Review on Quark masses

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6 known quarks

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...and 30 minutes to talk...

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$$\frac{30\,\text{minutes}}{\text{Up, Down, Strange, Charm, Bottom, Top}} = 5\,\text{Minutes per quark}$$

6 known quarks

...but we must fit in 5 minutes for questions!

6 known quarks

...but we must fit in 5 minutes for questions!

$$\frac{25\,\text{minutes}}{\text{Up, Down, Strange, Charm, Bottom, }\cancel{\textit{Top}}} = 5\,\text{Minutes per quark}$$

Introduction

FLAG II - arXiv:1310.8555

- Flag did a great job
- They already gave an average for $m_l = \frac{m_u+m_d}{2}$, m_s and m_d/m_u
- If they did the same for m_c and m_b, I would've been even more relaxed

Methods that I want to discuss

Instead of filling you with numbers, I prefer to discuss the following points:

- Strategies for heavy quarks
- Relevance of quark mass ratios
- Nonperturbative renormalization approaches

Quantitative results that I want to discuss

- Collect the contributions presented at this conference for all quark masses
- Update averages for m_c, m_b, m_d/m_u

Relevance of quark mass values

Input parameters for other computations

Countless phenomenological applications

Examples:

- charm effect in the loops to B-physics observables in FCNC processes
- cross section of the $H \to b\bar{b}$ decay, dominant mode for a $m_H = 126\,\mathrm{GeV}$ in SM

Consistency: Universality of continuum limit

- Quarks are confined, no comparison with m_q^{exp} available. Instead, comparison in a specific renormalization scheme and at a specific renormalization scale.
- Traditionally it is $\overline{\mathrm{MS}}$ and $\mu = 2$ GeV (except for c, b-quark), now moving to higher scales
- Higher scale $\to$ more accurate comparison with LQCD results obtained in non-MS schemes
- Increase in precision of the computation allows to check consistency of lattice methods

Flavor theory

- Grand Unified Theories predict quark masses in terms of other fundamental parameters
- Example SU(5): $m_e = m_d$, $m_\mu = m_s$, $m_\tau = m_b$
- We do not know the true Flavor model, so we can **test ability** of suggested models to reproduce quark mass hierarchy $\to$ provide bounds on GUT

Computing renormalized quark masses

Regularize the theory

$$\mathcal{L}_{QCD} = \sum_{f\in\{u,d,s,c\}} \bar{\psi}_f \left(\not{D} + m_f \right) \psi_f + \dots$$

- Introduce regulator: lattice scale a
- $N_f + 1$ parameters: 1 for each quark and absolute scale, related to Λ_{QCD}

Renormalize the theory

- Tune parameters to keep physics fixed while removing the cut-off
  - Appropriate choice: quantities strongly depending upon m_f
  - Typical choice: pseudoscalar meson masses and decay constants (recently also baryon masses)
- The procedure produces:
  - **bare** quark masses (parameters of the Action)
  - lattice spacings

Lattice quark masses and their ratios

Lattice quark masses

Every lattice computation must tune quark masses to reproduce QCD in the continuum limit

- Tune through some quantity, typically meson masses (combining with continuum limit)
- Bare parameters of the Lagrangian: am_q^{bare} available to everybody
- Knowledge of am_q^{bare} describing constant physics line **essential** to perform simulations
- But not useful to compare between different regularizations

Ratio of quark masses

As long as the quark mass is multiplicatively renormalizable

$$m_q^{ren} = Z_m m_q^{bare}$$

and in renormalization schemes in which Z_m does not depend upon m_q:

$$\partial_{m_q} Z_m = 0$$

ratio of **renormalized** quark masses can be computed through bare quark mass ratios:

$$\frac{m_{q_1}^{ren}}{m_{q_2}^{ren}} = \frac{Z_m m_{q_1}^{bare}}{Z_m m_{q_2}^{bare}} = \frac{am_{q_1}^{bare}}{am_{q_2}^{bare}}$$

Let us discuss a concrete example

Tuning $m_l = (m_u + m_d)/2$ and m_s bare masses - MILC collaboration

Light quark

At each β tune the ratio:

$$\frac{(a^2)\, M_\pi^2\,(am)}{(a^2)\, f_\pi^2\,(am)}$$

to reproduce $(M_\pi / f_\pi)^2_{exp}$ and learn:

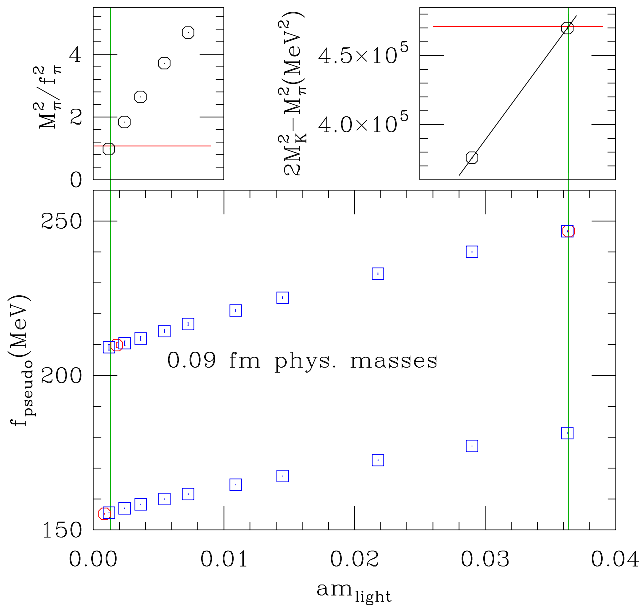
- am^{bare}_{light} from corresponding am
- a from $af_\pi \left(am^{bare}_{light} \right) / f_\pi^{exp}$

