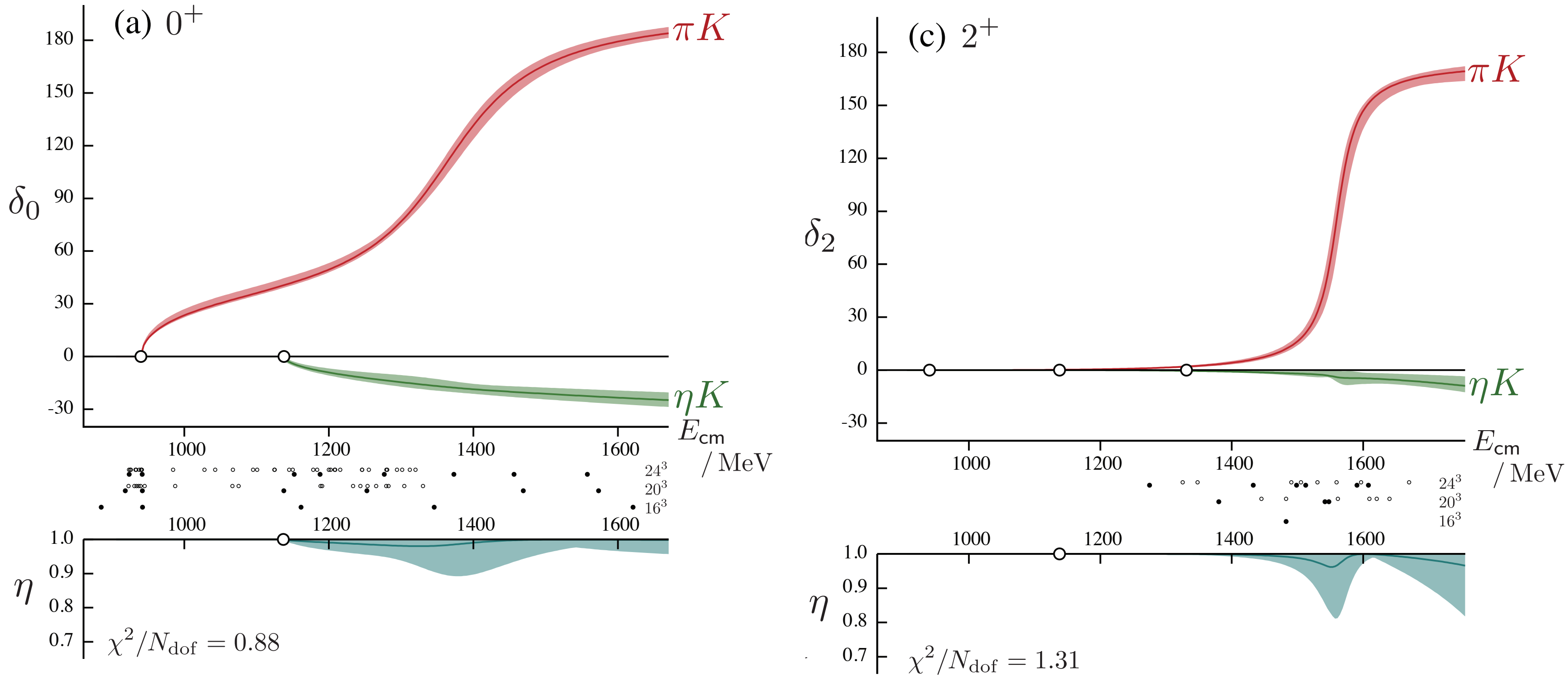
– $m_{K^*} < m_\pi + m_K$: bound state

Hadron Spectrum, arXiv:1406.4158

expect calculation at realistic kinematics within a few years

Phase shift III

$I = 1/2$ $K\pi$ S-wave and D-wave



$N_f = 2 + 1$ aniso. clover: $L|P|/2\pi = 0, 1, \sqrt{2}, \sqrt{3}, 2$, choose irreps

- $K\pi$, $K\eta$ coupled channel analysis [Talk:Wilson Mon 1B 15:35]
- $\delta_{K\pi}$, $\delta_{K\eta}$, η inelasticity

Hadron Spectrum, arXiv:1406.4158

- $m_\kappa < m_\pi + m_K$
- resonances corresponding to $K_0^*(K_2^*)$ in $l = 0(2)$

Comparison with HALQCD method

HALQCD method
Definition of potential

Ishii, Aoki, and Hatsuda, PRL99:022001(2007), Aoki, Hatsuda, and Ishii, PTP123:89(2010)

Nambu-Bethe-Salpeter (NBS) wave function for NN

$$\phi_n(\vec{r}) = \sum_{\vec{x}} \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | NN, W_n \rangle$$

$$\left(\frac{\nabla^2}{m_N} + \frac{p_n^2}{m_N} \right) \phi_n(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}\,') \phi_n(\vec{r}\,') \text{ where } W_n = 2\sqrt{m_N^2 + p_n^2}$$

NN 4-point function

$$C_{NN}(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | N(\vec{x}+\vec{r}, t) N(\vec{x}, t) \overline{NN}(0) | 0 \rangle = \sum_n A_n \phi_n(\vec{r}) e^{-W_n t}$$

Define $\overline{C}_{NN}(\vec{r}, t) = C_{NN}(\vec{r}, t) e^{2 m_N t}$

$$\left(\frac{\nabla^2}{m_N} + \frac{1}{4 m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right) \overline{C}_{NN}(\vec{r}, t) = \sum_n A_n \int d^3 r' U(\vec{r}, \vec{r}\,') \phi_n(\vec{r}\,') e^{-t(W_n - 2 m_N)}$$

HALQCD, PLB712:437(2012)

Assume $U(\vec{r}, \vec{r}\,') = V(\vec{r}) \delta(\vec{r} - \vec{r}\,') + O(\nabla^2)$

$$V(\vec{r}) = \frac{\left(\frac{\nabla^2}{m_N} + \frac{1}{4 m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right) \overline{C}_{NN}(\vec{r}, t)}{\overline{C}_{NN}(\vec{r}, t)}$$

$\rightarrow$ less sensitive to t than $C_{NN}(\vec{r}, t)$ at $t \gg 1$

HALQCD method

Strategy

– fit $V(r)$ with continuous functions, like Yukawa function

– solve Shrödinger equation to calculate $\delta(p)$ in any p

Assumptions

1. $U(\vec{r}, \vec{r}') = V(\vec{r})\delta(\vec{r} - \vec{r}') + O(\nabla^2)$ → small energy dependence of $V(\vec{r})$

   one small p: Murano *et al.*, PTP125:1225(2011)

2. $V(r)$ in infinite volume, if small finite volume effect of $V(r)$

   volume dependence at heavy m_π: HALQCD, PRL106:162002(2011)

3. $V(\vec{r})$ depends on sink operator of $C_{NN}(\vec{r}, t)$, but $\delta(p)$ does not

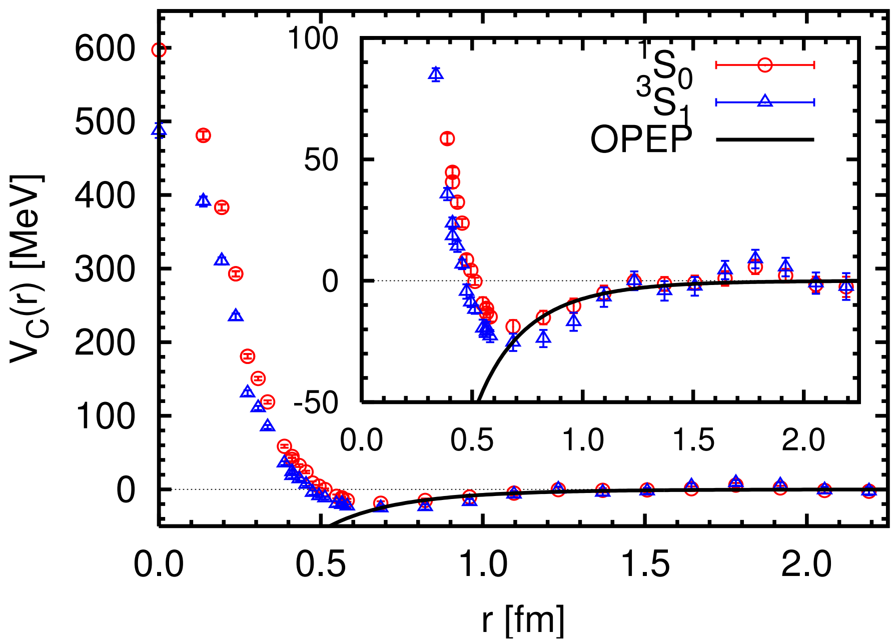
   c.f. operator dependence of $V(\vec{r})$ in SU(2) QCD

   Takahashi *et al.*, PRD82:094506(2010)

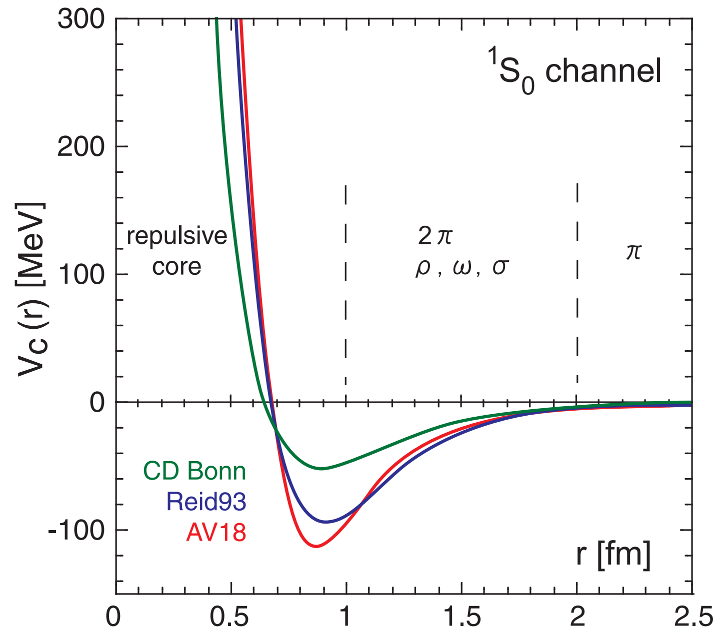
HALQCD method

$N_f = 0@m_\pi = 0.53\text{GeV}$ Ishii, Aoki, and Hatsuda, PRL99:022001(2007)

HALQCD



phenomenological



repulsive core + one-pion exchange potential

$\rightarrow$ Qualitatively consistent with phenomenological potential

HALQCD method
their works

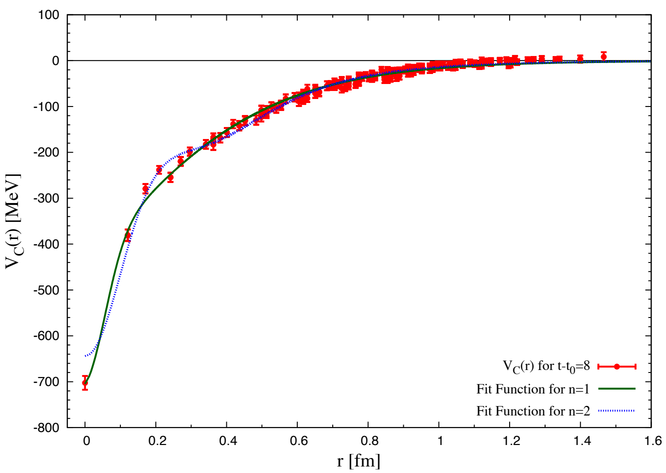
– Energy dependence of $V(\vec{r})$ Murano *et al.*, PTP125:1225(2011)

