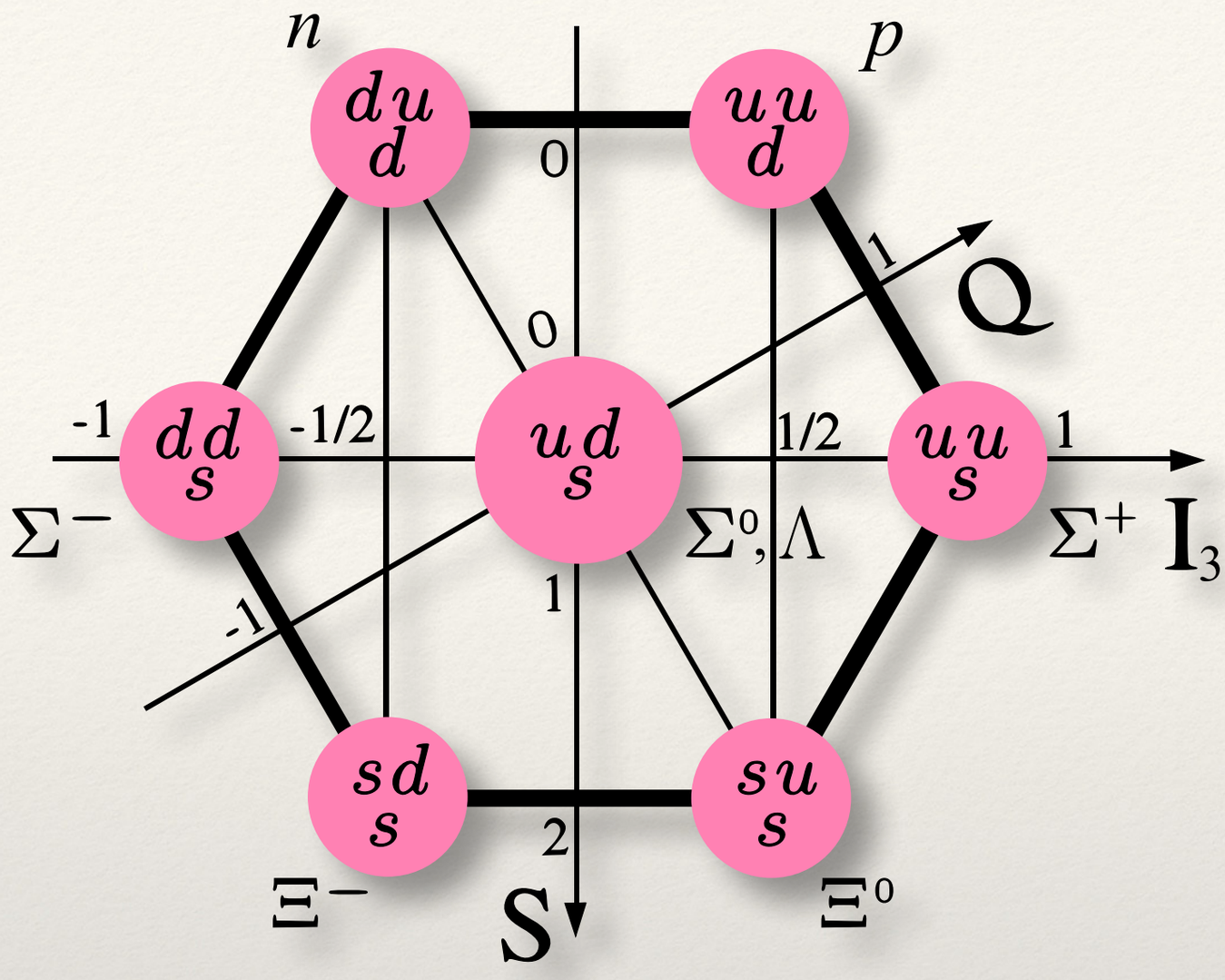

26th of June 2014 — Lattice 2014 (NYC, USA)

Inclusion of isospin breaking effects in lattice simulations

Antonin J. Portelli
(University of Southampton)

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- [Davoudi & Savage, 2014] — **[arXiv:1402.6741]**
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  - preliminary results for the baryon octet splittings
- [BMWc, 2014] — **[arXiv: 1406.4088]**
  - new set of $N_f = 1+1+1+1$ full QCD+QED simulations
  - extensive analytical/numerical study of finite-volume effects
  - high precision computation of the hadron spectrum splittings (continuum, infinite volume and physical point extrapolation)

- Motivations
- Update on electro-quenched results
- Lattice QED
- Full QCD+QED simulations
- Isospin splittings in the hadron spectrum
- Summary & outlook

Motivations

Isospin symmetry breaking

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	up	down
Mass (MeV)	$2.3(^{+0.7}_{-0.5})$	$4.8(^{+0.5}_{-0.3})$
Charge (e)	2/3	-1/3

source: [PDG, 2013]

Isospin symmetry breaking

- Isospin symmetric world: up and down quarks are particles with identical physical properties.
- Isospin symmetry is explicitly broken by:
  - the up and down **quark mass difference**

$$|m_u - m_d|/\Lambda_{\mathrm{QCD}} \simeq 0.01$$

  - the up and down **electric charge difference**

$$\alpha \simeq 0.0073$$

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- Well known experimentally:

$$\boxed{M_n - M_p = 1.2933322(4)\ \text{MeV}}$$

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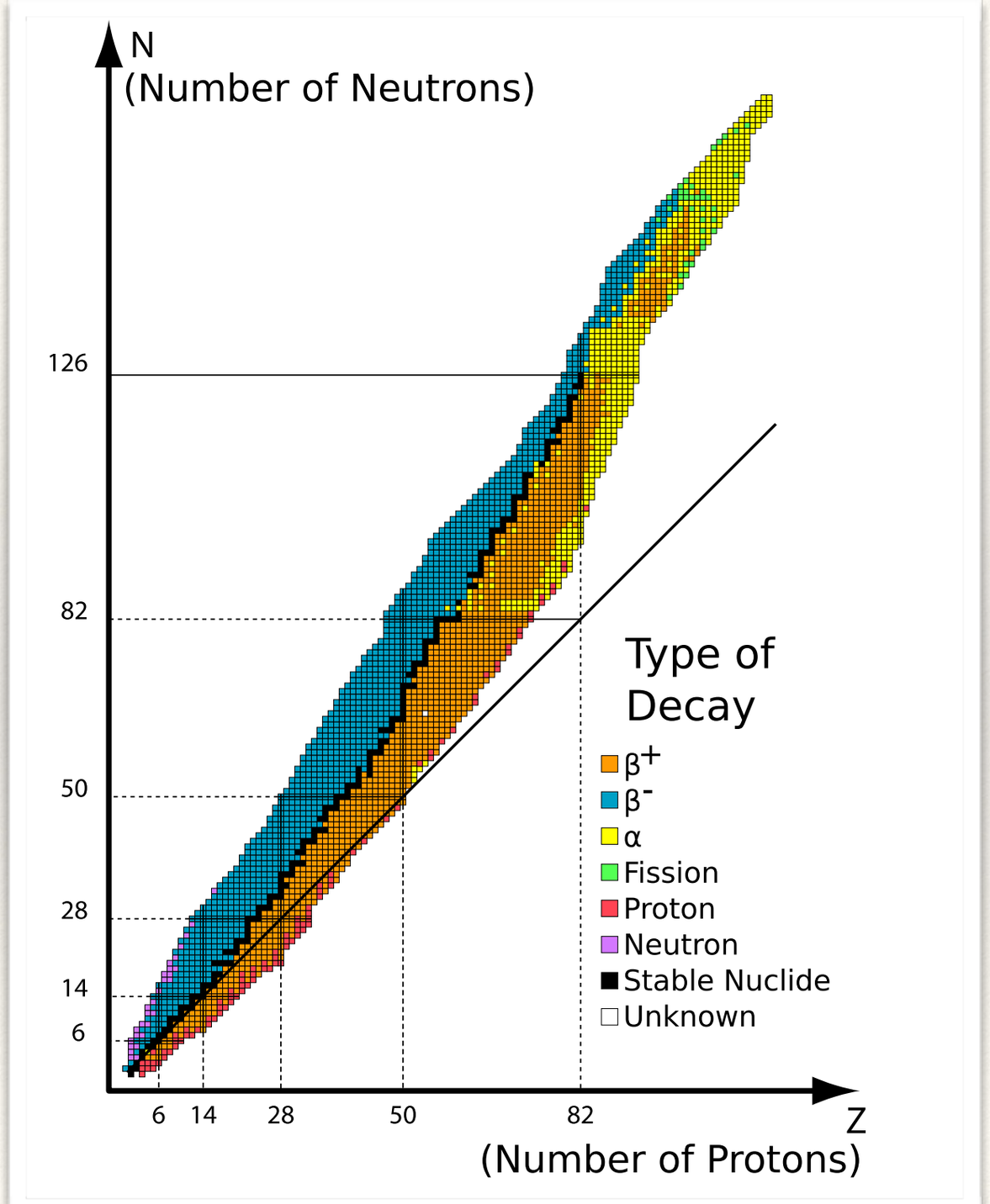

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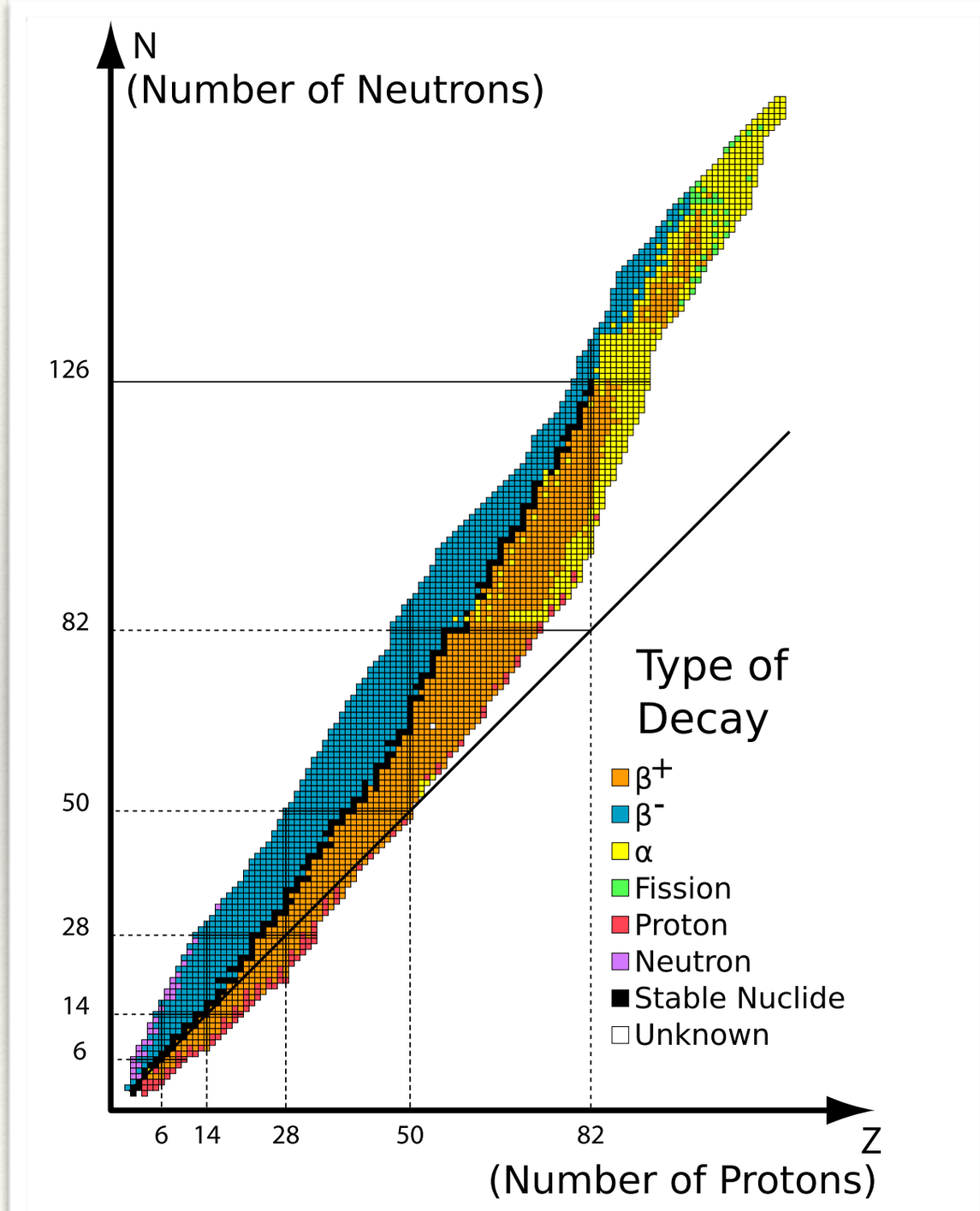
- initial condition for **Big-Bang nucleosynthesis**