Strange quark

Tune the quantity: $2M_K^2 - M_\pi^2$ (independent from m_{light} at LO) $\rightarrow$ learn m_s

Charm quark

Similarly, tune m_c to reproduce M_{D_s}



Javad Komijani talk, Wed. 25, 12.10

Extrapolating to the continuum m_s/m_l - MILC collaboration

Continuum limit

Once determined

$$[m_s/m_l]\,(a)$$

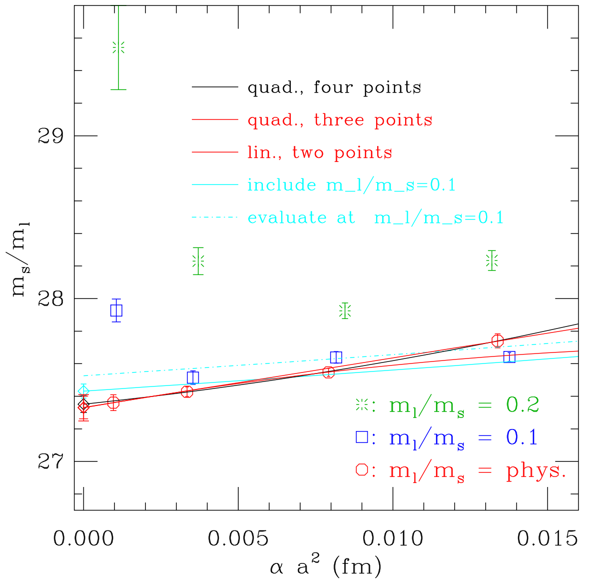
at each lattice spacing separately, the continuum limit must be taken

Renormalization Group Invariant?

- IF QED is not included or
- IF QED is included, for ratios of same charged quarks

Undervalued quantities!

- (Almost) Every lattice groups tuning to physical point is in the position to compute ratios
- But this information is scarcely emphasized



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Renormalization approaches

To give an absolute value for the renormalized quark mass we need to know Z_m

Non-Perturbative renormalization

- **Rome-Southampton** method or **Schroedinger functional**
- Then perturbatively matched to $\overline{\mathrm{MS}}$ (conventionally)

Perturbative renormalization

- Schroedinger functional **costly**
- Rome-Southampton not always easy to implement (e.g. Staggered quarks)

Forced to use perturbation theory to renormalize

- Convergence is quite unreliable and at least 2-loop perturbative correction is needed
- Difficult to go **beyond 2 loop** calculations on the lattice (but see: 3 loop stochastic computation by M.Brambilla et al., 1402.6581, cfr. talk by M. Brambilla, Fri 27, 16.50)

How to avoid the renormalization on the lattice?

- Compute a RGI quantity
- Match it to a continuum, perturbative computation in terms of $\overline{\mathrm{MS}}$ masses and coupling

Examples:

- Moments of the correlators
- Energy of the non-relativistic heavy meson

Learning Z_m from charm correlator moments

Starting point - HPQCD coll., Phys.Rev. D78 (2008) 054513

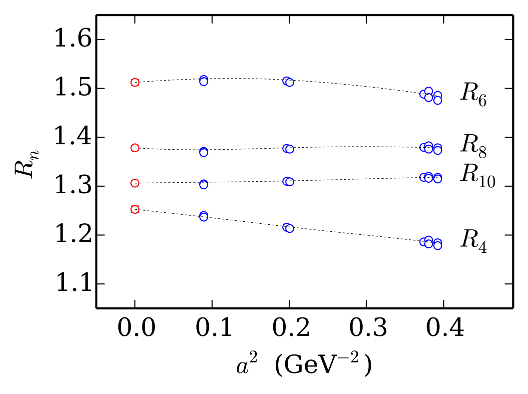
Adimensional moments of two points correlation function between charm currents:

$$G_n^{(j)} = \sum_t (t/a)^n G^{(j)}(t), \qquad G^{(j)}(t) = \left(am_c^{bare}\right)^2 \sum_{\vec{x}} \left\langle j^{ren}(\vec{x},t)\, j^{ren}\left(\vec{0},0\right)\right\rangle$$

Reduced moments

$$R_n^{(j)} = \frac{aM_{mes\,j}}{2am_c^0}\sqrt{\frac{G_n^{(j)}}{G_{n-2}^{(j)}}\frac{G_{n-2}^{(j0)}}{G_n^{(j0)}}}$$

