Review on
Quark masses

Francesco Sanfilippo

UNIVERSITY OF
Southampton
School of Physics
and Astronomy

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6 known quarks
Facts

6 known quarks

...and 30 minutes to talk...
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\[
\frac{30 \text{ minutes}}{\text{Up, Down, Strange, Charm, Bottom, Top}} = 5 \text{ Minutes per quark}
\]
6 known quarks

...but we must fit in 5 minutes for questions!
Facts

6 known quarks

...but we must fit in 5 minutes for questions!

\[
\text{25 minutes} \div (\text{Up, Down, Strange, Charm, Bottom, Top}) = 5 \text{ Minutes per quark}
\]
FLAG II - arXiv:1310.8555

- Flag did a great job
- They already gave an average for $m_l = \frac{m_u + m_d}{2}$, $m_s$, and $m_d/m_u$
- If they did the same for $m_c$ and $m_b$, I would’ve been even more relaxed

Methods that I want to discuss

Instead of filling you with numbers, I prefer to discuss the following points:

- Strategies for heavy quarks
- Relevance of quark mass ratios
- Nonperturbative renormalization approaches

Quantitative results that I want to discuss

- Collect the contributions presented at this conference for all quark masses
- Update averages for $m_c$, $m_b$, $m_d/m_u$
Relevance of quark mass values

Input parameters for other computations

Countless phenomenological applications
Examples:
- charm effect in the loops to B-physics observables in FCNC processes
- cross section of the $H \rightarrow b\bar{b}$ decay, dominant mode for a $m_H = 126$ GeV in SM

Consistency: Universality of continuum limit

- Quarks are confined, no comparison with $m_q^{\text{exp}}$ available. Instead, comparison in a specific renormalization scheme and at a specific renormalization scale.
- Traditionally it is $\overline{\text{MS}}$ and $\mu = 2$ GeV (except for c, b-quark), now moving to higher scales
- Higher scale $\rightarrow$ more accurate comparison with LQCD results obtained in non-MS schemes
- Increase in precision of the computation allows to check consistency of lattice methods

Flavor theory

- Grand Unified Theories predict quark masses in terms of other fundamental parameters
- Example SU(5): $m_e = m_d$, $m_\mu = m_s$, $m_\tau = m_b$
- We do not know the true Flavor model, so we can test ability of suggested models to reproduce quark mass hierarchy $\rightarrow$ provide bounds on GUT
Computing renormalized quark masses

Regularize the theory

\[ \mathcal{L}_{\overline{\text{QCD}}} = \sum_{f \in \{u, d, s, c\}} \bar{\psi}_f (D + m_f) \psi_f + \ldots \]

- Introduce regulator: lattice scale \( a \)
- \( N_f + 1 \) parameters: 1 for each quark and absolute scale, related to \( \Lambda_{\overline{\text{QCD}}} \)

Renormalize the theory

- Tune parameters to keep physics fixed while removing the cut-off
  - Appropriate choice: quantities strongly depending upon \( m_f \)
  - Typical choice: pseudoscalar meson masses and decay constants (recently also baryon masses)

- The procedure produces:
  - \textit{bare} quark masses (parameters of the Action)
  - lattice spacings
Lattice quark masses and their ratios

Lattice quark masses

Every lattice computation must tune quark masses to reproduce QCD in the continuum limit

- Tune through some quantity, typically meson masses (combining with continuum limit)
- Bare parameters of the Lagrangian: $a m_q^{\text{bare}}$ available to everybody
- Knowledge of $a m_q^{\text{bare}}$ describing constant physics line essential to perform simulations
- But not useful to compare between different regularizations

Ratio of quark masses

As long as the quark mass is multiplicatively renormalizable

$$m_q^{\text{ren}} = Z_m m_q^{\text{bare}}$$

and in renormalization schemes in which $Z_m$ does not depend upon $m_q$:

$$\partial_m Z_m = 0$$

ratio of renormalized quark masses can be computed through bare quark mass ratios:

$$\frac{m_{q_1}^{\text{ren}}}{m_{q_2}^{\text{ren}}} = \frac{Z_m m_{q_1}^{\text{bare}}}{Z_m m_{q_2}^{\text{bare}}} = \frac{a m_{q_1}^{\text{bare}}}{a m_{q_2}^{\text{bare}}}$$