Dashen's theorem

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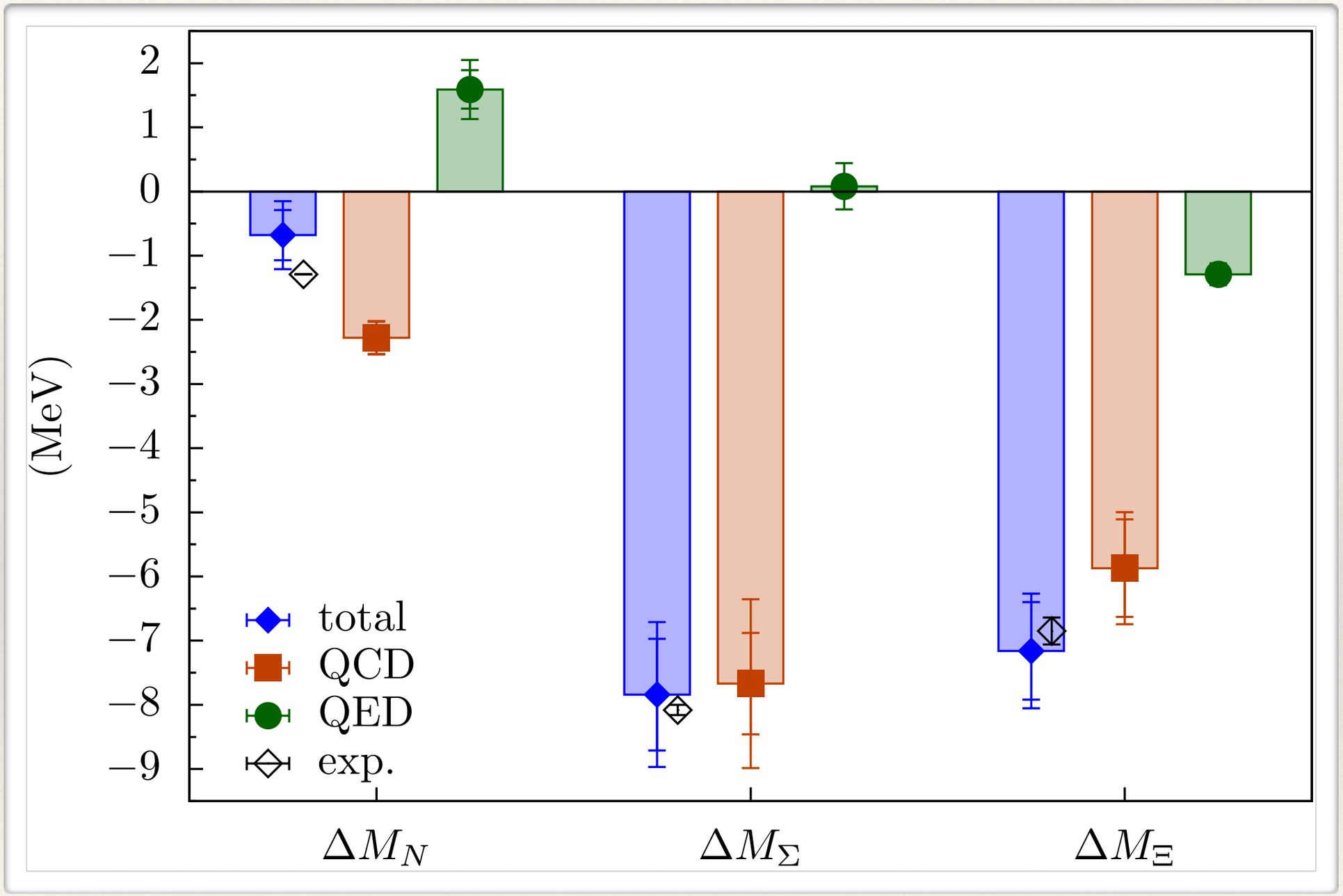
- ε is **important to determine light quark mass ratios**

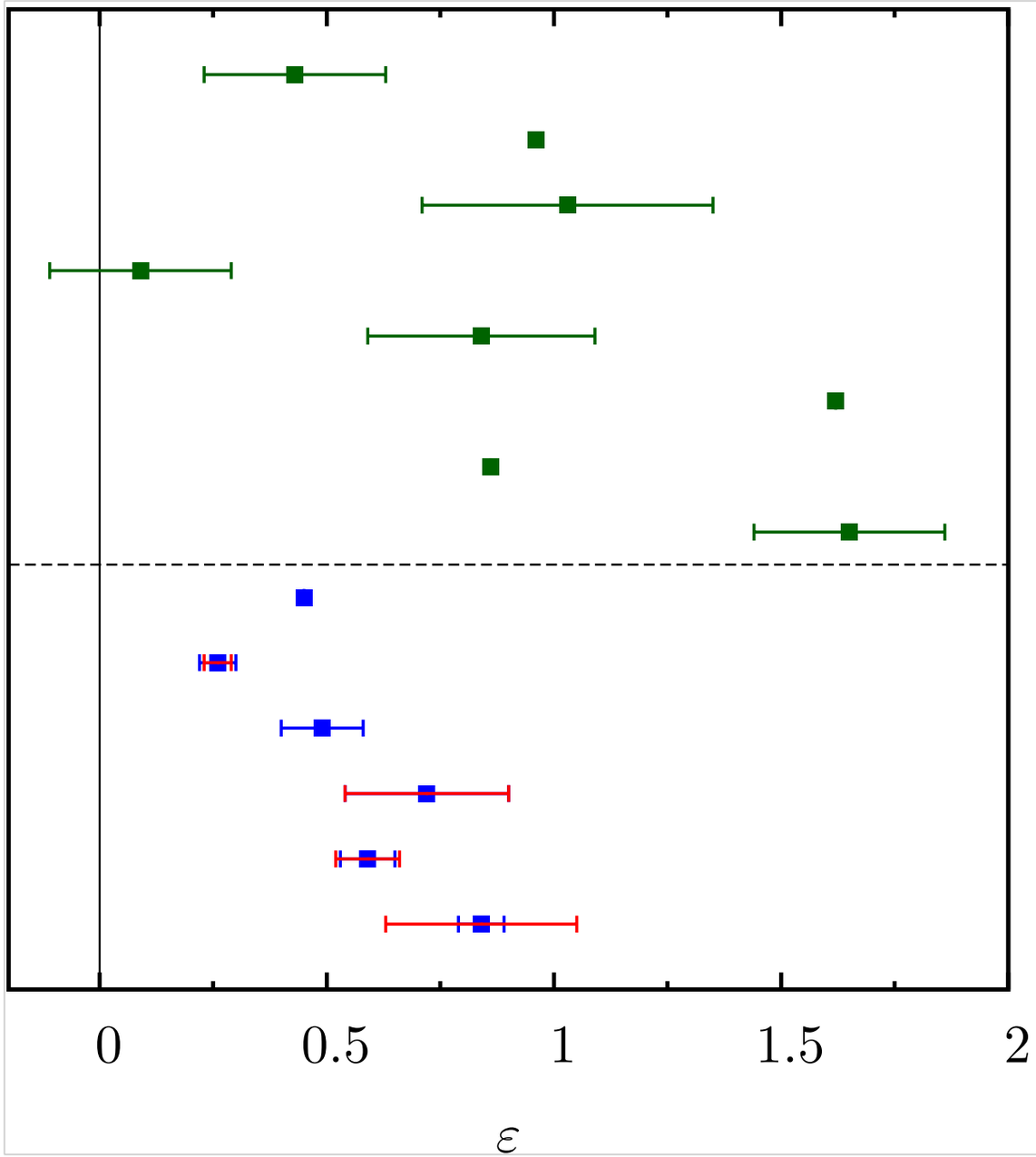
Update on electro-quenched results

EQ results for the baryon spectrum



[BMWc, 2013] (EQ): PRL 111(25) 252001, arXiv: 1306.2287

EQ results for ε



[Maltman and Kotchan, 1990]
[Donoghue *et al.*, 1993]
[Bijnens, 1993]
[Baur and Urech, 1996]
[Bijnens and Prades, 1997]
[Donoghue and Perez, 1997]
[Gao *et al.*, 1997]
[Moussallam, 1997]
[Duncan *et al.*, 1996] (quenched QCD)
[RBC-UKQCD, 2007]
[RBC-UKQCD, 2010]
[RM123, 2013]
[BMWc, Q, 2014] (preliminary)
[MILC, 2014] (preliminary)

EQ results for light quark masses

F. Sanfilippo plenary talk on quark masses: right after this talk

Lattice QED

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- Naively discretised **Maxwell action**:

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Zero-mode subtraction

Finite volume: **momentum quantisation**

$$\alpha \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2} \cdots \quad \longmapsto \quad \frac{\alpha}{V} \sum_k \frac{1}{k^2} \cdots$$

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$$\uparrow$$

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Contains a straight 1/0 !

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- Some more interesting that others

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- $\mathrm{QED}_{\mathrm{TL}}$ **does not have reflection positivity**

$\mathrm{QED}_{\mathrm{TL}}$ finite-volume effects

- Example — 1-loop $\mathrm{QED}_{\mathrm{TL}}$ [BMWc, 2014]:

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\left[1-\frac{\pi}{2\kappa}\frac{T}{L}\right]\right) - \frac{3\pi}{(mL)^3}\left[1-\frac{\coth(mT)}{2}\right] - \frac{3\pi}{2(mL)^4}\frac{L}{T}\right]\right\}$$

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- **Divergent finite volume effects** with $T \to \infty$, $L = \mathrm{cst.}$
- Same behaviour independently discovered by MILC