- Built of bare lattice quantities
- Automatically renormalized (simplified expression if PCAC holds)
- Can be extrapolated to the continuum limit
- Perturbative if n not too big (exponentially suppressed, $n-$power enhanced in t)



Update of C.McNeile et al., PRD82 (2010)

Learning Z_m from charm correlator moments

Lattice input

R_n^{LQCD} computed numerically:

- Interpolated to am_c^{bare} reproducing M_{η_c} (estimating *EM* & disconn. diagram)
- Extrapolated to the continuum and chiral limit

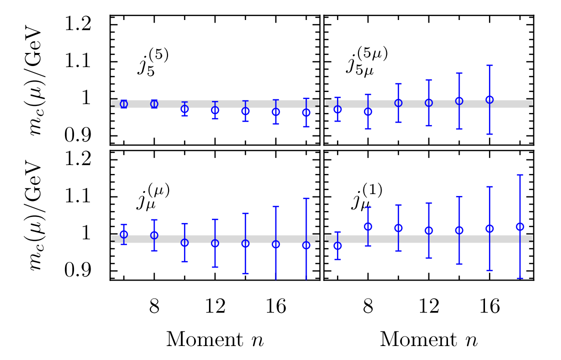
Continuum perturbation theory input

- Khum et al. Nucl.Phys. B778 (2007), at 3 order α_s

$$R_n^{PQCD} = \frac{r_n^{PQCD}\left(\alpha_{\overline{\mathrm{MS}}}, \mu/m_c\right)}{2m_c^{\overline{\mathrm{MS}}}\left(\mu\right)/M_{mes\,j}}$$

- Comparing R_n^{LQCD} and R_n^{PQCD}

$$m_c^{\overline{\mathrm{MS}}}\left(\mu\right) = \frac{M_{mes\,j}^{exp}}{2}\frac{r_n^{PQCD}}{R^{LQCD}}$$



In this way we learn $Z_m^{\overline{MS}}\left(\frac{1}{a}\right) = m_c^{\overline{\mathrm{MS}}}\left(\frac{1}{a}\right)/am_c^{bare}/a$ using a **physical input** ($M_{mes\,j}$)

Scared of non-perturbative effects?

Perturbativity issues

- HPQCD collaboration performed various checks:
  - stability of $m_c(\mu)$ as n is changed to probe perturbativity window of R_n
  - extending the analytic parametrization of R_n including condensates
- ETMC repeated this study and compared with Z_m^{RI-MOM} (M.Petschlies, Lattice 2011):
  - compatible with direct determination based upon Z_m^{RI-MOM} (preliminary!)
  - not clear advantage in terms of precision

Viability of the method

Is the method correct? Yes (for circumstantial evidence)

- Various internal consistency checks
- Results compatible with more traditional approaches

Is it useful? Yes and no

- Do not need to set-up Non Perturbative Renormalization program
- But it is subject to similar complications (α_s^m truncation, n-window, etc)

Not clearly superior, but a viable and interesting alternative

Future improvements and additional checks

HPQCD promised they will:

- check consistency with the RI-MOM-like determinations
- shift to determine Z_m from b quark in the future (more reliable perturbation theory)

Binding energy of non-relativistic heavy meson

Binding energy at finite lattice spacing

$$M_\Upsilon^{exp} = 2m_b^{pole} + \Delta M_\Upsilon, \quad \Delta M_\Upsilon = \mathrm{bind.\,energy}$$

- Non Relativistic QCD (NRQCD) is non-renormalizable
- m_b^{pole} can be determined by working at fixed lattice spacing
- Lattice-spacing-per-lattice-spacing: $\Delta M = a^{-1}\left(aE^{sim} - 2aE^0\right)$
- Relation between **divergent quantities** in the continuum limit

Ingredients

- tune $\overline{M_{b\bar{b}}} = a^{-1}\left(3aM_\Upsilon^{sim} + aM_{\eta_b}^{sim}\right)/4$ to its physical value, through kinetic energy M_{kin} extracted from dispersion relation of NRQCD meson $\rightarrow m_b^{bare}$
- compute ΔM_Υ subtracting (**power divergent** in a!) E^0 determined at 2 loops using automated perturbation theory & high β simulations (cfr. C.Monahan, Latt'13)

Determine $2m_b^{pole} = M_\Upsilon^{exp} - a^{-1}\left(aE_\Upsilon^{sim} - 2aE^0\right)$, cross-check using B_s

Compare different lattice spacing (<u>no continuum limit</u> can be taken)

Outcome - Phys. Rev. D 87, 074018 (2013), HPQCD coll.