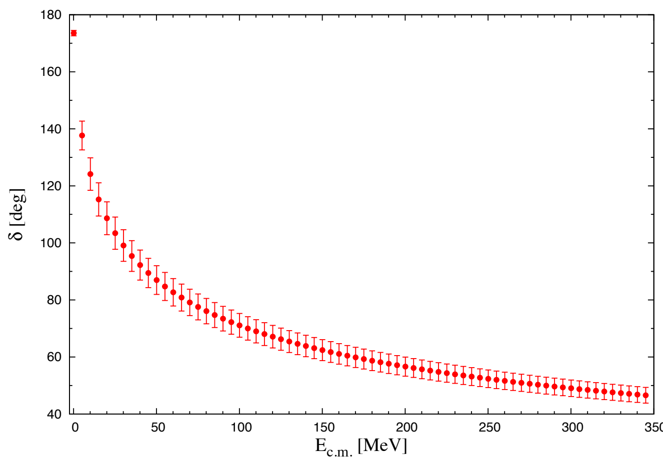
– LS force in odd parity sectors HALQCD, PoS(LATTICE 2013)234, 235

– Hyperon potentials HALQCD, PoS(LATTICE 2013)233

– charmed meson potentials HALQCD, PoS(LATTICE 2013)261

– spin-2 S-wave $N\Omega$ dibaryon HALQCD, arXiv:1403.7284





$$-\Delta E_{N\Omega} = 19(5)\left(^{12}_{2}\right) \text{ MeV}$$
$$a_{N\Omega} = -1.28(13)\left(^{14}_{15}\right) \text{ fm}$$
$$r_{N\Omega} = 0.499(26)\left(^{29}_{48}\right) \text{ fm}$$

– Ω-Ω potential HALQCD, PoS(LATTICE 2013)232 [Talk:Yamada Thu 7B 15:35]

c.f. $a_0 = -0.16(22)$ fm @ $m_\pi = 0.39$ GeV, Buchoff *et al.*, PRD85:094511(2012)

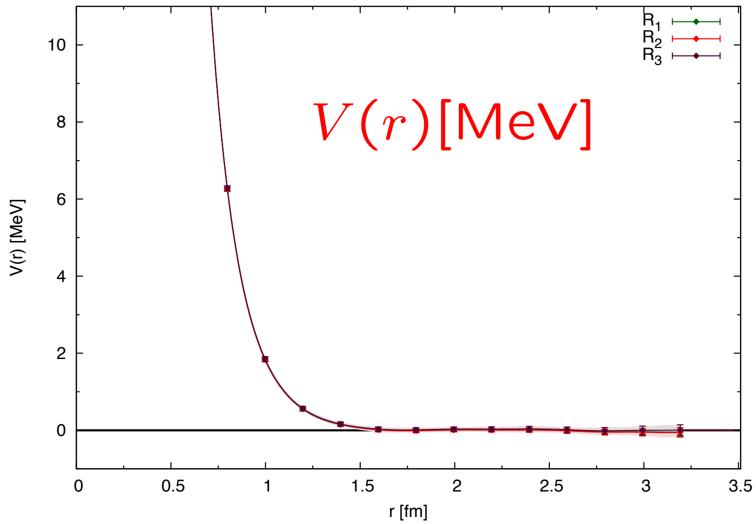
– three-nucleon force HALQCD, PTP127:723(2012)

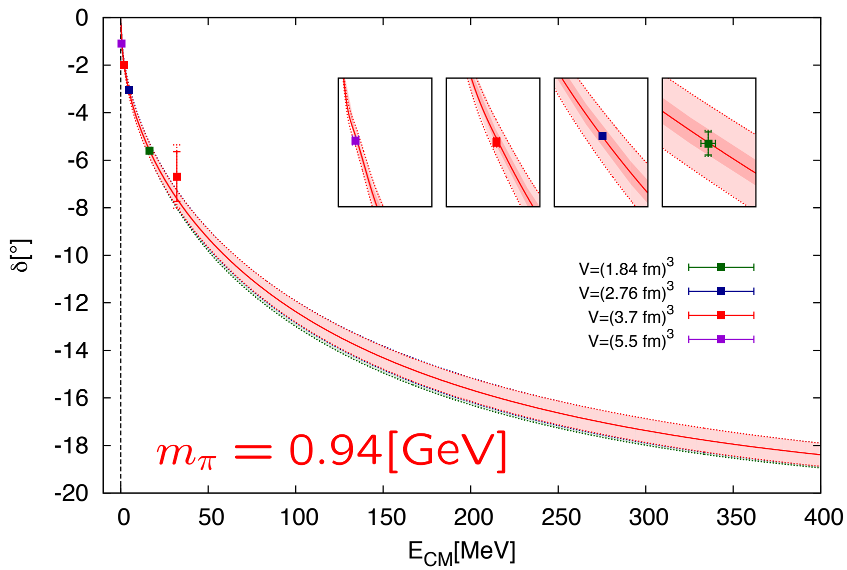
quark mass dependence of three-nucleon force [Talk:Doi Thu 7B 15:55]

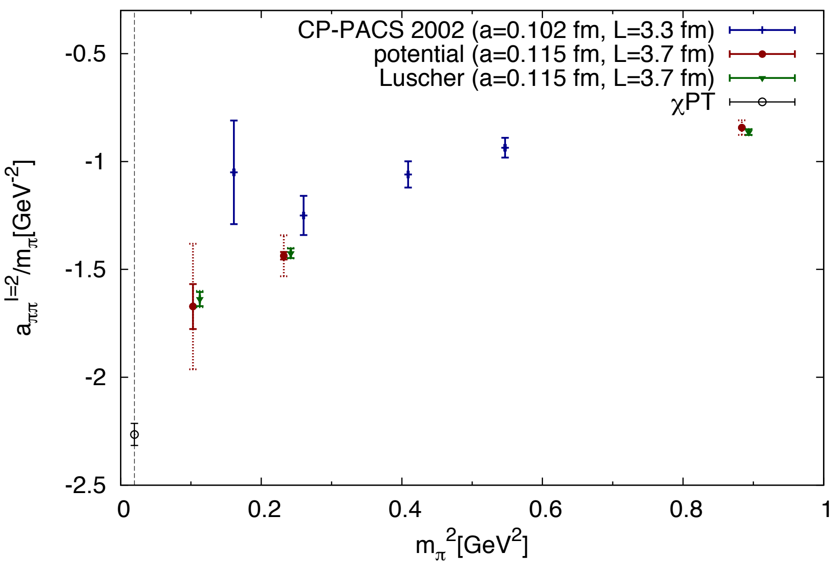
Comparison with HALQCD method

$I = 2\ \pi\pi$

Kurth *et al.*, JHEP012:015(2013)







Red line from HALQCD method

symbols from Lüscher's method

$\delta(p)$ at small p @ $m_\pi = 0.94$ GeV

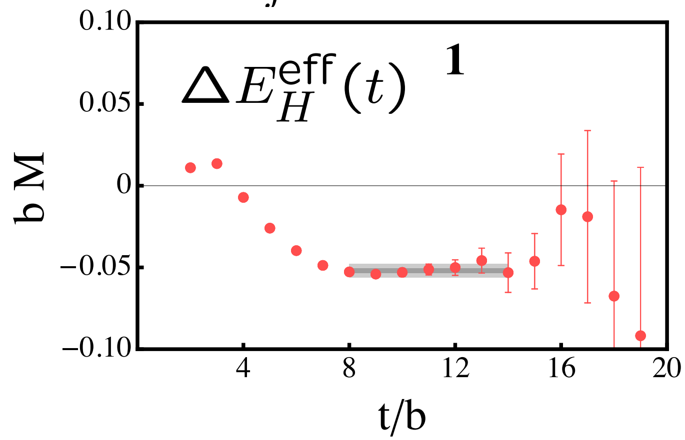
a_0/m_π @

$m_\pi = 0.94,\ 0.50,\ 0.33$ GeV

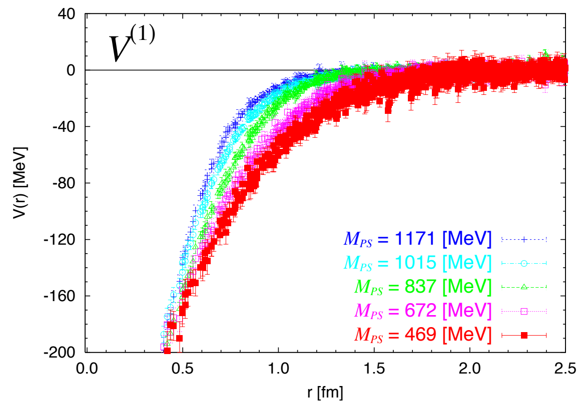
Quantitative agreement

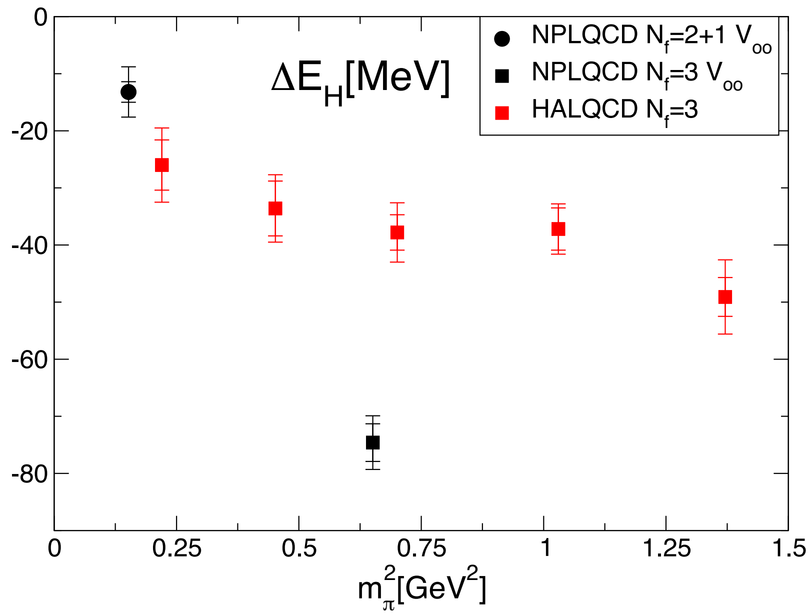
H dibaryon (bound state) $\Delta E_H = m_H - 2m_\Lambda$

NPLQCD $N_f = 3 @ m_\pi = 0.8$GeV



HALQCD $N_f = 3$





NPLQCD, PRL106:162001(2011)

PRD87:034506(2013)

HALQCD, PRL106:162002(2011)

NPA881:28(2012)

black:

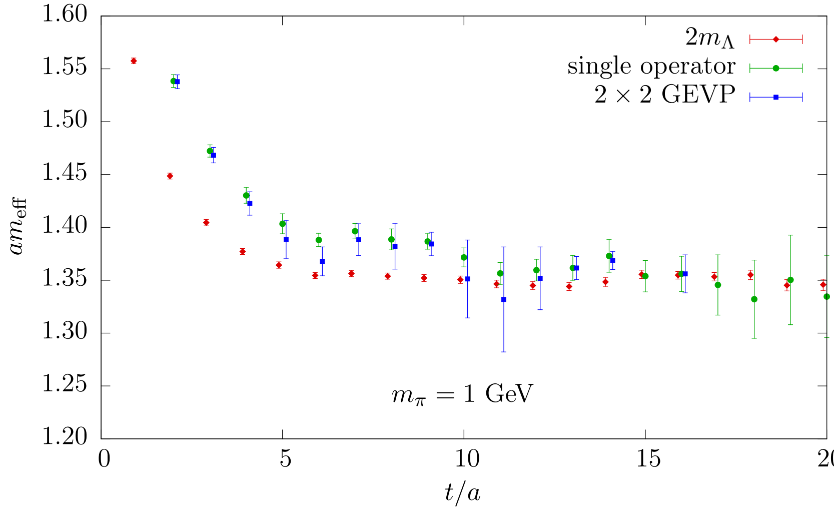
Lüscher's method ($L^3 \to \infty$)