Let us discuss a concrete example
Tuning $m_l = (m_u + m_d)/2$ and $m_s$ bare masses - MILC collaboration

**Light quark**

At each $\beta$ tune the ratio:

$$\frac{(a^2) M_{\pi}^2 (am)}{(a^2) f_{\pi}^2 (am)}$$

to reproduce $(M_{\pi}/f_{\pi})_\text{exp}^2$ and learn:
- $am^\text{bare}_{\text{light}}$ from corresponding $am$
- $a$ from $a f_{\pi} \left( am^\text{bare}_{\text{light}} \right) / f_{\pi}^\text{exp}$

**Strange quark**

Tune the quantity: $2M_K^2 - M_{\pi}^2$ (independent from $m_{\text{light}}$ at LO)

→ learn $m_s$

**Charm quark**

Similarly, tune $m_c$ to reproduce $M_{D_s}$

Javad Komijani talk, Wed. 25, 12.10
Extrapolating to the continuum $m_s/m_l$ - MILC collaboration

**Continuum limit**

Once determined

$$[m_s/m_l](a)$$

at each lattice spacing separately, the **continuum limit** must be taken

**Renormalization Group Invariant?**

- IF QED is not included or
- IF QED is included, for ratios of same charged quarks

**Undervalued quantities!**

- (Almost) Every lattice groups tuning to physical point is in the position to compute ratios
- But this information is **scarcely emphasized**
Renormalization approaches

To give an absolute value for the renormalized quark mass we need to know $Z_m$

### Non-Perturbative renormalization
- Rome-Southampton method or Schroedinger functional
- Then perturbatively matched to $\overline{MS}$ (conventionally)

### Perturbative renormalization
- Schroedinger functional **costly**
- Rome-Southampton not always easy to implement (e.g. Staggered quarks)

Forced to use **perturbation theory** to renormalize
- Convergence is quite unreliable and at least 2-loop perturbative correction is needed
- Difficult to go **beyond 2 loop** calculations on the lattice (but see: 3 loop stochastic computation by M.Brambilla et al., 1402.6581, cfr. [talk by M. Brambilla, Fri 27, 16.50](#))

### How to avoid the renormalization on the lattice?
- Compute a RGI quantity
- Match it to a continuum, perturbative computation in terms of $\overline{MS}$ masses and coupling

**Examples:**
- Moments of the correlators
- Energy of the non-relativistic heavy meson
Learning $Z_m$ from charm correlator moments


Adimensional moments of two points correlation function between charm currents:

$$G^{(j)}_n = \sum_t (t/a)^n G^{(j)}(t), \quad G^{(j)}(t) = \left( a m_c^{\text{bare}} \right)^2 \sum_{\vec{x}} \langle j^{\text{ren}}(\vec{x}, t) j^{\text{ren}}(\vec{0}, 0) \rangle$$

Reduced moments

$$R^{(j)}_n = \frac{a M_{\text{mes}j}}{2 a m_c^0} \sqrt{\frac{G^{(j)}_n G^{(j0)}_n}{G^{(j)}_{n-2} G^{(j0)}_{n-2}}}$$

- Built of bare lattice quantities
- Automatically renormalized (simplified expression if PCAC holds)
- Can be extrapolated to the continuum limit
- Perturbative if $n$ not too big (exponentially suppressed, $n-$power enhanced in $t$)

Update of C.McNeile et al., PRD82 (2010)
Learning $Z_m$ from charm correlator moments

**Lattice input**

$L_n^{\text{LQCD}}$ computed numerically:
- Interpolated to $am_c^\text{bare}$ reproducing $M_{\eta_c}$ (estimating $EM$ & disconn. diagram)
- Extrapolated to the continuum and chiral limit

**Continuum perturbation theory input**

- Khum et al. Nucl.Phys. B778 (2007), at 3 order $\alpha_s$
- Comparing $R_n^{\text{LQCD}}$ and $R_n^{\text{PQCD}}$

$$R_n^{\text{PQCD}} = \frac{r_n^{\text{PQCD}}(\alpha_{\text{MS}}, \mu/m_c)}{2m_c^{\text{MS}}(\mu)/M_{\text{mes}}^j}$$

In this way we learn $Z_m^{\text{MS}}(\frac{1}{a}) = m_c^{\text{MS}}(\frac{1}{a}) / am_c^\text{bare} / a$ using a **physical input** ($M_{\text{mes}}^j$)
Scared of non-perturbative effects?