- MILC 2+1 Asqtad ensembles, one-loop radiative corrected NRQCD action
- Convert m_b^{pole} to $m_b^{\overline{\mathrm{MS}}}(m_b) = 4.166(43)$ GeV for $N_f = 5$
- Improved over $m_b^{\overline{\mathrm{MS}}}(m_b) = 4.4(3)\ \mathrm{GeV}$ by A. Gray et al., PRD 72, ('05), including $\mathcal{O}\left(\beta^2\right)$

Alpha collaboration approach to heavy quarks - Phys.Lett. B730 (2014)

Matching *HQET* and *QCD*

After long efforts **Alpha** matched *HQET* to *QCD* at $\mathcal{O}\left(1/m_h\right)$

$$\mathcal{L}^{HQET} = \bar{\psi}_h\left[\left(D_0 + m^{bare}\right) - \omega_{\rm kin}\mathbf{D}^2 - \omega_{spin}\boldsymbol{\sigma}\cdot\mathbf{B}\right]\psi_h$$

by making use of Step Scaling method [cfr. JHEP 1209 (2012) 132]
The theory is renormalizable order by order

Observable of Expansion at $\mathcal{O}(1/m_h)$

Terms $\propto \omega_{kin}, \omega_{spin}$ are of $\mathcal{O}\left(1/m_h\right)$ and treated as **operator insertions**:

$$\langle O\rangle = \langle O\rangle_{stat} + \omega_{kin}\left\langle O\bar{\psi}_h\mathbf{D}^2\psi_h\right\rangle_{stat} + \omega_{kin}\left\langle O\bar{\psi}_h\boldsymbol{\sigma}\cdot\mathbf{B}\psi_h\right\rangle_{stat}$$

and similarly $$M_B = m^{bare} + E_{stat} + \omega_{kin}E_{kin} + \omega_{spin}E_{spin}\,,$$

E_{kin}, E_{spin} determined from time behavior of correlation functions with **operator insertions**

Determination of m_b^{ren}

Interpolate $M_B\left(m^{bare}\right)$ to the m_b^{bare} reproducing M_B^{exp} while:

- chirally and continuum extrapolating $M_B\left(m^{bare}, M_\pi, a\right)$ in HMChPT
- considering m^{bare} as a function of RGI mass as determined with Schroedinger Functional
- converting it to $m_b^{\overline{\rm MS}}$ using perturbation theory

✓ $N_f = 2$, improved wrt the quenched computation [M.Della Morte, JHEP 0701 (2007)]
✓ $1/m_h$ corrections turn out to be very small

ETM $N_f = 2+1+1$ determination (presented at Latt.'13)

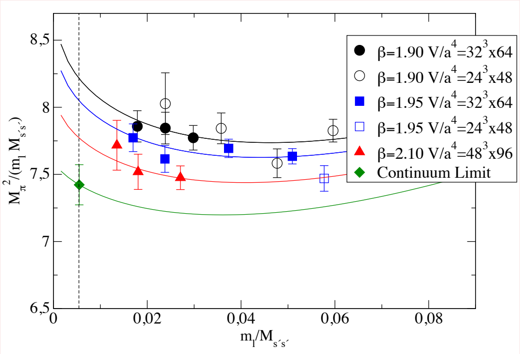
RI-MOM for $N_f = 2+1+1$ - 1403.4504

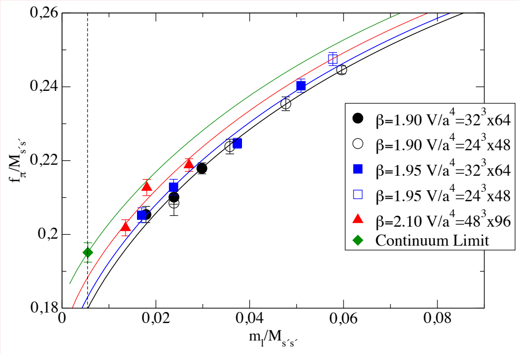
- Mass independent renormalization: all masses much smaller than μ
- Usual approach to match $\overline{\text{MS}}$: take chiral limit of Z as done for observables
- $N_f = 2+1+1$ simulations contain massive s and c quarks

ETM collaboration performed dedicated simulations with $N_f = 4$ light quarks

Cut-off effects

- Quark masses determined tuning f_π and pseudoscalar meson masses
- Reduce cut-off effects taking ratios between similar quantities (e.g. $M_\pi/M_{\bar{s}s}$, $M_{D_s}/M_{\bar{c}s}$)





ETM $N_f = 2+1+1$ determination of b quark mass

Extrapolating from c region

- The mass M_{hl} of a heavy-light meson diverges in the static limit: $\lim_{m_h \to \infty} \frac{M_{hl}}{m_h} = 1$
- Could be directly used to extrapolate $M_{hl}(m_h, m_l, a)$ from $h = c$ region

Ratio method [cfr. R.Frezzotti et al., JHEP 1004 (2010)]

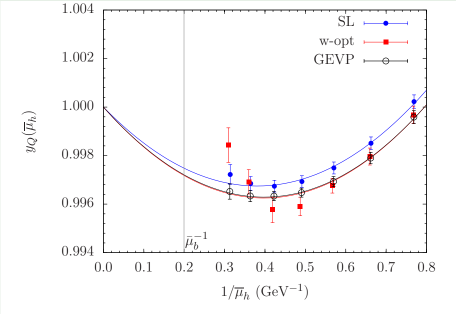
- Instead, consider a series of masses $m^{(0)} = m_c$, $m^{(1)} = \lambda m_c$, $\ldots$ $m^{(n)} = \lambda^n m_c$,

$$y\left(m_h^{(n)}, \lambda;\, m_l,\, a\right) = \lambda \frac{M_{hl}\left(m_h^{(n)}; m_l, a\right)}{M_{hl}\left(m_h^{(n)}/\lambda; m_l, a\right)} \xrightarrow{m_h \to \infty} 1$$