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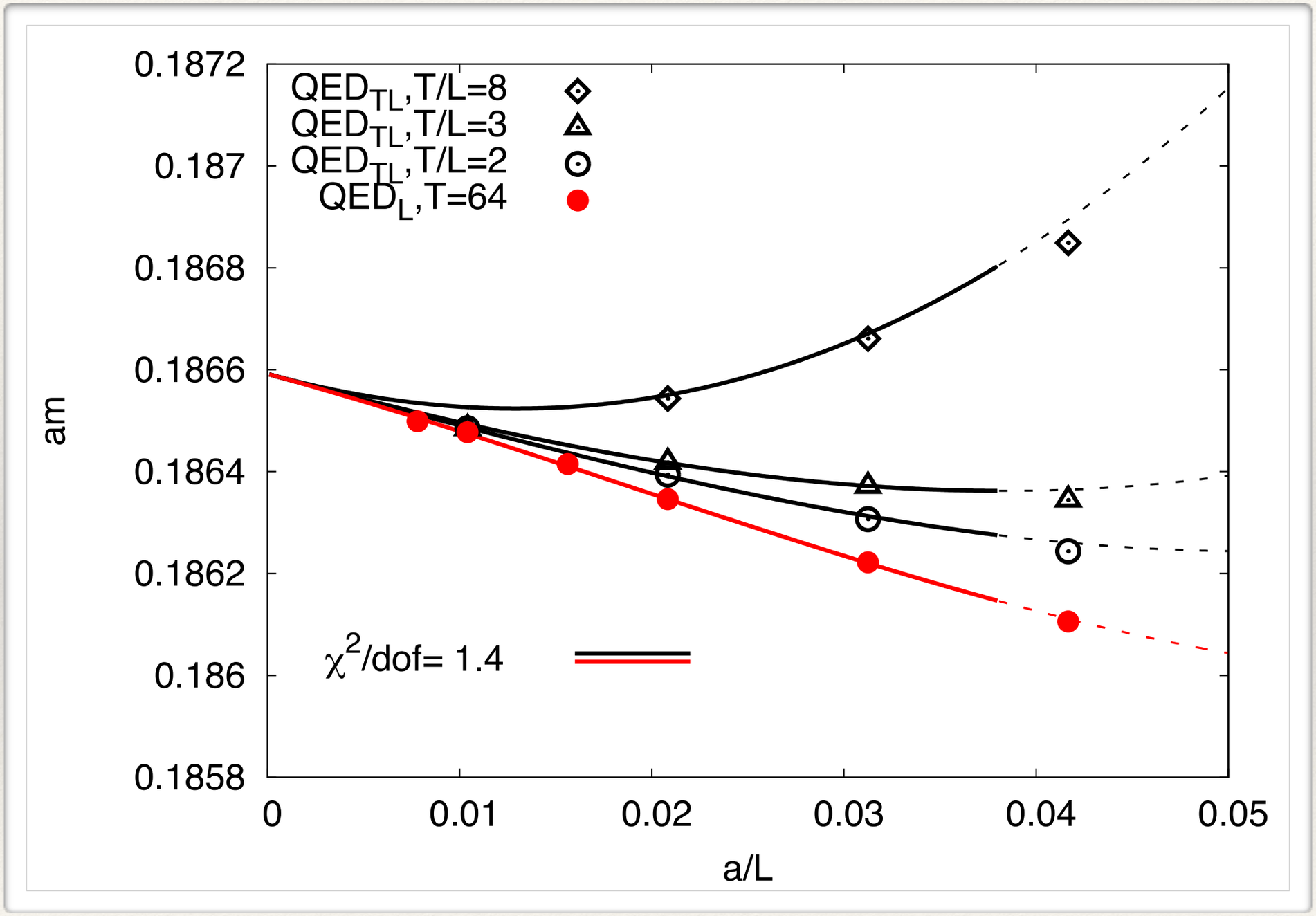
inverse powers of L, independent of T

Finite-volume effects



Pure QED simulations (quenched) from [BMWc, 2014]

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- parallel talk by **C. Lehner: tomorrow 15:35**

Full QCD+QED simulations

Full QCD + QED projects

	RBC-UKQCD	PACS-CS	**QCDSF-UKQCD**	**BMWc**
arXiv	1006.1311	1205.2961	**1311.4554 and Lat. 2014**	**1406.4088**
fermions	DWF	clover	**clover**	**clover**
N_f	2+1	1+1+1	**1+1+1**	**1+1+1+1**
method	reweighting	reweighting	**RHMC**	**RHMC**
$\min(M_\pi)$ (MeV)	420	135	**250**	**195**
a (fm)	0.11	0.09	**0.08**	**0.06 — 0.10**
$\#a$	1	1	**1**	**4**
L (fm)	1.8	2.9	**1.9 — 2.6**	**2.1 — 8.3**
$\#L$	1	1	**2**	**11**

Starting simulation program by MILC: **R. Zhou talk Monday 14:15**

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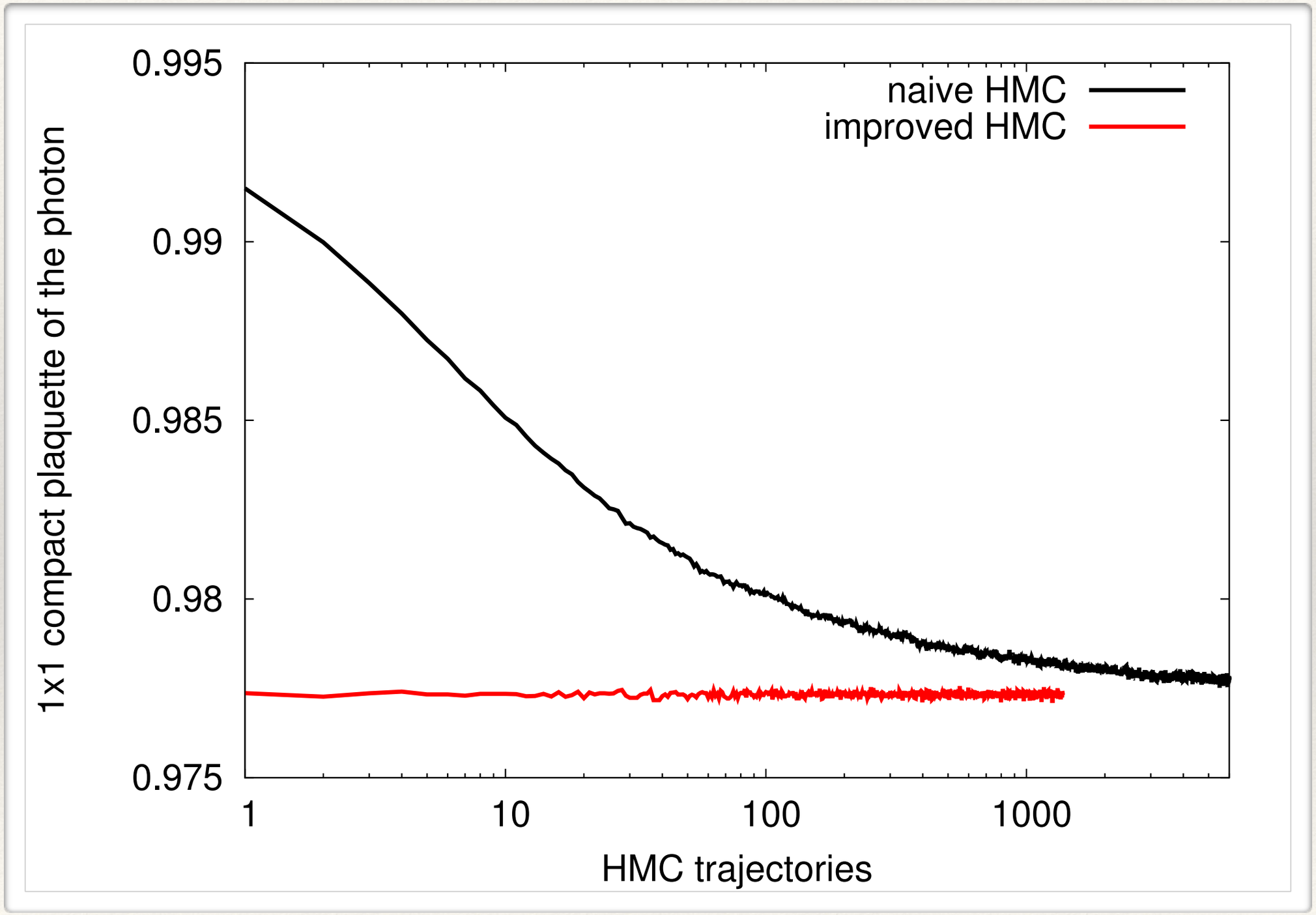
- Clover term greatly reduces discretisation errors

[BMWc, 2014]: QED simulations



Isospin splittings in the hadron spectrum

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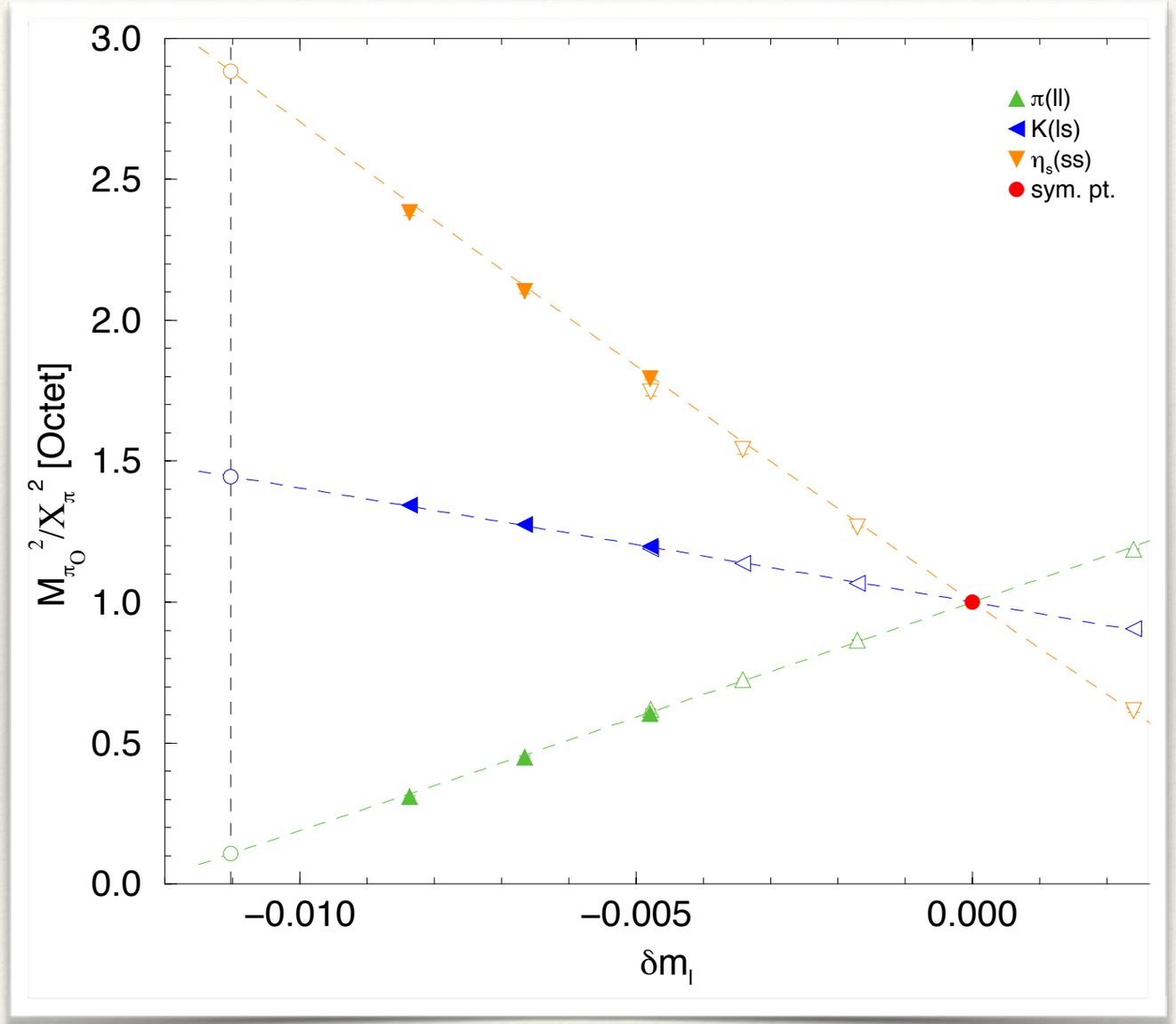