Perturbativity issues

- HPQCD collaboration performed various checks:
  - stability of $m_c(\mu)$ as $n$ is changed to probe perturbativity window of $R_n$
  - extending the analytic parametrization of $R_n$ including condensates
- ETMC repeated this study and compared with $Z^{RI-MOM}_m$ (M.Petschlies, Lattice 2011):
  - compatible with direct determination based upon $Z^{RI-MOM}_m$ (preliminary!)
  - not clear advantage in terms of precision

Viability of the method

Is the method correct? Yes (for circumstantial evidence)
- Various internal consistency checks
- Results compatible with more traditional approaches

Is it useful? Yes and no
- Do not need to set-up Non Perturbative Renormalization program
- But it is subject to similar complications ($\alpha_s^m$ truncation, $n$-window, etc)

Not clearly superior, but a viable and interesting alternative

Future improvements and additional checks

HPQCD promised they will:
- check consistency with the RI-MOM-like determinations
- shift to determine $Z_m$ from $b$ quark in the future (more reliable perturbation theory)
Binding energy at finite lattice spacing

\[ M_{\Upsilon}^{\text{exp}} = 2m_b^{\text{pole}} + \Delta M_{\Upsilon}, \quad \Delta M_{\Upsilon} = \text{bind. energy} \]

- Non Relativistic QCD (NRQCD) is non-renormalizable
- \( m_b^{\text{pole}} \) can be determined by working at fixed lattice spacing
- Lattice-spacing-per-lattice-spacing: \( \Delta M = a^{-1}(aE_{\text{sim}} - 2aE_0) \)
- Relation between **divergent quantities** in the continuum limit

**Ingredients**

- tune \( \overline{M}_{bb} = a^{-1}(3aM_{\Upsilon}^{\text{sim}} + aM_{\eta_b}^{\text{sim}}) / 4 \) to its physical value, through kinetic energy \( M_{\text{kin}} \) extracted from dispersion relation of NRQCD meson \( \rightarrow m_b^{\text{bare}} \)
- compute \( \Delta M_{\Upsilon} \) subtracting (**power divergent in a!**) \( E_0 \) determined at 2 loops using automated perturbation theory & high \( \beta \) simulations (cfr. C.Monahan, Latt’13)
  
  Determine \( 2m_b^{\text{pole}} = M_{\Upsilon}^{\text{exp}} - a^{-1}(aE_{\Upsilon}^{\text{sim}} - 2aE_0) \), cross-check using \( B_s \)
  
  Compare different lattice spacing (no **continuum limit** can be taken)

**Outcome - Phys. Rev. D 87, 074018 (2013), HPQCD coll.**

- MILC 2+1 Asqtad ensembles, one-loop radiative corrected NRQCD action
- Convert \( m_b^{\text{pole}} \) to \( m_b^{\text{MS}}(m_b) = 4.166(43) \) GeV for \( N_f = 5 \)
- Improved over \( m_b^{\text{MS}}(m_b) = 4.4(3) \) GeV by A. Gray et al., PRD 72, (’05), including \( \mathcal{O}(\beta^2) \)
Matching HQET and QCD

After long efforts **Alpha** matched **HQET** to **QCD** at $O(1/m_h)$

$$\mathcal{L}^{HQET} = \bar{\psi}_h \left[ \left(D_0 + m_{\text{bare}}\right) - \omega_{\text{kin}} D^2 - \omega_{\text{spin}} \sigma \cdot B \right] \psi_h$$

by making use of Step Scaling method [cfr. JHEP 1209 (2012) 132]

The theory is renormalizable order by order.