- Compute $y\left(m_h^{(n)}, \lambda; m_l,\, a\right)$, extrapolate to the continuum, and reconstruct $M_{hl}(m_h,\, m_l)$

Results for m_b

- Tune m_b to reproduce M_B
  [see: N.Carrasco et al., JHEP 1403 (2014)]
- Preliminary improvement:
  - Use GEVP
  - Adopt more sophisticated ratios y_Q



RBC/UKQCD Physical point computation

Physical point simulation

- $N_f = 2+1+1$ Möbius Domain Wall fermions,
- 2 lattice spacings: $a^{-1} = 2.358(7),\ 1.730(4)$ GeV
- Quark masses essentially at the physical point ($M_\pi = 139$ MeV)

Global fit

- How to re-tune to $M_{\pi_0} = 135$ MeV?
- Combine with heavier pion data to slightly extrapolate

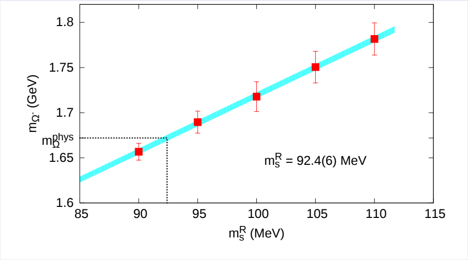
$$
\begin{aligned}
m_l^{\overline{\mathrm{MS}}}(3\,\mathrm{GeV}) &= 3.014(39)_{stat}(0)_{chir}(5)_{fse}(35)_{ren}\,\mathrm{MeV} \\
m_s^{\overline{\mathrm{MS}}}(3\,\mathrm{GeV}) &= 82.27(92)_{stat}(0)_{chir}(6)_{fse}(95)_{ren}\,\mathrm{MeV}
\end{aligned}
$$

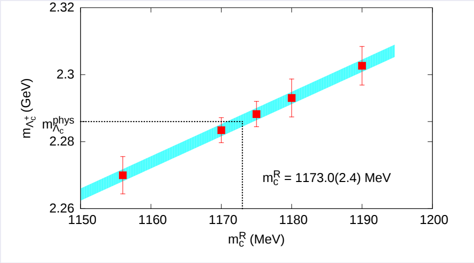
- Use many inputs in a **global fit** (M_K, M_π, M_Ω, f_K, f_π, etc.)

Tuning $m_{c,s}$ from baryon spectrum - C. Alexandrou et al., arXiv:1406.4310

Physical inputs

- Fix m_s from triply stranged baryon Ω
- Fix m_c from singly charmed baryon Λ_c





- Lattice spacings determined using Pion & Proton masses

Chiral and continuum extrapolation

$$M_\Omega = M_\Omega^{chir} + c_\Omega M_\pi^2 + d_\Omega a^2$$
$$M_{\Lambda_c} = M_{\Lambda_c}^{chir} + c_\Omega^{(2)} M_\pi^2 + c_\Omega^{(3)} M_\pi^3 + d_\Omega a^2$$

More challenging than meson analysis: less well founded Chiral theory and FSE guidance

Outcomes — Cfr. Ch.Kallidonis talk Wed. 25, 09:40

- Observed mild dependence on volume
- Reasonable agreement with determination obtained in meson sector

Electromagnetism

Hadron Self Energy

Correct inputs used to fix quark masses

- Neutral pseudo-Goldstone boson masses corrected only at $\mathcal{O}\left(e^2 m\right)$
- Compute electromagnetic contribution to meson masses: $\hat{M}_P = M_P - \Delta M^{QED}$
  - $\hat{M}^2_\pi$ and $\left[\hat{M}^2_{K^+} + \hat{M}^2_{K^0}\right]$ at LO independent of $m_u - m_d \to$ use to determine m_l and m_s
  - $\hat{M}^2_{K^+} - \hat{M}^2_{K^0} \propto B_2\left(m_d - m_u\right)$ at LO, use to determine $m_d - m_u$
- Note: separation of QED and QCD contributions requires defining a scheme