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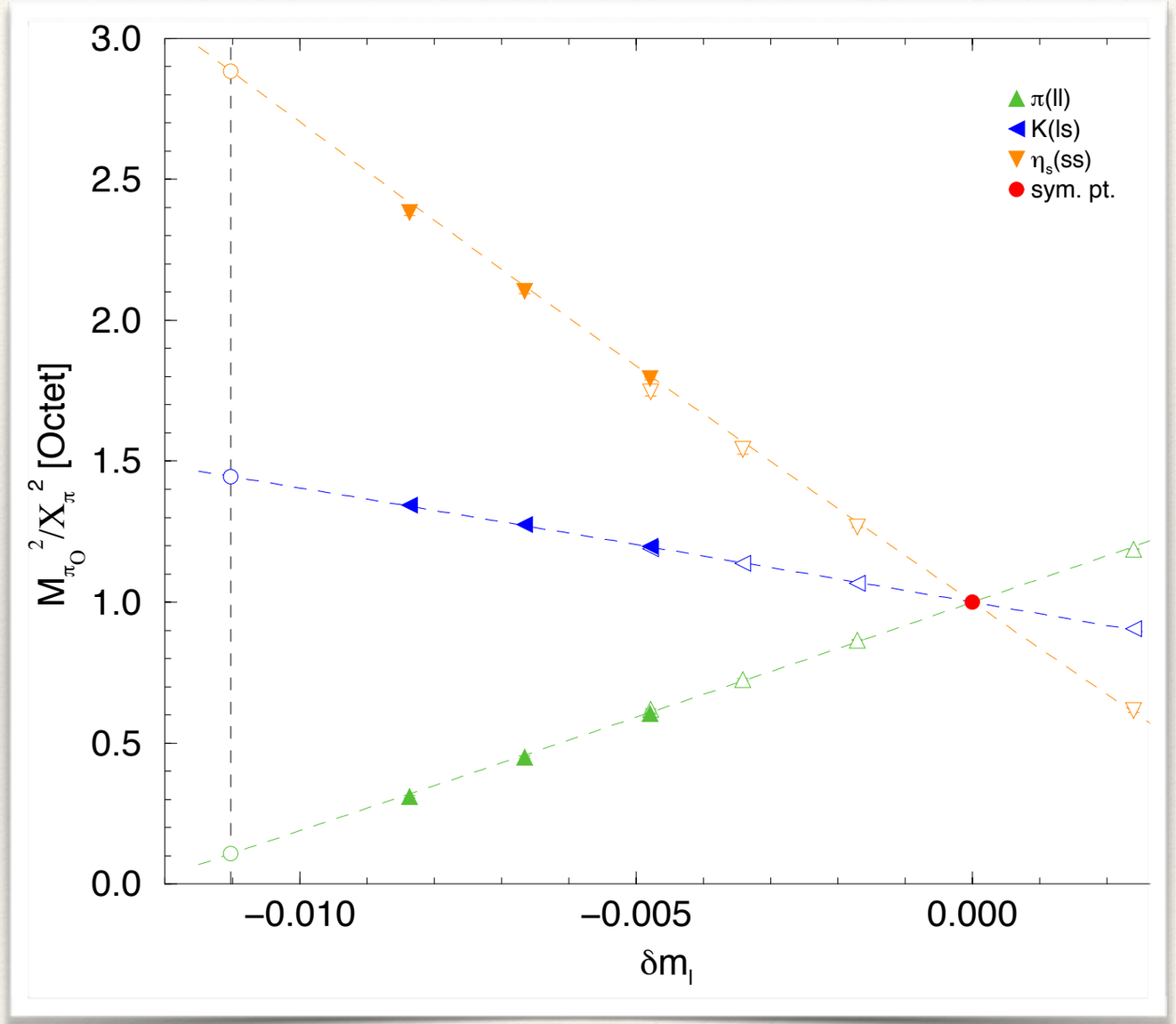

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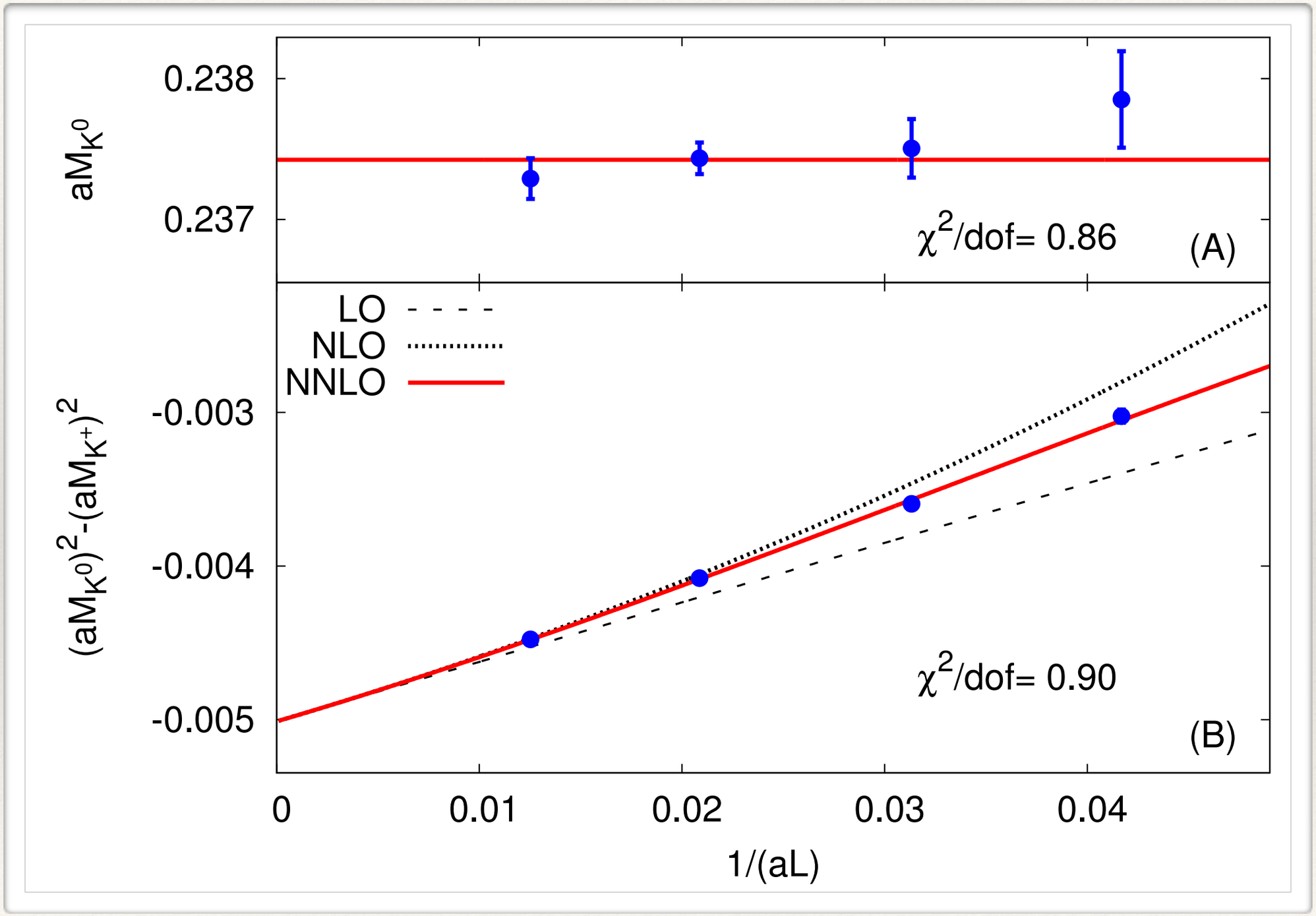
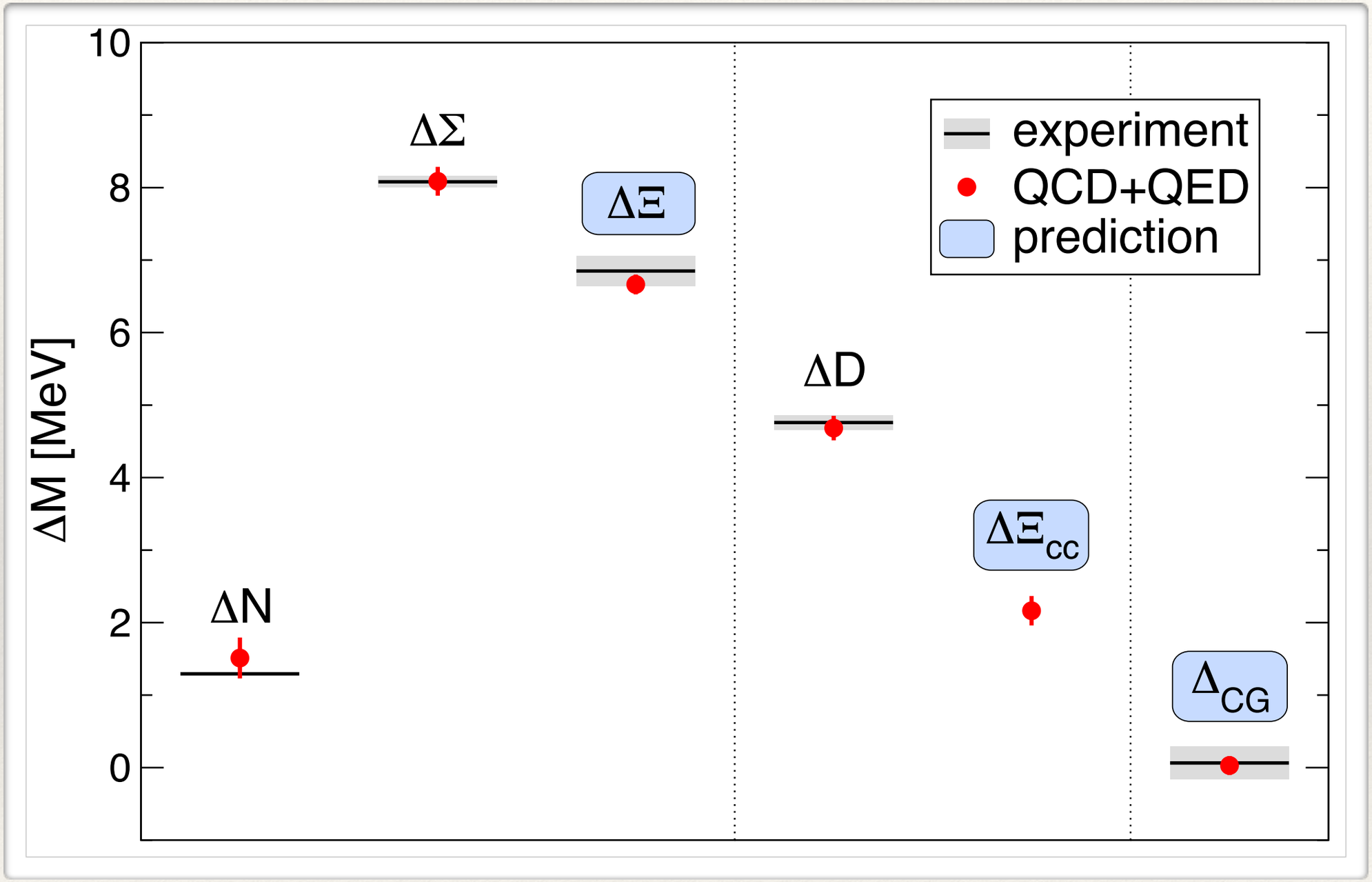
- **O(500) analyses per mass splitting**