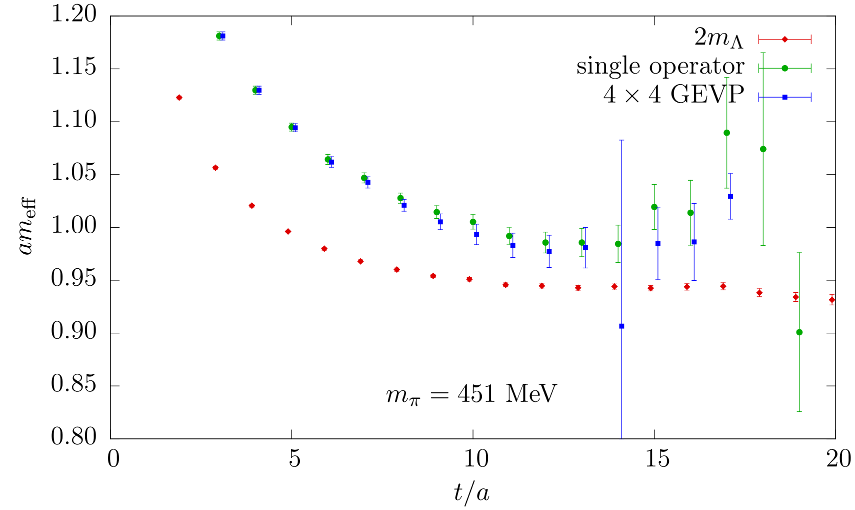
red: HALQCD method

Qualitative agreement: existence of H dibaryon

H dibaryon

Mainz group [Talk:Green Wed 6B 12:30]





update from Lat13 PoS(LATTICE 2013)440 using several improvements

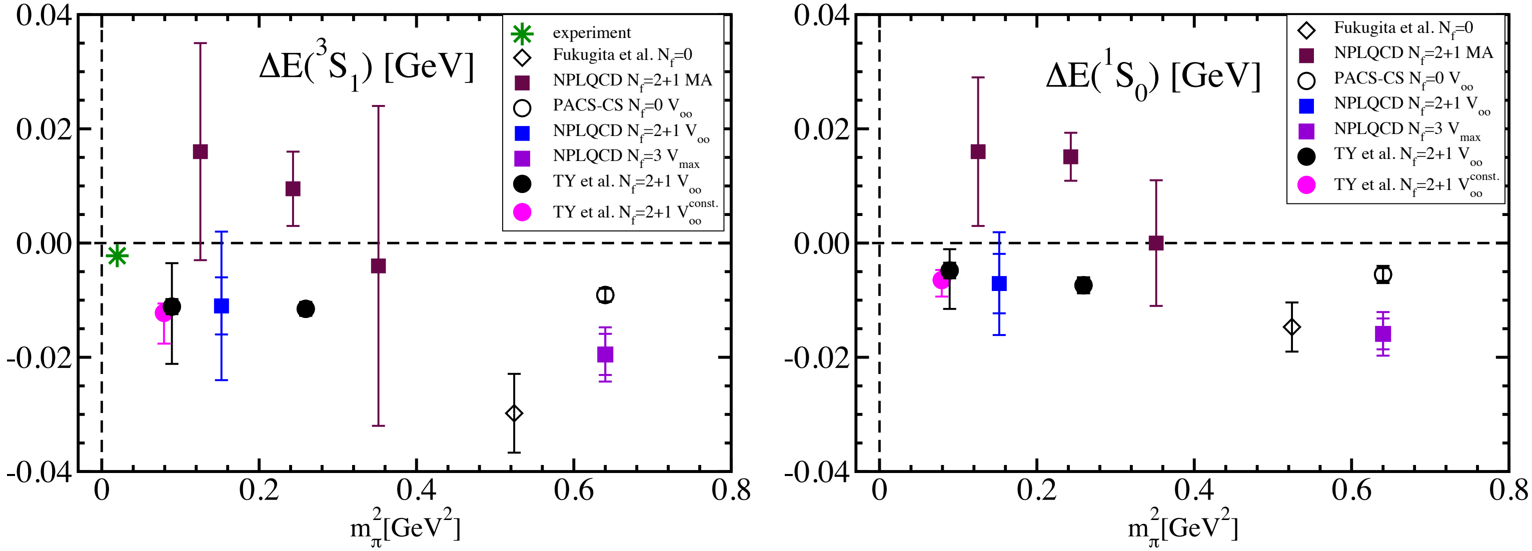
several local (smeared) six-quark operators with variational method

No signal of H dibaryon $E_0 > 2m_\Lambda$

Important to check: variational method including two-baryon operators

NN channels with Lüscher's method

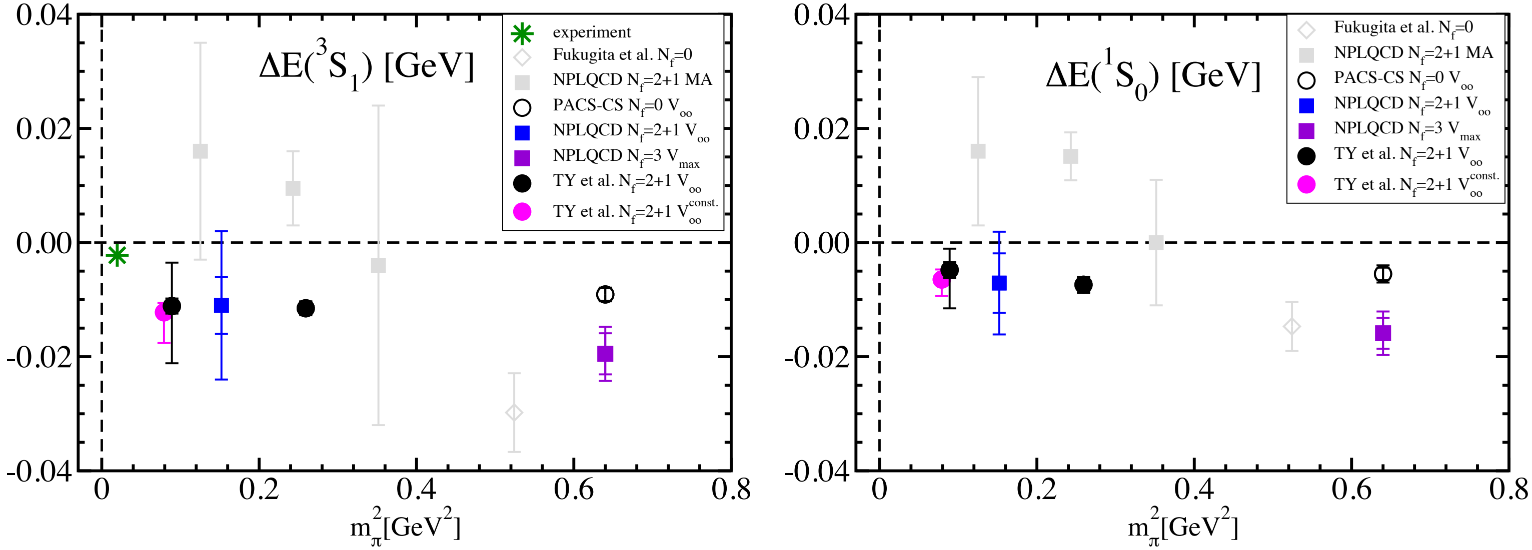
Current status from Lüscher's method



TY *et al.*, preliminary result@$m_\pi = 0.3$GeV with 2 volumes

NN channels with Lüscher's method

Current status from Lüscher's method



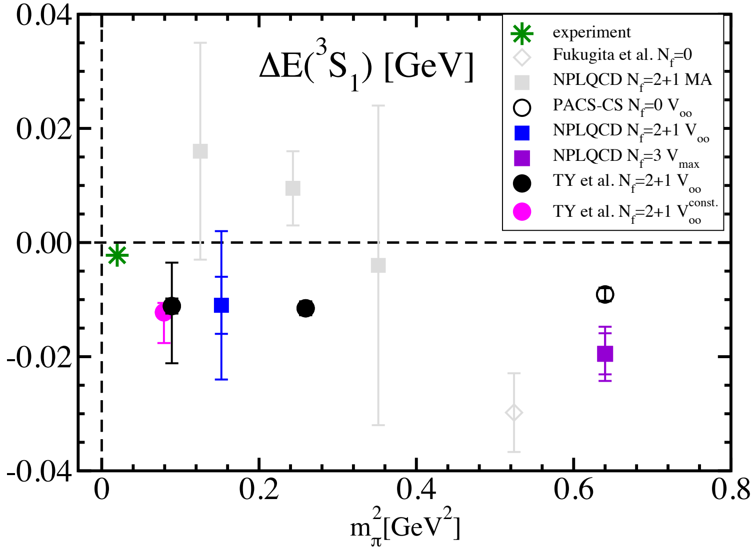
TY *et al.*, preliminary result@$m_\pi = 0.3$GeV with 2 volumes

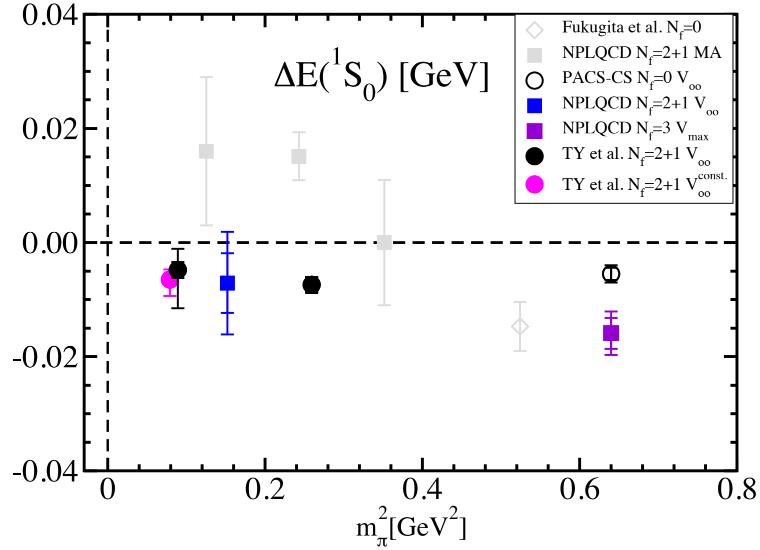
$L^3 \to \infty$: existence of bound states in ^{3}S$_1$ and ^{1}S$_0$

inconsistent with experiment due to larger m_π

NN channels with Lüscher's method

Current status from Lüscher's method





TY *et al.*, preliminary result@$m_\pi = 0.3$GeV with 2 volumes

$L^3 \to \infty$: existence of bound states in ^{3}S$_1$ and ^{1}S$_0$

inconsistent with experiment due to larger m_π

$a_0 < 0$ @ $m_\pi = 0.8$ GeV $\to$ bound state in each channel

c.f. Sasaki and TY, PRD74:114507(2006)