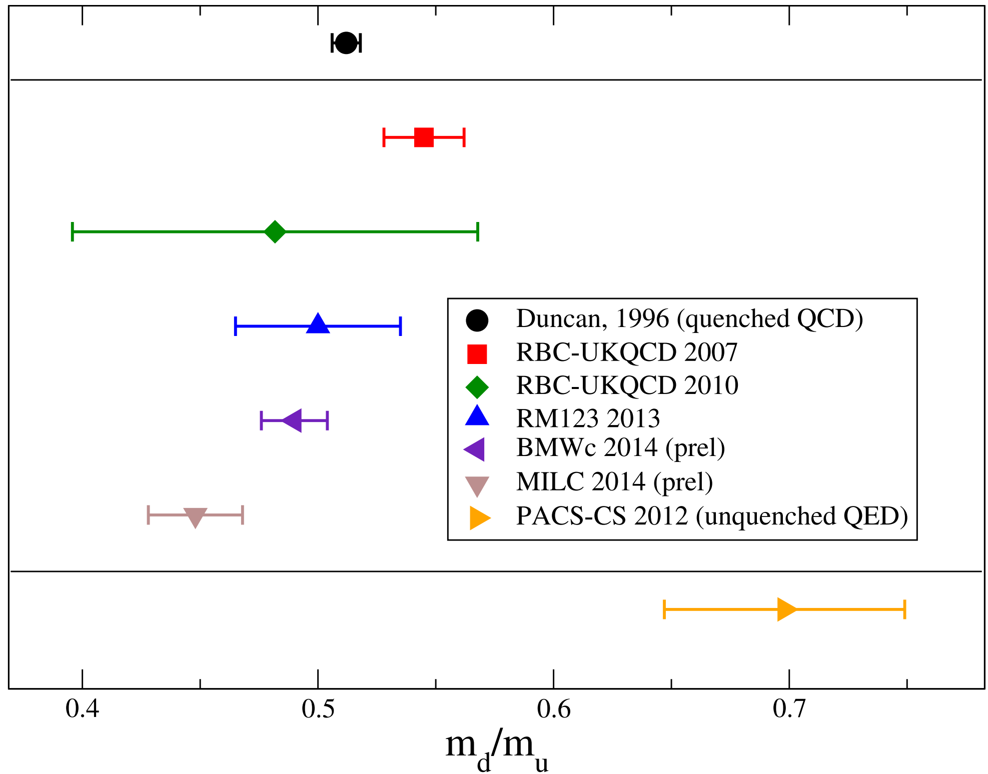
BMW results

- Electro-quenched simulations (not related to recent QCD+QED project 1406.4088)
- Determined from ChPT LEC B_2 (1310.3626) and Kaon mass difference PRL 111 (2013)
- **Preliminary**: $m^{\overline{\mathrm{MS}}}_{u,d}(2\,\mathrm{GeV}) = \{2.29(6)(5), 4.65(6)(5)\}$, $m_u/m_d = 0.49(1)(1)$

Other results

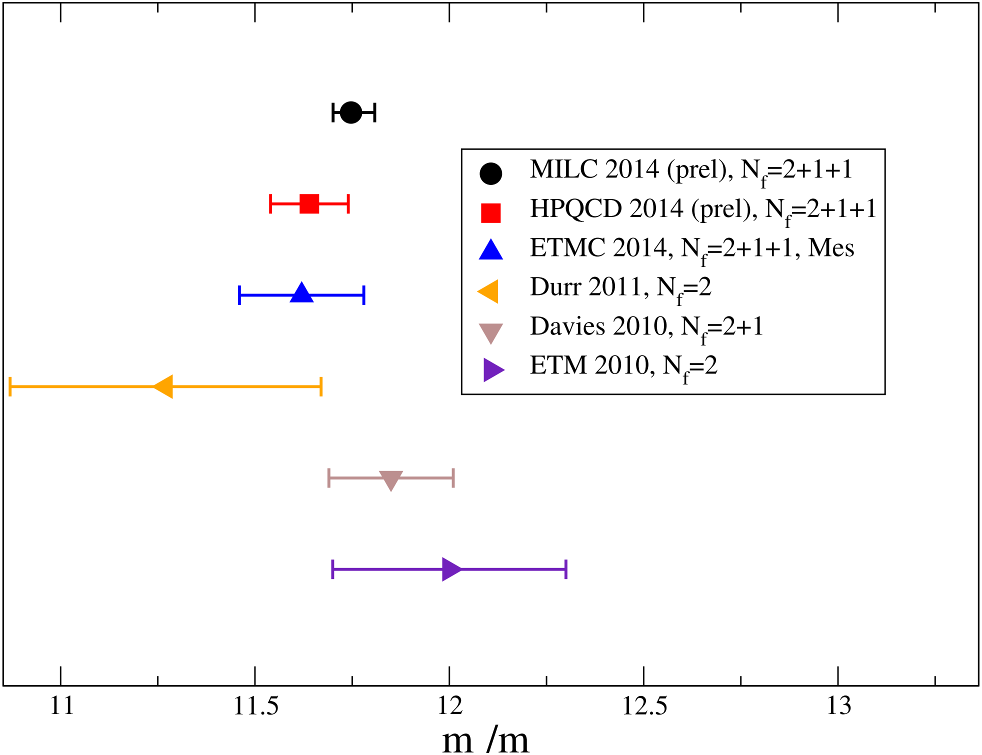
- ETM combining with RM123 results obtained expanding IB at first order PRD87 (2013)
- Fermilab: updating the Kaon mass splitting results of 1301.7137 combining with quark mass dependence found in decay constant analysis (cfr. talk by J.Komijani, Wed 26)