[BMWc, 2014]: finite-volume study



[BMWc, 2014]: result summary



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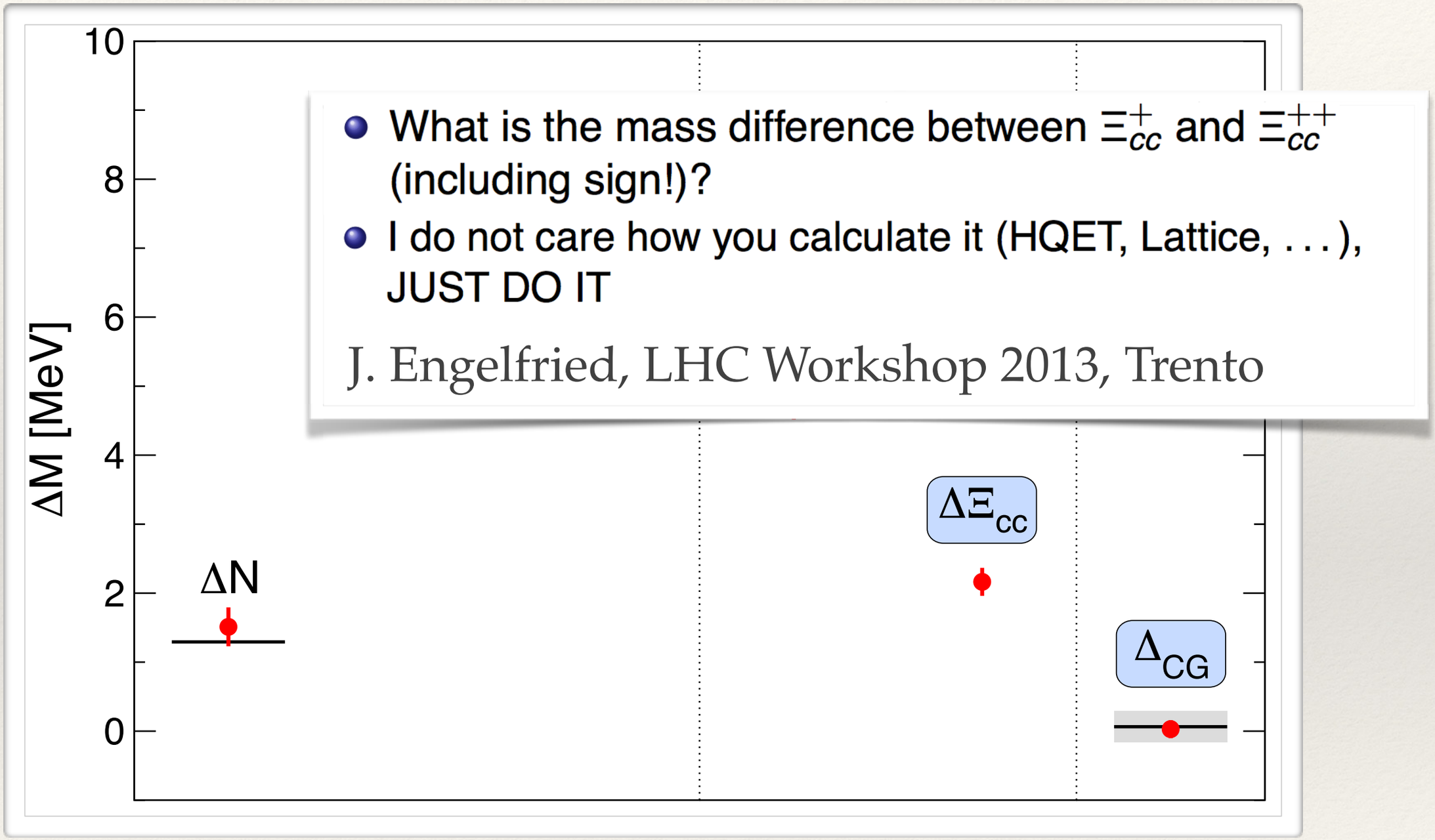

- What is the mass difference between Ξ_{cc}^{+} and Ξ_{cc}^{++} (including sign!)?
- I do not care how you calculate it (HQET, Lattice, . . .), JUST DO IT

J. Engelfried, LHC Workshop 2013, Trento

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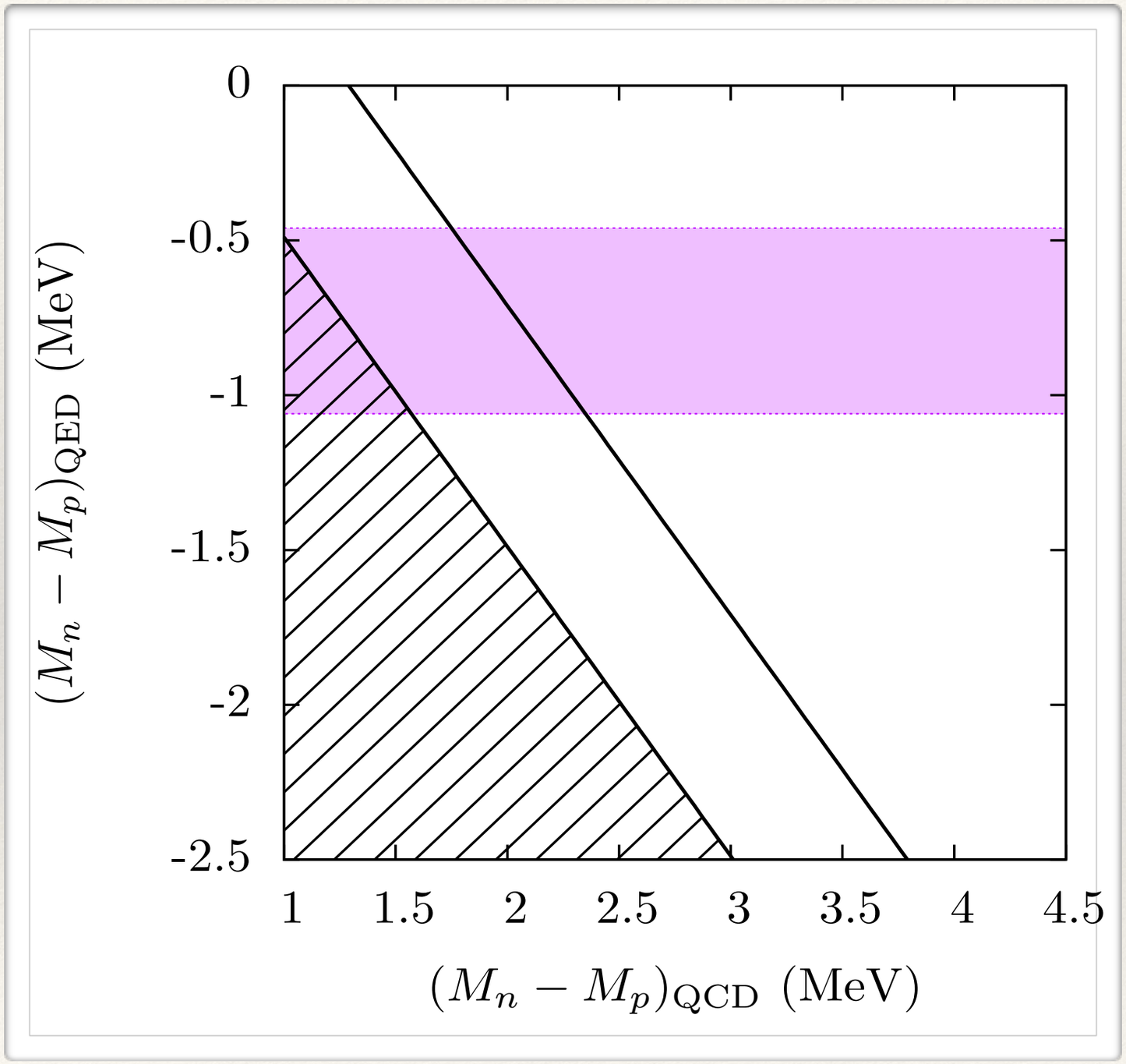
Results for the nucleon mass splitting



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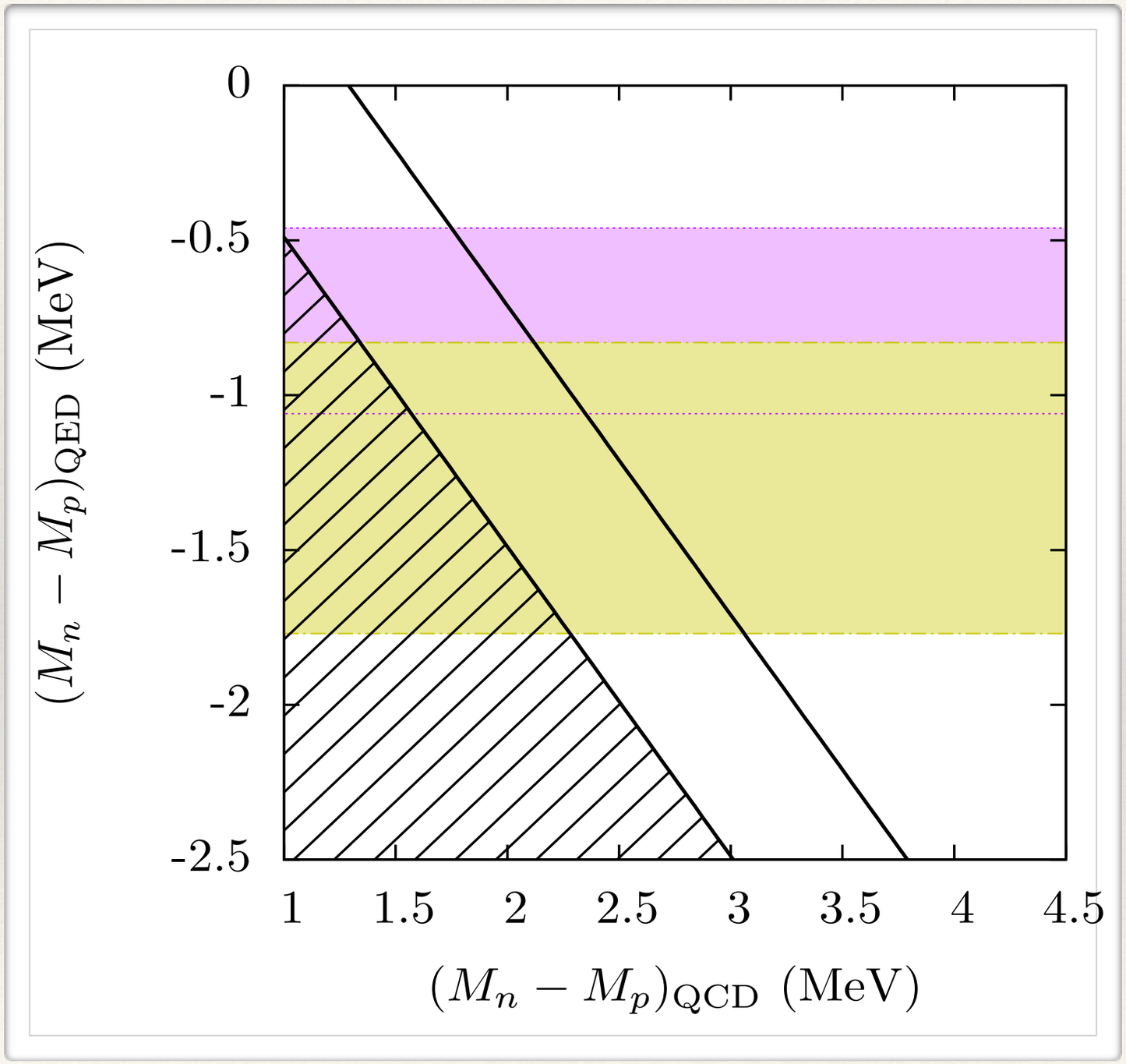