Observable of Expansion at $O(1/m_h)$

Terms $\propto \omega_{\text{kin}}, \omega_{\text{spin}}$ are of $O(1/m_h)$ and treated as **operator insertions**:

$$\langle O \rangle = \langle O \rangle_{\text{stat}} + \omega_{\text{kin}} \langle O \bar{\psi}_h D^2 \psi_h \rangle_{\text{stat}} + \omega_{\text{kin}} \langle O \bar{\psi}_h \sigma \cdot B \psi_h \rangle_{\text{stat}}$$

and similarly

$$M_B = m_{\text{bare}} + E_{\text{stat}} + \omega_{\text{kin}} E_{\text{kin}} + \omega_{\text{spin}} E_{\text{spin}},$$

$E_{\text{kin}}, E_{\text{spin}}$ determined from time behavior of correlation functions with **operator insertions**

Determination of $m^r_{b}$

Interpolate $M_B (m_{\text{bare}})$ to the $m_{\text{bare}}$ reproducing $M_B^{\text{exp}}$ while:

- chirally and continuum extrapolating $M_B (m_{\text{bare}}, M_\pi, a)$ in HMChPT
- considering $m_{\text{bare}}$ as a function of RGI mass as determined with Schröedinger Functional
- converting it to $m^\text{MS}_b$ using perturbation theory

✓ $N_f = 2$, improved wrt the quenched computation [M.Della Morte, JHEP 0701 (2007)]
✓ $1/m_h$ corrections turn out to be very small
**ETM** \( N_f = 2 + 1 + 1 \) determination (presented at Latt.’13)

**RI-MOM for** \( N_f = 2 + 1 + 1 \) - 1403.4504

- Mass independent renormalization: all masses much smaller than \( \mu \)
- Usual approach to match \( \overline{\text{MS}} \): take chiral limit of \( Z \) as done for observables
- \( N_f = 2 + 1 + 1 \) simulations contain massive \( s \) and \( c \) quarks

ETM collaboration performed dedicated simulations with \( N_f = 4 \) light quarks

**Cut-off effects**

- Quark masses determined tuning \( f_\pi \) and pseudoscalar meson masses
- Reduce cut-off effects taking ratios between similar quantities (e.g. \( M_\pi / M_\bar{s}s, M_D / M_\bar{c}s \))
ETM $N_f = 2 + 1 + 1$ determination of $b$ quark mass

Extrapolating from $c$ region

- The mass $M_{hl}$ of a heavy-light meson diverges in the static limit: $\lim_{m_h \to \infty} \frac{M_{hl}}{m_h} = 1$
- Could be directly used to extrapolate $M_{hl}(m_h, m_l, a)$ from $h = c$ region

Ratio method [cfr. R.Frezzotti et al., JHEP 1004 (2010)]

- Instead, consider a series of masses $m^{(0)} = m_c, m^{(1)} = \lambda m_c, \ldots m^{(n)} = \lambda^n m_c$,

$$y \left( m_h^{(n)}, \lambda; m_l, a \right) = \lambda \frac{M_{hl}(m_h^{(n)}; m_l, a)}{M_{hl}(m_h^{(n)}/\lambda; m_l, a)} \xrightarrow{m_h \to \infty} 1$$
- Compute $y \left( m_h^{(n)}, \lambda; m_l, a \right)$, extrapolate to the continuum, and reconstruct $M_{hl}(m_h, m_l)$

Results for $m_b$

- Tune $m_b$ to reproduce $M_B$ [see: N.Carrasco et al., JHEP 1403 (2014)]
- Preliminary improvement:
  - Use GEVP
  - Adopt more sophisticated ratios $y_Q$

![Figure 2: $c = 0.75$: $y_Q$ against $1/\mu_h$ at NLL using SL, w-opt and GEVP two-point correlation functions. The fit ansatz is of the form $y_Q(\mu_h) = 1 + \eta_1 \mu_h + \eta_2 \mu_h^2$. Input data of the $M^2$ type have been used.](image)
Physical point simulation

- $N_f = 2 + 1 + 1$ Möbius Domain Wall fermions,
- 2 lattice spacings: $a^{-1} = 2.358(7), 1.730(4)$ GeV
- Quark masses essentially at the physical point ($M_\pi = 139$ MeV)