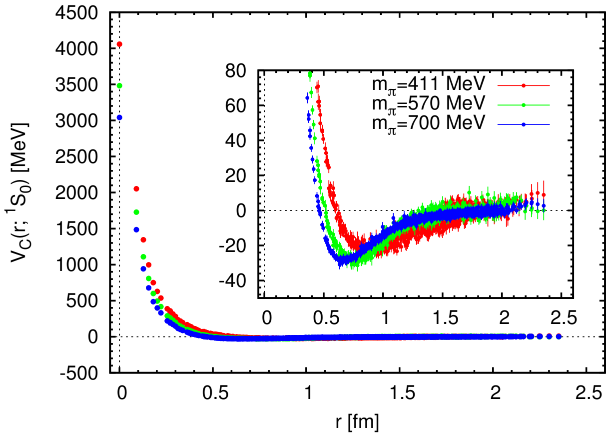
	PACS-CS, $N_f = 0^*$	NPLQCD, $N_f = 3$
$a_0^{^3S_1}$[fm]	$-1.05(24)\left(^{5}_{65}\right)$	$-1.82\left(^{14}_{13}\right)\left(^{17}_{12}\right)$
$a_0^{^1S_0}$[fm]	$-1.62(24)\left(^{1}_{75}\right)$	$-2.33\left(^{19}_{17}\right)\left(^{27}_{20}\right)$

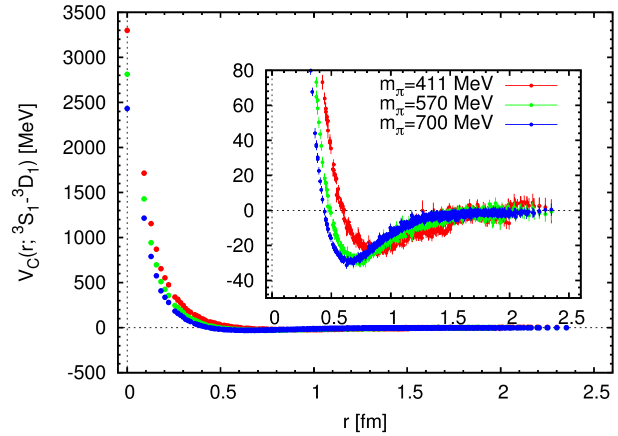
* from $L = 6.1$ fm PACS-CS, PRD84:054506(2011)

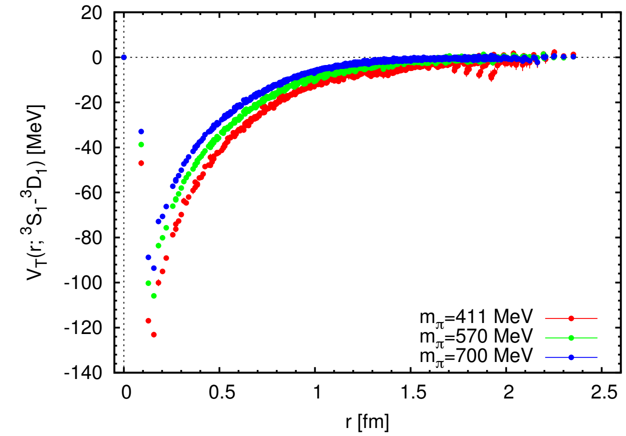
NPLQCD, PRD87:034506(2013)

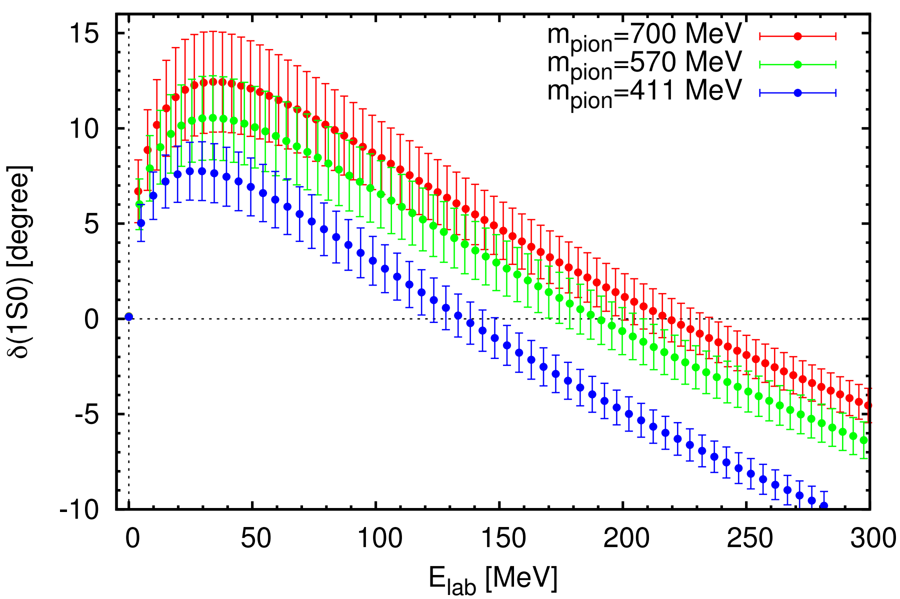
NN channels with HALQCD method

HALQCD, $N_f = 2+1$ $m_\pi = 0.41, 0.57, 0.70$ GeV PoS(CD12):025









No bound states

in both ${}^3S_1$ and ${}^1S_0$

inconsistent with experiment

due to larger m_π

Qualitative difference

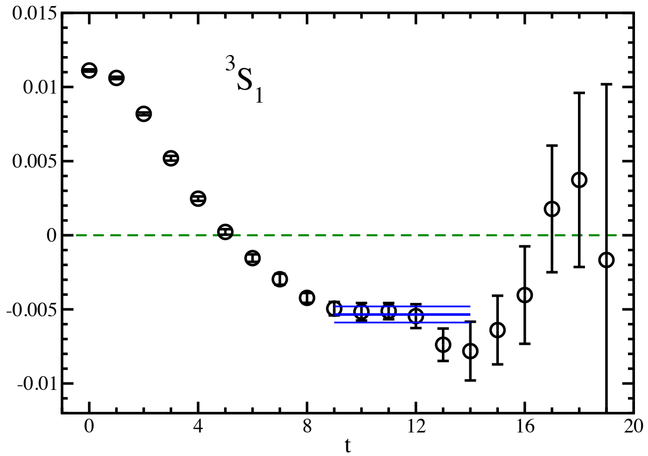
from Lüscher's method

Uncertainty of Lüscher's method

TY *et al.* and NPLQCD
Lüscher's method $\sim \Delta E$ of 0th state and $L^3 \to \infty$
$\to$ same as traditional method to obtain hadron mass

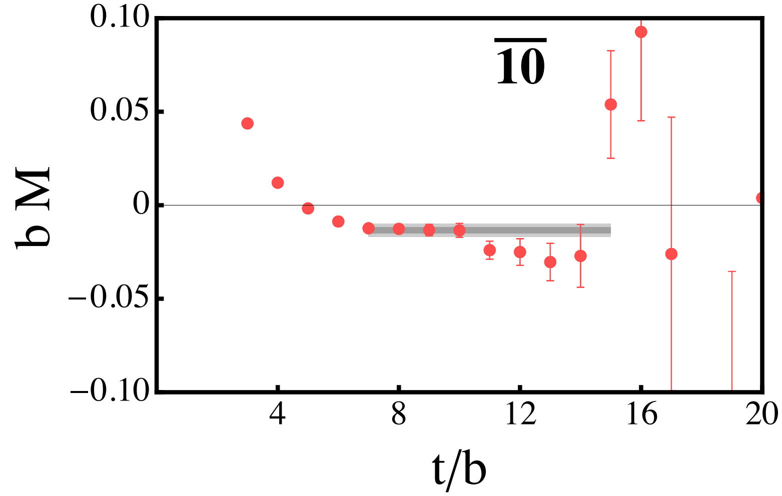
current study: smeared quark field + $C_{NN}(t)/(C_N(t))^2$ in large t
$\Delta E(t)$ from + to $-$ $\to$ plateau $\to$ large statistical fluctuation

TY *et al.*, $N_f = 2+1$ $m_\pi = 0.5$ GeV



PRD86:074514(2012)

NPLQCD, $N_f = 3$ $m_\pi = 0.8$ GeV



PRD87:034506(2013)

- 0th state energy from variational method

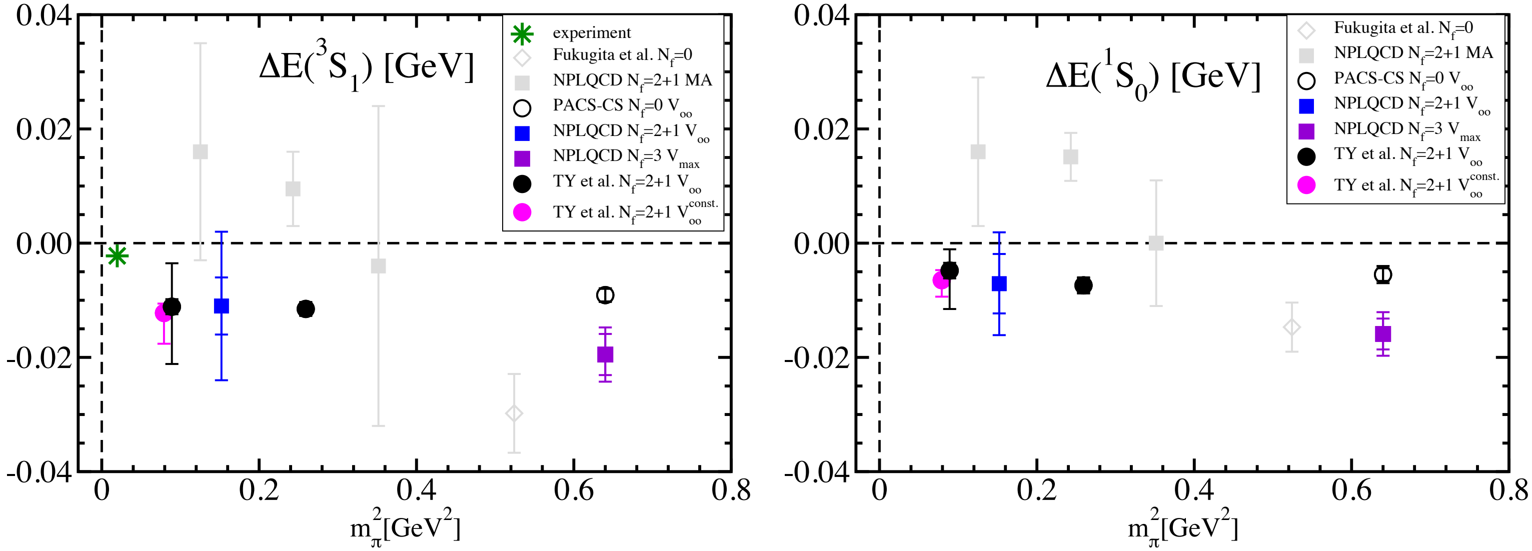
Uncertainty of Lüscher's method

TY *et al.* and NPLQCD

Lüscher's method $\sim \Delta E$ of 0th state and $L^3 \to \infty$

$\to$ same as traditional method to obtain hadron mass

- 0th state energy from variational method



- 2.9σ difference of ΔE at $m_\pi = 0.8$ GeV ($N_f = 0$ and $N_f = 3$)

- Investigation of m_π dependence

Bound state in $^1S_0$ vanishes at physical m_π?