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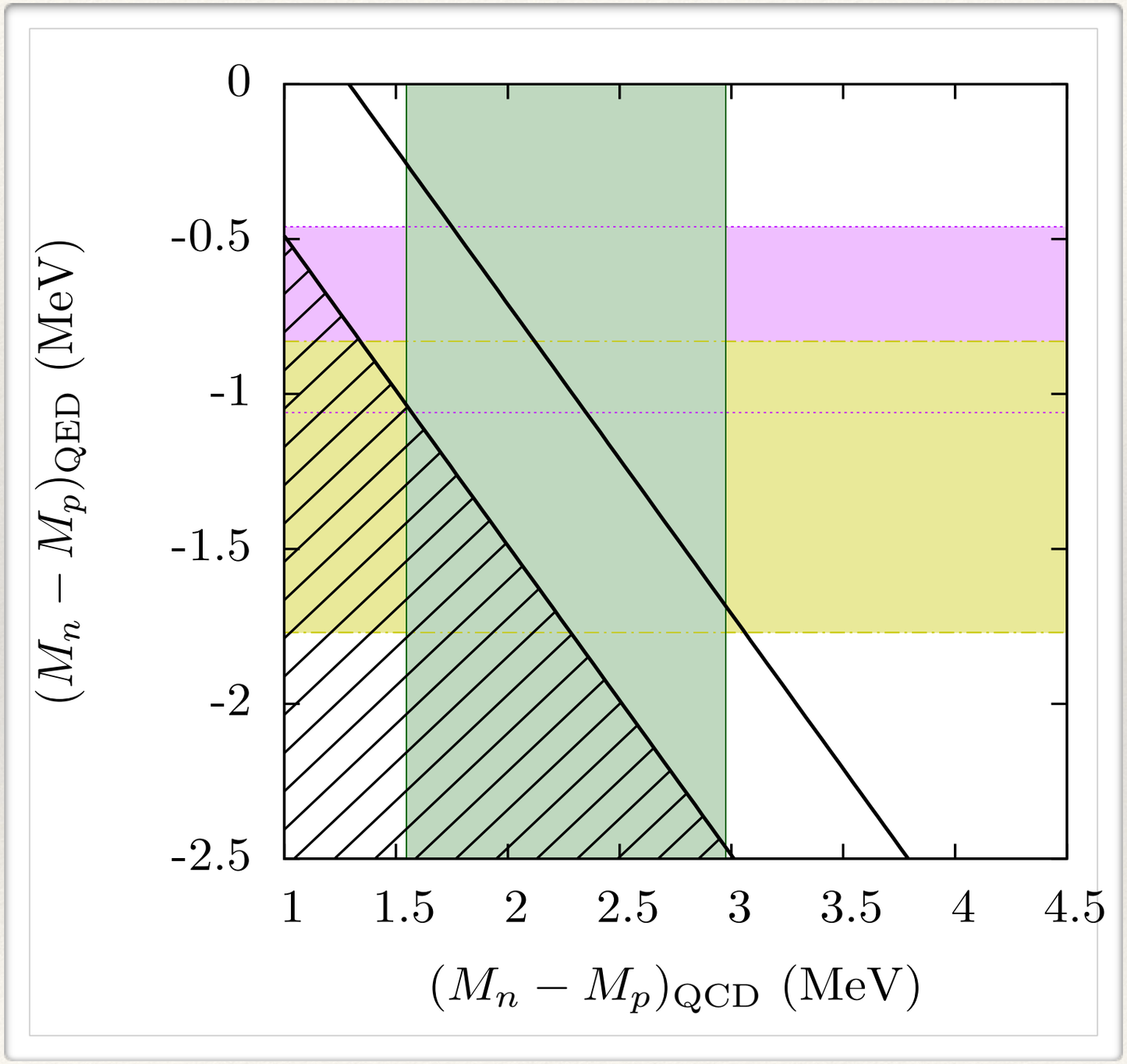
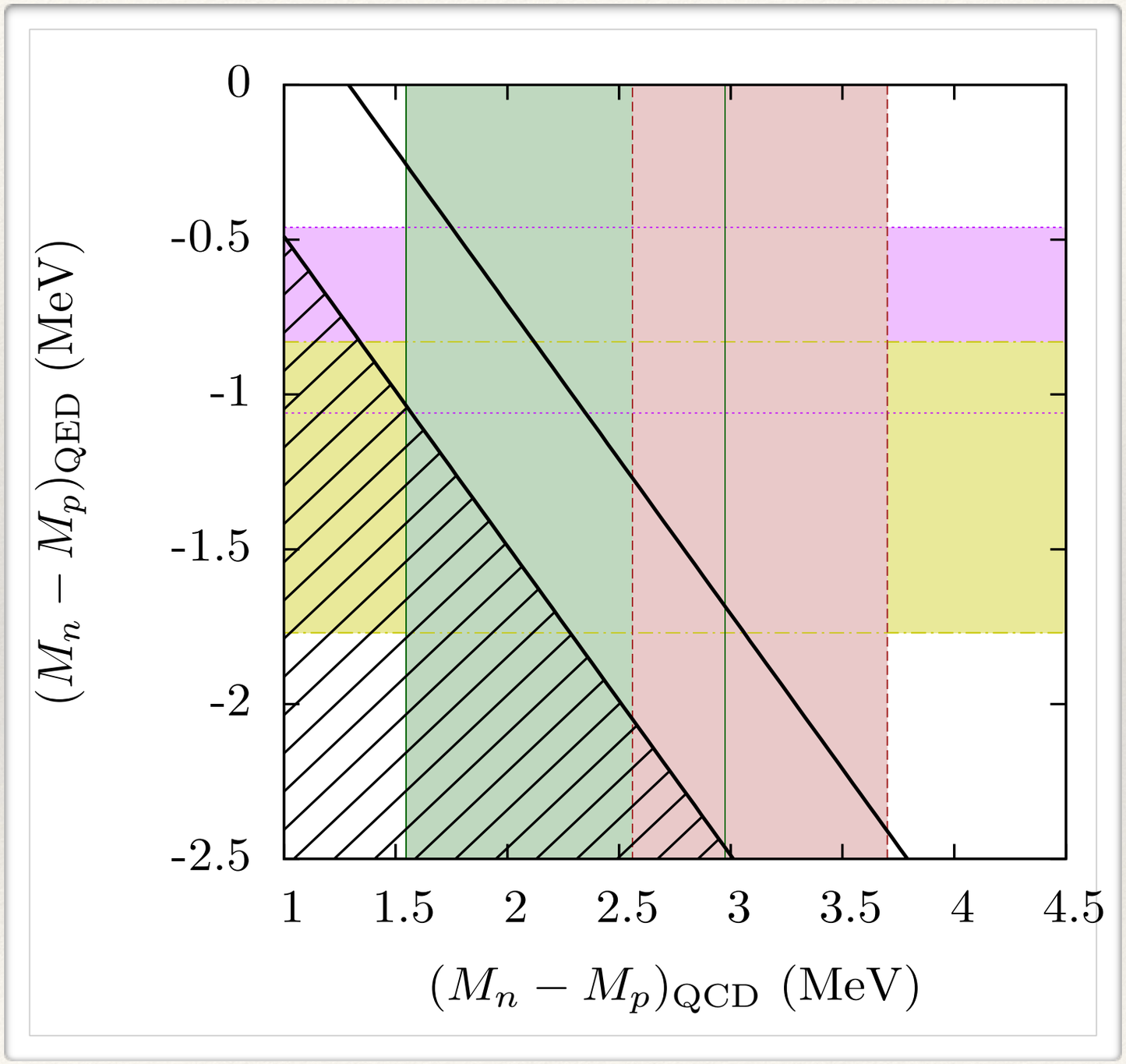


Results for the nucleon mass splitting



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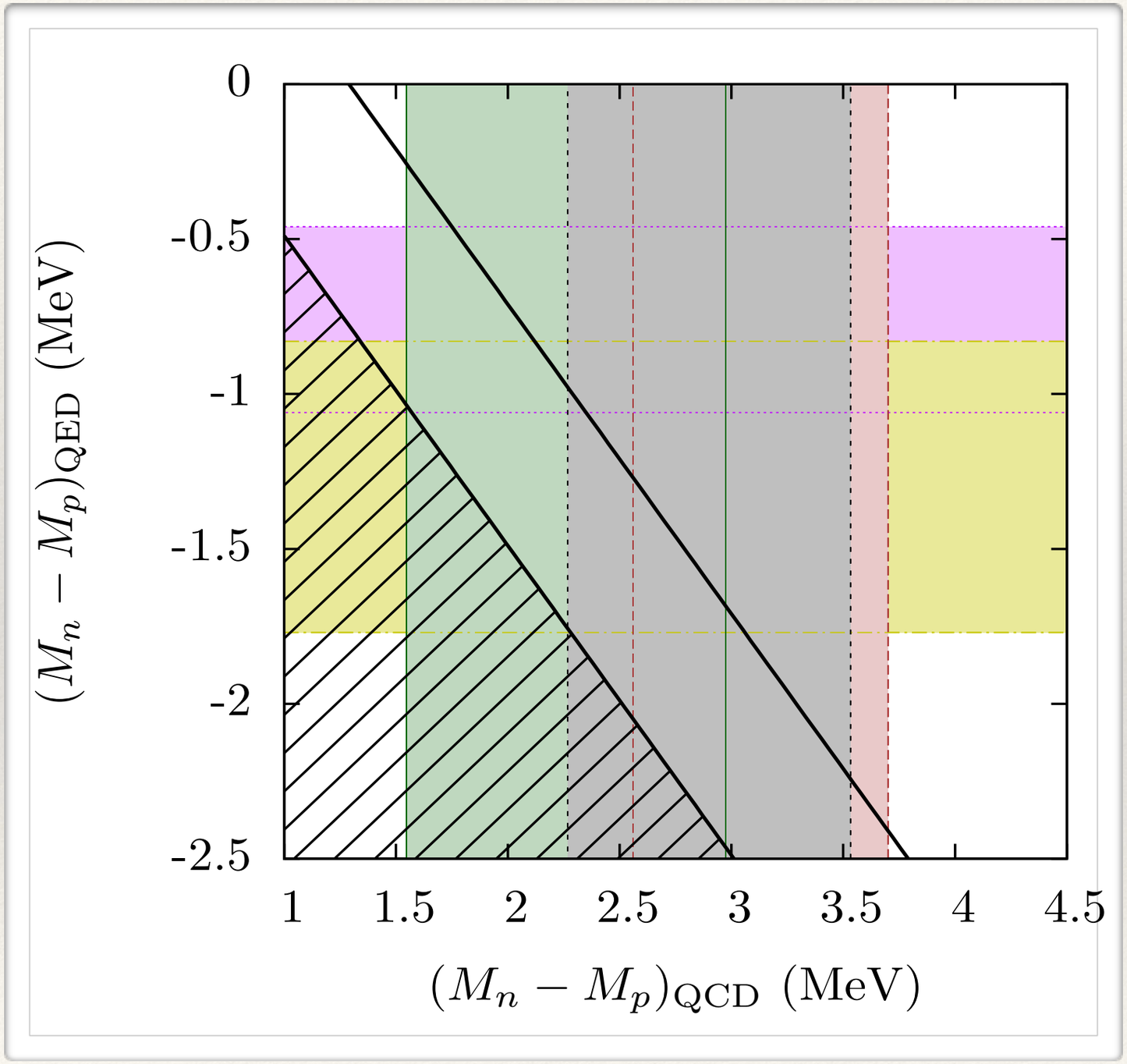

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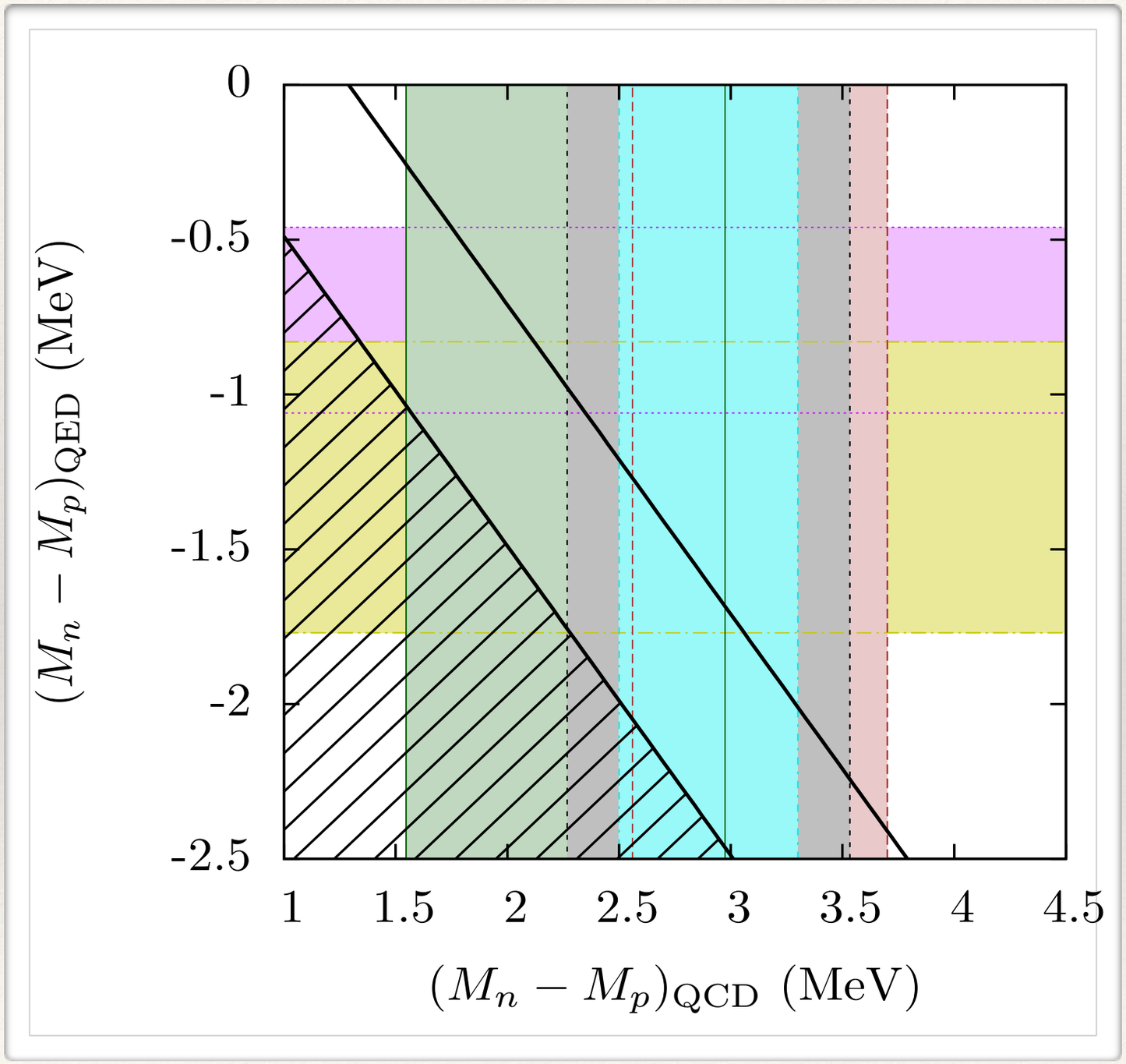