m_d/m_u



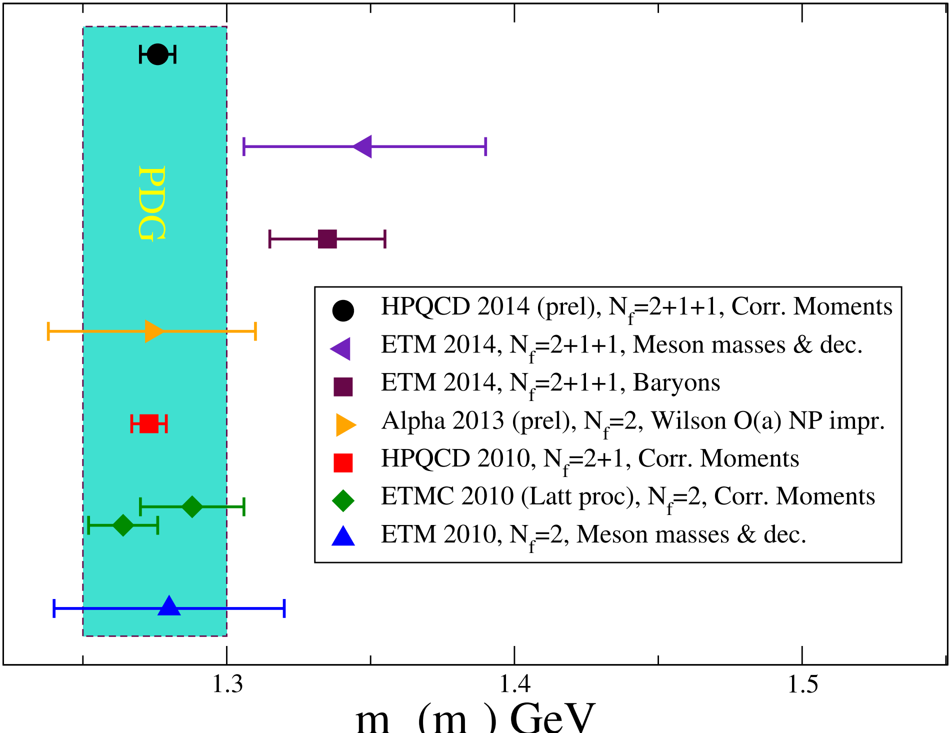
0.4
0.5
0.6
0.7
m_d/m_u
Duncan, 1996 (quenched QCD)
RBC-UKQCD 2007
RBC-UKQCD 2010
RM123 2013
BMWc 2014 (prel)
MILC 2014 (prel)
PACS-CS 2012 (unquenched QED)