Uncertainty of HALQCD method

HALQCD

Need to test validity of method

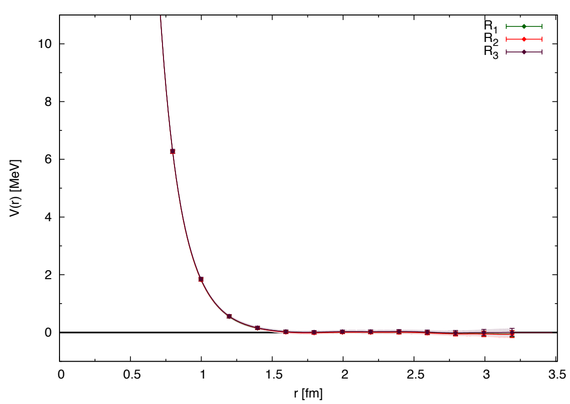
– $I = 2$ $\pi\pi$: quantitatively ok $\leftarrow$ only $\pi\pi$ and $V(r) \geq 0$ in all r

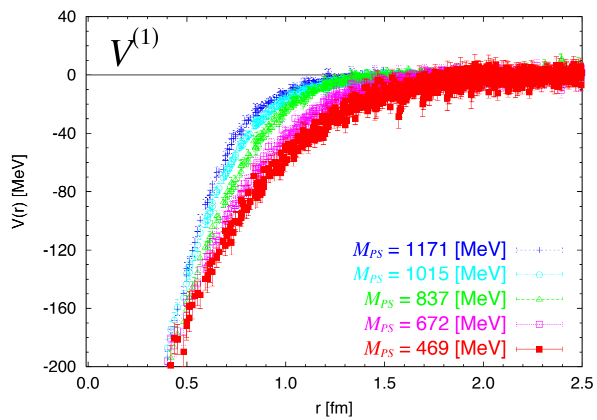
c.f. reasonable fit with $p \cot \delta(p) = 1/a_0$ Had. Spec., PRD86:034031(2012)

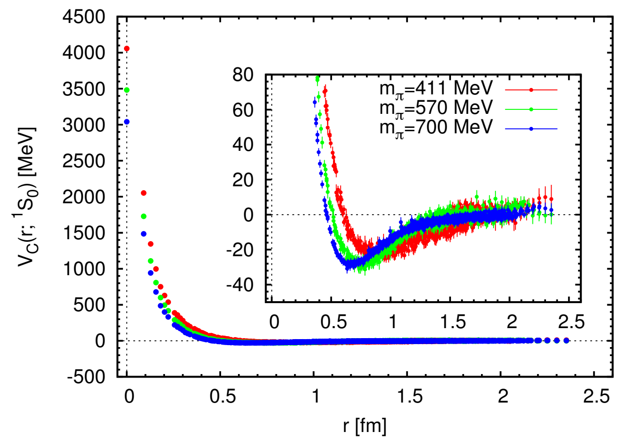
– Existence of H dibaryon: qualitatively ok $\leftarrow$ $V(r) \leq 0$ in all r

Agreements in simple systems (insensitive to $V(r)$ at small r)

$I = 2$ $\pi\pi$ | H dibaryon | NN







– NN channels: qualitative difference

complex $V(r)$ $\rightarrow$ positive and negative $V(r)$ depending on r

Important to study uncertainties of $V(r)$

Uncertainty of HALQCD method

HALQCD

– NN channels: qualitative difference

Possible uncertainties

1. Uncertainties of $V(r)$ in middle–large r

important for physical quantity in $p \sim 0$

$V(r) \sim 0 \to$ large relative error

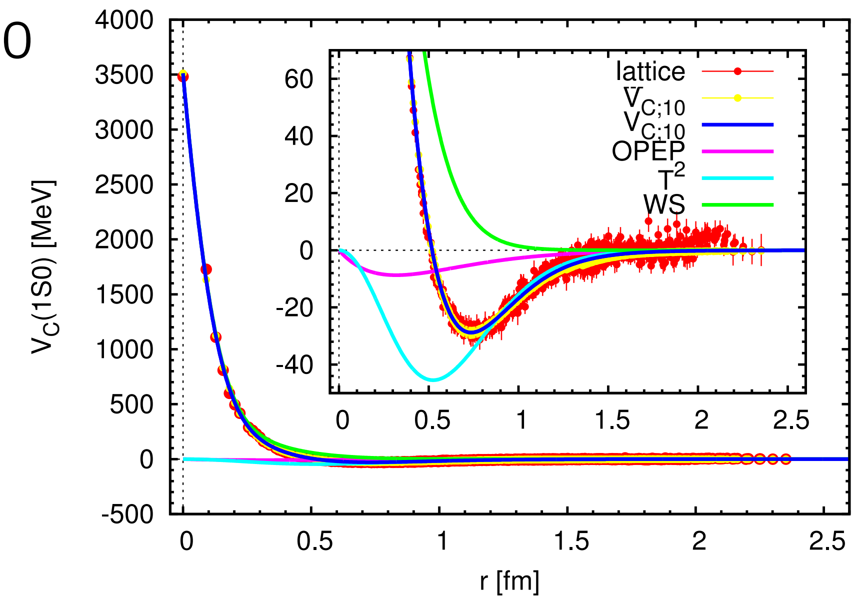
Statistics

HALQCD $\sim$ 12000meas@$m_\pi = 0.4$GeV

NPLQCD 150000meas@$m_\pi = 0.8$GeV

TY *et al.* 40000meas@$m_\pi = 0.5$GeV

meas $= N_{\text{conf}} \times N_{\text{src}}$



HALQCD, PoS(CD12)025

Uncertainty of HALQCD method

HALQCD

– NN channels: qualitative difference

Possible uncertainties

1. Uncertainties of $V(r)$ in middle–large r

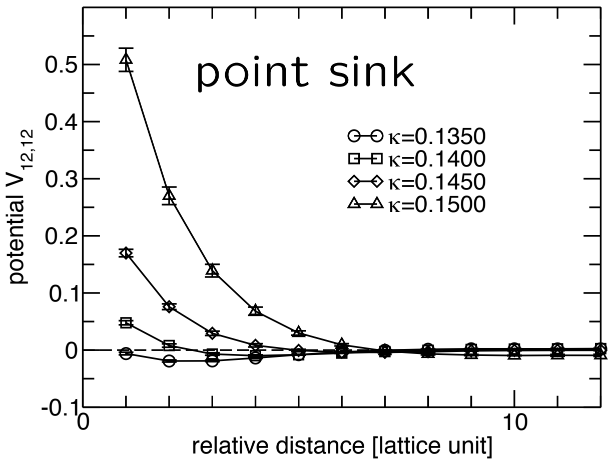
Statistics

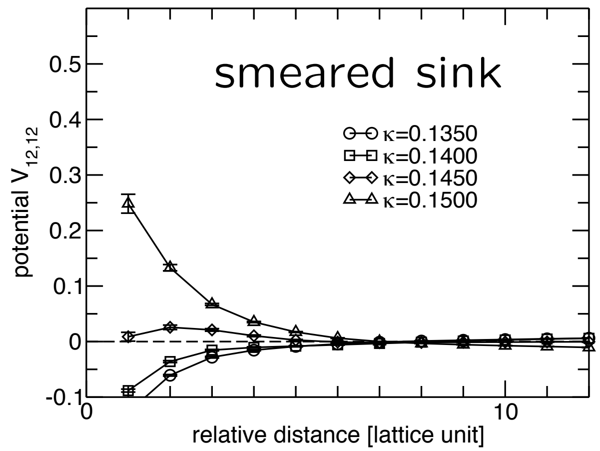
2. Uncertainties of $V(r)$ in small r

important for physical quantity in large p and also bound state

Finite a effect: affect large p HALQCD, PoS(LATTICE 2013)226

Sink operator dependence: SU(2) QCD, Takahashi *et al.*, PRD82:094506(2010)





large r insensitive, but small r sensitive to operator

Uncertainty of HALQCD method

HALQCD

– NN channels: qualitative difference

Possible uncertainties

1. Uncertainties of $V(r)$ in middle–large r

Statistics

2. Uncertainties of $V(r)$ in small r

Finite a effect: affect large p HALQCD, PoS(LATTICE 2013)226

Sink operator dependence: SU(2) QCD, Takahashi *et al.*, PRD82:094506(2010)

3. Uncertainties of m_N

Statistics and systematic $\to$ constant shift of $V(r)$

$$V(\vec{r}) \approx \frac{\left(\frac{\nabla^2}{m_N} - \frac{\partial}{\partial t}\right) C_{NN}(\vec{r},t)}{C_{NN}(\vec{r},t)} - 2m_N$$

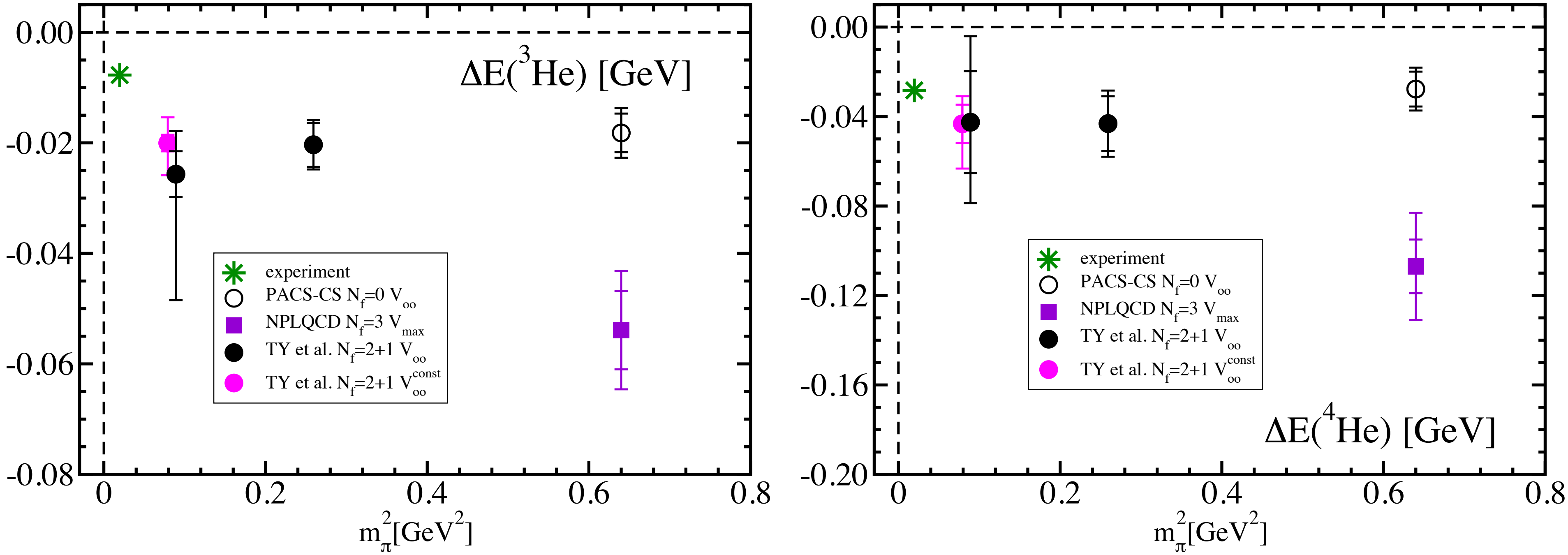
necessary detail investigation of uncertainties

Light nuclei

Light nuclei ^{3}He and ^{4}He

First calculation of ^{3}He and ^{4}He PACS-CS, PRD81:111504(R)(2010)

NPLQCD, PRD87:034506(2013), TY *et al.*, PRD86:074514(2012) and preliminary result@$m_\pi = 0.3$GeV