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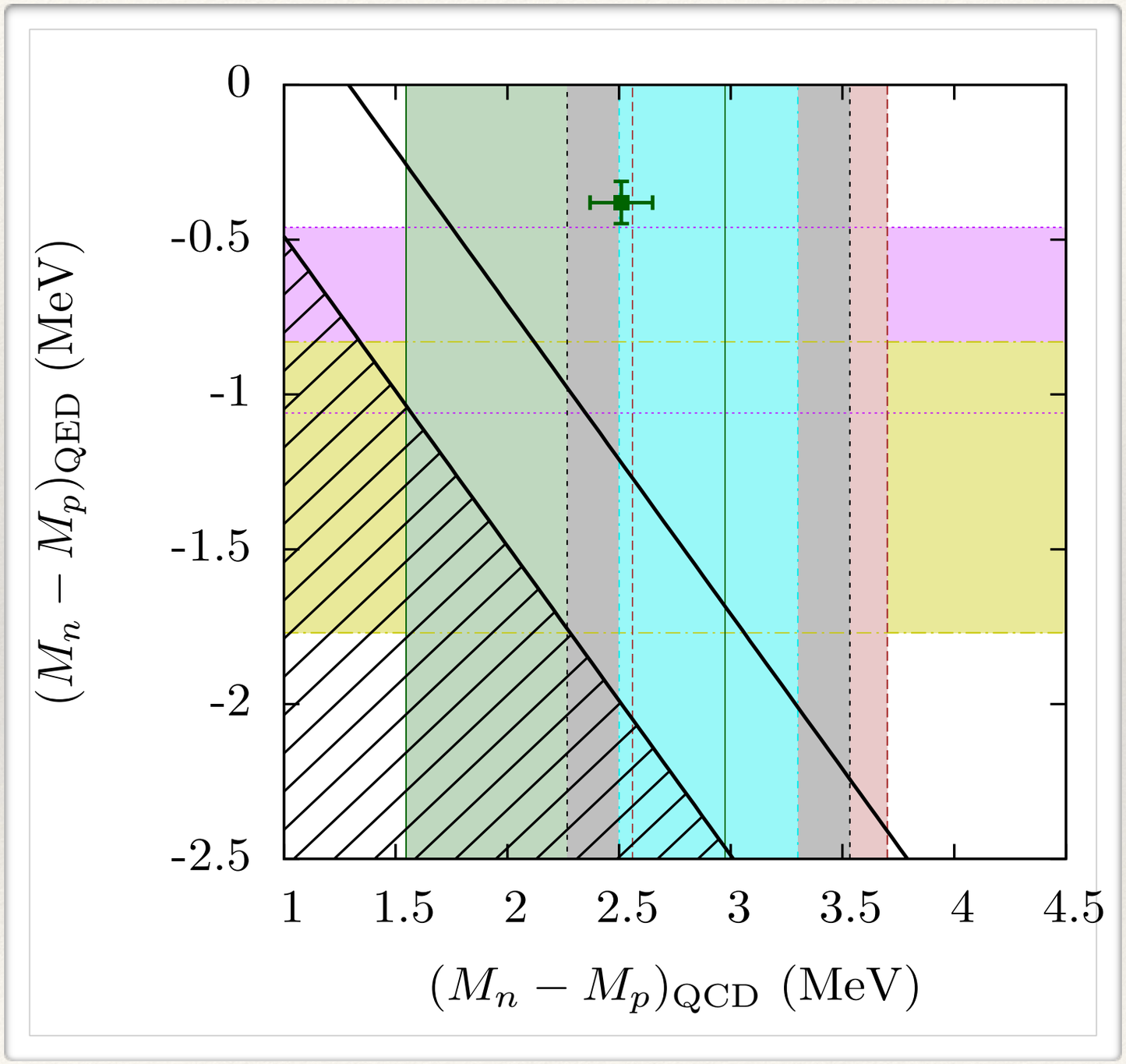

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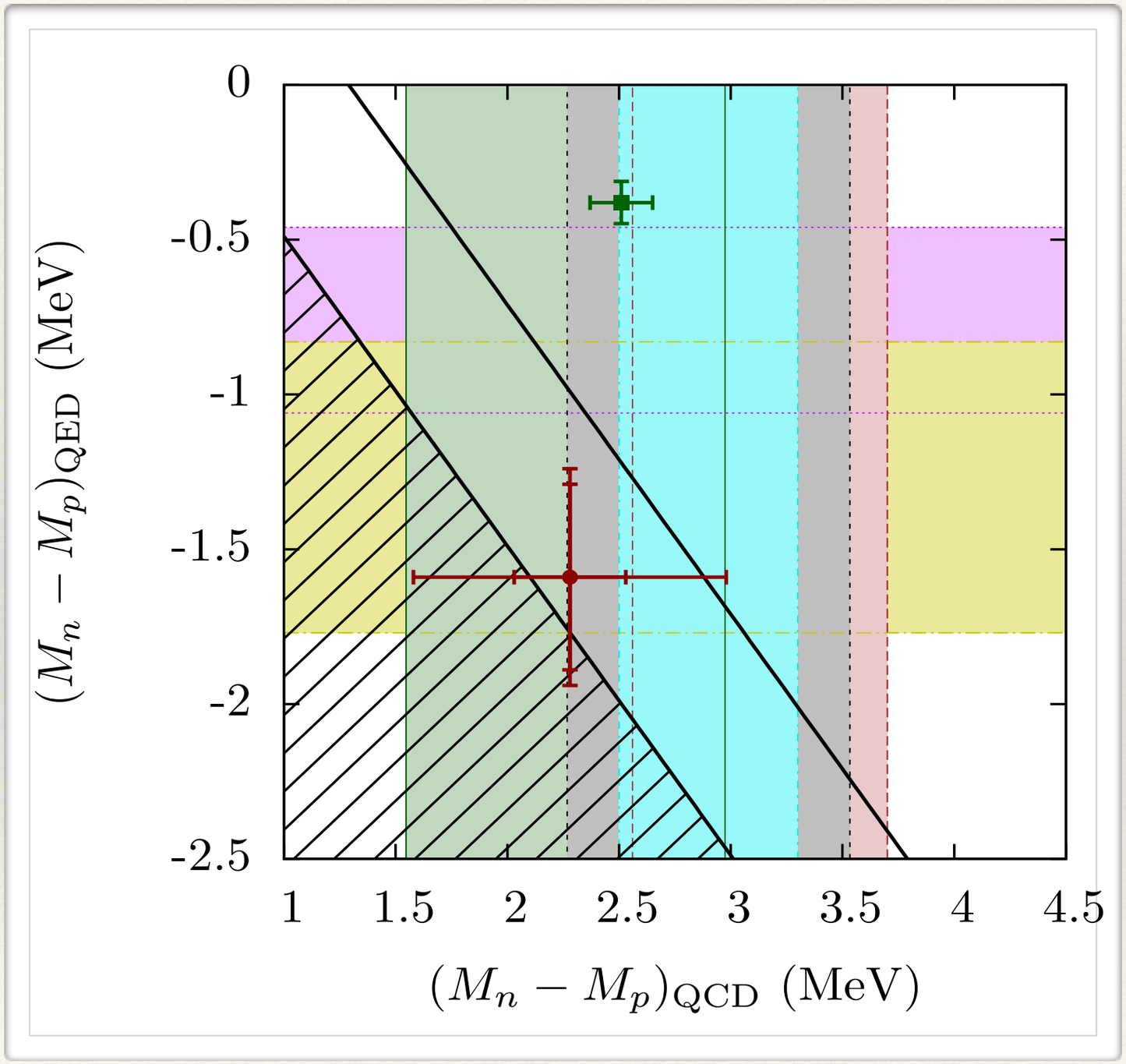

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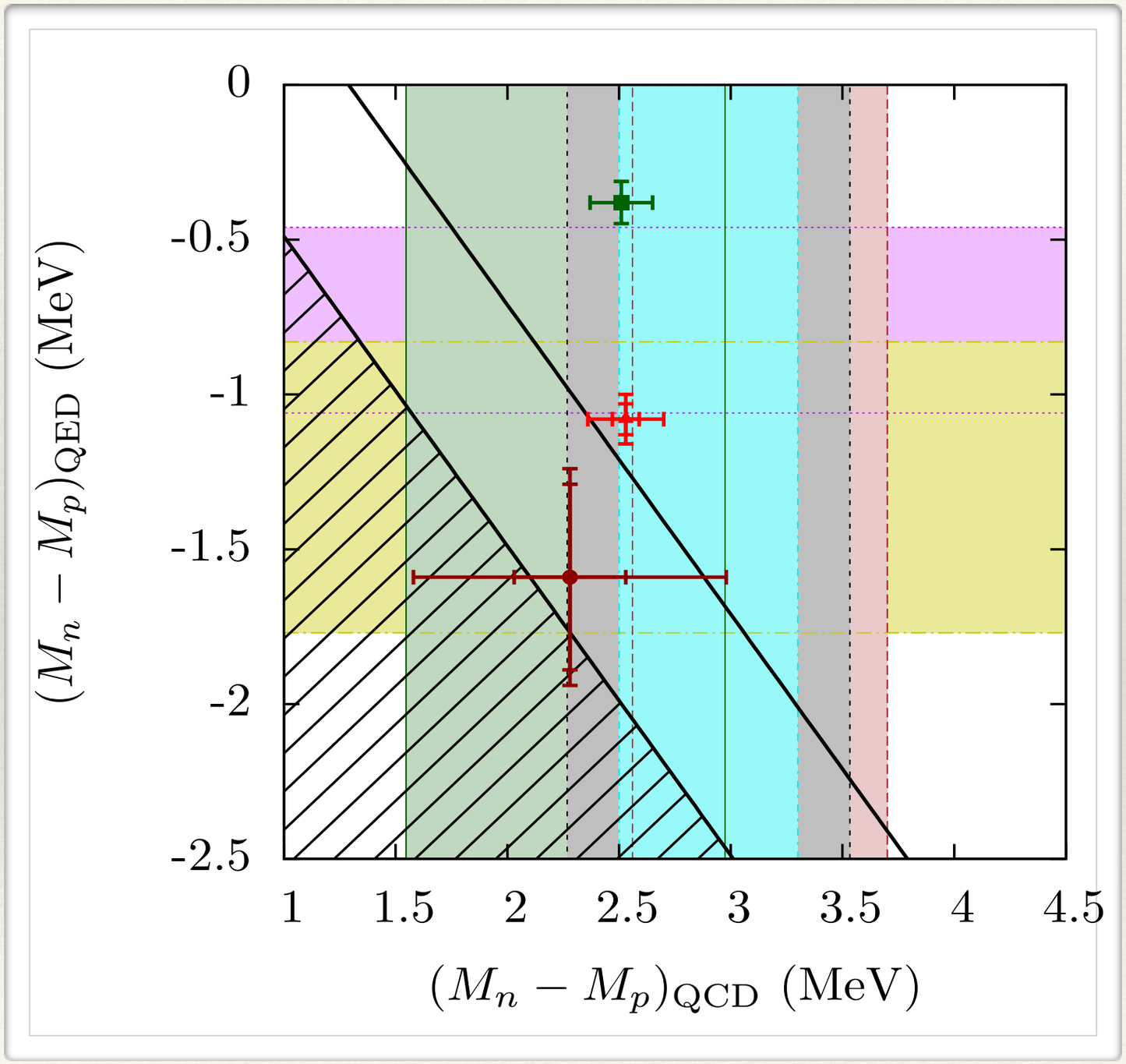