Global fit

- How to re-tune to $M_{\pi_0} = 135$ MeV?
- Combine with heavier pion data to slightly extrapolate

\[
m_l^{\overline{\text{MS}}} (3 \text{ GeV}) = 3.014(39)_{\text{stat}}(0)_{\text{chir}}(5)_{\text{fse}}(35)_{\text{ren}} \text{ MeV}
\]

\[
m_s^{\overline{\text{MS}}} (3 \text{ GeV}) = 82.27(92)_{\text{stat}}(0)_{\text{chir}}(6)_{\text{fse}}(95)_{\text{ren}} \text{ MeV}
\]

- Use many inputs in a global fit ($M_K$, $M_\pi$, $M_\Omega$, $f_K$, $f_\pi$, etc.)
**Physical inputs**

- **Fix** $m_s$ from triply stranged baryon $\Omega$

  ![Graph](image1.png)

- Fix $m_c$ from singly charmed baryon $\Lambda_c$

  ![Graph](image2.png)

- Lattice spacings determined using Pion & Proton masses

**Chiral and continuum extrapolation**

\[
\begin{align*}
M_\Omega &= M_{\Omega}^{\text{chir}} + c_\Omega M_\pi^2 + d_\Omega a^2 \\
M_{\Lambda_c} &= M_{\Lambda_c}^{\text{chir}} + c_\Omega^{(2)} M_\pi^2 + c_\Omega^{(3)} M_\pi^3 + d_\Omega a^2
\end{align*}
\]

More challenging than meson analysis: less well founded Chiral theory and FSE guidance

**Outcomes**

- Observed mild dependence on volume
- Reasonable agreement with determination obtained in meson sector

Cfr. Ch. Kallidonis talk Wed. 25, 09:40
**Electromagnetism**

### Hadron Self Energy

Correct inputs used to fix quark masses
- Neutral pseudo-Goldstone boson masses corrected only at $O(e^2 m)$
- Compute electromagnetic contribution to meson masses: $\hat{M}_P = M_P - \Delta M^{QED}$
  - $\hat{M}_\pi^2$ and $\left[ \hat{M}_{K^+}^2 + \hat{M}_{K^0}^2 \right]$ at LO independent of $m_u - m_d \rightarrow$ use to determine $m_l$ and $m_s$
  - $\hat{M}_{K^+}^2 - \hat{M}_{K^0}^2 \propto B_2 (m_d - m_u)$ at LO, use to determine $m_d - m_u$
- Note: separation of QED and QCD contributions requires defining a scheme

### BMW results

- Electro-quenched simulations (not related to recent QCD+QED project 1406.4088)
- Determined from ChPT LEC $B_2$ (1310.3626) and Kaon mass difference PRL 111 (2013)
- **Preliminary:** $m_{u,d}^{\overline{MS}}(2 \text{ GeV}) = \{2.29(6)(5), 4.65(6)(5)\}$, $m_u/m_d = 0.49(1)(1)$

### Other results

- ETM combining with RM123 results obtained expanding IB at first order PRD87 (2013)
- Fermilab: updating the Kaon mass splitting results of 1301.7137 combining with quark mass dependence found in decay constant analysis (cfr. talk by J.Komijani, Wed 26)
\( m_b^{\overline{\text{MS}}} (m_b) \) [GeV]

- **ETM 2014 (prel), \( N_f=2+1+1 \), Ratio method**
- **ETM 2013, \( N_f=2 \), Ratio method**
- **HPQCD 2013, \( N_f=2+1 \), NRQCD**
- **Alpha 2013, \( N_f=2 \), HQET**
- **HPQCD 2010, \( N_f=2+1 \), Corr.moments**

**PDG**
$m_b/m_c$

ETMC, $N_f=2+1+1$, prel
HPQCD 2010, $N_f=2+1$
# Conclusions

## Ratios of quark masses
- Renormalization constants cancel in ratios
- Many groups could contribute to estimate quantities such as $m_s/m_l$
- Please **come forward**...

## Absolute quark mass values
- Many ways to compute renormalized quark masses
- Only a few results currently available for heavy quarks

## Thanks a lot to...
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