$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments

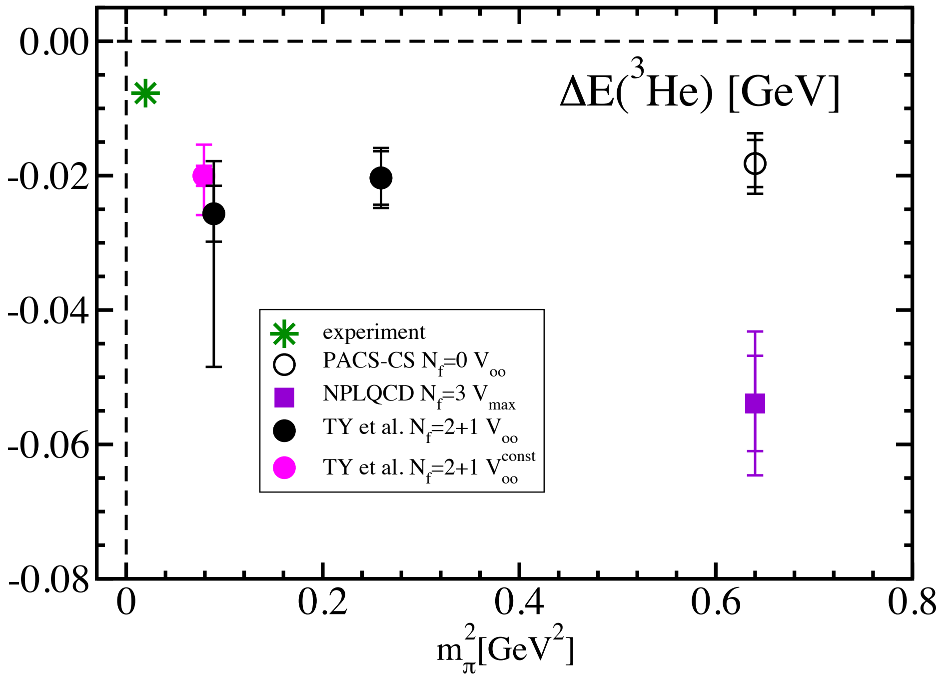
$A = 2, 3$ states in $J = 1$ bound in $N_f = 2$ SU(2) gauge theory

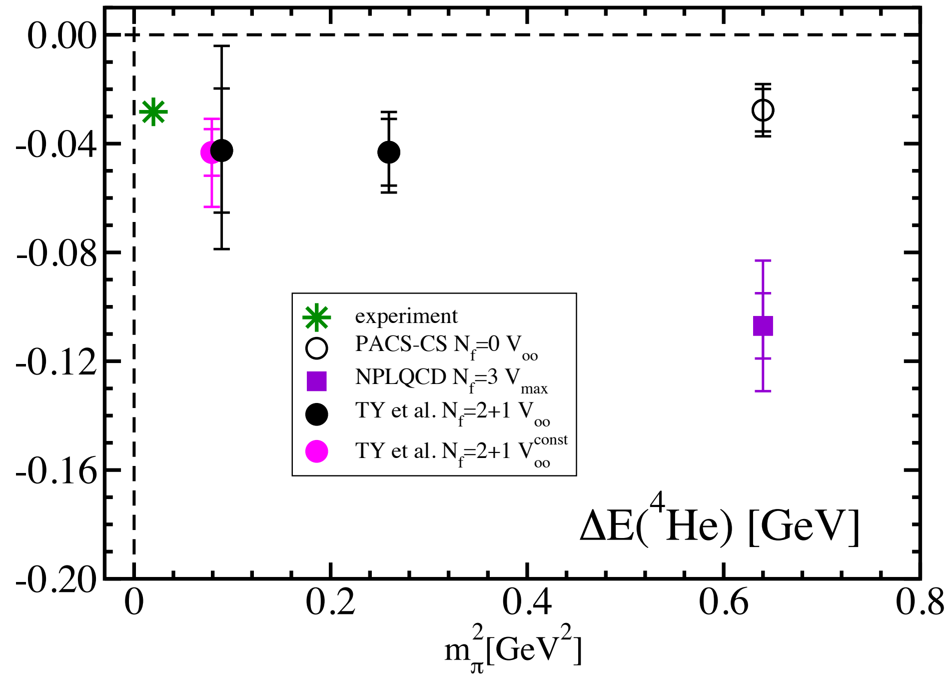
[Talk:Detmold Fri 8C 14:15] arXiv:1406.4116

Light nuclei ^{3}He and ^{4}He

First calculation of ^{3}He and ^{4}He PACS-CS, PRD81:111504(R)(2010)

NPLQCD, PRD87:034506(2013), TY *et al.*, PRD86:074514(2012) and preliminary result@$m_\pi = 0.3$GeV





$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments $\to$ relatively easier than NN

large $|\Delta E|$ less V dependence

touchstone of quantitative understanding of nuclei from lattice QCD

Investigations of m_π dependence $\to m_\pi = 0.14$ GeV @ $L \sim 10$ fm

Summary

Hadronic interactions

important to understand properties of hadrons and nuclei

various studies for hadronic scattering and decays

new ideas to overcome difficulties

exploratory study $\to$ precise measurement

comparison between Lüscher's method and HALQCD method

calculations with Lüscher's method need variational analysis

HALQCD method works well in $I = 2$ $\pi\pi$

$\to$ useful to calculate $\delta(p)$ in this channel

still need to comparison in NN channel

$\to$ investigation of uncertainties of $V(r)$

calculation of light nuclei (^{3}He and ^{4}He)

might be relatively easier than NN $\to$ touchstone of nuclei calculation

m_π dependence $\to$ $m_\pi = 0.14$ GeV @ $L \sim 10$ fm

precise measurement: $m_u \neq m_d$, EM effects,

and also property of single nucleon, e.g. g_A and form factors

Thank you for your attention

Thank you very much for sending results and comments
John Bulava, William Detmold, Jeremy Green, Yoshinobu Kuramashi, Christian Lang, Hidekatsu Nemura, Sasa Prelovsek, Akira Ukawa

Back up

Calculation of two-particle bound state

condition of bound state through $\delta(p)$ (pole of S matrix)

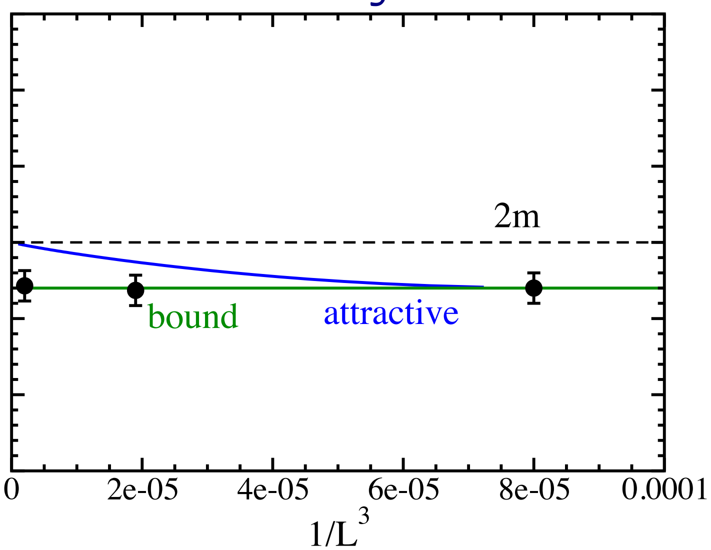
$$p \cot\delta(p) = -\gamma \text{ at } p^2 = -\gamma^2, \qquad p^2 = m_b^2/4 - m^2$$

finite volume correction of binding energy

$$\Delta E_L = \Delta E \left\{ 1 - \frac{C}{\gamma L} \sum_{\vec{n}}{}' \frac{\exp(-\gamma L \sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}} \right\}, \ \Delta E = m_b - 2m \approx -\frac{\gamma^2}{m_N}$$

Beane *et al.*, PLB585:106(2004), Sasaki and TY, PRD74:114507(2006)

Problem to identify bound state on finite volume



$$\Delta E_L = E_0(L) - 2m$$

Bound state

$$\Delta E_L = \Delta E + O\left(e^{-CL}\right) < 0$$

Attractive scattering state

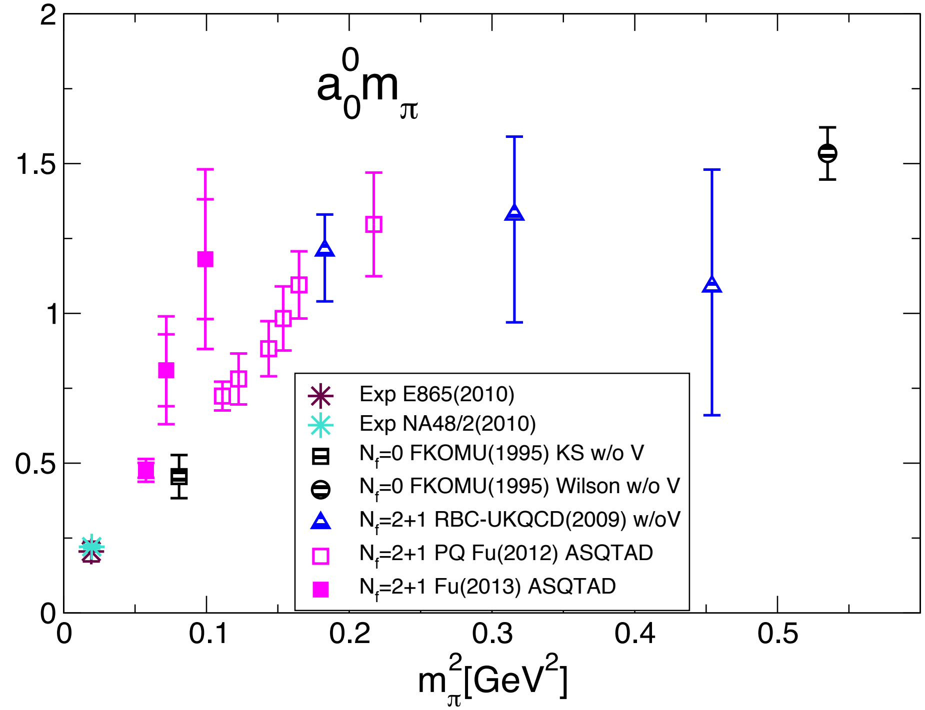
$$\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$$

necessary to take $L^3 \to \infty$

Scattering length III

$I = 0\ \pi\pi$ D, C, R, V necessary

Most difficult, but important in $\Delta I = 1/2$ $K \to \pi\pi$



$\bigcirc$ Wilson type; $\square$ ASQTAD; $\triangle$ DWF; PQ:partial quenched

Recent ASQTAD calculations reasonable errors w/ V diagram
However, V destroys signal in Wilson and DWF $\to \sim$ 100% error

necessary breakthrough for other actions than KS
and estimate of systematic error for KS