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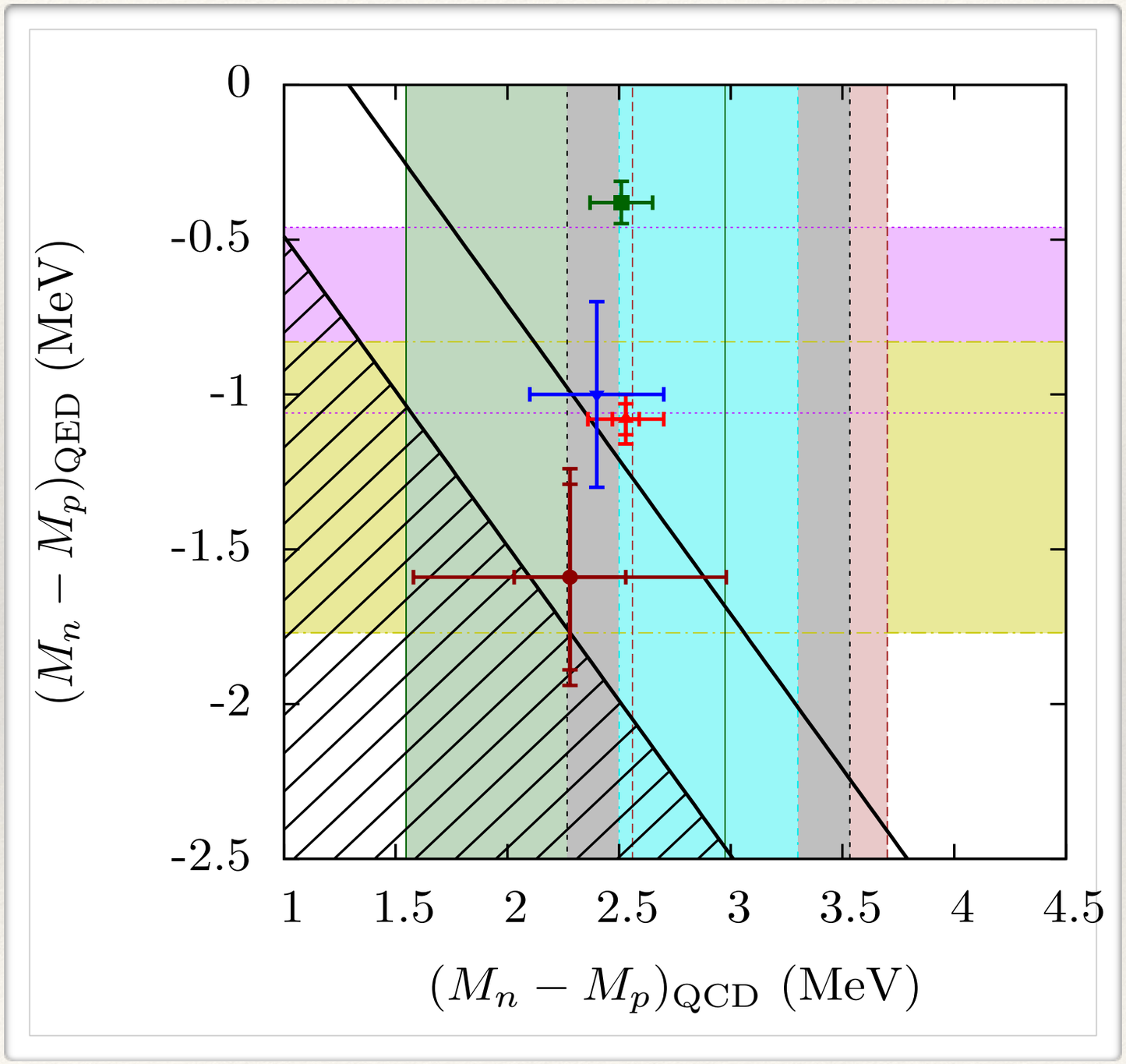

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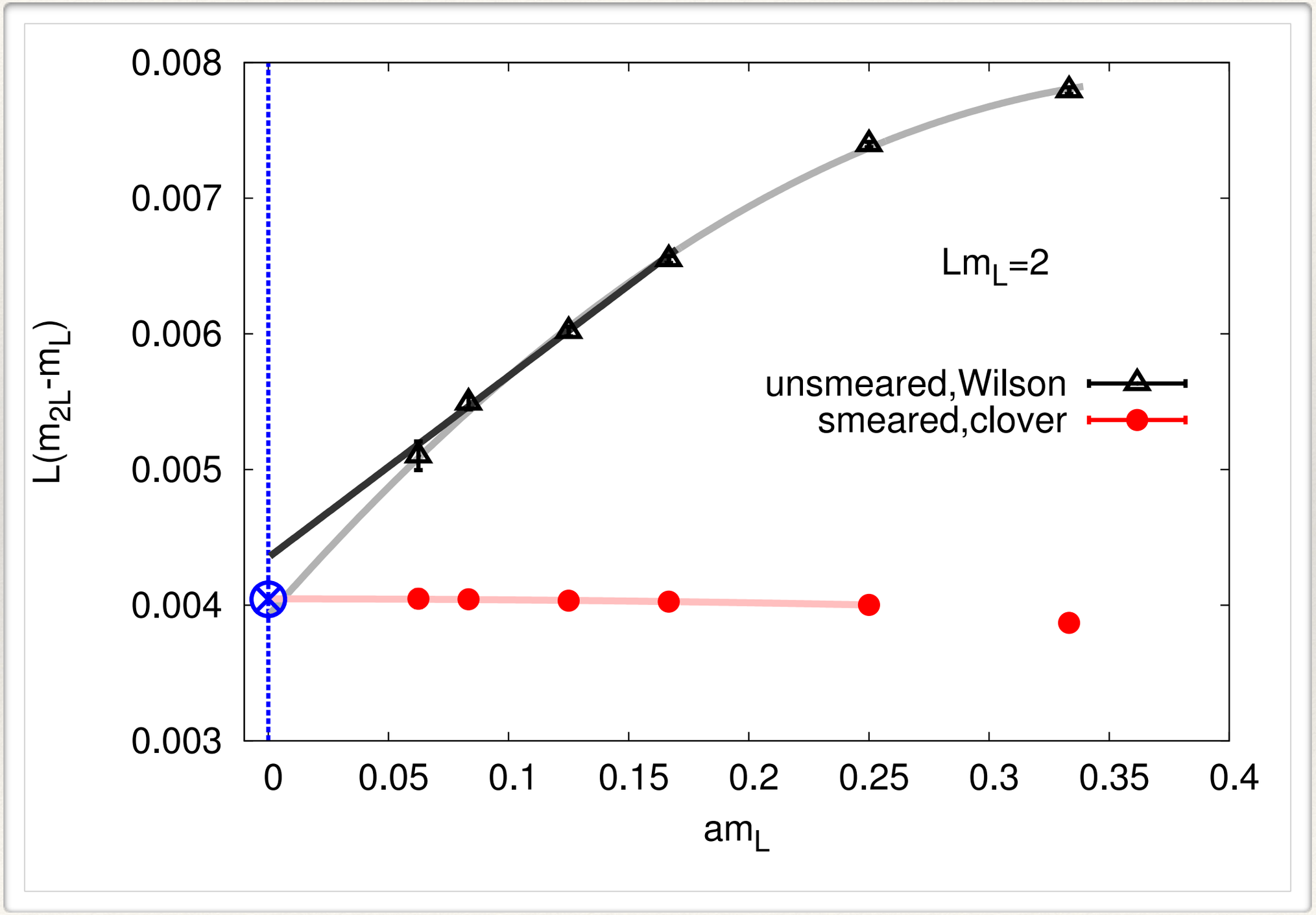
- QCD+QED to compute hadronic corrections to anomalous magnetic moments.

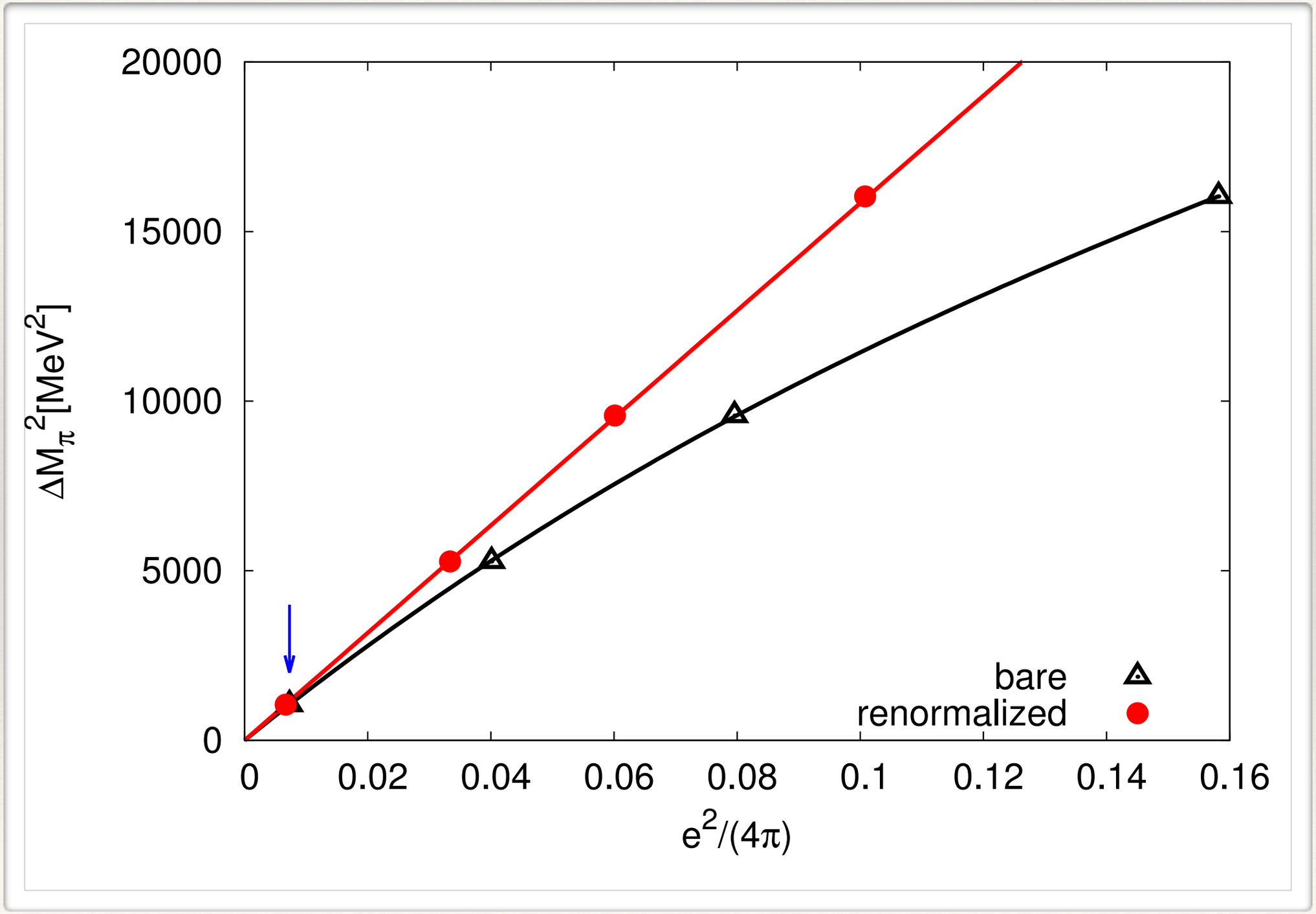
Thank you!

Backup

[BMWc, 2014]: QED simulations



[BMWc, 2014]: charge renormalisation



[BMWc, 2014]: charm discretisation effects