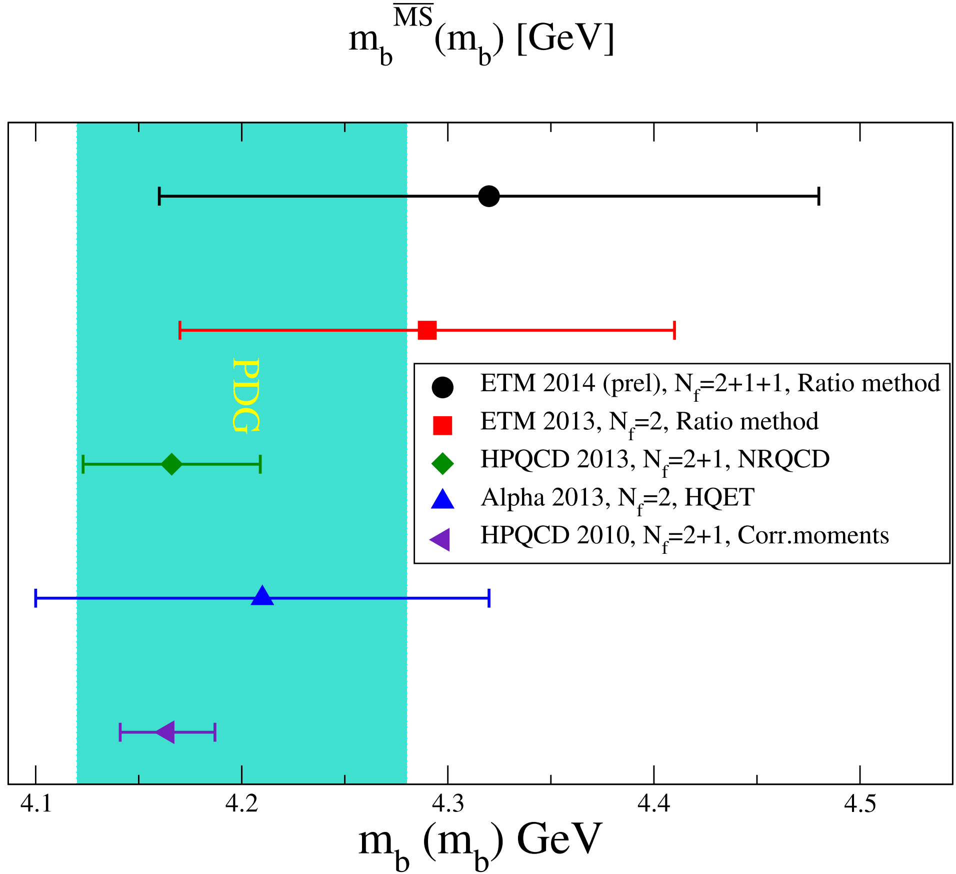
m_c/m_s



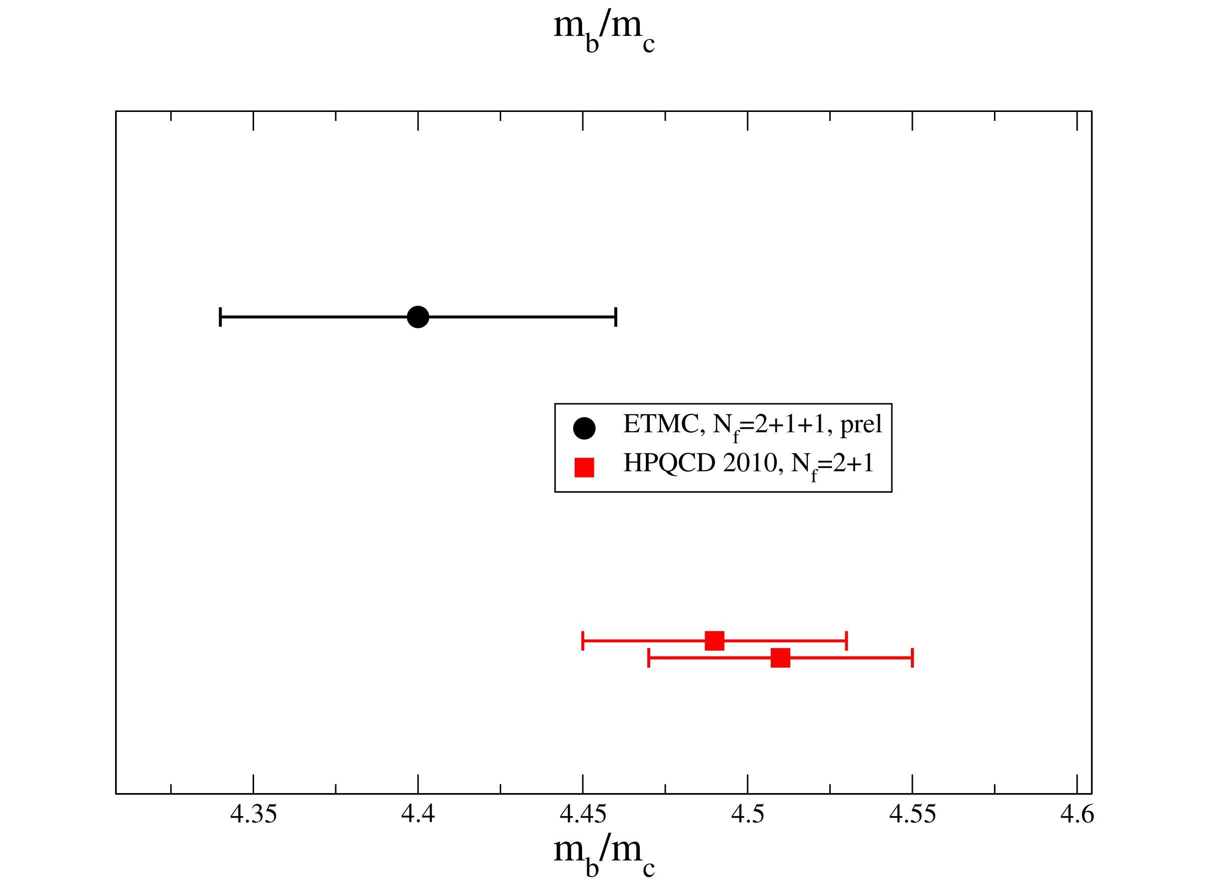
MILC 2014 (prel), N_f=2+1+1
HPQCD 2014 (prel), N_f=2+1+1
ETMC 2014, N_f=2+1+1, Mes
Durr 2011, N_f=2
Davies 2010, N_f=2+1
ETM 2010, N_f=2
11
11.5
12
12.5
13
m_c/m_s

$m_c^{\overline{MS}}(m_c)$ [GeV]





$m_b^{\overline{MS}}(m_b)$ [GeV]
PDG
ETM 2014 (prel), N_f=2+1+1, Ratio method
ETM 2013, N_f=2, Ratio method
HPQCD 2013, N_f=2+1, NRQCD
Alpha 2013, N_f=2, HQET
HPQCD 2010, N_f=2+1, Corr.moments
4.1
4.2
4.3
4.4
4.5
m_b (m_b) GeV



m_b/m_c
ETMC, N_f=2+1+1, prel
HPQCD 2010, N_f=2+1
4.35
4.4
4.45
4.5
4.55
4.6
m_b/m_c

Conclusions

Ratios of quark masses

- Renormalization constants cancel in ratios
- Many groups could contribute to estimate quantities such as m_s/m_l
- Please **come forward**...

Absolute quark mass values

- Many ways to compute renormalized quark masses
- Only a few results currently available for heavy quarks

Thanks a lot to...

- D.Becirevic
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- A.Portelli
- C.Sachrajda
- C.Tarantino

For **sending** their material and for the very **useful discussion!**