PACS-CS results

$N_f = 3$ NLO WChPT

$$a_0^2 m_\pi = \frac{m_\pi^2}{8\pi f_\pi^2}\left[-1 + \frac{32}{f_\pi^2}\left[m_\pi^2(L' - \frac{L_5}{2}) + \text{analytic} + \log\right]\right] - \frac{c_2 a^2}{8\pi f_\pi^2}$$

$$a_0^1 m_K = \frac{m_K^2}{8\pi f_K^2}\left[-1 + \frac{32}{f_K^2}\left[m_K^2(L' - \frac{L_5}{2}) + \text{analytic} + \log\right]\right] - \frac{c_2 a^2}{8 f_\pi f_K^2}$$

$$a_0^{3/2} \mu_{\pi K} = \frac{\mu_{\pi K}^2}{4\pi f_\pi f_K}\left[-1 + \frac{32}{f_\pi f_K}\left[m_\pi m_K L' - \frac{m_\pi^2 + m_K^2}{4} L_5 + \text{analytic} + \log\right]\right] - \frac{c_2 a^2}{4\pi f_\pi f_K}\frac{\mu_{\pi K}^2}{m_\pi m_K}$$

$$a_0^{1/2} \mu_{\pi K} = \frac{\mu_{\pi K}^2}{4\pi f_\pi f_K}\left[2 + \frac{32}{f_\pi f_K}\left[m_\pi m_K L' + \frac{m_\pi^2 + m_K^2}{2} L_5 + \text{analytic} + \log\right]\right] - \frac{c_2 a^2}{4\pi f_\pi f_K}\frac{\mu_{\pi K}^2}{m_\pi m_K}$$

$$L' = 2L_1 + 2L_2 + L_3 - 2L_4 - L_5/2 + 2L_6 + L_8$$

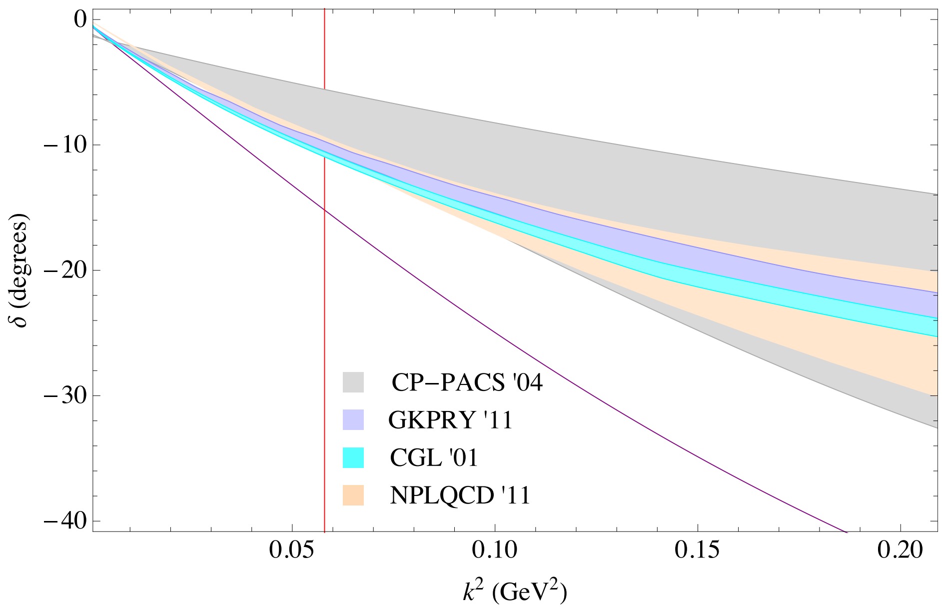
fit range:
a_0^2: $m_\pi \leq 0.41$GeV, $a_0^1, a_0^{3/2}, a_0^{1/2}$: $m_\pi \leq 0.30$GeV

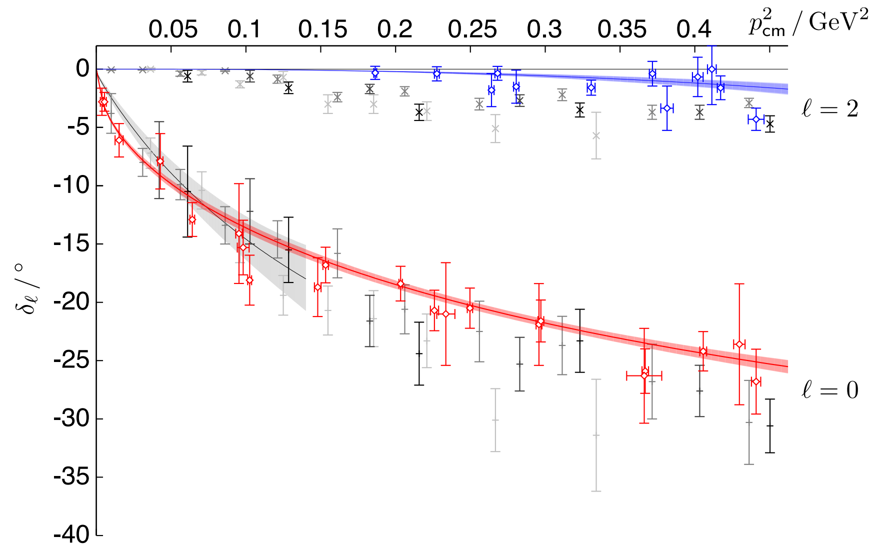
$c_2 = 0.089(24)\text{GeV}^4$, $L_5 = 2.1(1.1) \times 10^{-3}$, $L' = 0.83(64) \times 10^{-3}$
χ^2/d.o.f. $= 1.9(1.2)$

Phase shift I

$I = 2$ S-wave $\pi\pi$ Simplest scattering system

CM and Moving frames @ $m_\pi = 0.39$GeV





NPLQCD, PRD85:034505(2012)

NLO ChPT in $p \neq 0 \rightarrow$ physical m_π

m_π / f_π from MA calc.

Hadron Spectrum, PRD86:034031(2012)

S- and D-wave ($l = 0$ and 2)

other works: CP-PACS, PRD67:014502(2003), Kim, NPB(Proc.Suppl.)129:197(2004), CP-PACS, PRD70:074513(2004), CLQCD, JHEP06:053(2007), Sasaki and Ishizuka, PRD78:014511(2008), Kim and Sachrajda, PRD81:114506(2010), Hadron Spectrum, PRD83:071504(R)(2011)

Resonance from phase shift

Relativistic Breit-Wigner form for scattering amplitude

$$e^{i\delta(p)}\sin\delta(p) = \frac{-\sqrt{s}\Gamma_R(s)}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)}, \quad s = E_{\rm cm}^2$$

PDG, PRD:86.010001(2012)

Breit-Wigner form fit: P-wave $I = 1$ $\pi\pi \to \rho$ ($I = 1/2$ $K\pi \to K^*$)

$$M_R = m_\rho,\ \Gamma_R(s) = \frac{p^3}{s}\frac{g_{\rho\pi\pi}^2}{6\pi} \longrightarrow \frac{p^3}{\sqrt{s}}\cot\delta(p) = \frac{6\pi}{g_{\rho\pi\pi}^2}(m_\rho^2 - s)$$

Necessary condition of P-wave resonance (kinematics, calculation)

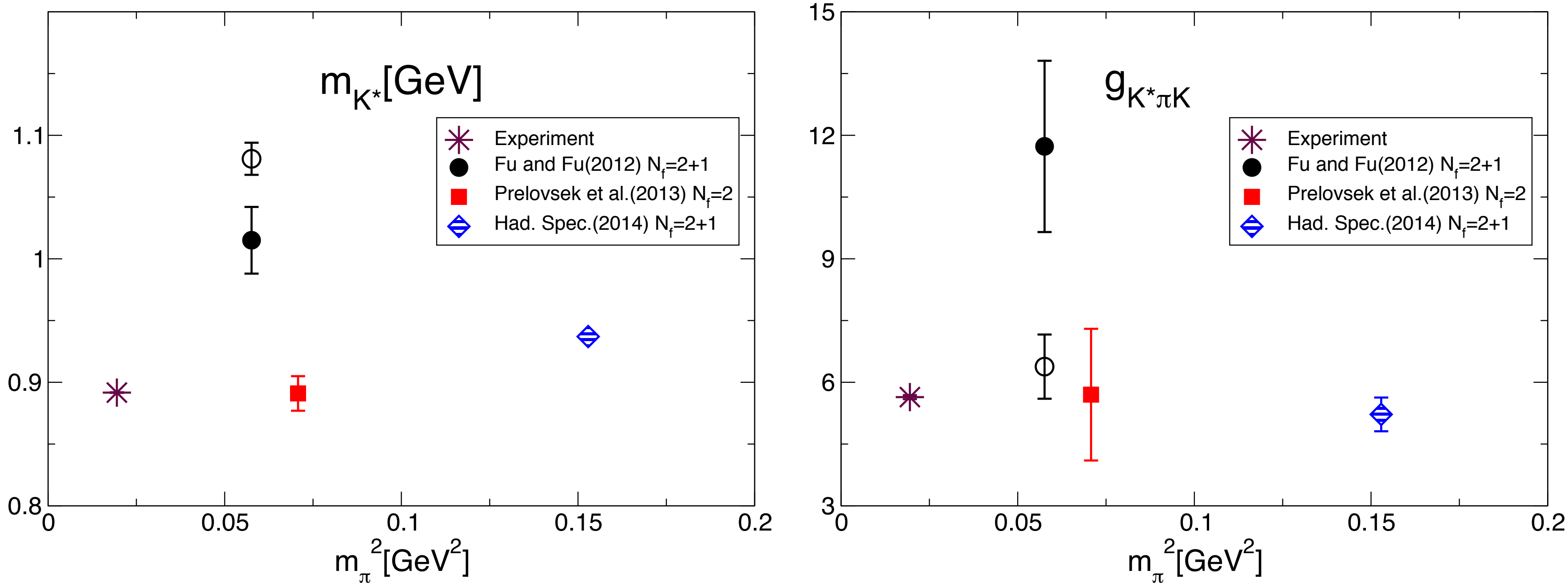
1. $m_\rho > 2m_\pi$
2. CM frame: $m_\rho > 2E_\pi(p) = 2\sqrt{m_\pi^2 + p^2}$, because $\rho \to \pi(p)\pi(-p)$
   Moving frames: e.g. $E_\rho(P) > m_\pi + E_\pi(P)$
3. ρ type and $\pi\pi$ operators in variational analysis

Amplitude (transfer matrix) method: Gottlieb *et al.*, PL134B:346(1984), Loft and DeGrand, PRD39:2692(1989), McNeile and Michael, PRD65:094505(2002), McNeile and Michael, PLB556:177(2003), McNeile and Michael, PRD73:074506(2006),
McNeile, Michael,and Urbach, PRD80:054510(2009), Alexandrou *et al.*, PRD88:031501(R)(2013)

Γ of decuplet baryons [Talk:Petschlies Thu 7B 14:35]

Phase shift III

$I = 1/2$ P-wave $K\pi$ D, C, R and T diagrams

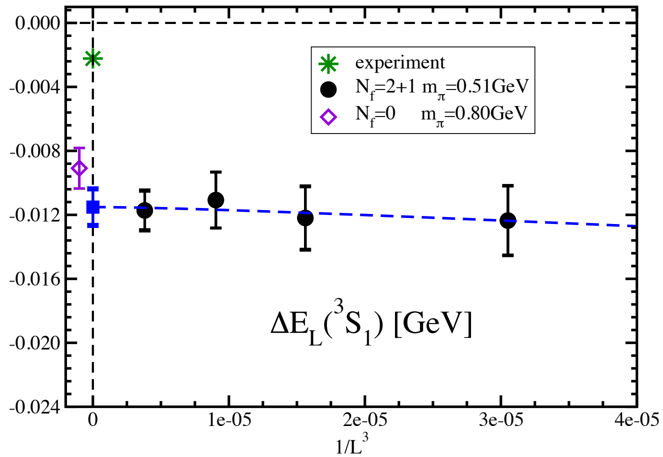


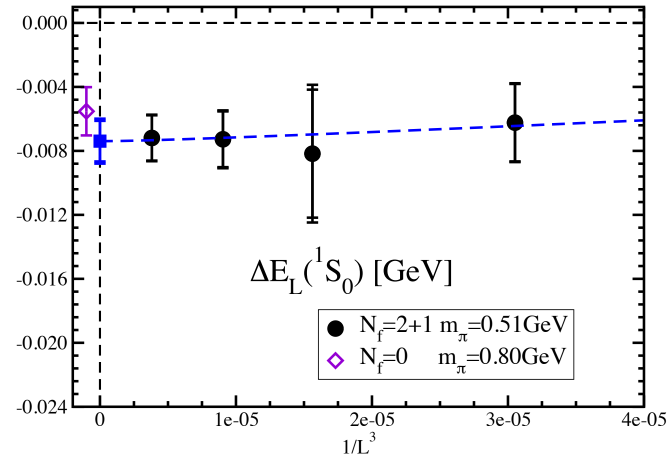
[Talk:Wilson Mon 1B 15:35]

Comparison with HALQCD method

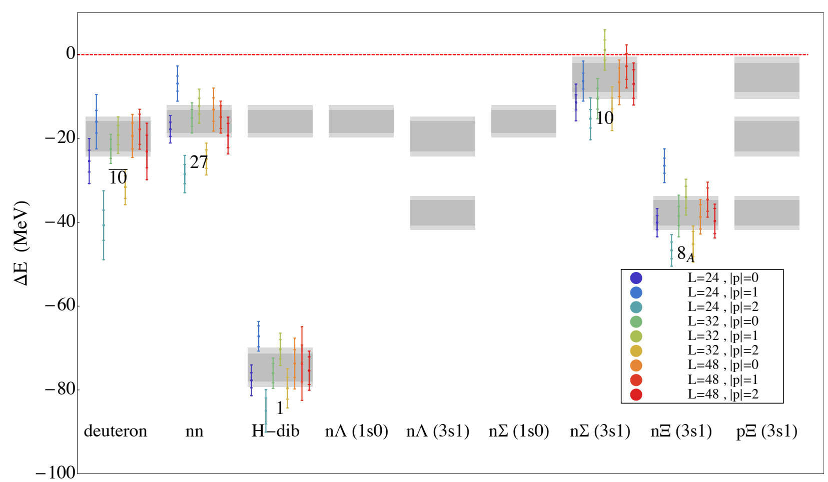
NN channels with Lüscher's method

TY *et al.*, $N_f = 2+1$ $m_\pi = 0.5$ GeV — PRD86:074514(2012)





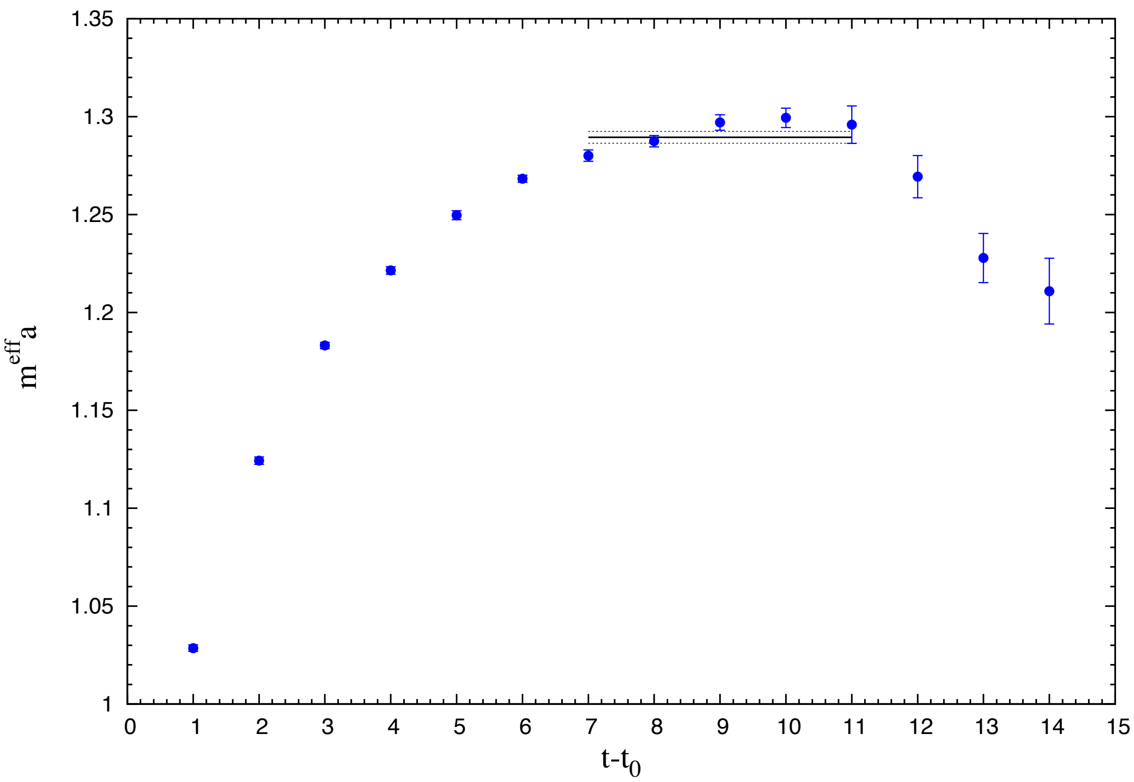
NPLQCD, $N_f = 3$ $m_\pi = 0.8$ GeV — PRD87:034506(2013)



Effective baryon mass

– spin-2 S-wave $N\Omega$ dibaryon HALQCD, arXiv:1403.7284

Effective m_Ω



Simulation parameters of TY *et al.*

$N_f = 2 + 1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV

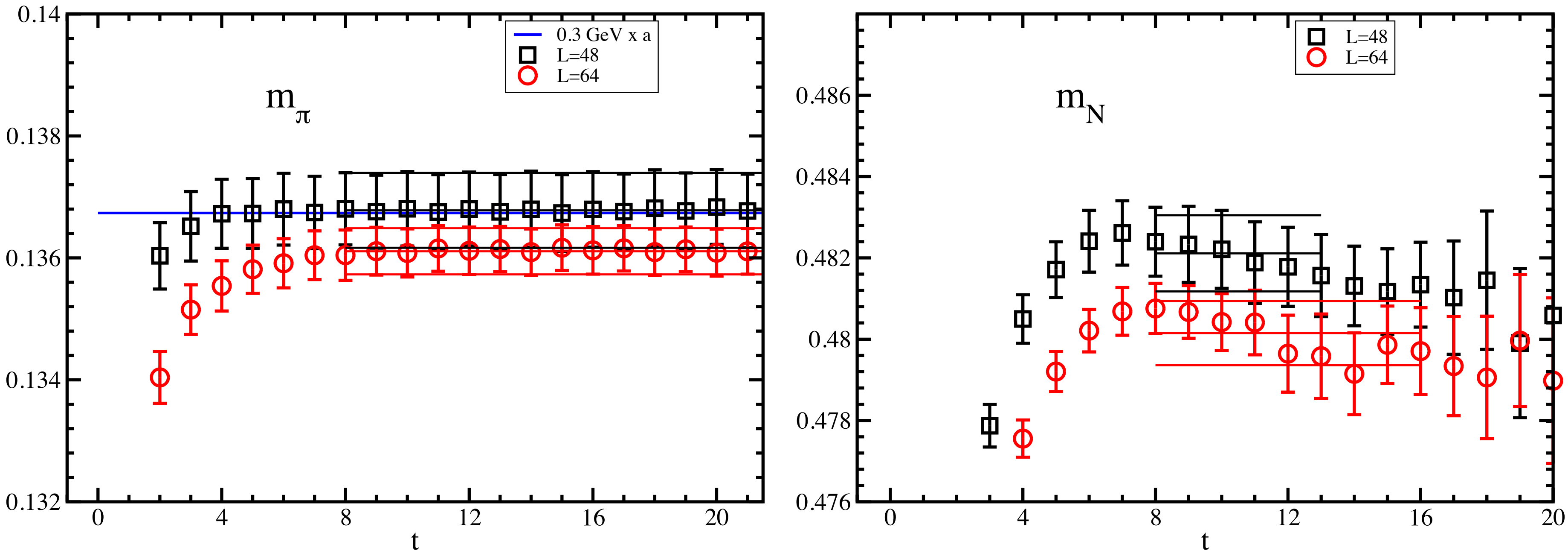
$m_s \sim$ physical strange quark mass

Finite volume dependence

		$m_\pi = 0.3$ GeV		$m_\pi = 0.5$ GeV	
L	L [fm]	N_{conf}	N_{meas}	N_{conf}	N_{meas}
32	2.9			200	192
40	3.6			200	192
48	4.3	400	1152	200	192
64	5.8	160	1536	190	256

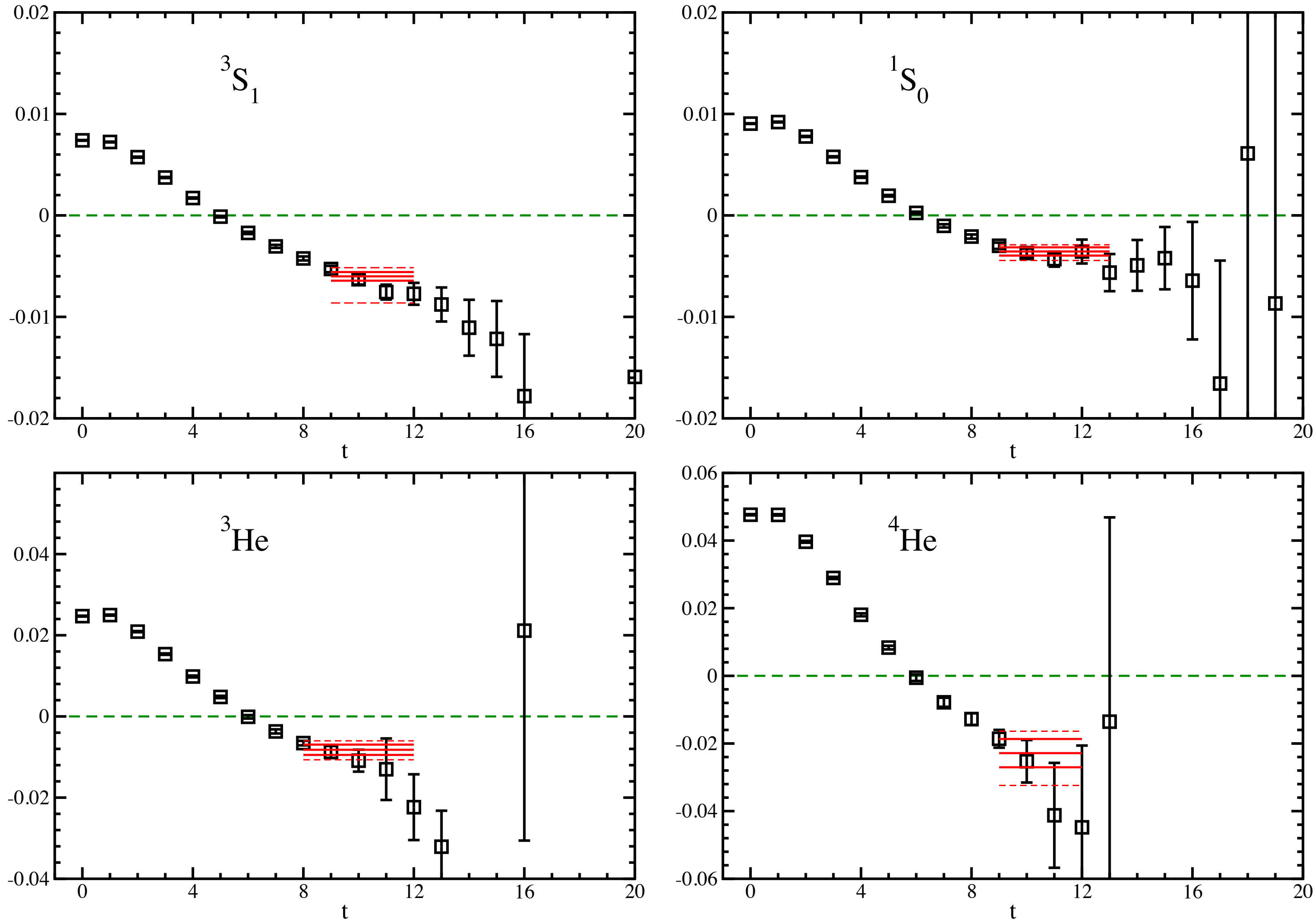
Effective mass @ $m_\pi = 0.3$GeV

Preliminary result of $N_f = 2 + 1$ TY *et al.*



effective ΔE_L @ $m_\pi = 0.3$GeV on $L = 48$

Preliminary result of $N_f = 2 + 1$ TY *et al.*



$L^3 \to \infty$ @ $m_\pi = 0.3$GeV

Preliminary result of $N_f = 2+1$ TY *et al.*